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USE OF A NESTED NUMERICAL SCHEME IN A SHALLOW WATER MODEL FOR THE COAST OF BANGLADESH

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ABSTRACT

The coast of Bangladesh is very irregular in shape and the off-shore region is full of small and big islands. In this paper a nested finite difference scheme is used to solve the vertically integrated shallow water equations. A fine mesh numerical scheme for the coastal belt of Bangladesh has been nested into a coarse mesh scheme covering up to 15° N of the Bay of Bengal. The fine mesh scheme incorporates the off-shore islands and bending of the coastline accurately. The coarse mesh scheme is completely independent and along the open boundaries of the fine mesh scheme, the parameters are prescribed from those obtained in the coarse mesh one in each time step of the solution process. The model is applied to estimate the water level due to tide and surge along the coast of Bangladesh and the results are found to be satisfactory.

Key words: Nested scheme, shallow water equations, coast, Bangladesh

I. INTRODUCTION

Among all the natural calamities, a tropical storm along with surge is the most devastating one along the coast of Bangladesh. Extreme bending of the coastline, shallowness of water, off-shore islands, huge discharge through the Meghna and other rivers, low lying islands and coastal regions are responsible for high surge even for a less intense storm. Moreover, the head Bay of Bengal is a large tidal range (difference between high and low tides) area with the highest range around the Sandwip Island. The water level due to tide-surge interaction becomes significantly high if a storm approaches the coast during a high tide period.

Analyses on prediction of tide, surge and their interaction have been made for the Atlantic [1-4], North Sea & North-West European Continental Shelf [5-9], Australian region [10-13] and Bay of Bengal [14-26]. The SLOSH model of Jelesnianski *et al.* [2] is used as the operational surge-forecasting model in USA. Thacker [3, 4] incorporated the curving coastline and island boundaries with irregular grid finite difference technique as a substitute of the finite element method. The studies of Arnold [10], Bills and Noye [11], and Tang and Grimshaw [13] for the Australian region are mainly based on investigation of various open sea boundary conditions.

Most of the works in Bay of Bengal [14-26] are based on vertically integrated shallow water equations. The studies of Das [14] and Flierl and Robinson [17] were based on linear equations and stair step representation of the coastline. The model of Das *et al.* [16], an extension of Das [14], was designed for the east coast of India and the cost of Bangladesh. The first nonlinear stair step model of Johns & Ali [20] for the coast of Bangladesh considered the dynamic effect of the major river system and islands. In a stair step model the coastal and island boundaries are approximated along the nearest finite difference grid lines. To represent the boundaries accurately the grid size must be very small, which is unnecessary away from the coast. Following Johns *et al.* [22], the study of Roy [24] is based on the nested numerical schemes, where a very fine mesh numerical scheme has been nested into a coarse mesh one. In the fine mesh scheme all the major islands of the Meghna estuary are included. In the present study, the work of Roy [24] has been extended where the fine mesh scheme is designed to cover the whole coast of Bangladesh instead of the Meghna estuary only.

II. MATHEMATICAL FORMULATION

A. Basic Shallow Water Equations

A rectangular Cartesian coordinates is considered with a horizontal *xy*- plane and *z*- axis in the upward direction. If the horizontal length scale is much larger than the vertical one, the *z* component of the momentum equation may be approximated by the hydrostatic equation. Moreover, if the density variation is negligible, the continuity equation reduces to the non-divergence of velocity. With these approximations and neglecting the molecular viscosity we obtain the basic shallow water equations with instantaneous velocity components as *u*, *v*, and *w*. Substituting

$$u = \langle u \rangle + u', \quad v = \langle v \rangle + v', \; w = \langle w \rangle + w'$$

and considering the time averages of fluctuations associated with turbulence as zero, the resulting equations can be averaged in time to get a simple partition between the mean flow and turbulent fields. Carrying out this averaging process we obtain the momentum equations where the eddy stress terms like gradient of $\langle u' w' \rangle$ retain and since the vertical gradients are much larger than the horizontal ones, we retain only the vertical ones. According to Prandtl's mixing length theory, each eddy stress term may be expressed as the gradient of mean flow. Thus we obtain the basic shallow water equations with averaged velocity components as (dropping the angle brackets)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau_x}{\partial z} \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} - fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + + \frac{1}{\rho}\frac{\partial \tau_y}{\partial z} \tag{3}$$

B. Vertically Integrated Equations

A system of rectangular Cartesian coordinates is used in which the origin, *O*, is in the undisturbed sea surface and *Oz* is directed vertically upwards. We consider the displaced position of the free surface as $z = \zeta(x, y, t)$ and the sea floor as $z = -h(x, y)$ so that, the total depth of the fluid layer is $\zeta + h$. Integrating vertically the hydrostatic equation from any depth z to $z = \zeta(x, y, t)$, we obtain

$$p = p_a + \rho g(\zeta - z) \tag{4}$$

Let us now integrate each term of the Eqs. (1) - (3) form $z = -h(x, y)$ to $z = \zeta(x, y, t)$ using (4), no slip bottom condition and the kinematical surface condition. We define the vertically integrated velocity components as

$$\bar{u} = \frac{1}{\zeta + h}\int_{-h}^{\zeta} u\,dz \;, \quad \bar{v} = \frac{1}{\zeta + h}\int_{-h}^{\zeta} v\,dz$$

Then the basic shallow water equations (1) - (3) become (dropping the over bars)

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}\left[(\zeta + h)u\right] + \frac{\partial}{\partial y}\left[(\zeta + h)v\right] = 0 \tag{5}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -g\frac{\partial \zeta}{\partial x} + \frac{T_x - F_x}{\rho(\zeta + h)} \tag{6}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -g\frac{\partial \zeta}{\partial y} + \frac{T_y - F_y}{\rho(\zeta + h)} \tag{7}$$

Parameterization of the bottom stress is made by the vertically integrated velocity components:

$$F_x = \rho\, C_f\, u\left(u^2 + v^2\right)^{1/2}$$

$$F_y = \rho\, C_f\, v\left(u^2 + v^2\right)^{1/2}$$

For numerical treatment it is convenient to express the Eqs. (6) & (7) in the flux form by using the Eq. (5). Using the above parameterization the Eqs. (5) - (7) may be expressed as

$$\frac{\partial \zeta}{\partial t} + \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0 \tag{8}$$

$$\frac{\partial \tilde{u}}{\partial t} + \frac{\partial(u\tilde{u})}{\partial x} + \frac{\partial(v\tilde{u})}{\partial y} - f\tilde{v} = -g(\zeta + h)\frac{\partial \zeta}{\partial x} + \frac{T_x}{\rho} - \frac{\rho C_f \tilde{u}\left(u^2 + v^2\right)^{1/2}}{\zeta + h} \tag{9}$$

$$\frac{\partial \tilde{v}}{\partial t} + \frac{\partial(u\tilde{v})}{\partial x} + \frac{\partial(v\tilde{v})}{\partial y} + f\tilde{u} = -g(\zeta + h)\frac{\partial \zeta}{\partial y} + \frac{T_y}{\rho} - \frac{\rho C_f \tilde{v}\left(u^2 + v^2\right)^{1/2}}{\zeta + h} \tag{10}$$

where, $(\tilde{u}, \tilde{v}) = (\zeta + h)(u, v)$

In the bottom stress terms of (9) & (10), u and v have been replaced by $\tilde{u}$ and $\tilde{v}$ in order to solve the equations in a semi-implicit manner. Here some terms have been omitted, as their contributions are negligible.

C. Boundary Conditions

Other than the coast of Bangladesh, the boundaries are considered as straight lines in the open sea. The western and eastern open sea boundaries lie parallel to x-axis and the southern open sea boundary lies parallel to y-axis. The radiation boundary conditions for the western, eastern and southern open sea boundaries, due to Johns *et al.* [21], are

$$v + (g/h)^{1/2}\zeta = 0 \tag{11}$$

$$v - (g/h)^{1/2}\zeta = 0 \tag{12}$$

$$u - (g/h)^{1/2}\zeta = 0 \tag{13}$$

This type of boundary condition allows the disturbance, generated within the model area, to go out through the open boundary. The coastal belts of the main land and islands are the closed boundaries where the normal components of the current are taken as zero.

D. Determination of the Wind stress

The wind stress is parameterized in terms of the wind filed associated with the storm by the conventional quadratic law:

$$\begin{aligned} T_x &= C_D \rho_a u_a \left(u_a^2 + v_a^2\right)^{1/2} \\ T_y &= C_D \rho_a v_a \left(u_a^2 + v_a^2\right)^{1/2} \end{aligned} \tag{14}$$

Generally, the storm information is available in terms of the maximum sustained wind speed (V_0) and the corresponding radius (R). The circulatory wind field is then generated by the empirical formula due to Jelesnianski [1], which is given by

$$\begin{aligned} V_a &= V_0 (r/R)^{3/2} \qquad \text{for } 0 \le r \le R \\ &= V_0 (R/r)^{1/2} \qquad \text{for } r \ge R \end{aligned} \tag{15}$$

At any radial distance r from the eye, u_a and v_a are derived from the above circulatory wind field.

III. NESTED NUMERICAL SCHEME

A. Set-Up of the Nested Schemes

In order to include the coastline and the island boundaries in the numerical scheme accurately, the mesh size should be very small whereas, this is unnecessary away from the coast. On the other hand, the model area should be considerably big so that a storm can move over it for two to three days before crossing the coast. The problems in taking small mesh size over the whole area are: (*i*) the number of grid points increases tremendously and thus, increases the computer overheads and (*ii*) smaller mesh size requires small time step in deep water to ensure the CFL stability criterion

$$\Delta t \le \frac{\Delta x \text{ or } \Delta y}{\sqrt{2gh}} \tag{16}$$

Smaller time step increases the number of steps of integration and thus requires much CPU time in the solution process. Considering the above facts, following Roy [24], a fine resolution numerical scheme (FNS) has been nested into a coarse mesh numerical scheme (CNS). The CNS covers the area between 15° *N* and 23° *N* and 85° *E* and 95° *E*. The mesh sizes along x and y - axes are 21.5 *km* and 22.5 *km*. The FNS covers the area between 21° *N* and 23° *N* lat. and 89° *E* and 92.25° *E*, which includes the coast of Bangladesh (Fig. 1). The mesh size along *x and y*- axes are 6.5 *km* and 6.5 *km*. There are 40 × 49 and 46 × 81grid points in the computational domains of CNS and FNS respectively. The CNS does not include any island whereas; Bhola, Hatiya and Sandwip islands are incorporated in the FNS. The time step of 120 seconds, that ensures stability of the numerical scheme, is considered for both the numerical schemes.

Both the schemes have the same dynamical equations (8)-(10) with different boundary conditions. The open boundary conditions for CNS are given by (11)-(13). The FNS is coupled with CNS in such a way that the latter is independent whereas along the open boundaries of the FNS ζ, u and v are prescribed from those obtained in the CNS in each time step. Along the northern boundary of FNS an open boundary segment of 20 *km*, breadth of the Meghna river, is considered between 90.4° *E* and 90.6° *E* long. At this segment, following Roy [24], the x-component of the velocity is given by

$$u_b = u + \frac{Q}{(\zeta + h)B} \tag{17}$$

where Q = the discharge of water through the Meghna river in $m^3 s^{-1}$.

B = breadth of the river in m.

At the closed boundaries of the main land as well as of the islands the normal components of the velocity are taken as zero.

B. Numerical Procedure

The equations for both CNS and FNS are solved by a conditionally stable semi-implicit finite difference method using staggered grid in which there are three distinct types of computational points, say, (i, j). With i even and j odd, the point is a ζ- point at which ζ is computed. If i is odd and j is odd, the point is a u- point. If i is even and j is even, the point is a v- point. We choose m, the total number of grid points in the x- direction, to be even so that at the southern boundary there are ζ- points and v - points. We choose n, the total number of grid points in the x- direction, to be odd thus ensuring only ζ- points and u - points along the western and eastern boundaries. The coastal and island boundaries are approximated by continuous segments either along the nearest y- directed odd grid line (i = odd) or along the nearest x- directed even grid line (j = even). Thus each boundary is represented by such a stair step that, at each segment there exists only the component of velocity that is normal to the segment. This is done in order to ensure the vanishing of the normal component of the current at boundary segment.

The governing equations are discretised by finite differences (forward in time and central in space) and are solved by a conditionally stable semi-implicit method as follows: in the bottom stress terms, $\tilde{\mathbf{V}}\left|\mathbf{V}\right|$ is discretised as $\tilde{\mathbf{V}}^{n+1}\left|\mathbf{V}^{n}\right|$, where $\mathbf{V}$ is the horizontal velocity vector and the superscripts indicate corresponding time level. Moreover, the CFL criterion has been followed in order to ensure the stability of the numerical scheme. Along the open boundaries of the CNS, ζ is computed from the boundary conditions (11)-(13). For example, the southern open sea boundary condition (13) is applied at (m–1, j) grid points and replacing ζ by $(\zeta_{m,j} + \zeta_{m-2,j})/2$ to compute ζ_{mj}. Along the northern open boundary of FNS, u_b at (1, j) is computed from (17) in which u on the RHS is replaced by u_{2j}. The initial values of ζ, u and v are taken as zero for both the schemes. Following Roy [24] the value of C_f is taken as 0.0026 in the model area except the region surrounding Sandwip where it is taken as 0.0015 and the value of C_D is taken as 0.0028.

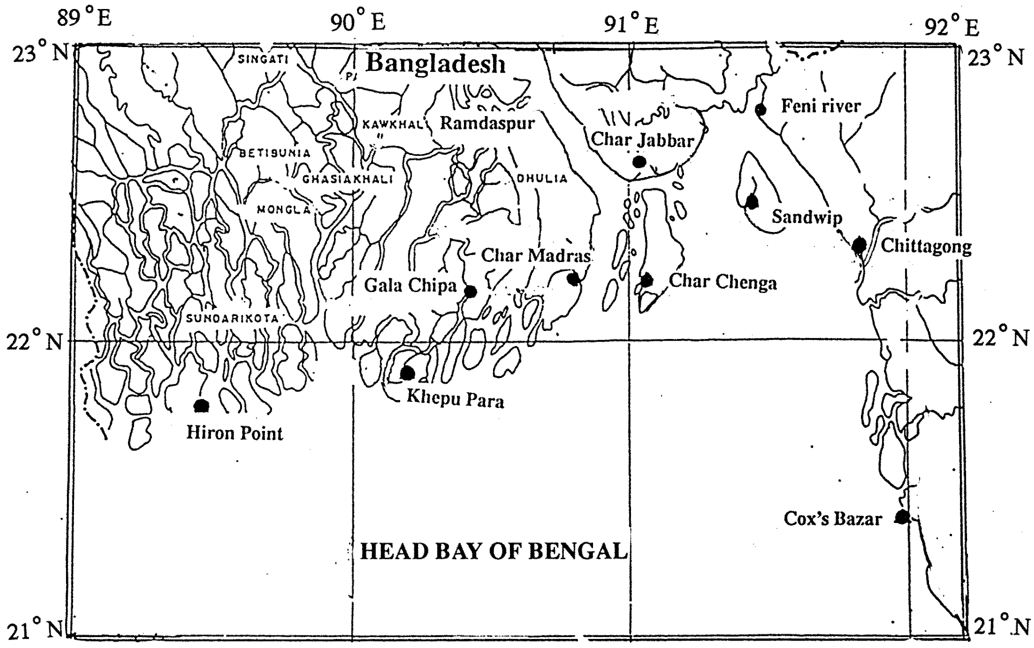

Fig. 1. Coastal boundary of Bangladesh and ten representative locations at which computed results are presented. The locations are Hiron Point, Khepu Para, Gala Chipa, Char Madras (Bhola), Char Chenga (Hatiya), Char Jabbar, Sandwip, Feni River, Chittagong and Cox's Bazar.

C. Tide and Surge Superposition

Since the astronomical tide is a continuous process in the sea, the tidal oscillation is the initial state of the sea for the tide-surge interaction phenomenon. But the difficulty in generating the tidal oscillation in a model lies with its variability in amplitude and phase. Since different constituents of tide interact among themselves, the interaction is characterized by (i) diurnal inequalities as to the high and low water, (ii) non-periodicity of the tidal oscillation and (iii) the variation of tidal amplitude in the spring-neap cycle. The complexity of the tidal phenomena is a great constraint in generating actual tidal oscillation in the model simulation process. Alternatively, the time series of tidal oscillation and computed surge at each location are superimposed linearly to obtain the total water level. The tidal information is available in the Bangladesh tide table four times a day as high and low values and the time series is generated through a cubic spline interpolation method.

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IV. MODEL VERIFICATION AND ANALYSIS OF RESULTS

For the purpose of verification and analysis, the computed results are presented at some locations associated with four storms that hit the coast in the past. For computing water levels due to superposition of tide and surge at a location, the tidal information of that location must be available; but this is not available at each location for every chosen storm period.

A. Analysis of Computed Water Levels due to Tide, Surge and Superposition

Figure 2 shows the tide (dotted line), computed surge (dash line) and superposition (solid line) associated with the November 1970 storm at Chittagong and Sandwip, and with April 1991storm at Chittagong and Hiron Point. In 1970, the maximum water levels at Chittagong due to tide, surge and superposition are respectively 2 *m*, 6.9 *m* and 8.75 *m*. According to the observation of the BMD, the water level near Chittagong was 6 to 9 *m*. At Sandwip, the maximum tide, surge and superposition are respectively 2.75 *m*, 7.25 *m* and 10 *m*, which are more than those at Chittagong. This is due to the fact that, (i) Sandwip is a large tidal range area, (ii) Sandwip is just at the right of the storm track and (iii) there is a sharp bending of the coastline at the Feni River area which is just north of Sandwip (Fig. 1).

For April 1991 storm, the computed water level at Hiron Point, which is far left of the track, goes below the minimum tide level (recession) during landfall of the storm (Fig. 2d). At that time the winds at Hiron Point were approximately northerly, because of the anti clock wise circulation, driving water from the shore towards the sea. The maximum surge at Chittagong is 6 *m*, which is 0.9 *m* less than that of 1970 though the 1991storm was more intense. This is attributed to the path, because in 1970 Chittagong was to the right of the track whereas in 1991 Chittagong was almost over the track.

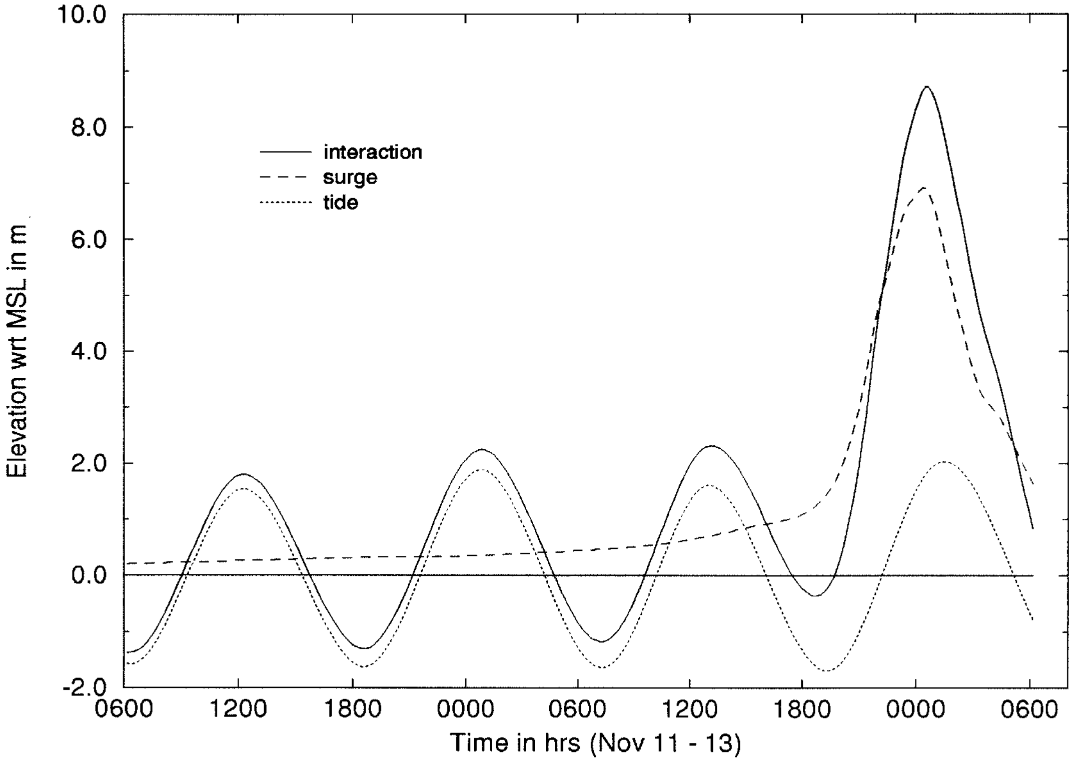
B. Comparison between Computed and Observed Time Series of Water Levels

No authentic observed water levels are available during the severe storm periods in Bangladesh. Some observed water level data, collected from BIWTA, were used by Roy [24] that are used here also. Figure 3 depicts the computed and observed water levels associated with December 1981 and May 1985 storms. For 1981 storm, there is a slight phase difference between computed and observed water levels at Sandwip. Also during first three peaks the computed water levels are found to be slightly lower and in the final peak it is about 0.5 *m* higher than that of observation. At Chittagong the computed water level is in general slightly lower than that of observation. For 1985 storm, there are differences both in amplitude and phase between computed and observed water levels at Char Chenga. At Chittagong the computed and observed water levels are in agreement except at the time of final peak where there is significant difference of about 1 *m*.

C. Analysis of Computed Water Levels due to Superposition

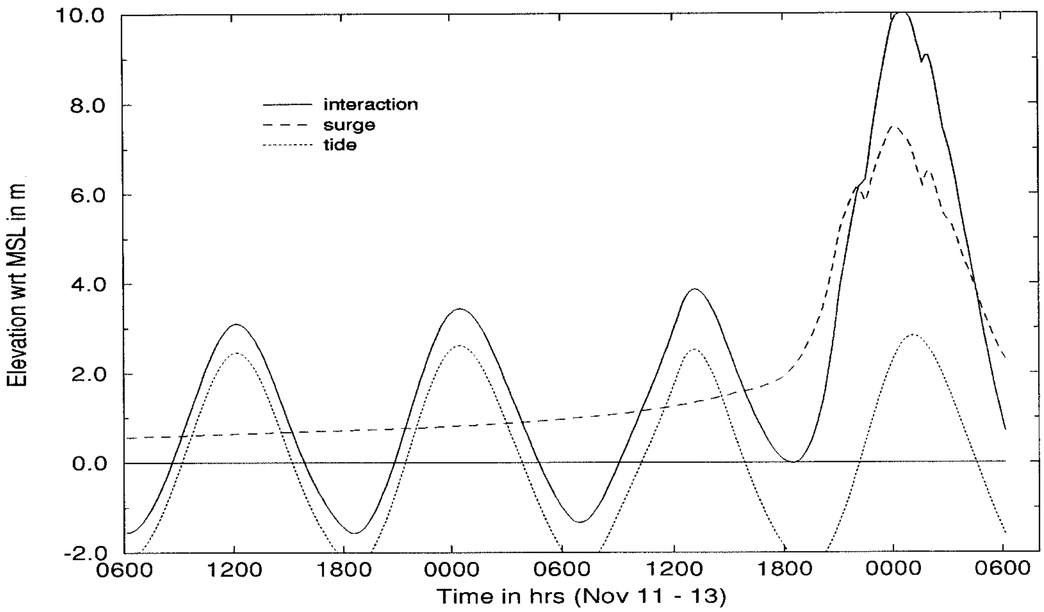
Figure 4 depicts the computed water levels due to superposition associated with the four storms. The water level is dominated by the tide during the early stage of the storm and dominated by surge when the storm is near the coast. The maximum water level is appearing at Sandwip during each storm. For 1985 and 1991 storms the recession occurred along the western stations, stronger in 1991. Although the intensity of 1991 storm is higher, the overall water levels of 1970 storm are more and this is attributed to the path of the storms.

(a)



10.0
8.0
6.0
4.0
2.0
0.0
-2.0
interaction
surge
tide
Elevation wrt MSL in m
0600
1200
1800
0000
0600
1200
1800
0000
0600
Time in hrs (Nov 11 - 13)

(b)



10.0
8.0
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interaction
surge
tide
Elevation wrt MSL in m
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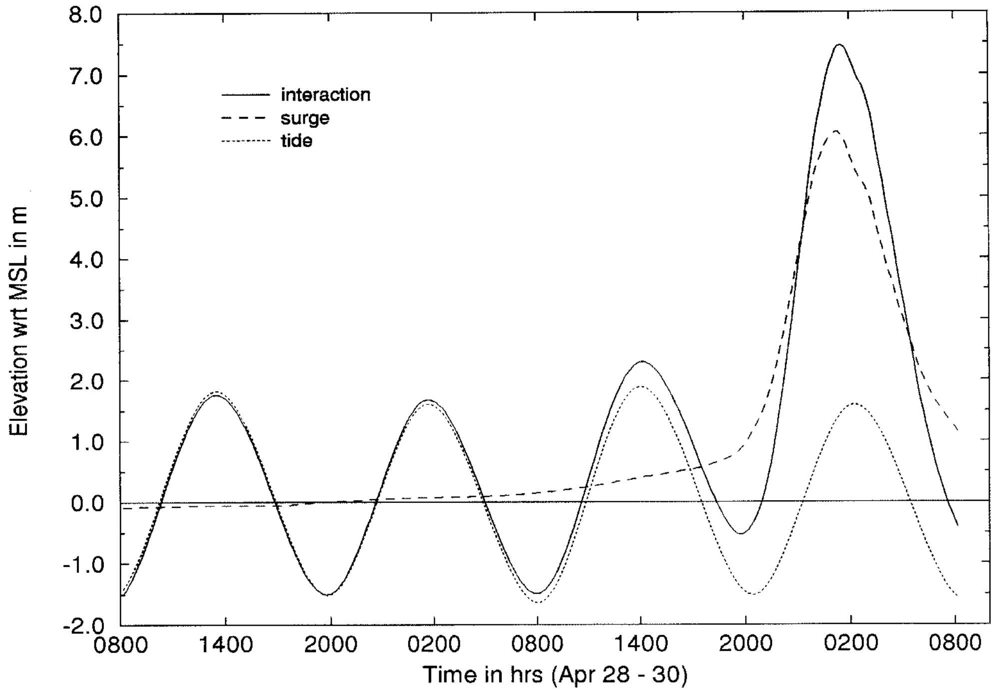
0600
Time in hrs (Nov 11 - 13)

(c)



(d)

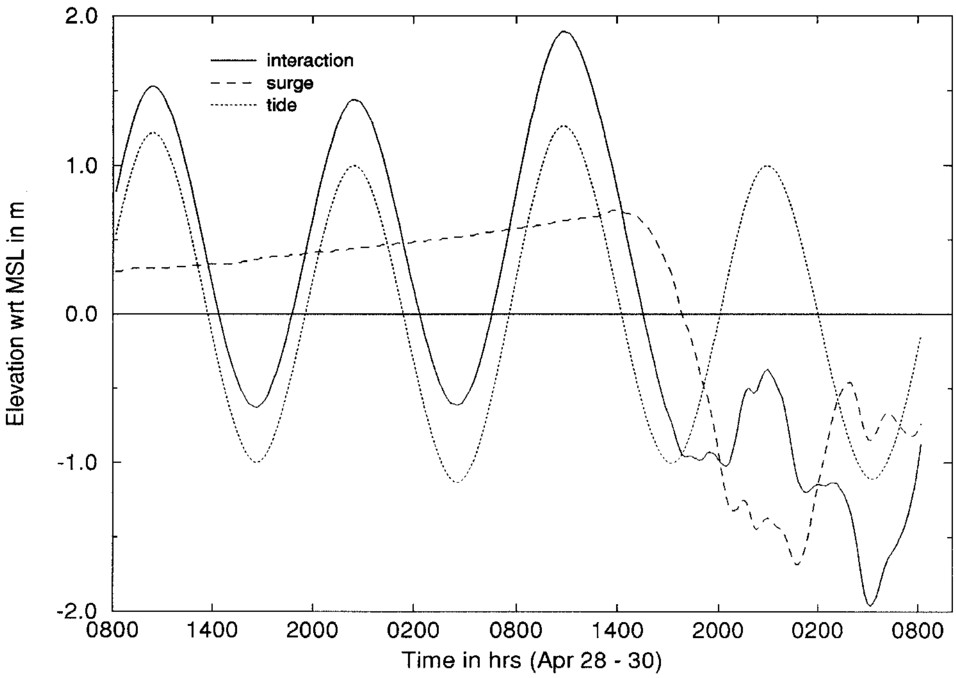


Fig 2. Computed water levels due to tide, surge and superposition associated with November 1970 and April 1991 storms; (a) at Chittagong for 1970, (b) at Sandwip for 1970, (c) at Chittagong for 1991 and (d) at Hiron Point for 1991.

D. Analysis of Computed Maximum Surge Response Along the Coastal Belt

Finally, the maximum surges associated with four chosen storms at nine locations from west to east along the coastal belt are shown in Fig. 5. For November 1970, the maximum surge increases from west to east up to Sandwip and then gradually decreases reaching minimum at Cox's Bazar. The same for 1991 storm is similar to that in 1970 but with less intensity all along except at the Cox's Bazar region. The path of the 1991 storm is almost parallel but to the right of 1970 storm. So, the surge response is sensitive to the path and landfall position, because, in general, the surge is more to the right of the track. Similar analysis can be made for the 1981 and 1985 storms.

CONCLUSION

In this study we have developed a nested numerical scheme for a Cartesian coordinate model that ensures fine resolution near the coast and coarse resolution in the deep sea. The scheme is found to be suitable for incorporating bending of the coastline and the island boundaries more accurately. The model is efficient to compute surge in the head Bay of Bengal, especially along the coast of Bangladesh. This model may be used to develop a operational forecasting model for the coast of Bangladesh and thus warning system may be improved during a storm period.

Acknowledgements

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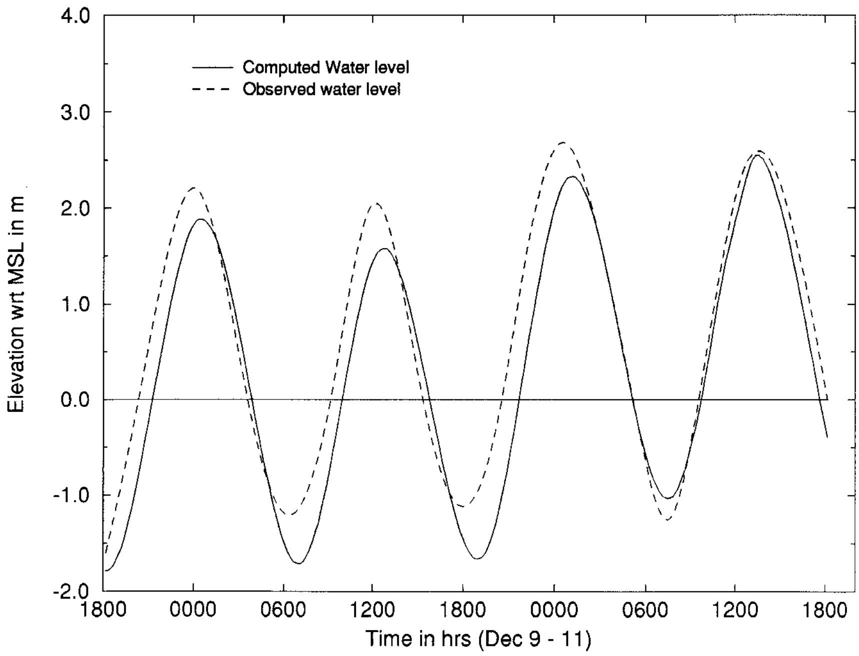
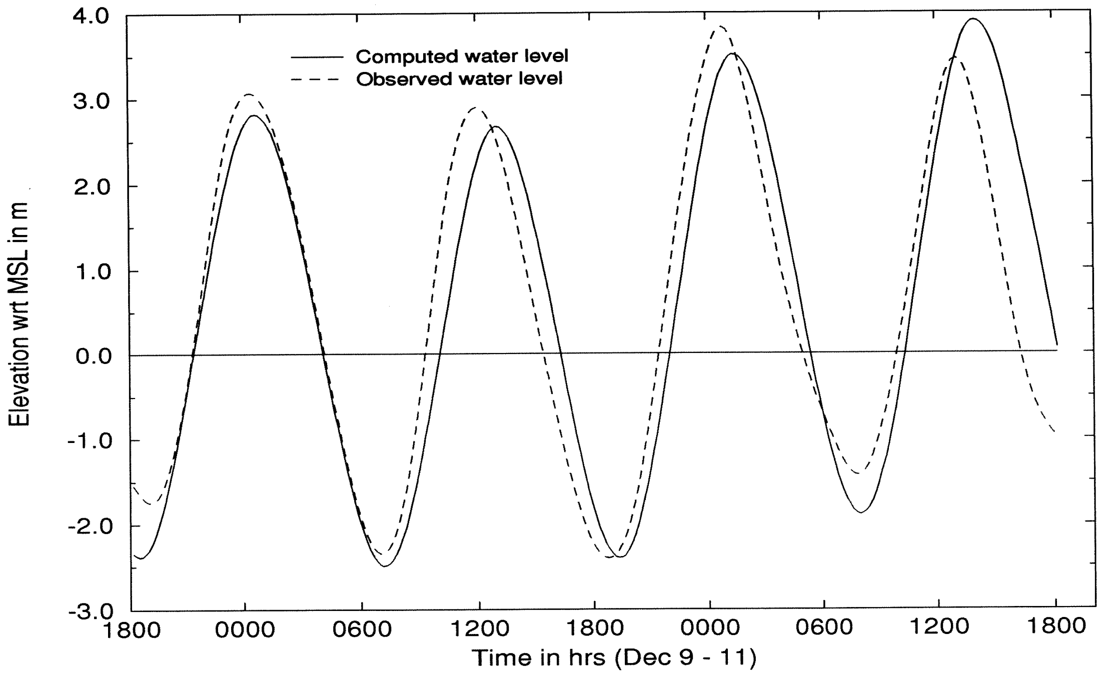
Nomenclature

C_D , C_f = the drag coefficient, the friction coefficient.
F_x , F_y = components of bottom friction, $N\,m^{-2}$
f = Coriolis parameter, s^{-1}
h = ocean depth from the mean sea level, m
m, n = no. of grid points along x- and y- axis
p, p_a = pressure, pressure at the sea surface, $N\,m^{-2}$
T_x , T_y = components of wind stress, $N\,m^{-2}$
u , v, w=velocity components of sea water, $m\,s^{-1}$
r = radial distance from the eye of a storm, m
R = radius of the maximum sustained wind, m
V_0 =maximum sustained wind speed of a storm, $m\,s^{-1}$
u_a , v_a,= components of the wind field, $m\,s^{-1}$
τ_x, τ_y = components of eddy stress, $N\,m^{-2}$
ζ= deviation from the mean sea level, m

(a)



(b)


Computed water level
Observed water level
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
Elevation wrt MSL in m
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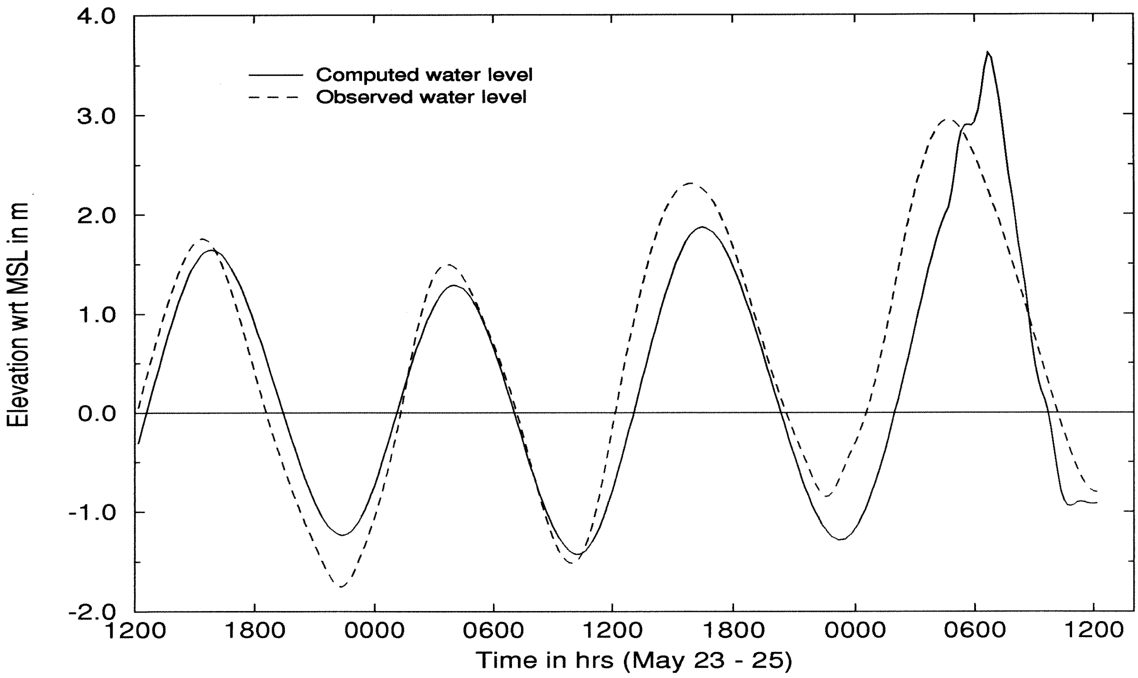
1800
Time in hrs (Dec 9 - 11)

(c)


Computed water level
Observed water level
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3.0
2.0
1.0
0.0
-1.0
-2.0
Elevation wrt MSL in m
1200
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Time in hrs (May 23 - 25)

(d)

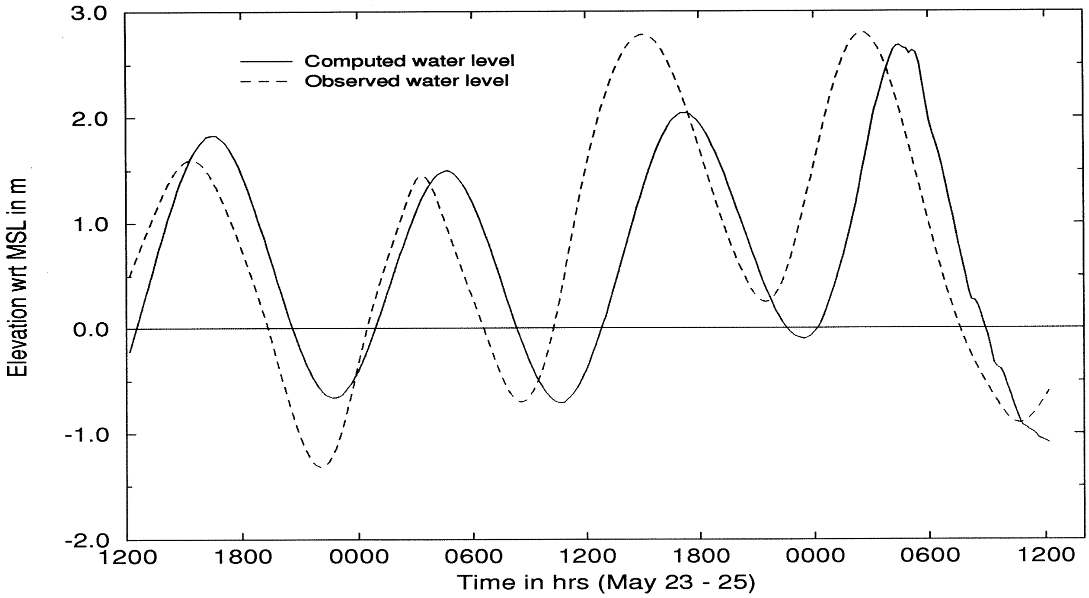
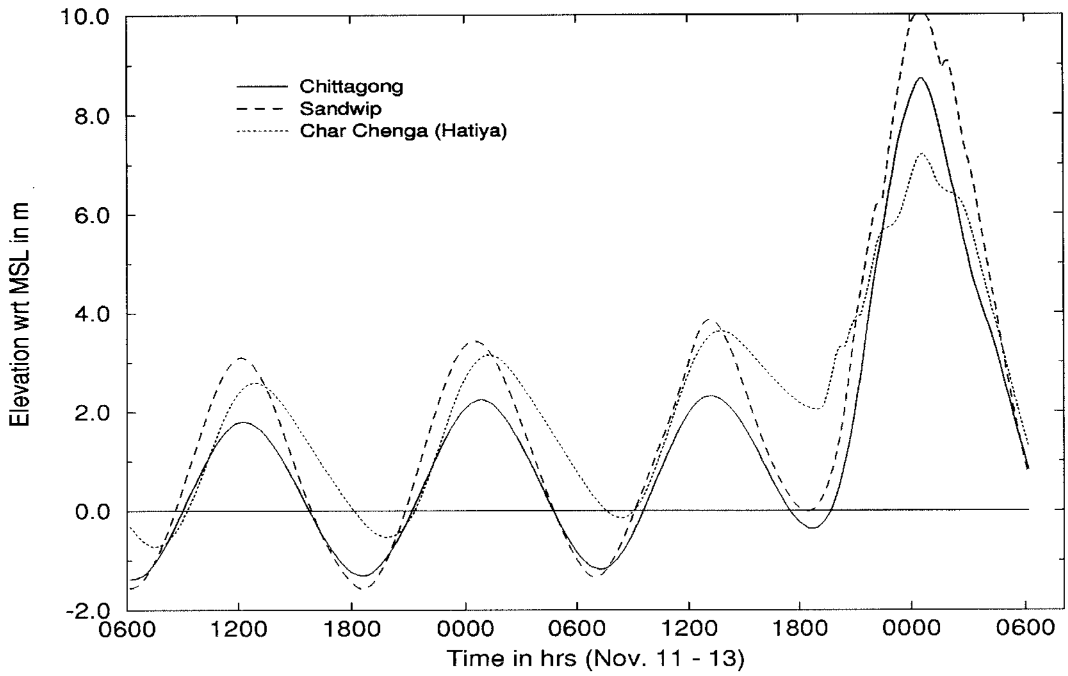
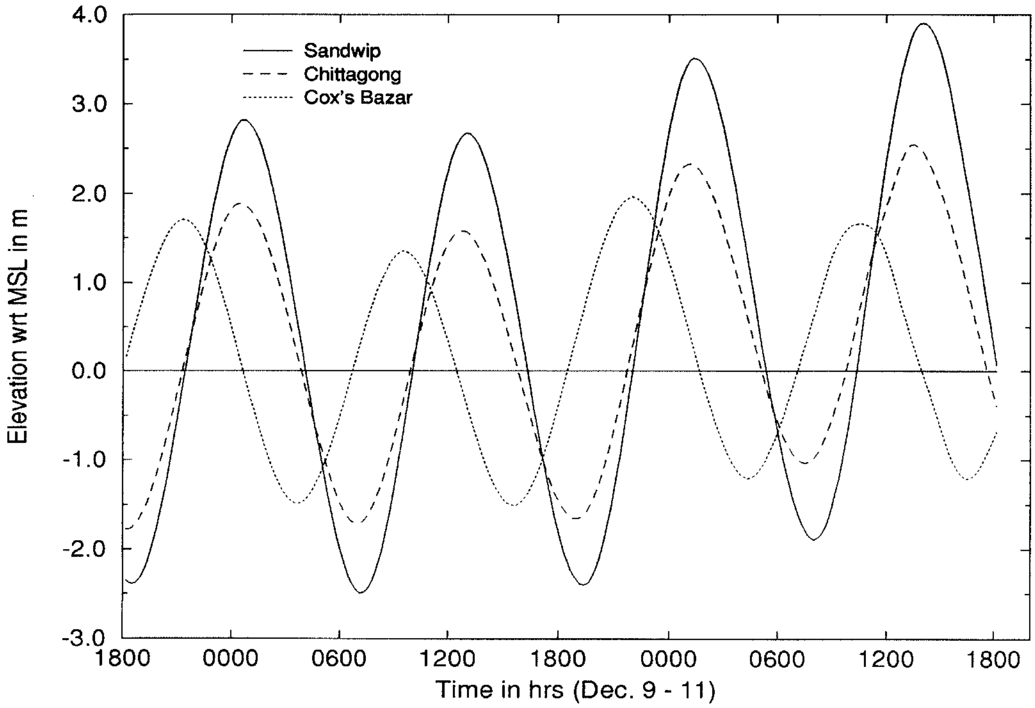


Fig. 3. Comparison between the computed (superposition) and observed water levels associated with December 1981 and May 1985 storms; (a) at Chittagong for 1981, (b) at Sandwip for 1981, (c) at Chittagong for 1985 and (d) at Char Chenga for 1985.

(a)



(b)


Sandwip
Chittagong
Cox's Bazar
Elevation wrt MSL in m
4.0
3.0
2.0
1.0
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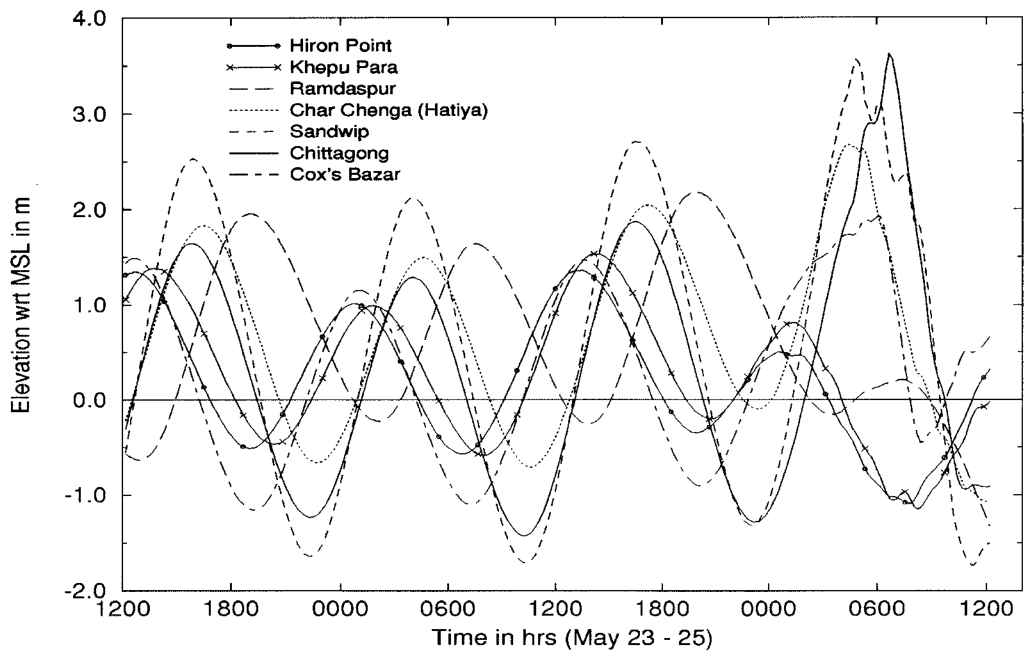
Time in hrs (Dec. 9 - 11)

(c)


Hiron Point
Khepu Para
Ramdaspur
Char Chenga (Hatiya)
Sandwip
Chittagong
Cox's Bazar
Elevation wrt MSL in m
4.0
3.0
2.0
1.0
0.0
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1200
1800
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Time in hrs (May 23 - 25)

(d)

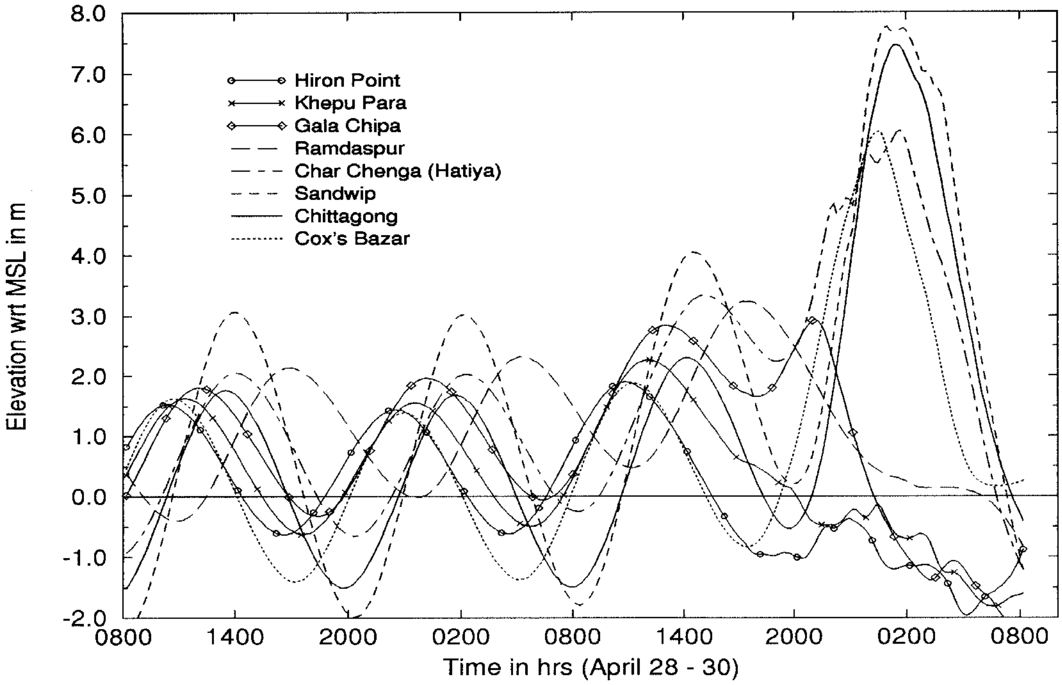

Fig. 4. Computed water levels due to tide and surge superposition associated with (a) November 1970, (b) December 1981, (c) May 1985 and (d) April 1991 storms.

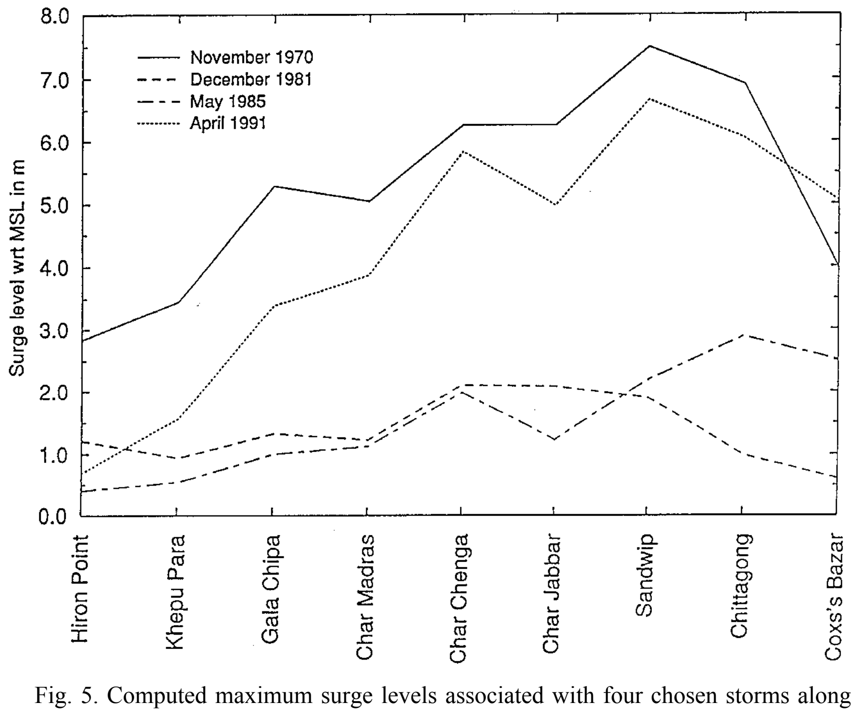

Fig. 5. Computed maximum surge levels associated with four chosen storms along the coastal belt between Hiron Point and Cox's Bazar.

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