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FREE CONVECTION AND MASS TRANSFER FLOW PAST AN INFINITE VERTICAL POROUS PLATE WITH DUFOUR AND SORET EFFECTS IN A POROUS MEDIUM

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ABSTRACT

A numerical solution of unsteady free convection and mass transfer flow is presented when a viscous, incompressible fluid flows along an infinite vertical porous plate embedded in a porous medium. Both the Dufour and Soret effects are considered when the fluid is not chemically reacting. Dimensionless velocity, temperature and concentration profiles are displayed graphically for different values of the parameters entering into the problem. The numerical values of the skin-friction coefficient, Nusselt and Sherwood numbers, which are of physical interest, are also tabulated.

Key words: Free Convection, Vertical Plate, Unsteady flow, Dufour Effect, Soret Effect.

I. INTRODUCTION

The subject of convective flow in porous media has attracted considerable attention in the last several decades and is now considered to be an important field of study in the general areas of fluid dynamics and heat transfer. This topic has important applications, such as heat transfer associated with heat recovery from geothermal systems and particularly in the field of large storage systems of agricultural products, heat transfer associated with storage of nuclear waste, exothermic reaction in packedded reactors, heat removal from nuclear fuel debris, flows in soils, petroleum extraction, control of pollutant spread in groundwater, solar power collectors and porous material regenerative heat exchangers, to name just a few applications. The growing volume of work devoted to this area is samply documented by the most recent books by Nield and Bejan [1] and Ingham and Pop [2, 3].

Combined heat and mass transfer driven by buoyancy due to temperature and concentration variations is also of great practical importance since there are many possible engineering applications, such as the migration of moisture through the air contained in fibrous insulation and grain storage installations, and

the dispersion of chemical contaminants through saturated soil. A comprehensive review on the phenomena has been recently provided by Trevisan and Bejan [4].

All of the above studies, the Dufour and Soret effects were neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. However, exceptions are observed therein. The Soret effect, for instance, has been utilized for isotope separation, and in mixture between gases with very light molecular weight (H_2, He) and of medium molecular weight (H_2 , air) the Dufour effect was found to be of order of considerable magnitude such that it cannot be neglected [5]. Very recently, Postelnicu [6] included the Dufour and Soret effects on steady MHD natural convection and mass transfer boundary layer flow. Hence, the objective of the present paper is to study the above-mentioned Dufour and Soret effects on unsteady free convection and mass transfer flow past an infinite vertical porous plate in a porous medium.

II. MATHEMATICAL ANALYSIS

Consider an unsteady free convection and mass transfer flow of an incompressible viscous fluid past an infinite vertical porous flat plate embedded in a porous medium. The flow is assumed to be in the x-direction, which is taken along the vertical plate in the upward direction and the y-axis is taken to be normal to it. Initially the plate and the fluid are at same temperature T_∞ in a stationary condition with concentration level C_∞ at all points. At time t>0, when the plate is heated isothermally, its temperature is raised to T_w and the concentration level at the plate is raised to C_w. Since the plate is considered to be of infinite extent, all derivatives with respect to x vanish. Then the problem is governed by the following boundary layer equations under the usual Boussinesq's approximation:

$$\frac{\partial v}{\partial y} = 0\,, \tag{1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\upsilon}{K'}u, \tag{2}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{D_m\, k_T}{c_s\, c_p}\frac{\partial^2 C}{\partial y^2}\,, \tag{3}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m\, k_T}{T_m}\frac{\partial^2 T}{\partial y^2}\,, \tag{4}$$

where u, v are the velocity components in the x and y directions respectively, ν is the kinematic viscosity, g is the acceleration due to gravity, ρ is the density of the fluid, β is the coefficient of volume expansion, β^* is the volumetric coefficient of expansion with concentration, T, T_w and T_∞ are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively, while C, C_w and C_∞ are the corresponding concentrations. Also, K' is the permeability of porous medium, α is the thermal diffusivity, D_m is the coefficient of mass diffusivity, c_p is the specific heat at constant pressure, T_m is the mean fluid temperature, k_T is the thermal diffusion ratio and c_s is the concentration susceptibility.

The boundary conditions for the above problem for $t > 0$ are:

$$u = 0, v = v(t), T = T_w, C = C_w \text{ at } y = 0\,, \tag{5a}$$

$$u = 0, v = 0, T = T_\infty, C = C_\infty \text{ as } y \to \infty\,. \tag{5b}$$

Now in order to obtain a local similarity solution of the problem under consideration, we introduce a time dependent length scale δ as $\delta = \delta(t)$. (6)

In terms of this length scale, a convenient solution of the equation (1) is considered to be in the following form:

$$v = v(t) = -v_0\frac{\upsilon}{\delta}\,, \tag{7}$$

where v_0 is the suction parameter.

We now introduce the following dimensionless variable:

$$\left.\begin{aligned} &\eta = \frac{y}{\delta},\\ &u = U_0 f(\eta),\\ &\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\\ &\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \end{aligned}\right\} \tag{8}$$

where U_0 is a constant uniform velocity. Then introducing the relations (6) - (8) into the equations (2), (3) and (4) respectively, we obtain the following ordinary differential equations:

$$f'' + \eta\left(\frac{\delta}{\upsilon}\frac{d\delta}{dt}\right)f' + v_0 f' + Gr\theta + Gm\phi - Kf = 0 \tag{9}$$

$$-\eta\left(\frac{\delta}{\upsilon}\frac{d\delta}{dt}\right)\theta' - v_0\theta' = \frac{1}{\Pr}\theta'' + Df\phi'' \tag{10}$$

$$-\eta\left(\frac{\delta}{\upsilon}\frac{d\delta}{dt}\right)\phi' - v_0\phi' = \frac{1}{Sc}\phi'' + Sr\theta'' \tag{11}$$

where

$\Pr = \dfrac{\upsilon}{\alpha}$ is the Prandtl number,

$Sc = \dfrac{\upsilon}{D_m}$ is the Schmidt number,

$K = \dfrac{\delta^2}{K'}$ is the Permeability parameter,

$Sr = \dfrac{D_m k_T (T_w - T_\infty)}{T_m \upsilon (C_w - C_\infty)}$ is the Soret number,

$Df = \dfrac{D_m k_T (C_w - C_\infty)}{c_s c_p \upsilon (T_w - T_\infty)}$ is the Dufour number,

$Gr = \dfrac{g\beta (T_w - T_\infty)\delta^2}{\upsilon U_0}$ is the local temperature Grashof number and $Gm = \dfrac{g\beta^* (C_w - C_\infty)\delta^2}{\upsilon U_0}$ is the local mass Grashof number.

The corresponding boundary conditions for $t > 0$ are obtained as:

$f = 0, \theta = 1, \phi = 1$ at $\eta = 0$ (12a)

$f = 0, \theta = 0, \phi = 0$ as $\eta \to \infty$ (12b)

Now the equations (9) - (11) are locally similar except the term $\left(\dfrac{\delta}{\upsilon}\dfrac{d\delta}{dt}\right)$, where t appears explicitly. Thus the local similarity condition requires that $\left(\dfrac{\delta}{\upsilon}\dfrac{d\delta}{dt}\right)$ in the equations (9) - (11) must be a constant quantity.

Hence following the works of Hasimoto [7], Satter and Hossain [8] and Satter [9], one can try a class of solutions of the equations (9) - (11) by assuming that

$$\left(\frac{\delta}{\upsilon}\frac{d\delta}{dt}\right) = \lambda \text{ (a contant).} \tag{13}$$

Integrating (13) we have

$$\delta = \sqrt{2\lambda\upsilon t}\,, \tag{14}$$

where the constant of integration is determined through the condition that $\delta = 0$ when $t = 0$. From (14) choosing λ = 2, the length scale $\delta(t) = 2\sqrt{\upsilon t}$ which exactly corresponds to the usual scaling factor for various unsteady boundary layer flows (Schlichting [10]). Since δ is a scaling factor as well as a similarity parameter, any value of λ in (13) would not change the nature of the solutions except that the scale would be different.

Now introducing (13) [with λ = 2] in the equations (9) - (11) respectively, we obtain the following dimensionless ordinary differential equations which are locally similar.

$$f'' + (2\eta + v_0)f' + Gr\theta + Gm\phi - Kf = 0 \tag{15}$$

$$\theta'' + \Pr(2\eta + v_0)\theta' + \Pr Df\phi'' = 0 \tag{16}$$

$$\phi'' + Sc(2\eta + v_0)\phi' + ScSr\theta'' = 0 \tag{17}$$

where primes denote partial differentiation with respect to the variable η.

Subject to the above formulation the boundary conditions follow from (12)

$$f = 0, \theta = 1, \phi = 1 \text{ at } \eta = 0\,, \tag{18a}$$

$$f = 0, \theta = 0, \phi = 0 \text{ as } \eta \to \infty\,. \tag{18b}$$

The system of equations (15) – (17) under the boundary conditions (18) have been solved numerically by applying Nachtsheim-Swigert [11] shooting iteration technique along with Runge-Kutta sixth order integration scheme. From the process of integration the skin-friction coefficient τ_x, Nusselt number Nu and Sherwood number Sh are also calculated out.

III. RESULTS AND DISCUSSION

Numerical calculations have been carried out for different values of v_0, K, Sr and Df and for fixed values of Pr, Sc, Gr and Gm. The values of Gr and Gm are taken to be both positive, since these values represent cooling of the plate. The value of Prandtl number (Pr) is taken to be 0.71, which corresponds to air, and the value of Schmidt number (Sc) is chosen to 0.22, which represents hydrogen at 25^0C and 1 atm. However, the values of v_0, K, Sr and Df are chosen arbitrarily. The numerical results for the velocity, temperature and concentration profiles are displayed in Figs.1-6.The effects of suction and permeability parameters on the velocity field are shown in Fig. 1 for cooling of the plate. It is seen from this figure that the velocity decreases with the increase of both suction and permeability parameters. In Fig.

2, the effects of Soret and Dufour numbers on the velocity field are shown. We observe that for cooling of the plate, the velocity increases with the increase of both Dufour and Soret number.

The temperature profiles are shown in Figs. 3 and 4 for cooling of the plate. From Fig. 3 we see that the temperature decreases with the increase of suction parameter. It is found from Fig. 4 that the temperature increases with the increase of Dufour number whereas it decreases with the increase of Soret number.

In Figs. 5 and 6, the concentration profiles are shown for cooling of the plate. It is observed from Fig. 5 that the velocity increases with the increase of suction parameter close to the wall (approx. $\eta < 0.60$) whereas for $\eta > 0.60$, the concentration decreases with the increase of suction parameter. In Fig. 6, the effects of Soret and Dufour numbers on the concentration profiles are seen. From this figure it is found that the concentration increases with the increase of both Dufour and Soret numbers.

Finally, the effects of Soret and Dufour numbers on the local skin-friction coefficients, rate of heat and mass transfer are shown in Table 1. These effects are in agreement with those seen from the velocity, temperature and concentration profiles. The conclusions and discussions regarding the behavior of these numbers on skin-friction and rate of heat and mass transfer coefficients are self evident from Table 1 and hence any further discussions about them seem to be redundant.

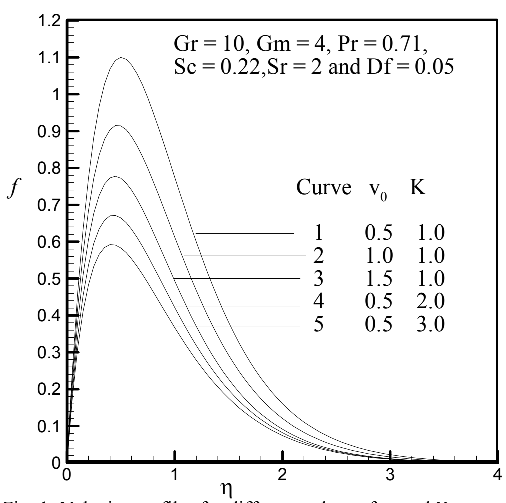

Fig. 1: Velocity profiles for different values of v_0 and K.

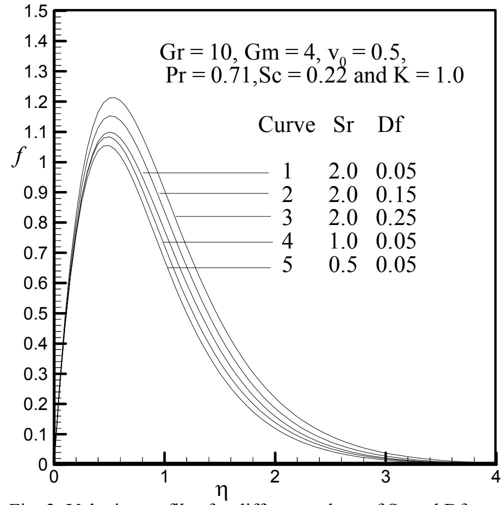

Fig. 2: Velocity profiles for different values of Sr and Df.

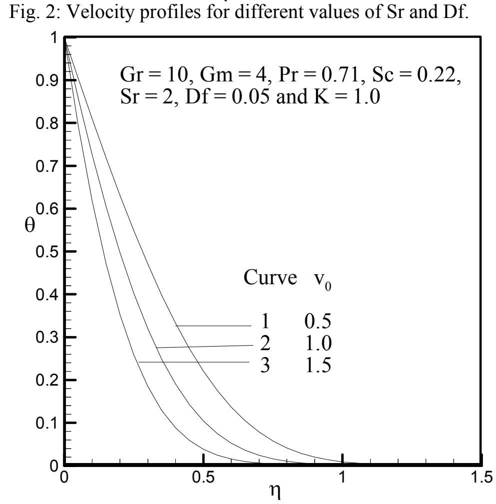

Fig. 3: Temperature profiles for different values of v_0.

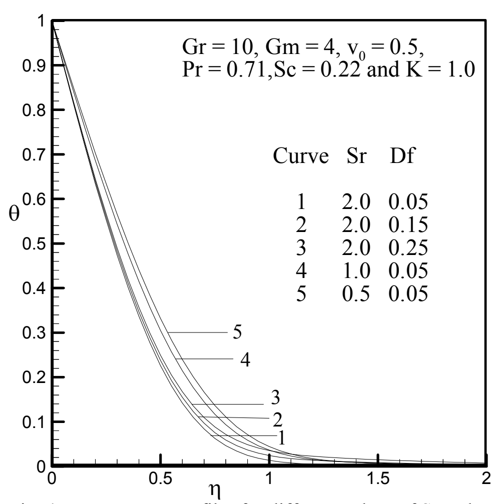

Fig. 4: Temperature profiles for different values of Sr and Df.

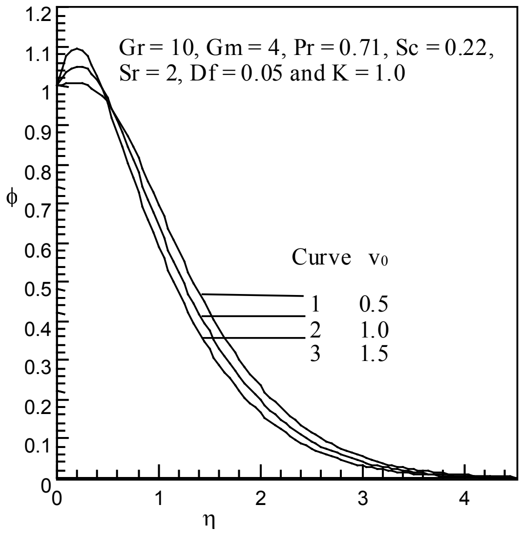

Fig. 5: Concentration profiles for different values of v_0.

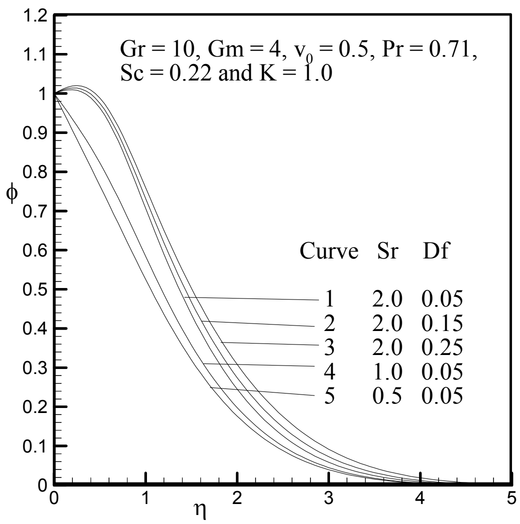

Fig. 6: Concentration profiles for different values of Sr and Df.

Table 1: Numerical values of τ_x, *Nu* and *Sh* for *Gr* = 10, *Gm* = 4, v_0 = 0.5, *K* = 1, *Pr* = 0.71 and *Sc* = 0.22.

Sr	*Df*	τ_x	*Nu*	*Sh*
2.0	0.05	5.371193	1.928019	-0.091517
2.0	0.15	5.513117	1.898647	-0.114106
2.0	0.25	5.667602	1.870944	-0.139538
1.0	0.05	5.298374	1.654820	0.316168
0.5	0.05	5.260280	1.559348	0.469570

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