# Performance Analysis of a Space Time Block Coded MIMO System over Faded Channel


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*"Anyone who stops learning is old, whether at twenty or eighty.
Anyone who keeps learning stays young. The greatest thing in life is to keep your mind young."*

*- Henry Ford*

## Declaration

We hereby declare that this thesis is based on the results found by ourselves. Materials of work found by other researchers are mentioned by reference. This thesis, neither in whole nor in part, has been previously submitted for any degree.

1. Abdur Rahman
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Signature of the Author

20/4/2011

Signature of the Supervisor

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# Abstract

Performance analysis will be carried out for a MIMO wireless link with space time block coding to find the performance over a Raleigh fading channel. The expression for the signal and SNR at the receiver output will be derived. The bit error rate (BER) will be found for maximal ratio combining technique and the performance results will be numerically evaluated for different system parameters. Performance improvement to space time block code will be determined at a given BER and to find the effect of carrier offset frequency on the link performance.

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# Contents

# List Of Figures

# CHAPTER 1

## INTRODUCTION

At present, wireless communication is experiencing an exponential growth rate. Providing adequate quality of service (QoS) under restricted bandwidth is now one of the greatest concerns [1], [5]. The next generation wireless systems are expected to meet the ever increasing demands, such as, high voice quality and bit rate, coverage, bandwidth and power efficiency, less effect of channel impairments, ability to be deployed in diverse environments, and so on. The remote units need to be small and lightweight to provide better service and work efficiently in any sort of environments [1]. At the same time the cost effectiveness should also be taken into account. To meet up all these ambitious objects we chose space time block code with MIMO system over a Rayleigh and Rician fading channel.

Space-time block coding (STBC) has been demonstrated to be a powerful diversity technique to combat channel fading in wireless communication [2]. In particular, the Alamouti code offers very simple encoding/decoding and is particularly suitable for future wireless systems [1]. Conventional STBC does not require channel knowledge at transmitter side. Since the BTS is transmitting to all users over the same frequency band at the same time, each user observes co-channel interference (CCI).

Fading is a common phenomenon which deteriorates the original signal while transmitted through wireless channel. This signal experiences both small-scale and large-scale fading. Small-scale fading is also known as Rayleigh Fading because if the multiple reflective paths are large in number and there is no line of sight signal component, the envelope of the received signal can statistically be described by a Rayleigh.

In most scattering environments, antenna diversity is a practical, effective and, hence, a widely applied technique for reducing the effect of Rayleigh fading [1]. The number of antennas at the transmitter or the receiver decides the type of the system that will finally be implemented. Space-time processing will either be receive diversity or transmit diversity. In receive diversity, the channel can be estimated and there can be multiple antennas at the receiver. The major problem with using the receive diversity approach is the cost, size, and power of the remote units. The use of multiple antennas and radio frequency (RF) chains (or selection and switching circuits) makes the remote units larger and more expensive. As a result, diversity techniques have almost exclusively been applied to base stations to improve their reception quality [3]. In this paper, we show two, four eight diversity techniques using STBC for multi-user MIMO systems over Rayleigh Fading channel condition.

# CHAPTER 2

# OVERVIEW of MIMO

## 2.1 INTRODUCTION TO MIMO

The invention of the radio telegraph by *Guglielmo Marconi* more than hundred years ago marks the commencement of wireless communications. In the last 20 years, the rapid progress in radio technology has activated a communications revolution. Wireless systems have been deployed through the world to help people and machines to communicate with each other independent of their location. *"Always best connected"* is one of the slogans for the fourth generation of wireless communications system (4G), meaning that your wireless equipment should connect to the network or system that at the moment is the "best" for you. Wireless communication is highly challenging due to the complex, time varying propagation medium. If we consider a wireless link with one transmitter and one receiver, the transmitted signal that is launched into wireless environment arrives at the receiver along a number of diverse paths, referred to as multipaths.

These paths occur from scattering and rejection of radiated energy from objects (buildings, hills, trees...) and each path has a different and time-varying delay, angle of arrival, and signal amplitude. As a consequence, the received signal can vary as a function of frequency, time and space. These variations are referred to as *fading* and cause deterioration of the system quality. Furthermore, wireless channels suffer of *cochannel interference* (CCI) from other cells that share the same frequency channel, leading to distortion of the desired signal and also low system performance. Therefore, wireless systems must be designed to mitigate fading and interference to guarantee a reliable communication.

Wireless systems of communication have recently turned to a strategy known as Multiple Input Multiple Output (MIMO) to improve the quality (bit-error rate) and data rate (bits/sec). This advantage can increase the quality of service and revenues of the operator. This is done by using multiple transmit and receive antennas, as well as appropriate coding techniques. They take advantage of spatial and temporal diversity to combat the random fading induced by multi-path propagation of the signal and maximize efficient use of bandwidth. There is also a fundamental gain in transmitting data over a matrix rather than vector channel. Transmission of data over MIMO channels has traditionally focused on data rate maximization or diversity maximization, and space-time codes were developed as a means to the latter.

## 2.2 MIMO SYSTEM MODEL

Figure (1) illustrates different antenna configurations used in defining space-time systems. Single-input single-output (SISO) is the well-known wireless configuration, single-input multiple-output (SIMO) uses a single transmitting antenna and multiple

(*MR*) receive antennas, multiple-input single-output (MISO) has multiple (*MT*) transmitting antennas and one receive antenna, MIMO has multiple (*MT*) transmitting antennas and multiple (*MR*) receive antennas and, finally, MIMO-multi-user (MIMO-MU), which refers to a configuration that comprises a base station with multiple transmit/receive antennas interacting with multiple users, each with one or more antennas.[4]

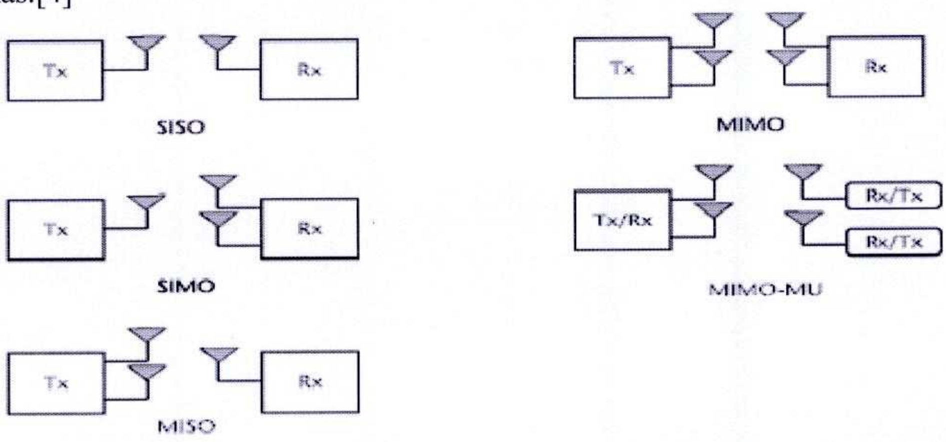



**Figure 2.1 Different antenna configurations in space-time systems**

## 2.3 APPLICATION OF MIMO

- Applications of MIMO Spatial multiplexing techniques makes the receivers very complex, and therefore it is typically combined with Orthogonal frequency-division multiplexing (OFDM) where the problems created by multi-path channel are handled efficiently. The IEEE 802.16e standard incorporates MIMO-OFDMA. The IEEE 802.11n standard, which is expected to be finalized soon, recommends MIMO-OFDM.
- MIMO is also planned to be used in Mobile radio telephone standards such as recent 3GPP and 3GPP2 standards. In 3GPP, High-Speed Packet Access plus (HSPA+) and Long Term Evolution (LTE) standards take MIMO into account. Moreover, to fully support cellular environments MIMO research consortia including IST-MASCOT propose to develop advanced MIMO techniques, i.e., multi-user MIMO (MU-MIMO).
- MIMO Enables the Digital Home MIMO delivers whole home coverage with the speed and reliability to stream multimedia applications.
- MIMO can reliably connect cabled video devices, computer networking devices, broadband connections, phone lines, music, storage devices, etc.
- MIMO is interoperable and can leverage the installed based of 802.11 wireless that is already deployed: computers, PDAs, handheld gaming devices, cameras, VoIP Phones, etc.

# CHAPTER 3

# FADING AND DIVERSITY

## 3.1 FADING

In wireless communications, fading is deviation of the attenuation that a carrier-modulated telecommunication signal experiences over certain propagation media. The fading may vary with time, geographical position and/or radio frequency, and is often modelled as a random process. A fading channel is a communication channel that experiences fading. In wireless systems, fading may either be due to multipath propagation, referred to as multipath induced fading, or due to shadowing from obstacles affecting the wave propagation, sometimes referred to as shadow fading.

### 3.1.1 RAYLEIGH FADING

Rayleigh fading is a statistical model for the effect of a propagation environment on a radio signal, such as that used by wireless devices. Rayleigh fading models assume that the magnitude of a signal that has passed through such a transmission medium (also called a communications channel) will vary randomly, or fade, according to a Rayleigh distribution — the radial component of the sum of two uncorrelated Gaussian random variables.

## 3.2 DIVERSITY TECHNIQUE

One of the most efficient and simple techniques to overcome the destructive effects of fading is Diversity. Diversity is an efficient technique to exploit the random nature of radio propagation by finding methods to generate and extract independent signal paths for communication. The concept behind diversity is relatively simple. If one signal path undergoes a deep fade at a particular point of time, another independent path may have a strong signal. By having more than one path to select from, both the instantaneous and average SNR can be improved in the receiver by a large amount. There are various types of diversity used in communication systems operating over fading channels [8]. They are:

- Space Diversity.
- Frequency Diversity.
- Time Diversity.
- Polarization Diversity.
- Multipath Diversity.

Whatever be the diversity technique employed, the receiver has to process the diversity signals obtained in a fashion that maximizes the power efficiency of the system. There are several possible diversity reception methods employed in communication receivers. The most common techniques are:

- Selection Diversity.
- Equal Gain Combining (EGC).

- Maximal Ratio Combining (MRC).

Among these three techniques we chose maximal ratio combining to combine received signals. Because MRC give better performance compare to the other technique.

## 3.3 MAXIMAL RATIO COMBINING (MRC)

In Maximal Ratio Combining (MRC), the signal all the branches are co-phased and individually weighed to provide the optimal SNR at the output. It can be shown that [7] the output SNR is maximized when the signals in each of the diversity branches are weighed by their own envelopes. In case of a two-fold diversity scheme, the combining equation is given by:

$$Z_k = r_{1K} Z_{1K} + r_{2K} Z_{2K}$$

Where, r1k and r2k represent the instantaneous envelopes of the signals received at each of the diversity branches. The SNR per bit at the output of the maximal ratio combiner (b) can be written as:

$$\gamma_b = \sum_{k=1}^{L} \gamma_k = \frac{E_b}{N_o} \sum_{k=1}^{L} R_k^2$$

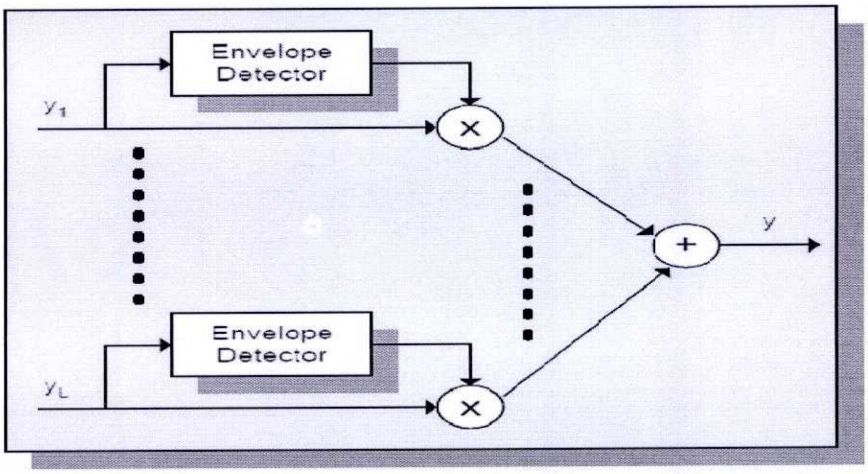



**Figure 3.1 Maximal ratio combining**

Where

$$\gamma_k = R^2(E_b/N_o)$$

is the instantaneous SNR in the k[th] diversity branch. The PDF of the output SNR can be written as [2]:

$$f_{\gamma_b}(\gamma_b) = \frac{1}{(L-1)!\,\gamma_c^L} \gamma_b^{L-1}\, e^{-\gamma_b/\bar{\gamma}_c}$$

Where $\bar{\gamma}_c$ is the average SNR per channel given by-

$$\bar{\gamma}_c = \frac{E_b}{N_o} E\{R^2\}$$

The final expression for the probability of error, i.e,

$$P_e = \int_0^\infty P_e(\gamma_b) f_{\gamma_b}(\gamma_b) d\gamma_b$$

A closed form expression does exist for this problem given by [9]:

$$P_e = \left(\frac{1-\mu}{2}\right)^L \sum_{k=0}^{L-1} \frac{(L-1+k)!}{(L-1)!\,k!} \left(\frac{1+\mu}{2}\right)^2$$

For large values of $\bar{\gamma}_c$ the above expression can be simplified to:

$$P_e \approx \left(\frac{1}{4\bar{\gamma}_c}\right)^L \frac{(2L-1)!}{(L-1)!\,L!}$$

We can also categorize diversity under the subheading of spatial diversity, based on whether diversity is applied to the transmitter or to the receiver. [8]

- **Receive diversity:** Maximum ratio combining is a frequently applied diversity scheme in receivers to improve signal quality. In cell phones it becomes costly and cumbersome to deploy. This is one of the main reasons transmit diversity became popular, since transmit diversity is easier to implement at the base station.[4]

- **Transmit diversity:** In this case we introduce controlled redundancies at the transmitter, which can be then exploited by appropriate signal processing techniques at the receiver. Generally this technique requires complete channel information at the transmitter to make this possible. But with the advent of space-time coding schemes like Alamouti's scheme [1], it became possible to implement transmit diversity *without* knowledge of the channel. This was one of the fundamental reasons why the MIMO industry began to rise. Space-time codes for MIMO exploit both transmit as well as receive diversity schemes, yielding a high quality of reception.[4]

## 3.4 SPATIAL MULTIPLEXING

Spatial multiplexing offers a linear (in the number of transmit-receive antenna pairs or min (*MR, MT*) increase in the transmission rate (or capacity) for the same bandwidth and with no additional power expenditure. It is only possible in MIMO channels. Consider the case of two transmits and two receive antennas. This can be extended to more general MIMO channels. The bit stream is split into two half-rate bit streams, modulated and transmitted simultaneously from both the antennas. The receiver, having complete knowledge of the channel, recovers these individual bit streams and combines them so as to recover the original bit stream. Since the receiver has knowledge of the channel it provides receive diversity, but the system has no transmit diversity since the bit streams are completely different from each other in that they carry totally different data. Thus spatial multiplexing increases the transmission rates proportionally with the number of transmit-receive antenna pairs. This concept can be extended to MIMO-MU. In such a case, two users transmit their respective information simultaneously to the base station equipped with two antennas. The base station can separate the two signals and can likewise transmit two signals with spatial filtering so that each user can decode his or her own signal correctly. This allows capacity to increase proportionally to the number of antennas at the base station and the number of users. [4]

# CHAPTER 4

# SPACE TIME BLOCK CODING

## 4.1 INTRODUCTION

Space-Time Codes (STCs) have been implemented in cellular communications as well as in wireless local area networks. Space time coding is performed in both spatial and temporal domain introducing redundancy between signals transmitted from various antennas at various time periods. It can achieve transmit diversity and antenna gain over spatially un-coded systems without sacrificing bandwidth. The research on STC focuses on improving the system performance by employing extra transmits antennas. In general, the designs of STC amounts to finding transmit matrices that satisfy certain optimality criteria. Constructing STC, researchers have to trade-off between three goals: simple decoding, minimizing the error probability, and maximizing the information rate. The essential question is: *How can we maximize the transmitted date rate using a simple coding and decoding algorithm at the same time as the bit error probability is minimized?*

## 4.2 SPACE TIME CODING MODEL

Suppose we have a MIMO system with n transmits antennas and m receives antennas. At the transmitter, information symbols belonging to a constellation set, such as QAM or HEX, are parsed into blocks: s(n) = [s(nK), . . . , s(nK + K − 1)]T of size K × 1. The block s(n) is encoded by the ST encoder which maps s(n) to column vectors in the following n× m ST code matrix

$$C=\begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1P} \\ C_{21} & C_{22} & \cdots & C_{2P} \\ \vdots & \vdots & \vdots & \vdots \\ C_{n1} & C_{n2} & \cdots & C_{nP} \end{bmatrix}$$

Where the coded symbol cij belong to the constellation set and P is the frame (block) length. At each time slot t, signals cit, i = 1, 2. . . n are transmitted simultaneously from the n transmit antennas [6]. Ultimately, each transmit antenna sees a differently encoded version of the same signal. Upon being received, these signals are resolved by the receiver into a single signal. This has the effect of combating multi-path fading that has occurred in the separate channels. There have been many approaches to STBCs, the scheme of Alamouti being the first [1]. Very recently [10] and [11] have developed what they call Perfect STCs.

## 4.3 ALAMOUTI STBC

A simple Space Time Code suggested by Mr. Siavash M Alamouti in his landmark October 1998 paper – A Simple Transmit Diversity Technique for Wireless Communication, offers a simple method for achieving spatial diversity with two transmit antennas. The scheme is as follows:

1. Consider that we have a transmission sequence, for example {S1, S2, S3, S4,….,Sn}
2. In normal transmission, we will be sending $S_1$ in the first time slot, $S_2$ in the second time slot, $S_3$ and so on.
3. However, Alamouti's suggested that we group the symbols into groups of two. In the first time slot, send $S_1$ and $S_2$ from the first and second antenna. In second time slot send -S2* and S1* from the first and second antenna. In the third time slot send S3 and S4 from the first and second antenna. In fourth time slot, send -S4* and S3* from the first and second antenna and so on.
4. Notice that though we are grouping two symbols, we still need two time slots to send two symbols. Hence, there is no change in the data rate.
5. This forms the simple explanation of the transmission scheme with Alamouti's Space Time Block coding.

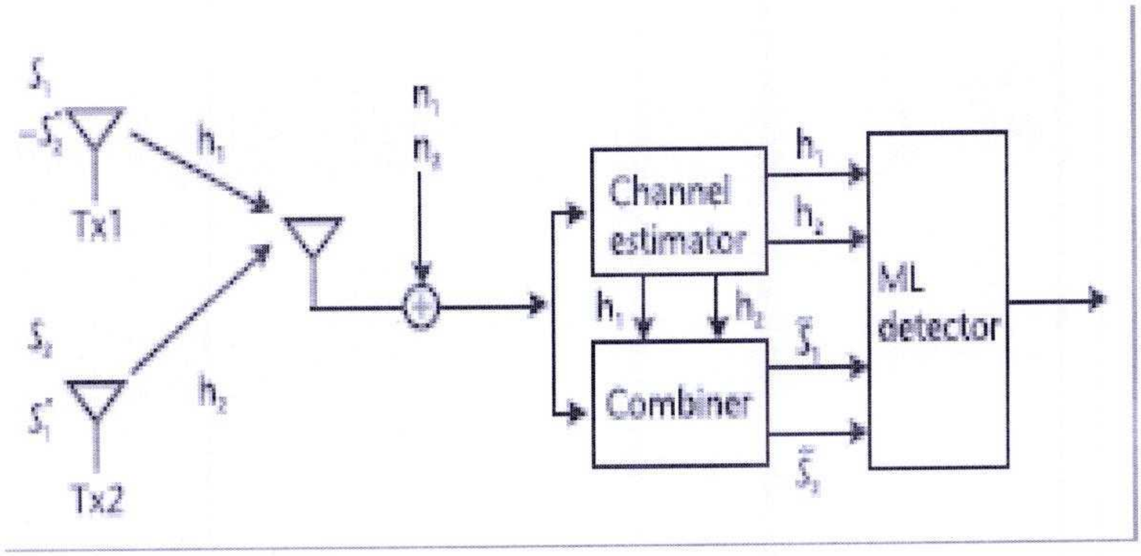



**Figure 4.3 Alamouti's two-antenna transmit diversity scheme.**

# CHAPTER 5

# PERFORMANCE ANALYSIS OF MIMO MODEL

## 5.1 INTRODUCTION

In MIMO channel, different factors or parameters are exist such as fading, noise, phase shift, co-channel interference (CCI), carrier frequency offset etc. In this chapter, we numerically derived and got SNR as well as BER expression for different antenna combination of MIMO model considering these effect.

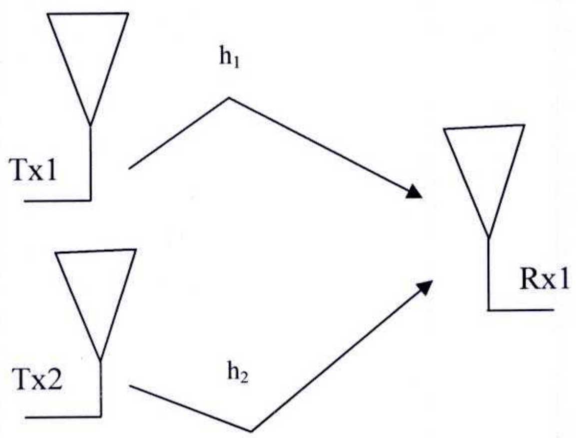
We consider various combination of transmit and receiving antenna for our analyses. Then numerically derived for every case and make equation which is applied in MATLAB for simulation. We divided it in different scheme which is given below:

## 5.2 MULTIPLE INPUT SINGLE OUTPUT (MISO)

In case of transmit diversity (MISO)-

1. Two transmit antenna, one receive antenna
2. Four transmit antenna, one receive antenna
3. Eight transmit antenna, one receive antenna

## 5.2.1 TWO TRANSMIT ANTENNA, ONE RECRIVE ANTENNA:




$$\text{Space antenna}\ \begin{array}{c|cc} & \multicolumn{2}{c}{\text{Time}} \\ \hline & x_1 & -x_2^* \\ & x_2 & x_1^* \end{array}$$

**Figure 5.1 2nd order transmit diversity with STBC and symbols distribution at transmission side**

First time slot, the received signal,

$y_1 = h_1x_1 + h_2x_2 + n_1$

$= [h_1 \quad h_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$

$2^{nd}$ time slot, the received signal,

$y_2 = -h_1x_2^* + h_2x_1^* + n_2$

$= [h_1 \quad h_2] \begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + n_2$

Where,

$y_1, y_2$ = Received symbol/signal on the $1^{st}$ and $2^{nd}$ time slot.

$h_1$ =Channel fading co-efficient associated with transmit antenna 1

$h_2$ = Channel fading co-efficient associated with transmit antenna 2

$x_1, x_2$ = Transmitted symbol.

$n_1, n_2$ =AWGN noise, respectively $1^{st}$ and $2^{nd}$ time slot with variance $\sigma_n^2 = N_o/2$ per real dimension.

So the received signal at two times instants, namely $r_1$ *and* $r_2$ is given by-

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2^* \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} \tag{1}$$

The received signal after maximum ratio combining is given by-

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} + \begin{bmatrix} r_1 \\ r_2^* \end{bmatrix}$$

$$= \begin{bmatrix} |h_1^2| + |h_2^2| & 0 \\ 0 & |h_1^2| + |h_2^2| \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} h_1^* n_1 + h_2 n_2^* \\ h_2^* n_1 - h_1 n_2^* \end{bmatrix} \tag{2}$$

So, SNR value for symbol $x_1$ is given by-

$$\text{SNR}= \frac{\left(|h_1^2| + |h_2^2|\right)^2 \frac{E_s}{2}}{\varepsilon\left(\left(h_1^* n_1 + h_2 n_2^*\right)\left(h_2^* n_1 + h_1 n_2^*\right)^*\right)}$$

$$= |h_1^2| + |h_2^2| \frac{E_s}{2N_o} \tag{3}$$

Here $E_s$ is the energy per transmitted symbol.

The received signal at the *k*th MS in the MU-STBC system with perfect channel knowledge at the transmitter side is given by [6]-

$$r_k(t) = \mathrm{H}_k \sum_{i=1}^{K} \mathrm{W}_i \mathrm{b}_i(t) + n_k(t) \tag{4}$$

However, when channel feedback error exits at the transmitter side, matrix $W_k$ is selected to be orthogonal to the erroneous channel matrix $\tilde{H}_k = H_k + E_k$ . As a result, the received signal at the $k$th terminal is given by [6]-

$$r_k(t) = \tilde{H}_k\, b_k(t) + \tilde{n}_k(t) \tag{5}$$

Where, $\tilde{n}_k(t)$ is the equivalent noise including both AWGN and the CCI from others users due to the imperfect cancellation.

$$\tilde{n}_k(t) = n_k(t) - x_k(t) \tag{6}$$

Let's consider the performance of MU-STBC system with one receive antenna per MS. Take the $k$th MS for example. For simplicity on the $k$th terminal is denoted as [6]

$$x_k(t) = E_k \sum_{i=1, i\neq k}^{K} W_i b_i(t) \tag{7}$$

Observing (6) and recalling that the entries of $\tilde{H}_k$, Gaussian random variables with zero-mean and unit variance, each MS observes a Rayleigh fading envelope from two transmit antennas, which is exactly the same form as represented in (2). The only difference is that that noise component in (2) is replaced by the combination of noise and CCI from other terminals. For the 1st symbol of $b_1(t)$, the equivalent noise after STBC decoding is derived as

$$\begin{aligned} s &= h_1^* \tilde{n}_k(1) + h_2\, \tilde{n}_k(2)^H \\ &= h_1^* n_k(1) + h_2\, n_k(1)^H - h_1^* x_k(1) - h_2 x_k(2)^H \end{aligned} \tag{8}$$

For simplicity, we approximate $x_k(2)$ with $x_k(1)$ thus the covariance of this equivalent noise component is given by-

$$\varepsilon(s^H s) \leq \left(| h_1^2 | + | h_2^2 |\right)\left(N_o + 2\varepsilon\left(| x_k(1) |^2\right)\right) \tag{9}$$

Referring to (3), the SNR after STBC decoding is obtained as

$$\begin{aligned} \text{SINR} &\leq \frac{\left(| h_1^2 | + | h_2^2 |\right)^2 \frac{E_s}{2}}{\left(| h_1^2 | + | h_2^2 |\right)\left(N_o + 2\varepsilon\left(| x_k(1) |^2\right)\right)} \\ &= \frac{\left(| h_1^2 | + | h_2^2 |\right)}{1 + 2\varepsilon\left(| x_k(1) |^2\right)/N_o} \times \frac{E_s}{2N_o} \end{aligned} \tag{10}$$

To simplify this result, we refer to (7) and obtain

$$\varepsilon\left(| x_k(1) |^2\right) = (K-1)\sigma_{MSE}^2 E_s \tag{11}$$

Substituting (11) into (10), the final result of SNR value for MU-STBC system, with channel estimation error at the transmitter side, is given by

$$\mathrm{SINR} \le \frac{\left(|h_1^2| + |h_2^2|\right)}{1+2(K-1)\sigma_{MSE}^2 E_s / N_o} \times \frac{E_s}{2N_o} \qquad (12)$$

The degraded SNR shown above is dependent on a number of system parameters, e.g., total user number $K$, channel estimation error $\sigma_{MSE}^2$ and system power level $E_s / N_o$. When there is only $K = 1$ user or the channel is perfect $\sigma_{MSE}^2 = 0$, a diversity gain of two is obtained.

The closed form BER performance associated with (12) is obtained by [2]

$$P_e = \left[\frac{1}{2}(1-\mu)\right]^2 \sum_{k=0}^{1} \frac{(k+1)!}{k!} \left[\frac{1}{2}(1+\mu)\right]^k$$

Where $\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$ and

Average SINR

$$\bar{\gamma} = \frac{E_s / 2N_o}{1+2(K-1)\sigma_{MSE}^2 E_s / N_o}$$

## 5.2.2 FOUR TRANSMIT ANTENNA, ONE RECRIVE ANTENNA

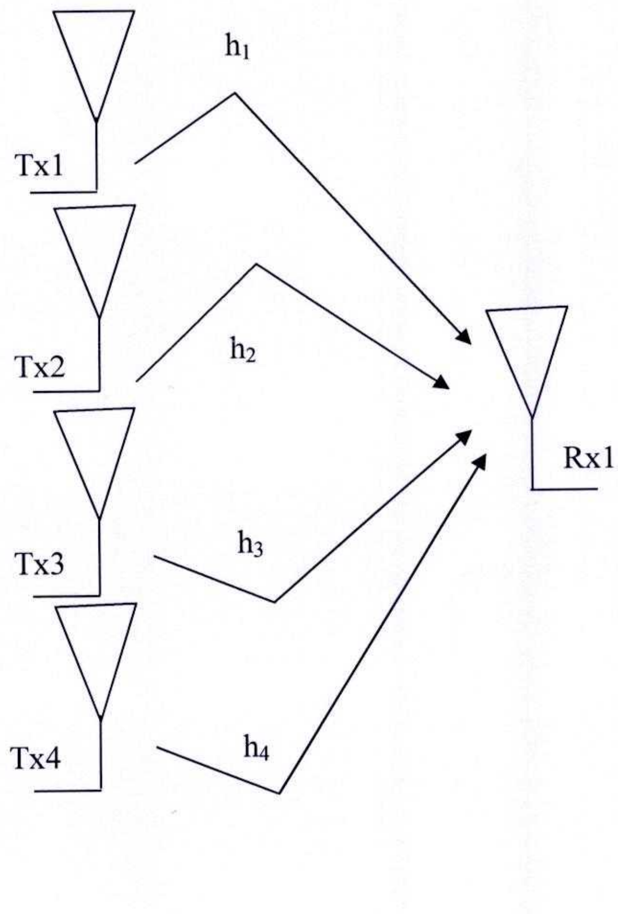


**Figure 5.2 4[th] order transmit diversity with STBC and symbols distribution at transmission side**

$$
\begin{array}{l|cccc}
 & \multicolumn{4}{l}{\text{Time}} \\
\hline
\text{Space} & x_1 & -x_2^* & x_3^* & -x_4 \\
\text{antenna} & x_2 & x_1^* & x_4^* & x_3 \\
 & x_3 & -x_4^* & -x_1^* & x_2 \\
 & x_4 & x_3^* & -x_2^* & x_1
\end{array}
$$

Similarly the SNR for two transmitter and one receiver antenna without CCI,

$$
\text{SNR} = \frac{\left(|h_1^2|+|h_2^2|+|h_3^2|+|h_4^2|\right)^2 E_s/4}{\varepsilon\left(\left(h_1^* n_1 + h_2 n_2^* + h_3 n_3^* + h_4 n_4\right)\left(h_1^* n_1 + h_2 n_2^* + h_3 n_3^* + h_4 n_4\right)^*\right)}
$$

$$
= \left(|h_1^2|+|h_2^2|+|h_3^2|+|h_4^2|\right)E_s/4N_o
$$

And with AWGN and CCI

$$
\text{SINR} \leq \frac{\left(|h_1^2|+|h_2^2|+|h_3^2|+|h_4^2|\right)}{1+4(K-1)\sigma_{MSE}^2 E_s/N_o} \times \frac{E_s}{4N_o}
$$

So the closed form BER performance

$$
P_e = \left[\frac{1}{2}(1-\mu)\right]^4 \sum_{k=0}^{3} \frac{(k+3)!}{k!\,3!}\left[\frac{1}{2}(1+\mu)\right]^k
$$

Where $\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$ and

Average SINR

$$
\bar{\gamma} = \frac{E_s/4N_o}{1+4(K-1)\sigma_{MSE}^2 E_s/N_o}
$$

## 5.2.3 EIGHT TRANSMIT ANTENNA, ONE RCEIVE ANTENNA

Similarly the derivation for eight transmit antenna and one receive antenna is same as two transmit antenna and one receive antenna.

So the closed form BER performance

$$P_e = \left[\frac{1}{2}(1-\mu)\right]^8 \sum_{k=0}^{7} \frac{(k+7)!}{k!\,7!} \left[\frac{1}{2}(1+\mu)\right]^k$$

Where $\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$ and

Average SINR

$$\bar{\gamma} = \frac{E_s / 8N_o}{1 + 8(K-1)\sigma_{MSE}^2 E_s / N_o}$$

## 5.3 SINGLE INPUT MULTIPLE OUTPUT (SIMO)

In case of receiving diversity (SIMO)-

1. One transmit antennas, two receive antenna
2. One transmit antennas, four receive antenna
3. One transmit antennas, eight receive antenna

## 5.3.1 ONE TRANSMIT ANTENNA, TWO RECEIVE ANTENNA

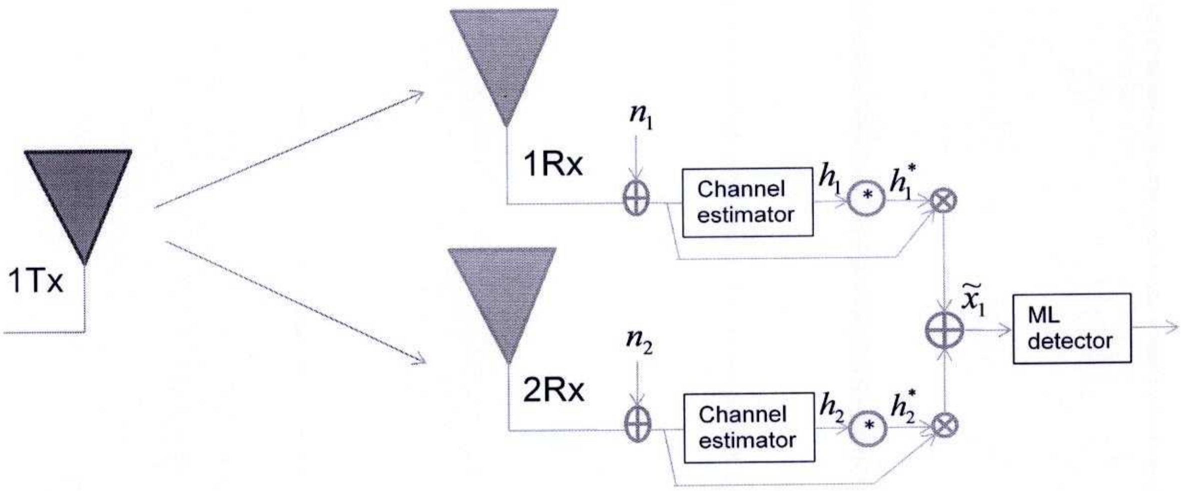



**Figure 5.3 Block diagram of second order receive diversity**

The received signals are

$$r_1 = h_1 x_1 + n_1$$
$$r_2 = h_2 x_1 + n_2$$

The receiver combining scheme for two-branch MRRC is as follows:

$$\widetilde{x}_1 = h_1^* x_1 + h_2^* r_2$$
$$= |h_1|^2 + |h_2|^2 + h_1^* n_1 + h_2^* n_2$$

The signal-to-noise ratio (SNR) becomes

$$\text{SNR}= \frac{(|h_1^2| + |h_2^2|)\ E_s}{N_0}$$

With channel feedback error or CCI SNR becomes

$$\text{SINR} \leq \frac{(|h_1^2| + |h_2^2|)}{1+2(K-1)\sigma_{MSE}^2 E_s / N_o} \times \frac{E_s}{N_o}$$

The bit error probability (BER) is given by

$$P_e = \left[\frac{1}{2}(1-\mu)\right]^4 \sum_{k=0}^{3} \frac{(k+3)!}{k!3!} \left[\frac{1}{2}(1+\mu)\right]^k$$

Where $\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$

And average SINR

$$\bar{\gamma} = \frac{E_s / N_o}{1+2(K-1)\sigma_{MSE}^2 E_s / N_o}$$

## 5.3.2 ONE TRANSMIT ANTENNA, FOUR RECEIVE ANTENNA

Similarly the derivation for one transmit antenna and four receive antenna is same as one transmit antenna and two receive antenna.

The bit error probability (BER) is given by

$$P_e = \left[\frac{1}{2}(1-\mu)\right]^8 \sum_{k=0}^{7} \frac{(k+7)!}{k!7!} \left[\frac{1}{2}(1+\mu)\right]^k$$

Where $\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$

And average SINR

$$\bar{\gamma} = \frac{E_s / N_o}{1 + 4(K-1)\sigma_{MSE}^2 E_s / N_o}$$

### 5.3.3 ONE TRANSMIT ANTENNA, EIGHT RECEIVE ANTENNA

Similarly the derivation for one transmit antenna and eight receive antenna is same as one transmit antenna and two receive antenna.

The bit error probability (BER) is given by

$$P_e = \left[\frac{1}{2}(1-\mu)\right]^{16} \sum_{k=0}^{15} \frac{(k+15)!}{k!15!} \left[\frac{1}{2}(1+\mu)\right]$$

Where $\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$

And average SINR

$$\bar{\gamma} = \frac{E_s / N_o}{1 + 8(K-1)\sigma_{MSE}^2 E_s / N_o}$$

# 5.4 PERFORMANCE EVALUATION

MATLAB simulation result provided in this section to demonstrate the performance of above schemes. In Fig. 5.4, we plot the bit error rate performance of the scheme against *EbN0* for various numbers of the transmit diversity $n_T$. From this figure, we can observe that at the BER of 10-4 the error performance is improved by about 6.5dB and 7dB, when the transmit diversity order is increased from two to four and four to eight, respectively. For a large number of diversity branches, the fading channel converges towards an AWGN channel, as the error performance curve for a large $n_T$ almost approaches the one for the AWGN channel.

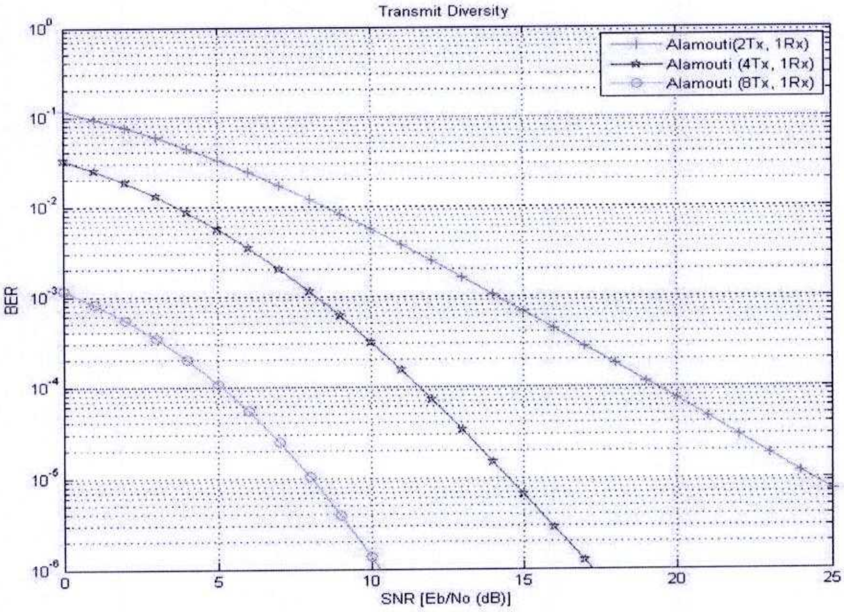



**Figure 5.4 BER performance of coherent BPSK on Rayleigh fading channels with MRC Transmit diversity; the top curve corresponds to the performance with 2 transmit antenna and 1 receive antenna; the other lower curves correspond to systems with 4 and 8 transmit antennas, respectively, starting from the top.**

The bit error rate curves for various numbers of receive antennas $nR$ are depicted in Fig.5.5. The receive antenna diversity dramatically improves the error performance compared to the case Transmit diversity. In particular, we observe that the error probability decreases inversely with the $nR$-th power of the SNR. For the same error rate of 10^-4, the MRC receive diversity technique reduces the transmission power by about 11 dB and 6.5 dB, when the number of receive antennas is increased from two to four and four to eight successively.

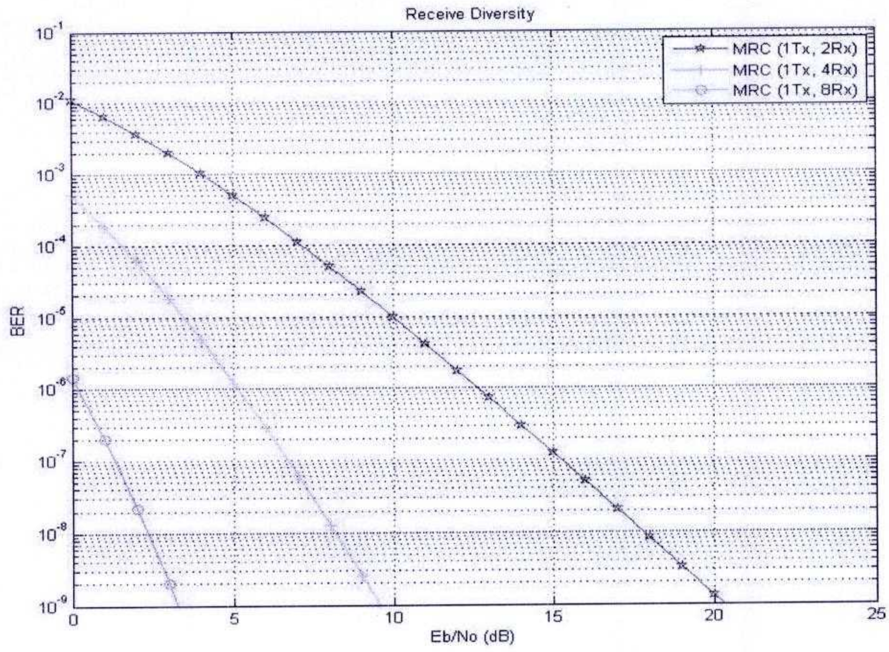



**Figure 5.5 BER performance of coherent BPSK on Rayleigh fading channels with MRC receive diversity; the top curve corresponds to the performance with 2 receive antenna and 1 transmit antenna; the other lower curves correspond to systems with 4 and 8 receive antennas, respectively, starting from the top.**

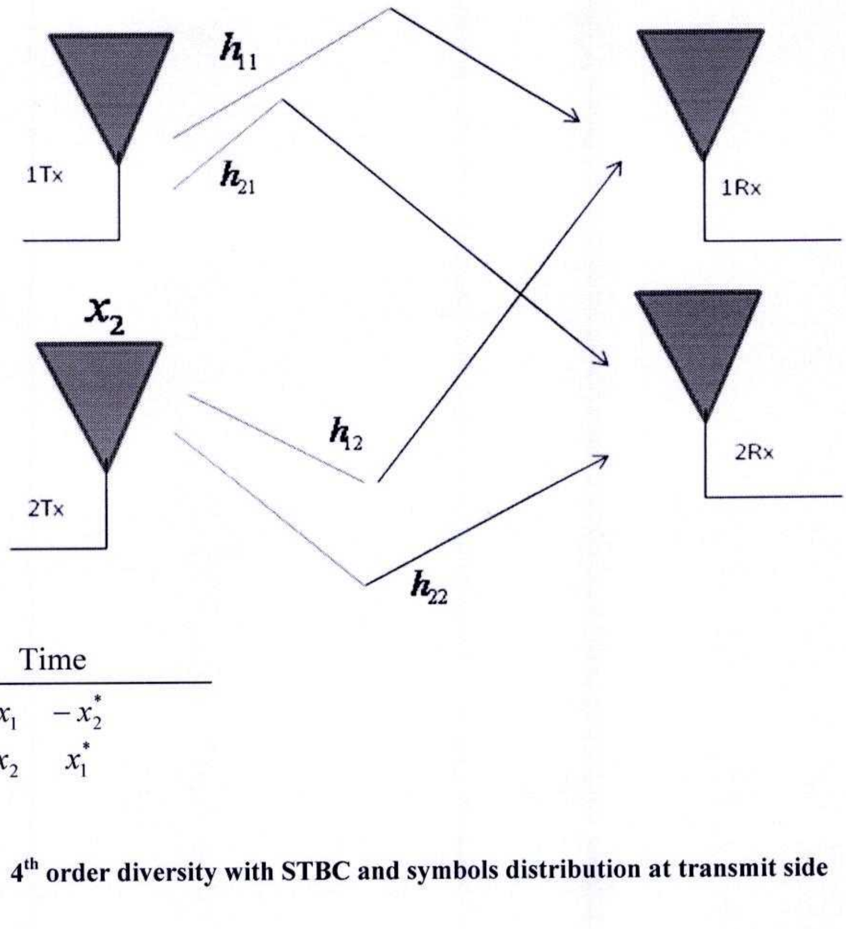
# CHAPTER 6

# MIMO MODEL

## 6.1 MIMO PERFORMANCE OVER DIFFERENT ANTENNA COMBINATION

Previously, we only consider either transmit diversity (MISO) or receive diversity (SIMO) for a single communication model. But now we combine these both case in same scenario which is also known as MIMO model. In this model, more than one antenna must exist in both sides. Below, the SNR and BER have been derived for different antenna combination with MATLAB simulation result.

### 6.1.1 TWO TRANSMIT ANTENNA, TWO RECRIVE ANTENNA:




$$\begin{array}{l|cc} & \text{Time} & \\ \hline \text{Space antenna} & x_1 & -x_2^* \\ & x_2 & x_1^* \end{array}$$

**Figure 6.1 4th order diversity with STBC and symbols distribution at transmit side**

First time slot, the received signal,

$$\begin{bmatrix} y_1^1 \\ y_2^1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1^1 \\ n_2^1 \end{bmatrix}$$

2nd time slot, the received signal,

$$\begin{bmatrix} y_1^2 \\ y_2^2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + \begin{bmatrix} n_1^2 \\ n_2^2 \end{bmatrix}$$

Where,

$\begin{bmatrix} y_1^1 \\ y_2^1 \end{bmatrix}$ = received information at time slot 1 on receive antenna 1, 2 respectively

$\begin{bmatrix} y_1^2 \\ y_2^2 \end{bmatrix}$ =received information at time slot 2 on receive antenna 1, 2 respectively

$x_1, x_2$ = Transmitted symbol.

$\begin{bmatrix} n_1^1 \\ n_2^1 \end{bmatrix}$ =noise at time slot 1 on receive antenna 1, 2 respectively

$\begin{bmatrix} n_1^2 \\ n_2^2 \end{bmatrix}$ =noise at time slot 2 on receive antenna 1, 2 respectively

Combining the equations at time slot 1 and 2

$$\begin{bmatrix} y_1^1 \\ y_2^1 \\ y_1^{2^*} \\ y_2^{2^*} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12}^* & -h_{11}^* \\ h_{22}^* & -h_{21}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1^1 \\ n_2^1 \\ n_1^{2^*} \\ n_2^{2^*} \end{bmatrix} \tag{1}$$

The received signal after maximum ratio combining is given by-

$$\begin{bmatrix} |h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2 & 0 \\ 0 & |h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} h_{11}^* n_{11} + h_{12} n_{12} \\ h_{12}^* n_{11} - h_{11} n_{12} \\ h_{21}^* n_{11} + h_{22} n_{22} \\ h_{22}^* n_{11} - h_{21} n_{22} \end{bmatrix} \tag{2}$$

So, SNR value for symbol $x_1$ is given by-

$$\text{SNR}=|h_{11}|^2+|h_{21}|^2+|h_{12}|^2+|h_{22}|^2*\frac{E_s}{2N_0} \tag{3}$$

Here $E_s$ is the energy per transmitted symbol.

Referring to (3), the SNR after STBC decoding is obtained as

$$\text{SINR}=\frac{\left(|h_{11}|^2+|h_{21}|^2+|h_{12}|^2+|h_{22}|^2\right)}{1+4\varepsilon\left(|x_k(1)|^2\right)/N_o}\times\frac{E_s}{2N_o} \tag{4}$$

To simplify this result, let consider the performance of MU-STBC system with one receiver per MS. Take the Kth MS for example,

$$\varepsilon\left(|x_k(1)|^2\right)=(K-1)\sigma_{MSE}^2E_s \tag{5}$$

Substituting (5) into (4), the final result of SNR value for MU-STBC system, with channel estimation error at the transmitter side, is given by

$$\text{SINR}\le\frac{\left(|h_{11}|^2+|h_{21}|^2+|h_{12}|^2+|h_{22}|^2\right)}{1+4(K-1)\sigma_{MSE}^2E_s/N_o}\times\frac{E_s}{2N_o} \tag{6}$$

The degraded SNR shown above is dependent on a number of system parameters, e.g., total user number $K$, channel estimation error $\sigma_{MSE}^2$ and system power level $E_s/N_o$. When there is only $K=1$ user or the channel is perfect $\sigma_{MSE}^2=0$, a diversity gain of two is obtained.

The closed form BER performance associated with (6) is obtained by [2]

$$P_e=[\frac{1}{2}(1-\mu)]^4\sum_{k=0}^{3}\frac{(k+3)!}{k!}[\frac{1}{2}(1-\mu)]^k$$

Where $\mu=\sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$ and

Average SINR

$$\bar{\gamma}=\frac{E_s/2N_o}{1+4(K-1)\sigma_{MSE}^2E_s/N_o}\times4\sigma_\alpha^2$$

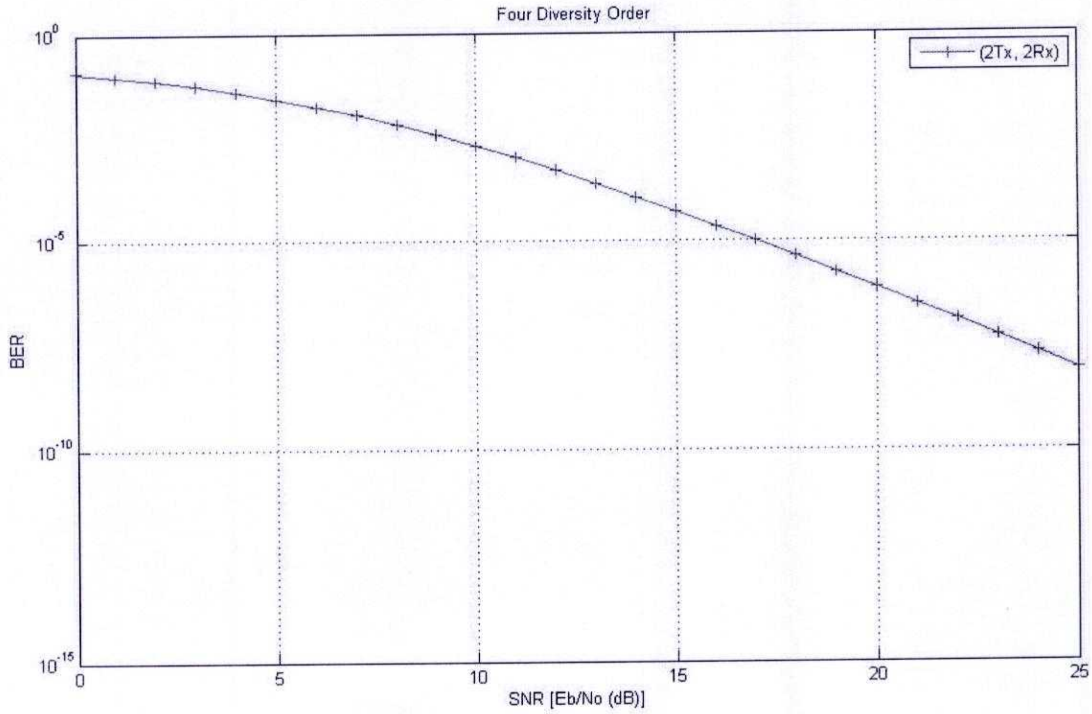



**Figure 6.2. Performance curves for 4th diversity order a fast and block Rayleigh fading channel with maximum ratio diversity combining**

## 6.1.2 TWO TRANSMIT ANTENNA, FOUR RECEIVE ANTENNA

The derivation of SINR and BER for two transmit antenna and four receive antenna are same as the derivation for two transmit antenna and two receive antenna. The difference is the number of channel will be increased which is the multiplication of transmit and receive antenna of an antenna combination model.
After deriving the closed form BER performance expression is,

$$P_e = [\frac{1}{2}(1-\mu)]^8 \sum_{k=0}^{7} \frac{(k+7)!}{k!} [\frac{1}{2}(1-\mu)]^k$$

Where $\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$ and

Average SINR

$$\bar{\gamma} = \frac{E_s / 2N_o}{1 + 8(K-1)\sigma_{MSE}^2 E_s / N_o} \times 8\sigma_\alpha^2$$

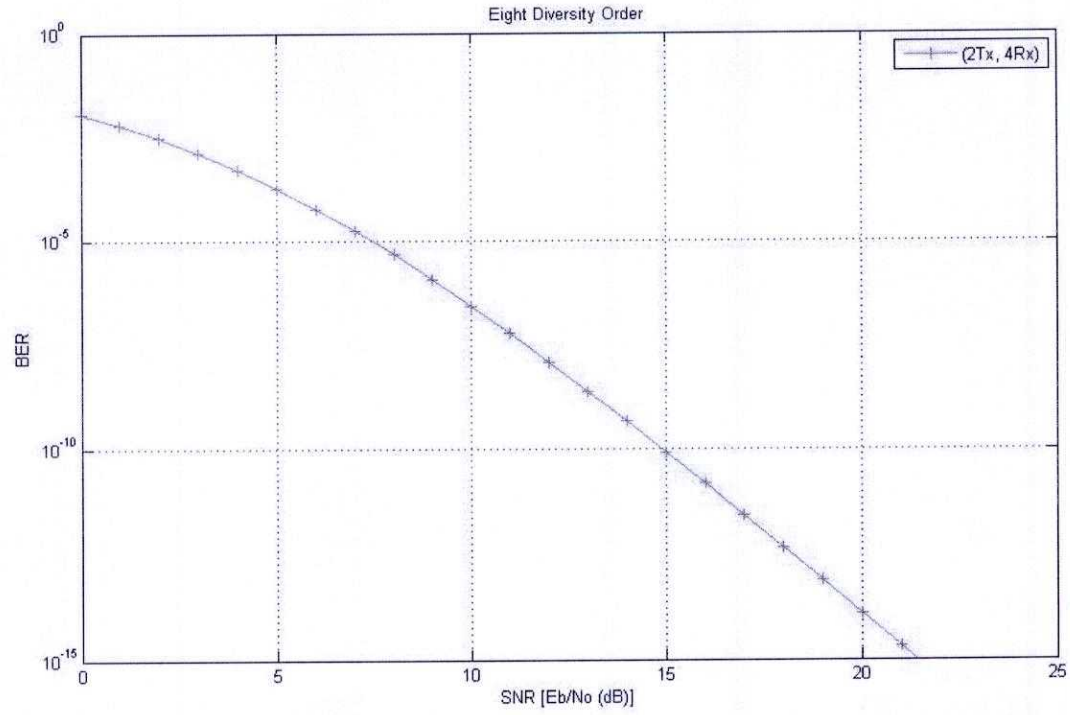



**Figure 6.3 Performance curves for 8th diversity order a fast and block Rayleigh fading channel with maximum ratio diversity combining**

## 6.1.3 TWO TRANSMIT ANTENNA, EIGHT RECRIVE ANTENNA:

The derivation of SINR and BER for two transmit antenna and eight receive antenna are same as the derivation for two transmit antenna and two receive antenna. The difference is the number of channel will be increased which is the multiplication of transmit and receive antenna of an antenna combination model.
After deriving the closed form BER performance expression is,

$$P_e = \left[\frac{1}{2}(1-\mu)\right]^{16} \sum_{k=0}^{15} \frac{(k+15)!}{k!} \left[\frac{1}{2}(1+\mu)\right]^{k}$$

Where $\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$ and

Average SINR

$$\bar{\gamma} = \frac{E_s / 2N_o}{1+16(K-1)\sigma_{MSE}^2 E_s / N_o} \times 16\sigma_{\alpha}^2$$


**Figure 6.4** **Performance curves for 16th diversity order a fast and block Rayleigh fading channel with maximum ratio diversity combining**

## 6.1.4 PERFORMANCE COMPARISON FOR 2Tx: 2Rx, 4Rx AND 8Rx

Here we compare the performance of two transmit antenna and two receive antenna model with the performance of two transmit antenna and four receive antennas as well as two transmit antenna and eight receive antenna model in a same graph. We see that highest number of receiving antenna give better performance than other.

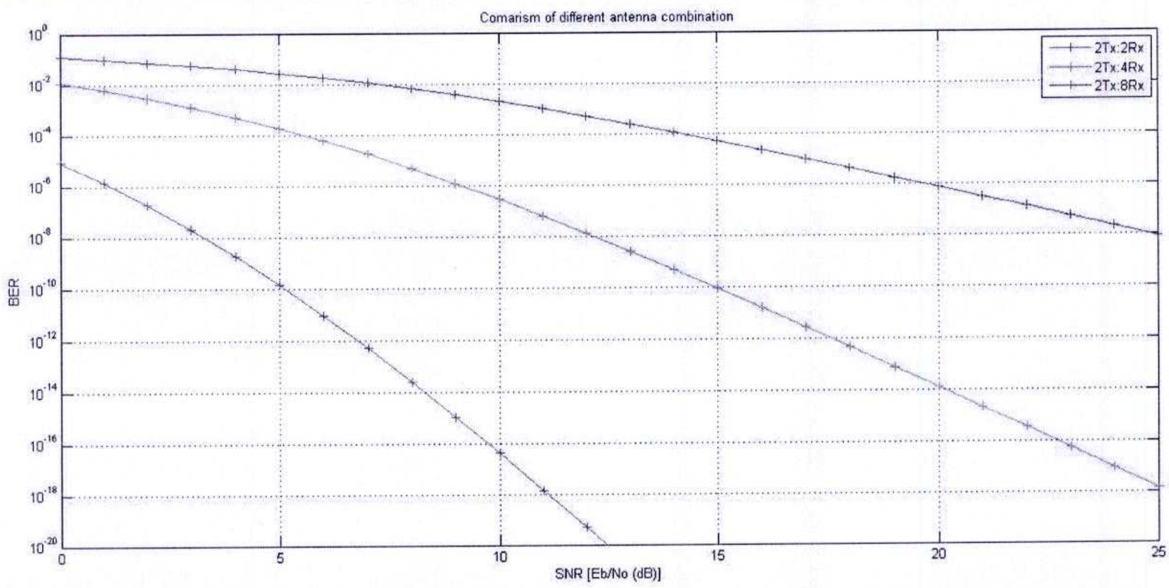



**Figure 6.5** **Performance comparison of 4,8 &16 diversity order a fast and block Rayleigh fading channel with maximum ratio diversity combining**

## 6.2 FOUR TRANSMIT ANTENNA, TWO RECRIVE ANTENNA

The derivation of SINR and BER for four transmit antenna and two receive antenna are same as the derivation for two transmit antenna and two receive antenna. The difference is the number of channel will be increased which is the multiplication of transmit and receive antenna of an antenna combination model.
After deriving the closed form BER performance expression is,

$$P_e = \left[\frac{1}{2}(1-\mu)\right]^8 \sum_{k=0}^{7} \frac{(k+7)!}{k!} \left[\frac{1}{2}(1+\mu)\right]^k$$

Where $\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$ and

Average SINR

$$\bar{\gamma} = \frac{E_s / 4N_o}{1 + 8(K-1)\sigma_{MSE}^2 E_s / N_o} \times 8\sigma_\alpha^2$$


**Figure 6.6 Performance curves for 8th diversity order a fast and block Rayleigh fading channel with maximum ratio diversity combining**

### 6.2.1 FOUR TRANSMIT ANTENNA, FOUR RECRIVE ANTENNA

The derivation of SINR and BER for four transmit antenna and four receive antenna are same as the derivation for two transmit antenna and two receive antenna. The difference is the number of channel will be increased which is the multiplication of transmit and receive antenna of an antenna combination model.

After deriving the closed form BER performance expression is,

$$P_e = \left[\frac{1}{2}(1-\mu)\right]^{16} \sum_{k=0}^{15} \frac{(k+15)!}{k!} \left[\frac{1}{2}(1+\mu)\right]^k$$

Where $\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$ and

Average SINR

$$\bar{\gamma} = \frac{E_s / 4N_o}{1+16(K-1)\sigma_{MSE}^2 E_s / N_o} \times 16\sigma_\alpha^2$$

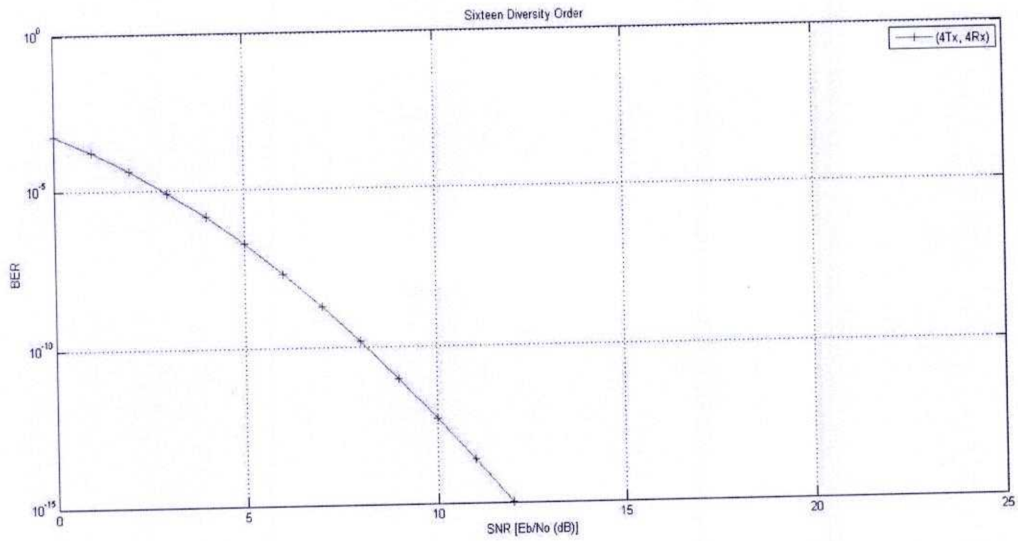



**Figure 6.7 Performance curves for 16 diversity order a fast and block Rayleigh fading channel with maximum ratio diversity combining**

## 6.2.2 FOUR TRANSMIT ANTENNA, EIGHT RECRIVE ANTENNA

The derivation of SINR and BER for four transmit antenna and eight receive antenna are same as the derivation for two transmit antenna and two receive antenna. The difference is the number of channel will be increased which is the multiplication of transmit and receive antenna of an antenna combination model.
After deriving the closed form BER performance expression is,

$$P_e = \left[\frac{1}{2}(1-\mu)\right]^{32} \sum_{k=0}^{31} \frac{(k+31)!}{k!} \left[\frac{1}{2}(1+\mu)\right]^k$$

Where $\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$ and

Average SINR

$$\bar{\gamma} = \frac{E_s / 4N_o}{1 + 32(K-1)\sigma_{MSE}^2 E_s / N_o} \times 32\sigma_\alpha^2$$

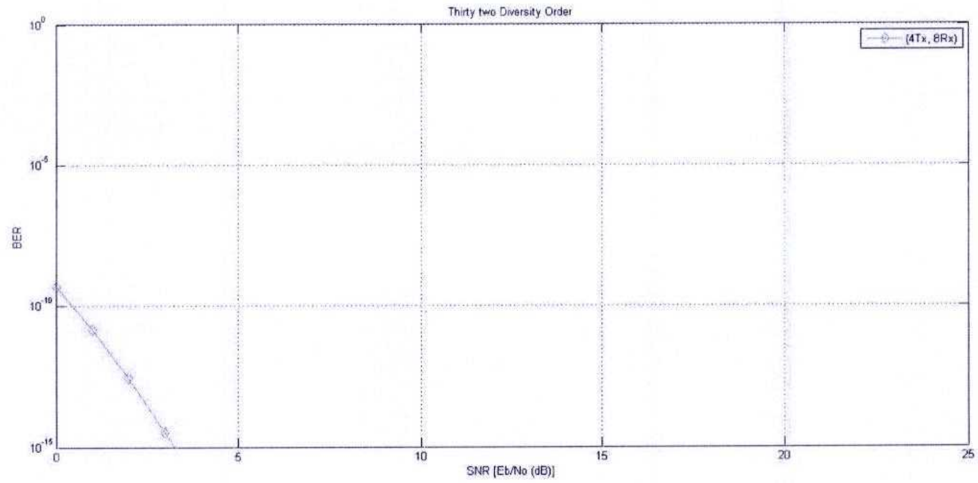



**Figure 6.8 Performance curves for 32 diversity order a fast and block Rayleigh fading channel with maximum ratio diversity combining**

## 6.2.3 PERFORMANCE COMPARISON FOR 4Tx: 2Rx, 4Rx AND 8Rx ANTENNA

Here we compare the performance of four transmit antenna and two receive antenna model with the performance of four transmit antenna and four receive antennas as well as four transmit antenna and eight receive antenna model in a same graph. We see that highest number of receiving antenna give better performance than other.

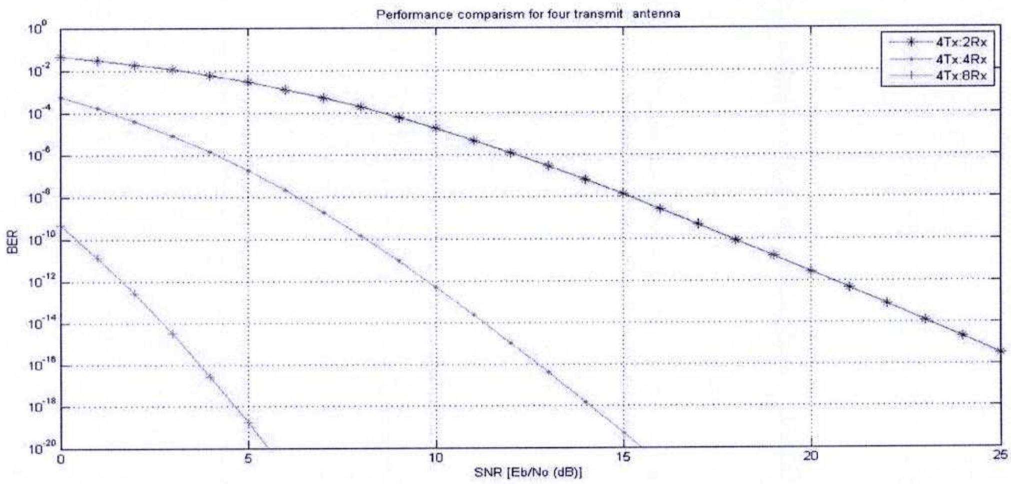



**Figure 6.9 Performance comparison of 8, 16 &32 diversity order a fast and block Rayleigh fading channel with maximum ratio diversity combining**

# CHAPTER 7

# SNR IMPROVEMENT OVER DIFFERENT ANTENNA COMBINATION

We also show SNR improvement curve by increasing receiving antenna keeping constant of transmit antenna at BER=10^-10. Simultaneously, we increase transmit antenna for another antenna combination model and plot SNR improvement graph with the increase of receiving antenna. Below SNR improvement graph is given for different antenna combination.

## 7.1 TWO TRANSMIT ANTENNA & N NUMBER OF RECEIVING ANTENNA

Below the graph, each plotting curve represents the performance for a certain number of receiving antenna with two transmit antenna. The upper most curve represents for two receiving antenna. Sequentially, the lower most curve represent for twelve receiving antenna.

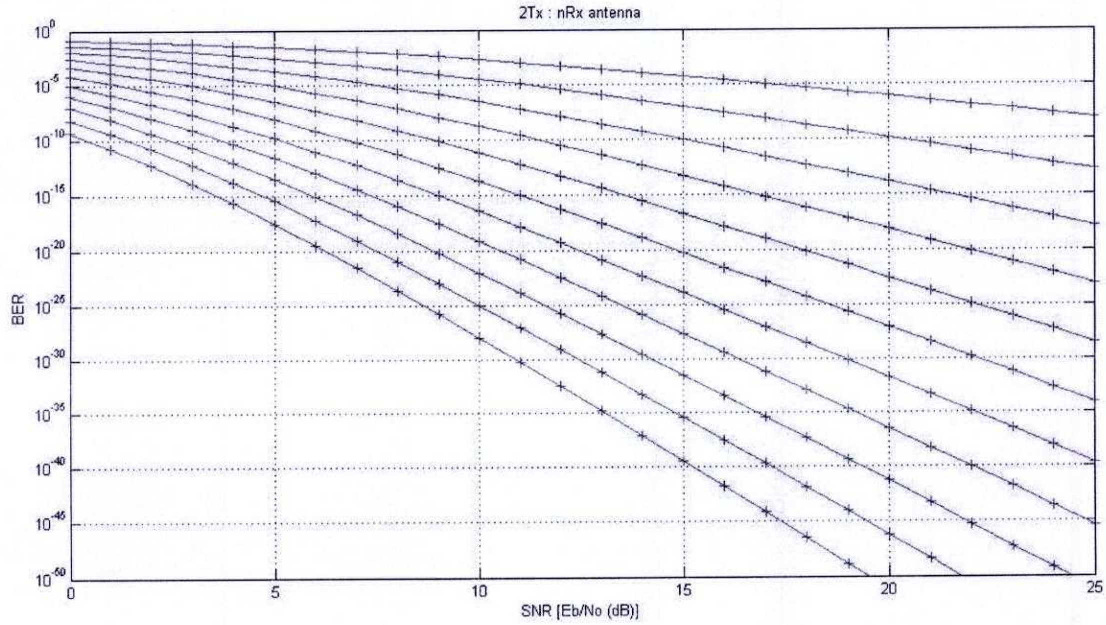



**Figure 7.1 BER performance of STBC on Rayleigh fading channels with receive diversity; the top curve corresponds to 2 transmit & 2 receive antennas, and the curves below it correspond to systems with 3, 4, 5, 6, 7, 8, 9,10, 11 and 12 receive antennas, respectively, starting from the top**

## Table of SNR improvement for N number of receiving antenna:

We obtain the table below from the upper figure, where we take SNR of two transmit antenna at BER=10^-10 as reference and find SNR improvement for other receiving antenna

| No. of Rx antenna | SNR at BER=10^-10 | SNR improvement |
|---|---|---|
| 2 | 25(reference) | 0 |
| 3 | 20 | 5 |
| 4 | 14 | 9 |
| 5 | 11 | 14 |
| 6 | 8 | 17 |
| 7 | 6 | 19 |
| 8 | 5 | 20 |
| 9 | 3 | 22 |
| 10 | 2 | 23 |
| 11 | 1 | 24 |
| 12 | 0 | 25 |

Then we plot the curve for receiving antenna vs. SNR improvement at BER=10^-10 which is also called performance improvement curve,

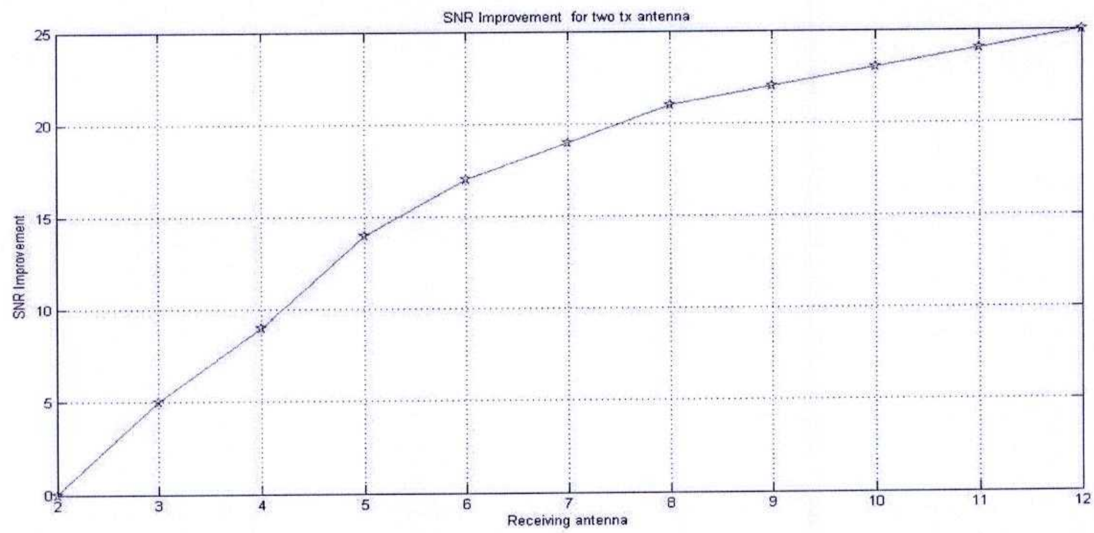



**Figure 7.2 SNR improvement curve on STBC of Rayleigh fading channel with two transmit antenna and the increase of receiving antenna**

## 7.2 THREE TRANSMIT ANTENNA & N NUMBER OF RECEIVING ANTENNA

Below the graph, each plotting curve represents the performance for a certain number of receiving antenna with three transmit antenna. The upper most curve represents for three receiving antenna. Sequentially, the lower most curve represent for twelve receiving antenna.

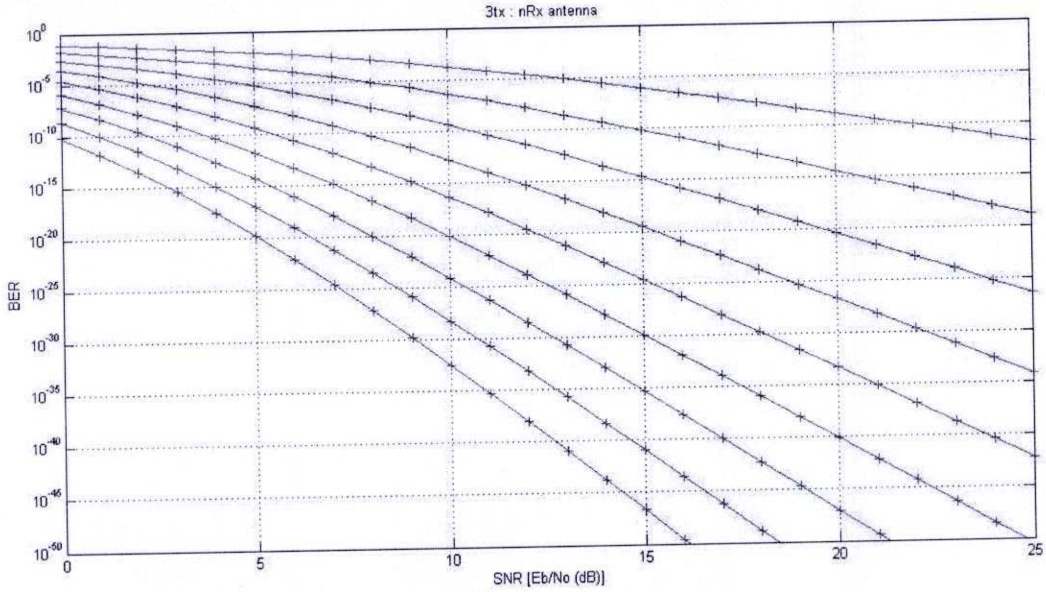



**Figure 7.3 BER performance of STBC on Rayleigh fading channels with receive diversity; the top curve corresponds to 3 transmit & 2 receive antennas, and the curves below it correspond to systems with 3, 4, 5, 6, 7, 8, 9 and 10 receive antennas, respectively, starting from the top**

## Table of SNR improvement for N number of receiving antenna

We obtain the table below from the upper figure, where we take SNR of three transmit antenna at BER=10^-10 as reference and find SNR improvement for other receiving antenna

| No. of Rx antenna | SNR at BER=10^-10 | SNR improvement |
|---|---|---|
| 2 | 21(reference) | 0 |
| 3 | 14 | 7 |
| 4 | 10 | 11 |
| 5 | 7 | 14 |
| 6 | 5 | 16 |
| 7 | 3 | 18 |
| 8 | 2 | 19 |
| 9 | 1 | 20 |
| 10 | 0 | 21 |

Then we plot the curve for receiving antenna vs. SNR improvement at BER=10^-10 which is also called performance improvement curve,

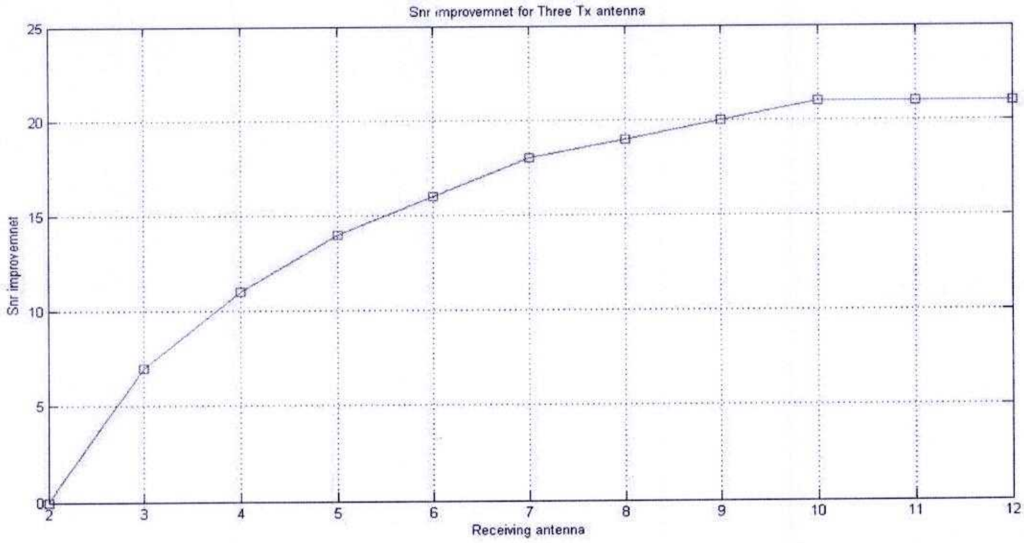



**Figure 7.4 SNR improvement curve on STBC of Rayleigh fading channel with three transmit antenna and the increase of receiving antenna**

# 7.3 FOUR TRANSMIT ANTENNA & N NUMBER OF RECEIVING ANTENNA

Below the graph, each plotting curve represents the performance for a certain number of receiving antenna with four transmit antenna. The upper most curve represents for four receiving antenna. Sequentially, the lower most curve represent for twelve receiving antenna

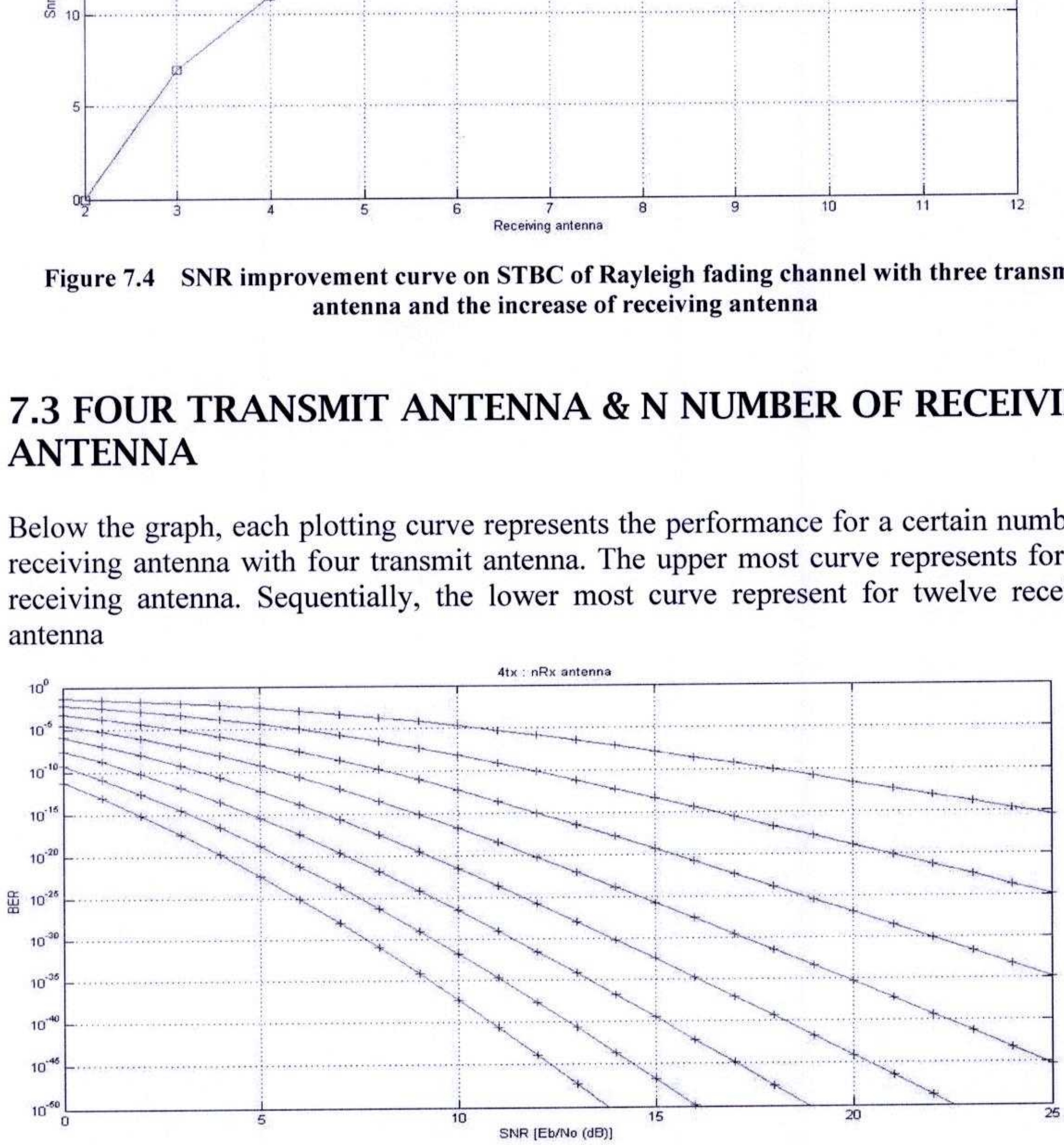



**Figure 7.5 BER performance of STBC on Rayleigh fading channels with receive diversity; the top curve corresponds to 4 transmit & 2 receive antennas, and the curves below it correspond to systems with 3, 4, 5, 6, 7, 8 and 9 receive antennas, respectively, starting from the top**

## Table of SNR improvement for N number of receiving antenna

We obtain the table below from the upper figure, where we take SNR of four transmit antenna at BER=10^-10 as reference and find SNR improvement for other receiving antenna

| No. of Rx antenna | SNR at BER=10^-10 | SNR improvement |
|---|---|---|
| 2 | 18(reference) | 0 |
| 3 | 12 | 6 |
| 4 | 8 | 10 |
| 5 | 6 | 12 |
| 6 | 4 | 14 |
| 7 | 2 | 16 |
| 8 | 1 | 17 |
| 9 | 0 | 18 |

Then we plot the curve for receiving antenna vs. SNR improvement at BER=10^-10 which is also called performance improvement curve,

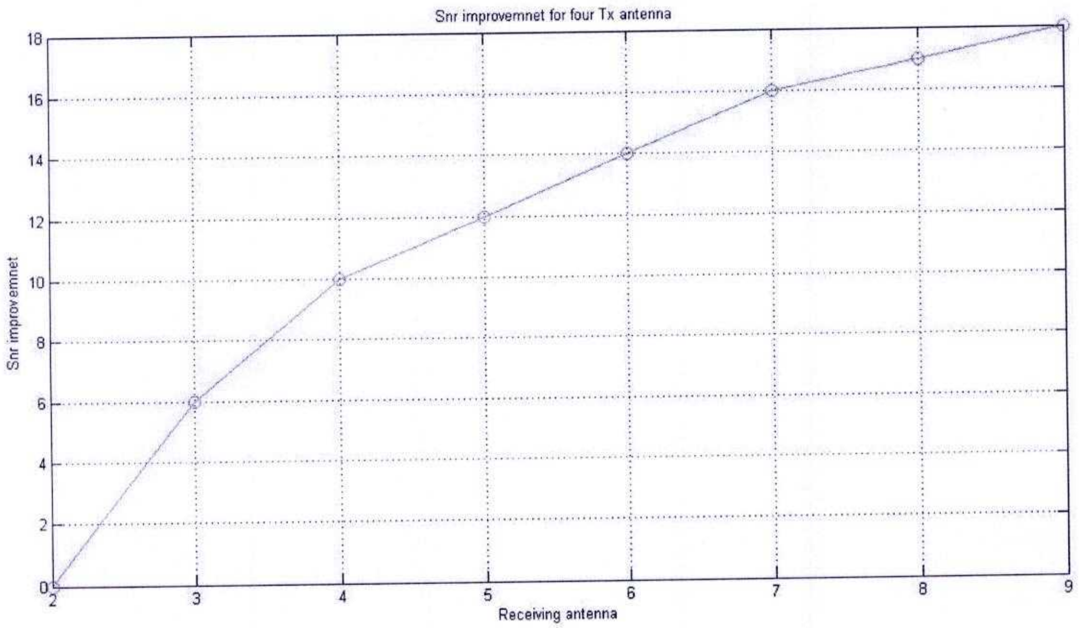



**Figure 7.6 SNR improvement curve on STBC of Rayleigh fading channel with four transmit antenna and the increase of receiving antenna**

# 7.4 FIVE TRANSMIT ANTENNA & N NUMBER OF RECEIVING ANTENNA

Below the graph, each plotting curve represents the performance for a certain number of receiving antenna with two transmit antenna. The upper most curve represents for two receiving antenna. Sequentially, the lower most curve represent for twelve receiving antenna

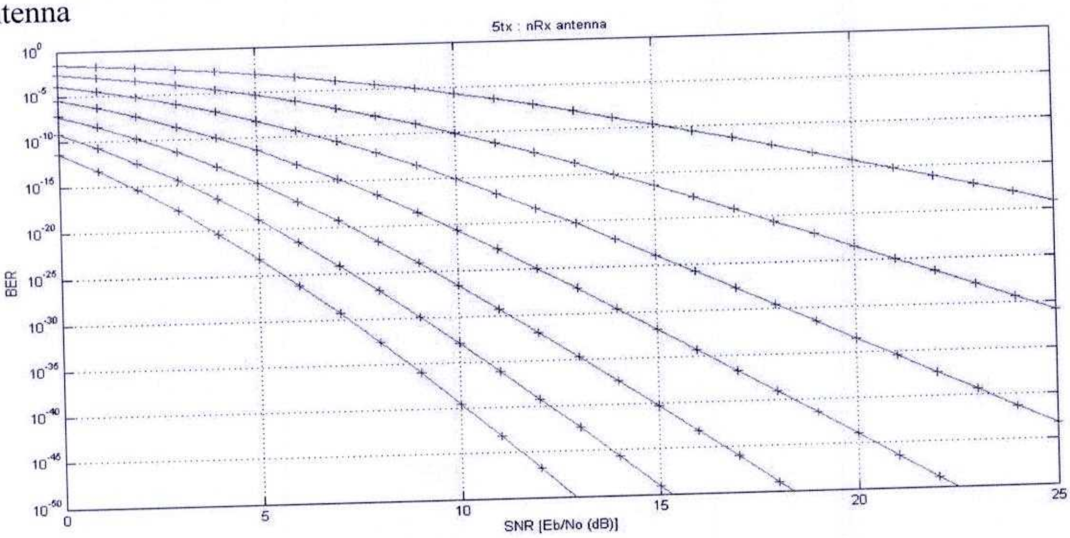



**Figure 7.7 BER performance of STBC on Rayleigh fading channels with receive diversity; the top curve corresponds to 5 transmit & 2 receive antennas, and the curves below it correspond to systems with 3, 4, 5, 6, 7 and 8receive antennas, respectively, starting from the top**

## Table of SNR improvement for N number of receiving antenna

We obtain the table below from the upper figure, where we take SNR of two transmit antenna at BER=10^-10 as reference and find SNR improvement for other receiving antenna,

| No. of Rx antenna | SNR at BER=10^-10 | SNR improvement |
|---|---|---|
| 2 | 15(reference) | 0 |
| 3 | 10 | 5 |
| 4 | 6 | 9 |
| 5 | 4 | 11 |
| 6 | 2 | 13 |
| 7 | 1 | 14 |
| 8 | 0 | 15 |

Then we plot the curve for receiving antenna vs. SNR improvement at BER=10^-10 which is also called performance improvement curve.

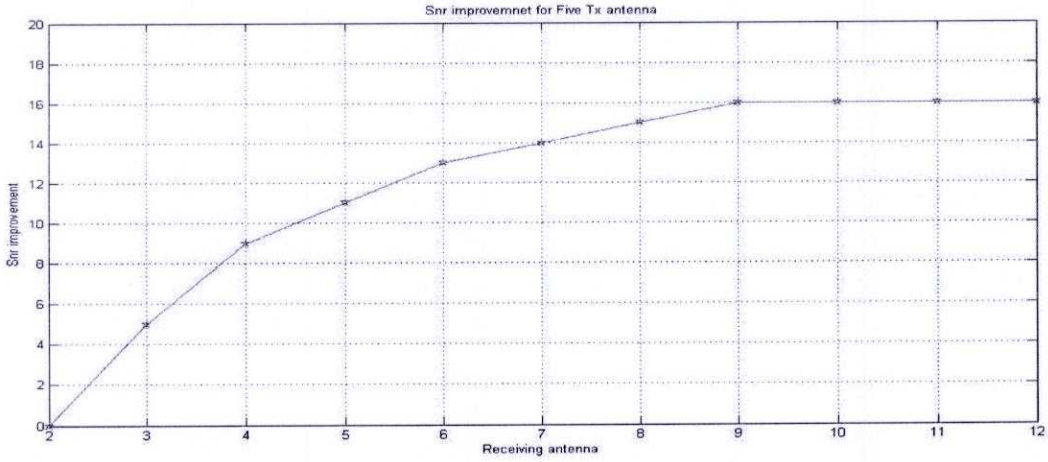



**Figure 7.8 SNR improvement curve on STBC of Rayleigh fading channel with five transmit antenna and the increase of receiving antenna**

## 7.5 SIX TRANSMIT ANTENNA & N NUMBER OF RECEIVING ANTENNA

Below the graph, each plotting curve represents the performance for a certain number of receiving antenna with six transmit antenna. The upper most curve represents for six receiving antenna. Sequentially, the lower most curve represent for twelve receiving antenna

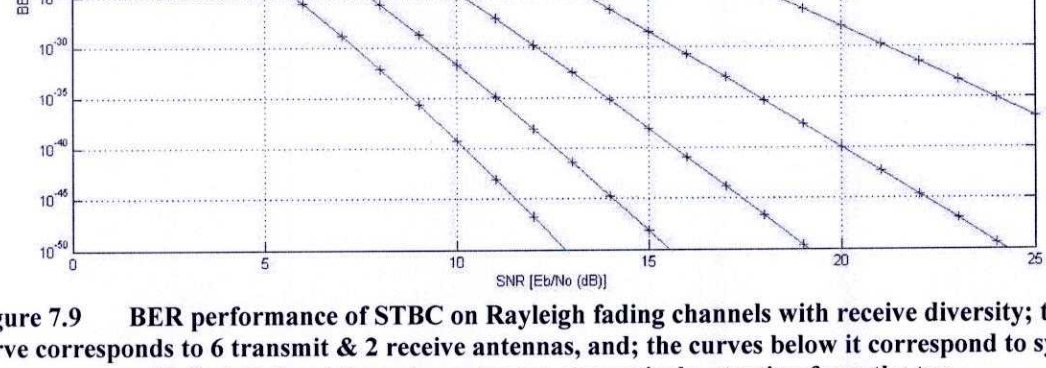



**Figure 7.9 BER performance of STBC on Rayleigh fading channels with receive diversity; the top curve corresponds to 6 transmit & 2 receive antennas, and; the curves below it correspond to systems with 3, 4, 5, 6 and 7 receive antennas, respectively, starting from the top**

## Table of SNR improvement for N number of receiving antenna

We obtain the table below from the upper figure, where we take SNR of six transmit antenna at BER=10^-10 as reference and find SNR improvement for other receiving antenna

| No. of Rx antenna | SNR at BER=10^-10 | SNR improvement |
|---|---|---|
| 2 | 13(reference) | 0 |
| 3 | 8 | 5 |
| 4 | 5 | 8 |
| 5 | 3 | 10 |
| 6 | 1 | 11 |
| 7 | 0 | 13 |

Then we plot the curve for receiving antenna vs. SNR improvement at BER=10^-10 which is also called performance improvement curve,

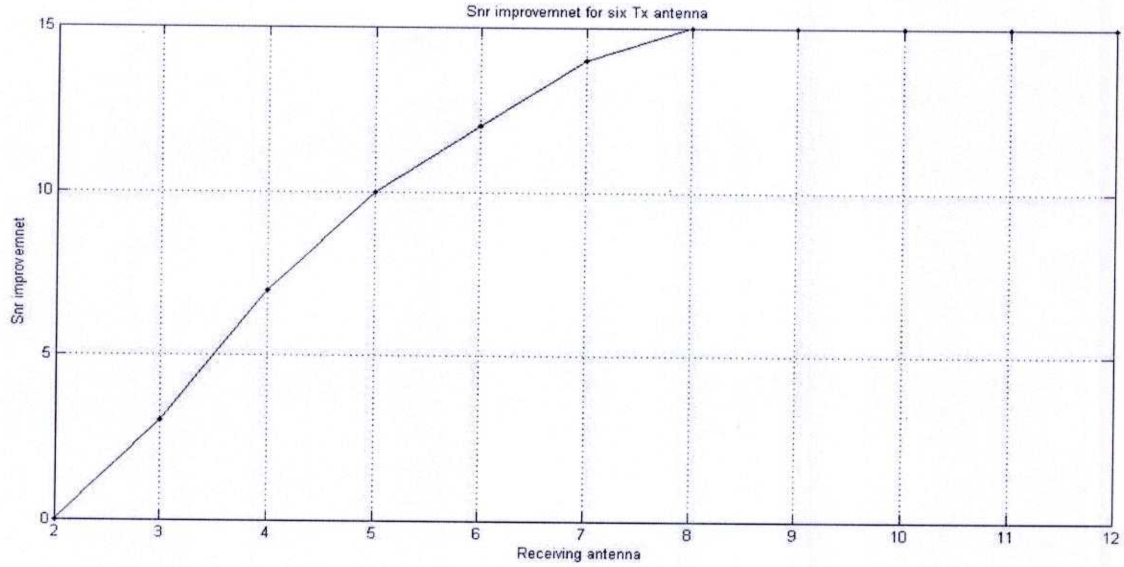



**Figure 7.10 SNR improvement curve on STBC of Rayleigh fading channel with six transmit antenna and the increase of receiving antenna**

## 7.6 PERFORMENCE COMPARISON OF SNR IMPROVEMENT AT GIVEN BER FOR TWO, FOUR AND SIX TRANSMIT ANTENNA

Below the figure represent the performance improvement with the increasing number of receiving antennas. The upper most curve represent two transmit antenna with N number of receiving antenna. The middle curve represent four transmit antenna and the lowest curve is for six transmit antenna

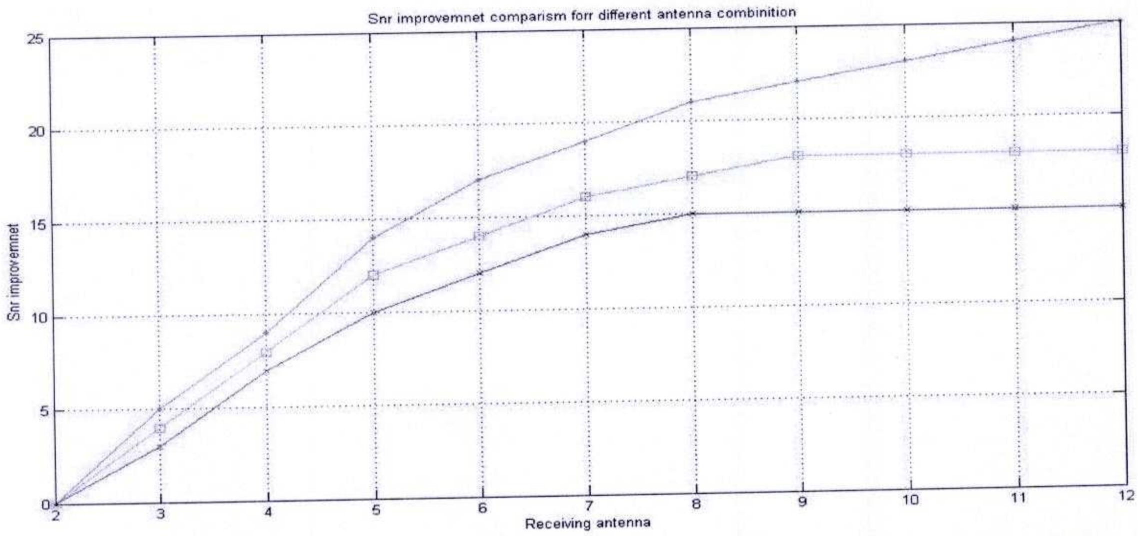



**Figure 7.11 SNR improvement curves of STBC Rayleigh fading channel with 2,4 and 6 transmit antenna from top to bottom curve respectfully**

Above the figure, the highest SNR improvement is obtained from two transmit antenna and twelve receiving antenna which is about 25 dB. Second highest SNR improvement, which is approximately 17 dB, is obtained from four transmit antenna and nine receive antenna combination. And the lowest SNR improvement, 14 dB, is found from two transmit antenna and six receiving antenna combination. So from this figure, we conclude that to get best SNR with two transmit antenna we need to use twelve receiving antenna, for four transmit antenna we can use up to nine receiving antenna and for six transmit antenna, the SNR improvement will be found up to eight receiving antenna.

# CHAPTER 8

# CARRIER FREQUENCY OFFSET IN MIMO MODEL

## 8.1 CARRIER FREQUENCY OFFSET

In a common system, CFO refers to the difference in carrier frequency at transmitter and receiver. The transmitter (Tx) would transmit at the nominal carrier frequency. At the receiver (Rx) the un-modulated frequency is required for reception; and is usually generated in the Rx with crystals or so. However, it is not physical possible to make the Rx frequency exactly match the Tx frequency. This offset is termed CFO.

The input carrier frequency at the receiver can vary due to Doppler Effect, caused by relative motion between Tx and Rx and is another source of CFO.

In both cases, it is required to reduce this offset to a minimum for proper performance.

Expression for the CFO [29]

$$fo = \frac{1}{\sqrt{2\pi\sigma_{\varepsilon}^{2}}}\exp(-\frac{\varepsilon^{2}}{2\sigma_{\varepsilon}^{2}})$$

Where $\sigma_{\varepsilon}^{2}$ is the variance of $\varepsilon$
And $\varepsilon$ is the normalized CFO of the channel.

So the expression of Signal to Interference plus Noise Ratio (SINR) for 2Tx:2Rx MIMO signal with CFO is given by-

$$\bar{\gamma} = \frac{(E_s / 2N_o) * fo}{1 + [4(K-1)\sigma_{MSE}^{2} E_s / N_o] * fo} \times 4\sigma_{\alpha}^{2}$$

In presence of combined degrading effect of CFO and Rayleigh fading, expression of BER becomes,

$$P_e = \left[\frac{1}{2}(1-\mu)\right]^{2} \sum_{k=0}^{1} \frac{(k+1)!}{k!} \left[\frac{1}{2}(1+\mu)\right]^{k}$$

Where, $\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$

## 8.2 Performance Evaluation:

The performance above scheme on slow Rayleigh fading channels with CFO is evaluated by simulation. Eventually another curve is plotted in the same graph where CFO effect is omitted. In the simulations, it is assumed that fading from each transmit antenna to each receive antenna is mutually independent and that the receiver has the perfect knowledge of the channel coefficients.

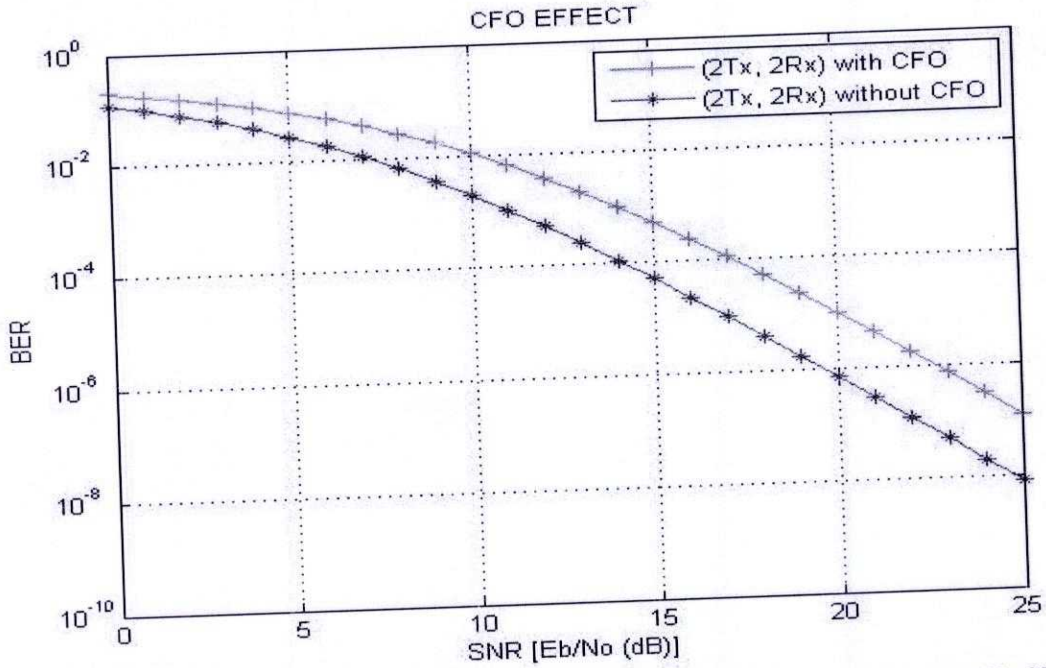



**Figure 8.1 BER performance comparison for 2 transmit and 2 receive antennas with CFO and without CFO**

In fig 8.1, the top curve of the graph is representing the performance for 2 transmit and 2 receive antenna with CFO effect where the bottom curve the performance curve is without CFO effect. It can be observed that without CFO effect, the performance is comparatively better than with CFO effect performance curve.

Similarly when we plot the BER performance curves with CFO and without CFO for 2Tx:4Rx, 4Tx:2Rx and 4Tx:4Rx antenna combination we get following fig respectively

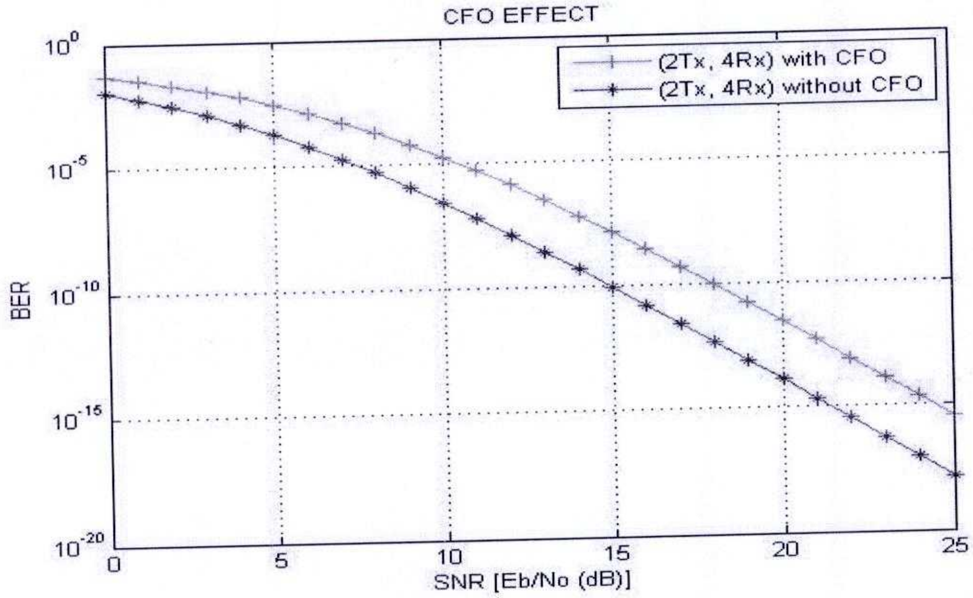



**Figure 8.2 BER performance comparison for 2 transmit and 4 receive antennas with CFO and without CFO**

From fig 8.1 to fig 8.4, in every case, when the CFO effect is add with any antenna combination of MIMO system ,the performance is degraded comparatively to CFO effect free performance curve. At the receiver, additive white Gaussian noise (AWGN) is added which has a equal impact on both performance link. Channel knowledge is unknown to the transmitter in each link system

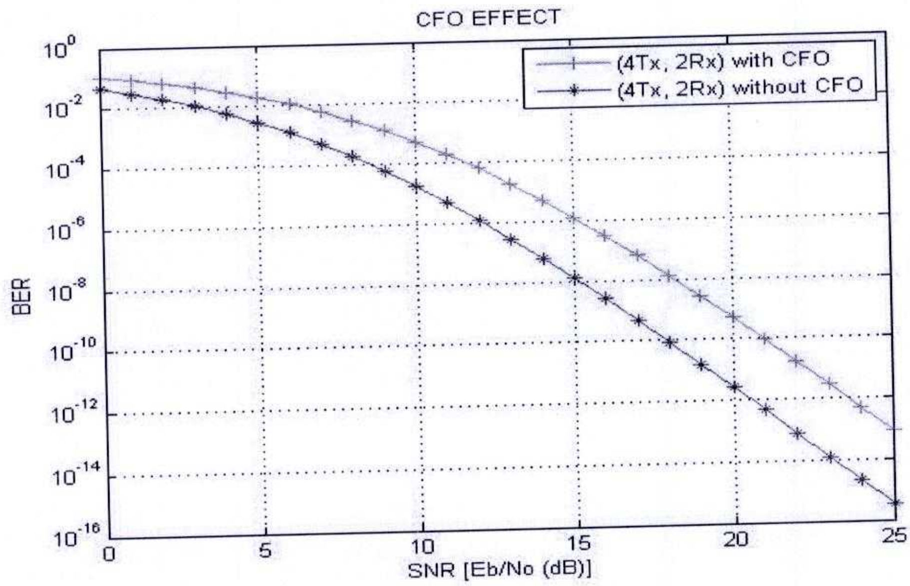



**Figure 8.3 BER performance comparison for 4 transmit and 2 receive antennas with CFO and without CFO**

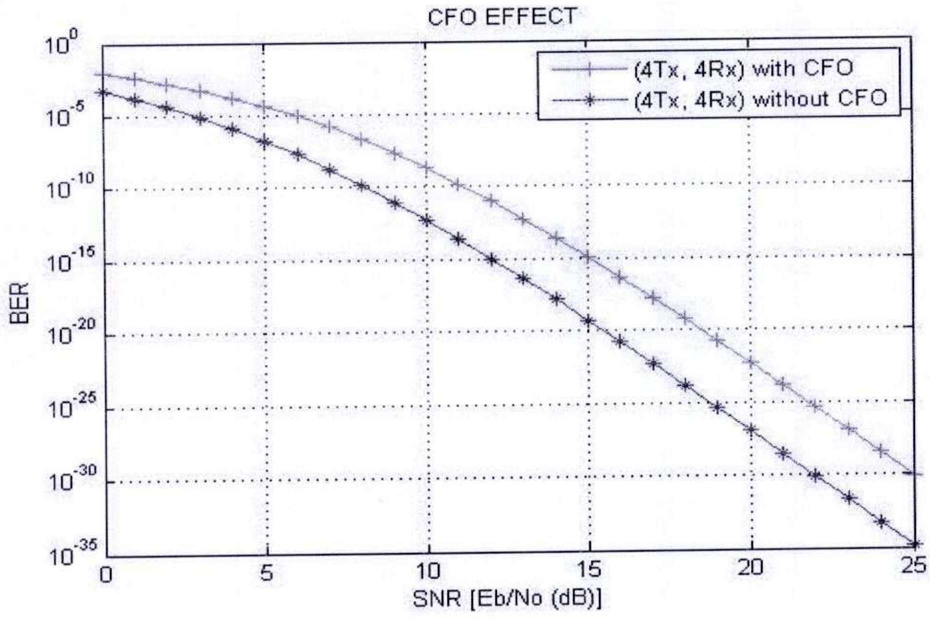



**Figure 8.4 BER performance comparison for 4 transmit and 4 receive antennas with CFO and without CFO**

When CFO effect are add with system above figures, the degradation of SNR, at BER 10^-5 are 3 dB, 2.5 dB, 3.5 dB and 3 dB. It clearly indicates that CFO effect significantly decline the systems link performance.

## 8.3 SNR improvement comparison:

This section provides SNR improvement comparison along with CFO effect and without CFO effect. The primary effect of carrier frequency offset (CFO) is inter-carrier interference (ICI) that leads to a reduction in SNR and ultimately to an irreducible error floor in performance [30]. As much effort has gone into quantifying the effects of carrier frequency offset, certainly more has been spent in mitigating its effects [31], for instance, presents various such techniques. When CFO effect is added with a system, the link performance is improve slowly compare to the link of a system without CFO effect.

. Fig 8.5 represent the comparison of SNR improvement of two transmit antenna with CFO and two transmit antenna without CFO with the increase of receiving antenna. The top curve is the SNR improvement without CFO where as the bottom curve showing SNR improvement without CFO. When increasing receiving antenna CFO combined link, most of the case SNR are comparative less improved then CFO effect free link

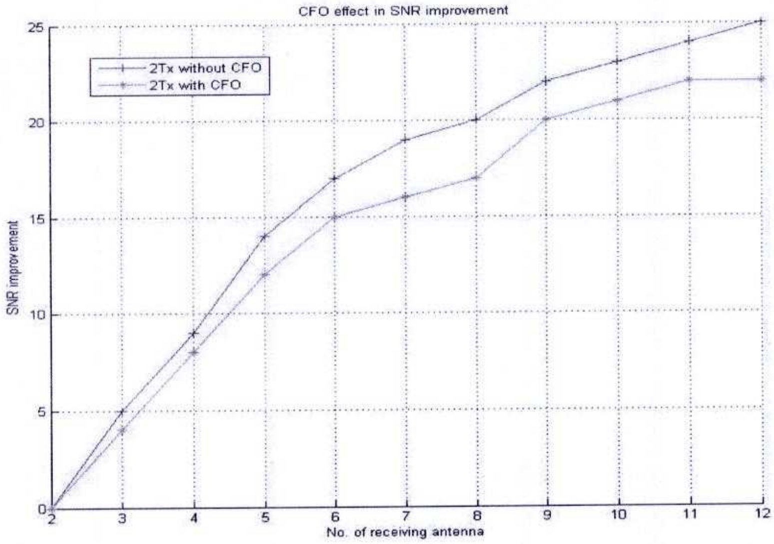



**Figure 8.5 SNR improvement comparison 2 transmit and increasing number of receiving antennas along with CFO and without CFO**

In figure 8.6, increasing receiving antenna CFO combined link, most of the case SNR are comparative less improved then CFO effect free link. When receiver are increased two to three, CFO free links SNR improves 7dB where as CFO effected link's SNR improve 5dB

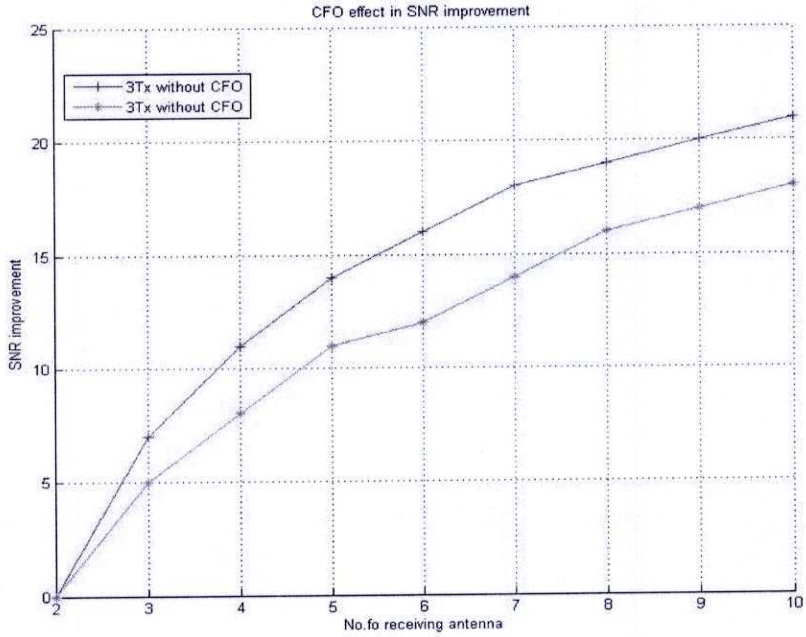



**Figure 8.6 SNR improvement comparison 3 transmit and increasing number of receiving antennas along with CFO and without CFO**

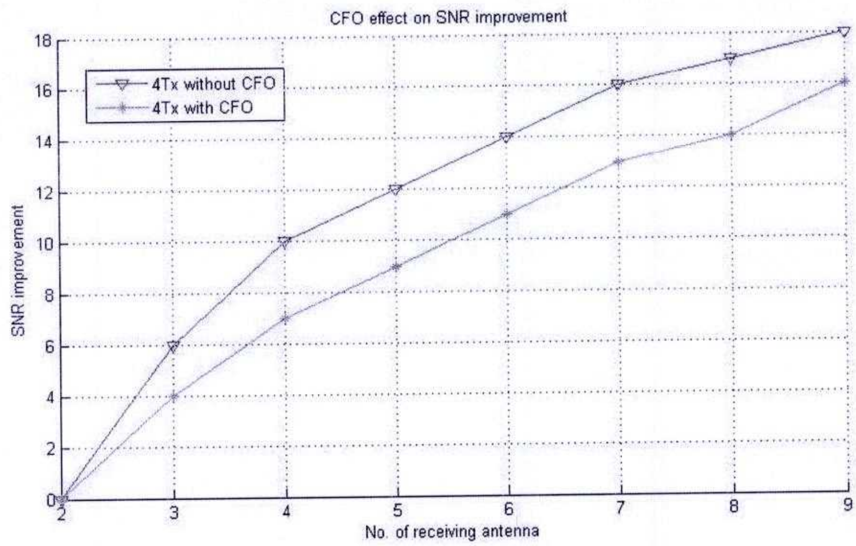



**Figure 8.7 SNR improvement comparison 4 transmit and increasing number of receiving antennas along with CFO and without CFO**

In figure 8.7 and figure 8.8, both cases SNR are slowly improved at CFO affected link comparatively to CFO effect free link with the increase of receiving antenna. Thus, CFO not only degraded the performance of the link but also it reduce the SNR improvement rate of MIMO channel of a system.

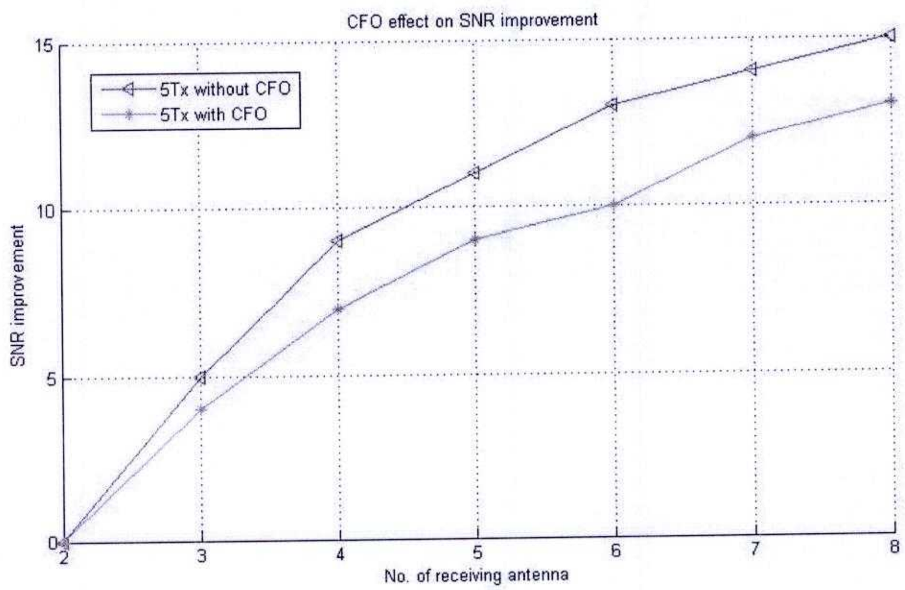



**Figure 8.8 SNR improvement comparison 5 transmit and increasing number of receiving antennas along with CFO and without CFO**

# Chapter 9

# CONCLUSION

This thesis is devoted to space-time coding for multiple- input/multiple-output (MIMO) systems. The concept of space-time coding is explained in a systematic way. The performance of space-time codes for wireless multiple-antenna systems with and without channel state information (CSI) at the transmitter has been also studied.

In this work it is found that the BER performance is degrades severely when the channel is Rayleigh faded. Considering the poor and unacceptable performance in Rayleigh fading environment, it has been shown that quality of service (QOS) can be improved by employing diversity schemes. Relative merits and demerits of various combining techniques are analyzed for Rayleigh faded channel. Effects of frequency offset are also considered during the analysis of diversity schemes. Analytical BER performance results are evaluated which show that by increasing receiver antenna number and transmitter antenna number, the system performance can be significantly improved even in the presence of fading and CFO. It also proved that the probability of getting poor SNR over Rayleigh faded channel reduces for multiple receive antennas rather than multiple transmit antenna. We also shown SNR improvement graph from where we found the maximum number of receiver antenna for a fixed number of transmitter antenna to achieve maximum performance at particular BER in MIMO Channel.

One common aspect of STBC design is that it is assumed that no channel information is available at the transmitter. However, the performance of multiple antennas can be improved if channel state information obtained at the receiver is fed back to the transmitter. Exploiting partial channel knowledge at the transmitter, two simple channel adaptive transmission schemes, namely, channel adaptive code selection and channel adaptive transmit antenna selection have been proposed in this thesis.

In this work new useful STBCs have been proposed which can be a good candidate for future wireless communication systems. These useful STBCs provide high transmission rate with simple decoding algorithms and perform very well on i.i.d. MIMO channels as well on realistic MIMO channels. The drawback of these codes, a diversity loss, can be avoided by simple closed-loop transmission schemes which only require a small amount of feedback bits. Partial channel knowledge at the transmitter increases the diversity of STBCs and orthogonalizes the STBCs opening the way to simple decoding algorithms that offer a good trade-off between performance and complexity. Furthermore, STBCs using partial channel information make a MIMO system more robust against the negative influences of the wireless channel environment, e.g. high antenna correlations.

# APPENDIX

# MATLAB CODING

```
%transmitter diversity%
%for two transmit and one receive antenna%
clc;
k=1;
sig=.1;
E_N_db=0:25;
E_N=10.^(E_N_db/10);
gama=(E_N/2)./(1 + 2*(k-1)*sig^2.*E_N);
mu=sqrt(gama./(1+gama));
sum_p=zeros(1,length(mu));
for j=1:length(mu)
for i=0:1
    P=(.5*(1-mu(j)))^2*(factorial(i+1)/factorial(i))*(.5*(1+mu(j)))^i;
    sum_p(j)=sum_p(j)+P;
end
end
disp(sum_p)
semilogy(E_N_db,sum_p,'r+-')
hold on
legend('Alamouti (2Tx, 1Rx)')
title('Transmit Diversity');
grid on
xlabel('SNR [Eb/No (dB)]')
ylabel('BER')



%receive diversity%
%for one transmit and two receive antenna%
clc;
k=1;
sig=.1;
E_N_db=0:25;
E_N=10.^(E_N_db/10);
gama=(E_N)./(1 + 2*(k-1)*sig^2.*E_N);
mu=sqrt(gama./(1+gama));
sum_p=zeros(1,length(mu));
for j=1:length(mu)
for i=0:3
    P=(.5*(1-
mu(j)))^4*(factorial(i+3)/(factorial(i)*factorial(3)))*(.5*(1+mu(j)))^i
;
    sum_p(j)=sum_p(j)+P;
end
end
disp(sum_p)
semilogy(E_N_db,sum_p, 'bp-')
hold on;
legend('MRC (1Tx, 2Rx)')
title('Receive Diversity');
grid on
xlabel('Eb/No (dB)')
ylabel('BER')
```

---

```
%carrier frequency offset combined without MIMO
clc;
n_t=2;
n_r=2;
k=1;
sig_alpha_sqr=.1;   %channel co-efficient assume rayleigh fading
sig=.1;             %channel estimation error
E_N_db=0:25;
E_N=10.^(E_N_db/10);
gama=(((E_N.*(n_t*n_r*sig_alpha_sqr))./n_t))./(1 + (n_t*n_r*(k-
1)*sig^2.*E_N));
mu=sqrt(gama./(1+gama));
sum_p=zeros(1,length(mu));
for j=1:length(mu)
for i=0:((n_t*n_r)-1)
    P=(.5*(1-mu(j)))^(n_t*n_r)*(factorial((n_t*n_r)-
1+i)/(factorial(i)*factorial((n_t*n_r)-1)))*(.5*(1+mu(j)))^i;
    sum_p(j)=sum_p(j)+P;
end
end
disp(sum_p)
semilogy(E_N_db,sum_p,'b-*')
hold on
legend('(2Tx, 2Rx) without CFO');
title('CFO EFFECT');
grid on
xlabel('SNR [Eb/No (dB)]')
ylabel('BER')
```

---

```
%carrier frequency offset combined with MIMO
clc;
n_t=2;
n_r=2;
k=1;
sig_ep_sqr=2^-1;   %variance of ep.
ep_sqr=(.5);
of=(1/sqrt(2*pi*sig_ep_sqr))*(exp(-ep_sqr/2*sig_ep_sqr));%carrier
frequency offset
sig_alpha_sqr=.1;   %channel co-efficient assume rayleigh fading
sig=.1;             %channel estimation error
E_N_db=0:25;
E_N=10.^(E_N_db/10);
gama=(((E_N.*(n_t*n_r*sig_alpha_sqr))./n_t)*of)./(1 + (n_t*n_r*(k-
1)*sig^2.*E_N)*of);
mu=sqrt(gama./(1+gama));
sum_p=zeros(1,length(mu));
for j=1:length(mu)
for i=0:((n_t*n_r)-1)
    P=(.5*(1-mu(j)))^(n_t*n_r)*(factorial((n_t*n_r)-
1+i)/(factorial(i)*factorial((n_t*n_r)-1)))*(.5*(1+mu(j)))^i;
    sum_p(j)=sum_p(j)+P;
end
end
```

```
disp(sum_p)
semilogy(E_N_db,sum_p,'r+-')
hold on
legend('(2Tx, 2Rx) with CFO');
title('CFO EFFECT');
grid on
xlabel('SNR [Eb/No (dB)]')
ylabel('BER')
```

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