



Inspiring Excellence

**On Some General Relativistic Compact
Anisotropic Charged Stellar Models**

By
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A thesis submitted in partial fulfillment for the
degree of Bachelor of Science in Physics

Department of Mathematics and Natural Sciences

BRAC University

July 2015

Declaration of Authorship

I, Tazkera Haque, declare that this thesis titled, 'On Some General Relativistic Compact Anisotropic Charged Stellar Models ' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in requirement for a partial fulfillment of a Bachelor's degree in Physics at BRAC University.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help. This thesis has been done based on the work previously done by M. H. Murad and S. Fatema.

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"The increase of disorder or entropy is what distinguishes the past from the future, giving a direction to time."

Stephen Hawking, A Brief History of Time

BRAC University

Abstract

Physics Undergraduate Program

Department of Mathematics and Natural Sciences

Bachelor of Science in Physics

by Tazkera Haque

With the presence of electric charge and pressure anisotropy some anisotropic stellar models have been developed. An algorithm presented by Herrera et al. (Phys. Rev. D 77, 027502 (2008)) to generate static spherically symmetric anisotropic solutions of Einsteins equations has been used to derive relativistic anisotropic charged fluid spheres. In the absence of pressure anisotropy the fluid spheres reduce to some well-known Generalized Tolman IV exact metrics. The astrophysical significance of the resulting equations of state (EOS) for a particular case (Wyman-Leibovitz-Adler) for the anisotropic charged matter distribution has been discussed. The interior matter pressure, energy-density, and the adiabatic sound speed are expressed in terms of simple algebraic functions. The constant parameters involved in the solution have been set so that certain physical criteria are satisfied. Physical analysis shows that the relativistic stellar structure obtained in this work may reasonably model an electrically charged compact star, whose energy-density associated with the electric fields is of the same order of magnitude as the energy density of fluid matter itself like electrically charged bare strange quark stars.

Acknowledgements

I would like to give special thanks to God Almighty, without whom I would not have had the motivation and strength to complete this thesis. I would also like to thank my family for their moral support and encouragement. I thank my respected faculty member Mohammad Hassan Murad for giving me the opportunity to work under his research project and for providing me the papers to help writhing this thesis. I am also grateful to him for helping me in every aspect to complete the whole dissertation written in L^AT_EX. I thank the entire stellar models research group whose papers I have frequently used and consulted, and in particular Late Prof. Dr. Mofiz Uddin Ahmed and Prof. Dr. A.A. Ziauddin Ahmad for their continuous support, advice, and help in the writing of this thesis. I also express my gratitude to BRAC University for giving me the opportunity of availing its resources and enhancing my knowledge. The Department of Mathematics and Natural Sciences has been supportive in its unique manner in all academic and official procedures. I am indeed lucky to have an affectionate Chairperson Prof. Dr. A.A. Ziauddin Ahmad whose constant guidance and concern had encouraged me to do well in my studies. The fulfillment of the undergraduate studies would not have been possible without the supervision of my course teachers - Mr. Lutfor Rahman, Mr. Mahabobe Shobahani, Dr. Md. Firoze H. Haque, Ms. Amel Chowdhury, and Ms. Sharmina Hussain - I am grateful for their effort in making me capable. Thanks to you all and God bless.

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Physical Constants

Speed of Light	$c = 2.997\,924\,58 \times 10^8 \text{ ms}^{-2} \text{ (exact)}$
Gravitational constant	$G = 6.67384 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$
vacuum permittivity	$\epsilon_0 = 8.854\,187\,817 \times 10^{-12} \text{ F/m}$
Solar mass	$M_\odot = 1.486 \text{ km} = 2 \times 10^{30} \text{ kg}$
Surface density	$\rho_s = 4.68 \times 10^{14} \text{ g cm}^{-3}$
Bag constant	$B = 60 \text{ MeV fm}^{-3}$
Strong interaction coupling constant	$\alpha_c = 0.17$

In loving memory of
Prof. Dr. Mofiz Uddin Ahmed
who left us forever on January 23, 2015

Chapter 1

Introduction

The study of regular relativistic charged fluid spheres obtained from the analytic solutions of Einstein-Maxwell gravitational field equations with certain geometry that satisfy physical constraints have remained the subject of considerable interest to the researchers [1–8]. Such solutions are also important in relativistic astrophysics because they enable to study the highly concentrated distribution of matter, up to the order of $10^{15} \text{ g cm}^{-3}$ in the interior of compact astrophysical stellar objects, to be modeled in terms of simple algebraic relations between the anisotropic pressure and its energy density. The Reissner-Nordström solution at the pressure free interface is interesting to observe that (present charge) the gravitational collapse of a spherical symmetric distribution of the matter to a point singularity may be avoided if the matter distribution acquires large amount of electric charge [9–11]. Any compact star is not composed of charged perfect fluid but may be used to make a suitable model of compact object with charge matter for the numerical study of the stellar structures [12, 13]. One of the two classes of known analytic solutions of Einstein's gravitational field equations describes self-bound stars for which density at the surface where the pressure vanishes, is finite, about 2-3 times the normal nuclear matter saturation density [14, 15]. The study of charged relativistic fluid spheres attracts the interest of researchers of physics and astrophysics due to some of the following reasons [16–20]:

- A spherical body can remain in equilibrium under its own gravity and electric repulsion if the matter present in the sphere carries certain modest electric charge density [9, 21]. The problem of the stability of a homogeneous distribution of matter containing a net surface charge was considered by Stettner [22]. He showed that a fluid sphere of uniform density with a modest surface charge has more stability than the same system without charge. His solution is also stable towards an increase in the net surface charge. The electric charge weakens gravity to the

extent of turning it into a repulsive field, as happens in the vicinity of a Reissener-Nordström singularity. Thus the gravitational collapse of a charged fluid sphere to a point singularity may be avoided [23].

- Charged-dust (CD) models and electromagnetic mass models are expected to provide some clues about the structure of an electron [24–28].
- Several static fluid spheres, which do not satisfy some or all the relevant physical conditions (e.g., monotone decrease in the subluminal adiabatic sound speed) become relevant when they are charged (See Table 4 of [29], and Table 1 of [30]).
- Charged fluid sphere may be used to model electrically charged compact stars, (e.g. electrically charged strange star). The electric charge distribution can have a significant impact on the structure of quark stars. The energy density associated with such huge electric fields ($E \approx 10^{19-20}$ V/cm) is of the same order of magnitude as the energy density of strange matter itself. This acts as an additional energy-momentum source in relativistic gravity [7, 31–40].
- A compact, electrically charged object with density and mass comparable to those of a neutron star can impart a redshift as great as quasars do. Therefore, if quasars are dense, massive, and electrically charged objects, their large redshifts do not imply that all quasars are extremely distant; thus, the luminosity of quasars and their association with low-redshift galaxies can be understood. This interpretation does not conflict with big bang cosmology, because the *electric redshifts* are negligible for normal stars, galaxies, and large-scale matter, which are not dense and electrically charged [41].

The possibility that 3-flavored strange quark matter (SQM, for short), made up of roughly equal numbers of unconfined up (u), down (d), and strange (s) quarks, may be the absolute ground state of the strong interaction is known as Bodmer-Witten hypothesis or the *strange quark matter hypothesis* [42]. There is still no sound scientific basis, both theoretical and experimental, on which one could either confirm or reject the hypothesis so that it remains a serious possibility of fundamental significance for physics and astrophysics [39, 43–45].

One very striking consequence of the hypothesis is the prediction of the existence of a new class of compact stars called strange (quark) stars [46]. The massive members of this hypothetical family of compact stars have masses and radii almost similar to those of neutron stars, however, strange stars would form a distinct and disconnected branch of compact stars and are not part of the continuum of equilibrium configurations that include white dwarfs and neutron stars [45, 47]. Compact objects like neutron stars or strange stars may be classified on the basis of mass-radius relation [48]. Apart from

the internal constituents, another most striking distinction between a strange star and a conventional normal matter neutron star is the surface electric field associated with the former one [45]. Since strange matter is, by the above hypothesis, absolutely stable, it must be so even at the surface, where the pressure $P = 0$, and this raises the exciting possibility that the surface of stars may, in fact, be exposed quark matter. Such a bare quark surface is expected to have most unusual properties: the density drops abruptly from the SQM density to zero and the thickness of the quark surface would be ≈ 1 fm, the range of strong interactions. These types of stars are known as *bare strange star* (BSS for short). A bare strange star have own ultra-strong electric fields on their surface which is around 10^{18} V/cm [49] and for color superconducting strange matter is 10^{20} V/cm.

First models of quark star, built exclusively of quark matter and possessing the surface density $2.28 \times 10^{14} \text{ gcm}^{-3}$, were considered by Brecher and Caporaso [50]. Using the simplest SQM equation of state (EOS) of massless non-interacting quarks (strange quark mass $m_s = 0$, and strong interaction (QCD) coupling constant $\alpha_c = 0$), they obtained $M_{\text{max}} = 2.8M_{\odot}$, much higher than for the EOSs of baryon matter available at that time. Their bag constant was about half of the today standard value, which easily explains why they obtained such a large M_{max} for their quark stars.

Eight years later, in 1984, the topic of quark stars was reconsidered by Witten [51], who also used the simplest model (in the limit $m_s \rightarrow 0$, $\alpha_c \rightarrow 0$) of the three-flavor quark matter with the EOS of the form

$$P = \frac{1}{3}(\rho - 4B)$$

in his approximate calculation of strange star mass-radius relation. He showed that for the bag constant $B^{\frac{1}{4}} = 145 \text{ MeV}$ ($B \approx 57.5 \text{ MeV fm}^{-3}$), the parameters of the maximum-mass configuration for strange stars are similar to those for realistic stars built of baryonic matter ($M_{\text{max}} = 2M_{\odot}$, $R_{\text{max}} = 11.1 \text{ km}$). The first detailed models of strange stars, based on a more realistic EOS of strange quark matter, taking into account strange quark mass and the lowest order QCD interactions, were constructed by Haensel et al. [46], who considered also specific features of accretion on strange stars. Similar results were presented somewhat later by Alcock et al. [49], who additionally proposed scenarios of the formation of strange stars and analyzed strange stars with the normal crust. These papers summarized the basic properties of strange stars and their astrophysical manifestations. For review see [44, 46, 48, 49, 52–55].

A recent calculation for cold and dense QCD strange quark matter indicates that massive compact stars with mass $\geq 2M_{\odot}$ up to maximal masses $\approx 2.75M_{\odot}$ would be interpreted as possible candidates for strange quark stars and that compact stars with observed masses up to $\approx 2M_{\odot}$ would be identified with hybrid stars, neutron stars with a central

core made up of deconfined quark matter [45]. Kurkela et al. [56] argued that a confirmed determination of a compact star with a mass in excess of $2M_{\odot}$ would seem to strongly favor—rather than rule out—the existence of quark matter in astrophysical objects.

Usov [52, 57], Negreiros et al. [58] and Ray et al. [59] first inquired the influence of energy density of ultra-high electric fields on the bulk properties of compact stars. Weber [39, 60] also have proposed that the existence of super-high electric fields of the magnitude 10^{20}C , generated by the charge distributions located near the surfaces of strange quark stars, increase the stellar mass by up to 30% depending on the strength of the electric field. This also may destroy the isotropy of the stellar configuration making the radial component of the pressure different from the angular ones. Taking the charged and anisotropic matters separately into considerations the generalization of Tolman type solutions were extensively studied. Maurya and Gupta also studied the isotropic and anisotropic charged generalizations of the same class of fluid spheres. Electrically neutral/charged anisotropic stellar models of strange quark stars within the framework of linear equation of state (EOS) based on MIT bag model [8, 25, 38] together with a particular choice of metric potentials/mass function have been taken into consideration in the following work [40, 54, 61, 62]. However some recent studies pointed out that the viability of nonlinear equation of state in the description of such compact stars cannot be ruled out. In a very recent past a polytropic quark star model has been suggested [53, 63] in order to establish a general framework in which theoretical quark star models could be tested by observations. The key difference between polytropic quark stars and the polytropic model studied previously for normal (i.e., non-quarkian) stars is that the quark star models with non-vanishing density at the stellar surface may not be avoidable due to the strong interaction between quarks which is relevant to the effect of color confinement. As discussed in [53], the polytropic equations of state are stiffer than the conventional realistic models (e.g. the MIT bag model) for quark matter, and pulsar-like stars calculated with a polytropic equation of state could then have high maximum masses $\geq 2M_{\odot}$. In this framework of polytropic model a very low massive quark star can also be, and be still gravitationally stable even if the polytropic index, $n > 3$. The charged analogues of Tolman V-VI models obtained by [64–67] are not physically viable in the description of compact astrophysical objects as the infinite values of central density and pressure. However, the charged analogues of Tolman IV and VII models obtained by [68–70], as the neutral ones, exhibit the physical features required for the construction of physically realizable relativistic compact stellar structure. However, various authors usually have chosen $2 \times 10^{14} \text{ g cm}^{-3}$ as stellar surface density to calculate the mass and radius of the charged fluid spheres which may have given rise to the stellar configuration as massive as $4 - 6M_{\odot}$ with much lower central density. Such massive configuration may not serve as a realistic model for a strange quark star. This choice is, therefore, is not a physical one. Modeling a compact (quark) star requires the use of a

higher surface density. Certainly, the value of the surface density affects the calculated value of the stellar mass—to see this, we observe that the method employed by [30, 68] can obtain arbitrarily large maximum mass just by inserting vanishing small surface density (e.g., $0.1 - 1 \text{ g cm}^{-3}$) to model a thin crust. In our model calculation, the density at the stellar radius is a value taken from the work of Sharma et al. [6] within the range $4 - 10 \times 10^{14} \text{ g cm}^{-3}$ [44] and drops abruptly to zero, as with all stellar models matching an interior metric to the external Reissner-Nordström form. This sharp drop in density is a reasonable model approximation since the thickness of the quark surface is of order 1 fm, a negligibly small dimension compared to the stellar radius.

Being inspired by the aforementioned recent works we develop some new analytical relativistic anisotropic charged stellar models by using Generalized Tolman IV model following the algorithm presented by Herrera et al. [71, 72]. Our analysis depends on several mathematical key assumptions. The form of metric potential ensures that the metric function is non-singular, continuous, and well behaved in the interior of the star. This is one of the desirable features for the model on physical grounds. Further, we assume some particular forms of electric charge distribution function and pressure anisotropy. The solutions obtained in this work are expected to provide simplified but easy to mathematically analyze stellar models with nonzero super-high surface density which could reasonably model the stellar core of an electrically charged bare strange quark star by satisfying applicable physical boundary conditions.

Chapter 2

Non-singular Anisotropic Charged Stellar Models

2.1 Einstein-Maxwell field equations

Compact astrophysical stellar objects were usually assumed to be composed of perfect fluid. Since the work of Schwarzschild¹ [73], Tolman, Oppenheimer and Volkoff [74, 75], the subject of modeling relativistic compact stellar objects through the analytical solution of Einsteins gravitational field equations has become one of the key issue in relativistic astrophysics. Such findings are significant because due to the absence of reliable information about the behavior of matter at such ultrahigh density near the central core, insight into the structure can be obtained by reference to applicable analytic solutions which enable the distribution of matter in the interior of stellar object to be modeled in terms of simple algebraic relations.

For a static spherically symmetric mass distribution, whose interior metric is given in Schwarzschild coordinates $x^\mu = (t, r, \theta, \phi)$ may be written as [74] :

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) \quad (2.1.1)$$

The functions ν and λ satisfy the Einstein-Maxwell field equations:

$$G^\mu_\nu = R^\mu_\nu - \frac{1}{2} R \delta^\mu_\nu = \kappa(T^\mu_\nu + E^\mu_\nu) \quad (2.1.2)$$

¹ Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie, Sitzungsberichte der Kniglich Preuischen Akademie der Wissenschaften (Berlin), 1916, Seite 189-196.

where we have chosen the units so that $c = G = 1$, $\kappa (= 8\pi)$ is Einstein's constant. R^μ_ν and δ^μ_ν are Ricci tensor and Ricci scalar respectively. T^μ_ν and E^μ_ν are the energy-momentum tensor of perfect fluid and electromagnetic field defined by the following [76]:

$$\begin{aligned} T^\mu_\nu &= (\rho + P)v^\mu v_\nu - P\delta^\mu_\nu \\ E^\mu_\nu &= \frac{1}{4\pi} \left(-F^{\mu m} F_{\nu m} - \frac{1}{4}\delta^\mu_\nu F^{mn} F_{mn} \right) \end{aligned}$$

and ρ , P and v^μ denote energy density, isotropic fluid pressure and the velocity vector respectively. F_{ik} denotes the anti-symmetric electromagnetic field strength tensor defined by:

$$F_{ik} = \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k} \quad (2.1.3)$$

which satisfies Maxwell equations:

$$F_{;k}^{ik} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^k} (\sqrt{-g} F^{ik}) = -4\pi j^i \quad (2.1.4a)$$

$$F_{ik;l} = 0 \quad (2.1.4b)$$

where g , the determinant of the quantities g_{ij} in eq. (2.1.1), is defined by,

$$g = \begin{vmatrix} e^\nu & 0 & 0 & 0 \\ 0 & -e^\lambda & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2 \theta \end{vmatrix} = -e^{\nu+\lambda} r^4 \sin^2 \theta$$

A_i is four potential with components $A_0 = \phi(r)$, $A_1 = A_2 = A_3 = 0$ and j^i is the four current vector defined by the expression,

$$j^i = \frac{\rho_{ch}}{\sqrt{g_{00}}} \frac{dx^i}{dx^0}$$

where ρ_{ch} denotes the charge density.

For static stellar configurations, which are considered in this paper, the only surviving component of the four-current is j^0 . Because of symmetry reasons, the four-current component is only a function of radial distance, r , and all components of the electromagnetic field tensor vanish, with the exception of F^{01} and F^{10} , related by $F^{01} = -F^{10}$, which describe the radial component of the electric field. From eq. (2.1.4a) one obtains

the following expression for the electric field:

$$\begin{aligned}\frac{1}{\sqrt{-g}} \frac{\partial}{\partial r} (\sqrt{-g} F^{01}) &= -4\pi j^0 \\ F^{01} &= -e^{\frac{(\nu+\lambda)}{2}} \frac{q(r)}{r^2}\end{aligned}$$

where $q(r)$ represents the total charge contained within the sphere of radius r defined by [2, 12, 13, 32]

$$q(r) = 4\pi \int_0^r e^{\frac{\lambda}{2}} \rho_{ch} u^2 du \quad (2.1.5)$$

which can be interpreted as the relativistic version of Gauss's law.

If the matter within the star is assumed to be perfect fluid in nature, then for the metric eq. (2.1.1), the Einstein field equations can be expressed as the following system of ordinary differential equations [24, 74, 75]

$$\frac{\nu'}{r} e^{-\lambda} - \frac{(1 - e^{-\lambda})}{r^2} = \kappa P \quad (2.1.6)$$

$$\left(\frac{\nu''}{2} - \frac{\nu' \lambda'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r} \right) e^{-\lambda} = \kappa P \quad (2.1.7)$$

$$\frac{\lambda'}{r} e^{-\lambda} - \frac{(1 - e^{-\lambda})}{r^2} = \kappa \rho \quad (2.1.8)$$

where prime ($'$) denotes d/dr .

By defining a quantity, $m(r)$, which represents the gravitational mass contained in a sphere of radius r , by the following expression,

$$e^{-\lambda} = 1 - \frac{2m(r)}{r} \quad (2.1.9)$$

and using eqs. (2.1.8), (2.1.6), and (2.1.7), respectively, one can arrive at the following equations:

$$m(r) = \frac{\kappa}{2} \int \rho r^2 dr \quad (2.1.10)$$

$$\nu' = \frac{(\kappa r P + \frac{2m}{r^2})}{1 - \frac{2m}{r}} \quad (2.1.11)$$

$$\frac{dP}{dr} = -\frac{(P + \rho)}{2} \nu' \quad (2.1.12)$$

where R is the radius of the fluid distribution determined by the condition $P(r = R) = 0$. Finally combining eqs. (2.1.11) and (2.1.12), one gets the well-known Tolman-Oppenheimer-Volkoff (TOV) equation of hydrostatic equilibrium for stellar configuration

[75]

$$\frac{dP}{dr} = -\frac{(P + \rho)}{2} \frac{(\kappa r P + \frac{2m}{r^2})}{1 - \frac{2m}{r}} \quad (2.1.13)$$

The necessary boundary conditions to solve eqs. (2.1.10)-(2.1.13) are: $P(r = 0) = P_c$, $m(r = 0) = 0$, $P(r = R) = 0$, $m(r = R) = M$, $\rho(r = 0) = \rho_c$, where M represents the total gravitational mass inside the fluid sphere. Equation (2.1.13) may also be obtained by requiring the conservation of mass-energy, $T_{;\nu}^{\mu\nu} = 0$ as that determines the global structure of stars. In order to obtain a realistic stellar model one can start with an explicit EOS and then integrating the TOV equation. The integration starts at the center of the star with a prescribed central pressure and ends where the pressure drops to zero, indicating the surface of the star. This approach requires numerical and graphical techniques and hence analytical specification of EOS does not always lead to closed-form solutions of gravitational field equations. Tolman first showed that instead of solving eq. (2.1.13) to obtain e^ν and e^λ , for any specified EOS, one can integrate the following equation of pressure isotropy, which follows from eqs. (2.1.7) and (2.1.8):

$$\left(\frac{\nu''}{2} - \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r} \right) e^{-\lambda} + \frac{(1 - e^{-\lambda})}{r^2} = 0 \quad (2.1.14)$$

and extract EOS using the system (2.1.6) to (2.1.8) by making some extra ad hoc assumptions on the metric functions ν or λ . Equation (2.1.14) is a second-order nonlinear (Riccati) differential equation in ν but first-order linear in λ . Following his approach, Tolman solved (2.1.14) under various assumptions on $\nu(r)$ and/or $\lambda(r)$ and presented eight models.

For example, model VI was obtained by setting $e^{\nu(r)} = \text{constant} = 2 - n^2$ into eq. (2.1.14) which yields $e^{\nu(r)} = (Ar^{1-n} - Br^{1+n})^2$. Later Wyman showed that this solution can be further generalized [77]. Unfortunately Tolman approach might lead to the unrealistic situation more frequently than to physical ones.

By introducing the transformation $e^{-\lambda} = Z$ into eq. (2.1.14), one can easily obtain

$$\frac{dZ}{dr} + P(r)Z = Q(r) \quad (2.1.15)$$

with the solution:

$$Z = \exp\left(-\int P(r)dr\right) \left[\int \left(\exp\left(\int P(r)dr\right) Q(r) \right) dr + A \right] \quad (2.1.16)$$

where A is a constant of integration and

$$P(r) = \frac{2r^2\nu'' + r^2\nu'^2 - 2r\nu' - 4}{r^2\nu' + 2r} \quad (2.1.17)$$

$$Q(r) = -\frac{4}{r^2\nu' + 2r} \quad (2.1.18)$$

Following the approach of Tolman, it has been shown by Korkina [78], Durgapal [79], Ishak et al. [80], and Lake [81] that the ansatz for the metric function

$$e^\nu = B_N(1 + Cr^2)^N \quad (2.1.19)$$

where N is a positive integer and $B_N, C > 0$ are constants, generates physically interesting solutions. Spherically symmetric analytic solutions of Einstein's gravitational field equations are usually applied to construct two different classes of stellar models.

- The first class that describes normal matter neutron stars for which density vanishes at the surface where the pressure vanishes.
- The second class that describes self-bound stars for which density is significantly finite, about 2-3 times the normal nuclear matter saturation density, at the surface where the pressure vanishes.

The Tolman VII solution with vanishing surface energy density falls into the first class and hence is useful approximation to realistic neutron star models and the second class includes models generated by (2.1.19), known as *Generalized Tolman IV* solution. Equation (2.1.19) produces an infinite family of analytic solutions of the self-bound type. Few of these were previously known. For example, $N = 1$ corresponds to Tolman IV model, and $N = 2, 3, 4, 5$ correspond to Adler [82], Heintzmann [83], and Durgapal [79] models, respectively.

The theoretical investigations on realistic stellar models strongly suggest that the matter distribution in such massive stellar objects may be *locally* anisotropic, the radial pressure may not be equal to the tangential pressure, at least in certain very high density ranges ($\rho > 10^{15} \text{ g cm}^{-3}$). The analytic solution of Bowers and Liang [84] of anisotropic spheres with uniform energy density showed that anisotropy could also play an important role in describing the high redshift objects like quasars. Anisotropy in pressure could significantly affect the physical parameters like maximum compactness, mass and radius of star. And since after then the pioneering work of Bowers and Liang an extensive literature has been devoted to the study the behavior of anisotropic spherically symmetric static general relativistic configurations and their stability. The Maharaj-Maartens [85] solution of anisotropic spheres with uniform energy density does not reduce to Schwarzschild interior solution in the absence of anisotropy.

If the matter within the star is assumed to be locally anisotropic charged perfect fluid in nature and T_ν^μ and E_ν^μ denote the energy-momentum tensor of anisotropic fluid distribution and electromagnetic field, then for the metric eq. (2.1.1), the Einstein-Maxwell field equations can be expressed as the following system of ordinary differential equations [86]:

$$\frac{\nu'}{r}e^{-\lambda} - \frac{(1 - e^{-\lambda})}{r^2} = \kappa P_r - \frac{q^2}{r^4} \quad (2.1.20)$$

$$\left(\frac{\nu''}{2} - \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r} \right) e^{-\lambda} = \kappa P_t + \frac{q^2}{r^4} \quad (2.1.21)$$

$$\frac{\lambda'}{r}e^{-\lambda} - \frac{(1 - e^{-\lambda})}{r^2} = \kappa \rho + \frac{q^2}{r^4} \quad (2.1.22)$$

By defining the quantity, $m(r)$, similar to the uncharged case, by the following expression,

$$e^{-\lambda} = 1 - \frac{2m(r)}{r} + \frac{q^2(r)}{r^2} \quad (2.1.23)$$

and using eqs. (2.1.22), (2.1.20), and (2.1.21) respectively, one can arrive at the following equations:

$$m(r) = \frac{\kappa}{2} \int \rho r^2 dr + \frac{Q^2}{2R} + \frac{1}{2} \int \frac{q^2}{r^2} dr \quad (2.1.24)$$

$$\nu' = \frac{\kappa r P_r + 2m/r^2 - 2q^2/r^3}{1 - 2m/r + q^2/r^2} \quad (2.1.25)$$

$$\frac{dP_r}{dr} = -\frac{(P_r + \rho)}{2} \nu' + \frac{q}{4\pi r^4} \frac{dq}{dr} + 2 \frac{(P_t - P_r)}{r} \quad (2.1.26)$$

where, $q(r = R) = Q$ represents the total electric charge inside the fluid sphere. Finally combining eqs. (2.1.25) and (2.1.26), we get the anisotropic charged generalization of TOV equation [74],

$$\frac{dP_r}{dr} = -\frac{(P_r + \rho)}{2} \frac{(\kappa r P_r + 2m/r^2 - 2q^2/r^3)}{(1 - 2m/r + q^2/r^2)} + \frac{q}{4\pi r^4} \frac{dq}{dr} + 2 \frac{(P_t - P_r)}{r} \quad (2.1.27)$$

The necessary boundary conditions to solve eqs. (2.1.24)-(2.1.27) are:

$$P_r(r = 0) = P_t(r = 0) = P_c, \quad P_r(r = R) = 0, \quad m(r = 0) = 0$$

$$m(r = R) = M, \quad q(r = 0) = 0, \quad q(r = R) = Q$$

Equation (2.1.27) may also be obtained by requiring the conservation of mass-energy, $T_{;\nu}^{\mu\nu} = 0$, as that determines the global structure of charged stars. It is to be noted that the presence of an additional term, $2(P_t - P_r)/r$, represents force, due to the pressure anisotropy, which is directed outward when $P_t > P_r$ and inward when $P_t < P_r$.

The existence of repulsive force allows the construction of more compact distribution when using anisotropic fluid than when using isotropic fluid. The equation of pressure anisotropy, which follows from eqs. (2.1.20) and (2.1.21) may be written as:

$$\left(\frac{\nu''}{2} - \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r} \right) e^{-\lambda} + \frac{(1 - e^{-\lambda})}{r^2} = \kappa(P_t - P_r) + \frac{2q^2}{r^4} \quad (2.1.28)$$

Now eqs. (2.1.17) and (2.1.18) become:

$$P(r) = \frac{2r^2\nu'' + r^2\nu'^2 - 2r\nu' - 4}{r^2\nu' + 2r} \quad (2.1.29)$$

$$Q(r) = \frac{4r^2\kappa(P_t - P_r) + 8Cq^2/r^2 - 4}{r^2\nu' + 2r} \quad (2.1.30)$$

With reference to eqs. (2.1.19), (2.1.29), (2.1.30), eq. (2.1.16) yields the following solution [1, 18, 87-89]:

$$\begin{aligned} Z &= \frac{x}{(1+x)^{N-2}[1+(1+N)x]^{\frac{2}{1+N}}} \\ &\times \int \frac{(1+x)^{N-1}[1+(1+N)x]^{\frac{1-N}{1+N}}}{x^2} \left(\frac{2Cq^2}{x} + \Delta x \right) dx \\ &+ \frac{1}{(1+x)^{N-2}} - \frac{1}{2} \binom{N-1}{2} \frac{x}{(1+x)^{N-2}} \\ &- \sum_{i=0}^{N-4} \sum_{j=0}^{i+1} \frac{(-1)^j}{(N+1)^{i+2}} \binom{N-1}{i+3} \binom{i+1}{j} \frac{x[1+(1+N)x]^{i-j+1}}{(i-j+1)(1+x)^{N-2}} \\ &+ A_N \frac{x}{(1+x)^{N-2}[1+(1+N)x]^{\frac{2}{1+N}}}; N \geq 4 \end{aligned} \quad (2.1.31)$$

where, $x = Cr^2$, $\Delta = \frac{\kappa}{C}(P_t - P_r)$ is the measure of pressure anisotropy and A_N is the constant of integration, depending on N , may be determined by imposing appropriate physical boundary conditions. Once the metric function $Z = e^{-\lambda}$ is obtained the remaining physical quantities P_r , P_t , and ρ follow from the following equations:

$$\frac{\kappa}{C}P_r = \frac{[1+(2N+1)x]}{x(1+x)}Z - \frac{1}{x} + \frac{Cq^2}{x^2} \quad (2.1.32)$$

$$\frac{\kappa}{C}P_t = \frac{\kappa}{C}P_r + \Delta \quad (2.1.33)$$

$$\frac{\kappa}{C}\rho = -2\frac{dZ}{dx} - \frac{Z}{x} + \frac{1}{x} - \frac{Cq^2}{x^2} \quad (2.1.34)$$

2.2 Models of electric charge distribution and pressure anisotropy

It seems reasonable to propose that due to electrical repulsion, the charge distribution should be weighted toward the surface. Due to the lack of information on how the electric charges are exactly distributed inside the fluid sphere [90], one can imagine several plausible forms of electric charge distribution, $\frac{2Cq^2}{x^2}$, to integrate the eq. (2.1.31). Various authors presented variety of solutions previously for different suitable choices of charge distributions with isotropic pressure [2, 29, 64, 74, 88, 89, 91–98]. In this work we make the following integrable choices on the forms of electric charge distribution,

$$\frac{2Cq^2}{x^2} = Kx^{n+1}(1+mx)^p(1+x)^{1-N}[1+(1+N)x]^{\frac{N-1}{N+1}} \quad (2.2.1)$$

and pressure anisotropy,

$$\Delta = \delta x^{n+2}(1-2ax)^p(1+x)^{1-N}[1+(1+N)x]^{\frac{N-1}{N+1}} \quad (2.2.2)$$

where $K, \delta \in [0, \infty)$, n, a are non negative integers, m is nonzero real number and p is positive integer and $a, K, \delta, m \in (-\infty, +\infty)$. All of these constants will be called *adjustable parameters*. It must be emphasized that these hypothetical models of electric charge distribution and pressure anisotropy are preferred for mathematical convenience, as these allow us to integrate and express eq. (2.1.31) explicitly.

The physical reasons behind these choices are that the electric field intensity and anisotropy vanish at the center and remain continuous and bounded in the interior of the star for a wide range of values of the adjustable parameters. Furthermore, the factor $(1-2ax)$, $a > 0$ and $\delta \in (-\infty, +\infty)$ in eq. (2.2.2) make the pressure anisotropy increase to some extent and then decrease to the boundary and due to the presence of strong electric field may be nonzero at the surface. Hence, these choices may be physically reasonable and useful in the study of the gravitational behavior of anisotropic charged stellar objects.

After feeding eqs. (2.2.1) and (2.2.2) into (2.1.31), one obtains:

$$\begin{aligned}
 Z = & K \sum_{i=0}^p \binom{p}{i} m^i \frac{x^{i+n+1}}{(1+x)^{N-2} [1+(1+N)x]^{\frac{2}{1+N}} (i+n+1)} \\
 & + \delta \sum_{j=0}^p \binom{p}{j} (-1)^j (2a)^j \frac{x^{j+n+1}}{(1+x)^{N-2} [1+(1+N)x]^{\frac{2}{1+N}} (j+n+1)} \\
 & + \frac{1}{(1+x)^{N-2}} - \frac{1}{2} \binom{N-1}{2} \frac{x}{(1+x)^{N-2}} \\
 & - \sum_{i=0}^{N-4} \sum_{j=0}^{i+1} \frac{(-1)^j}{(N+1)^{i+2}} \binom{N-1}{i+3} \binom{i+1}{j} \frac{x[1+(1+N)x]^{i-j+1}}{(i-j+1)(1+x)^{N-2}} \\
 & + A_N \frac{x}{(1+x)^{N-2} [1+(1+N)x]^{\frac{2}{1+N}}}; N \geq 4
 \end{aligned} \tag{2.2.3}$$

And feeding eqs.(2.2.3) and (2.2.1) into eqs. (2.1.32) and (2.1.33) one obtains the following:

$$\begin{aligned}
 \frac{\kappa}{C} P_r = & \frac{[1+(2N+1)x]}{(1+x)^{N-1}} \left[K \sum_{i=0}^p \binom{p}{i} m^i \frac{x^{i+n+1}}{(i+n+1)[1+(1+N)x]^{\frac{2}{1+N}}} \right. \\
 & + \delta \sum_{j=0}^p \binom{p}{j} (-1)^j (2a)^j \frac{x^{j+n+1}}{(j+n+1)[1+(1+N)x]^{\frac{2}{1+N}}} + \frac{1}{x} - \frac{1}{2} \binom{N-1}{2} \\
 & + \sum_{i=0}^{N-4} \sum_{j=0}^{i+1} \frac{(-1)^j}{(N+1)^{i+2}} \binom{N-1}{i+3} \binom{i+1}{j} \frac{[1+(1+N)x]^{(i-j+1)}}{(i-j+1)} \\
 & \left. + \frac{A_N}{[1+(1+N)x]^{\frac{2}{1+N}}} \right] - \frac{1}{x} \\
 & + \frac{K}{2} x^{n+1} (1+mx)^p (1+x)^{1-N} [1+(1+N)x]^{\frac{N-1}{N+1}}
 \end{aligned} \tag{2.2.4}$$

$$\begin{aligned}
 \frac{\kappa}{C} P_t = & \frac{[1+(2N+1)x]}{(1+x)^{N-1}} \left[K \sum_{i=0}^p \binom{p}{i} m^i \frac{x^{i+n+1}}{(i+n+1)[1+(1+N)x]^{\frac{2}{1+N}}} \right. \\
 & + \delta \sum_{j=0}^p \binom{p}{j} (-1)^j (2a)^j \frac{x^{j+n+1}}{(j+n+1)[1+(1+N)x]^{\frac{2}{1+N}}} + \frac{1}{x} - \frac{1}{2} \binom{N-1}{2} \\
 & + \sum_{i=0}^{N-4} \sum_{j=0}^{i+1} \frac{(-1)^j}{(N+1)^{i+2}} \binom{N-1}{i+3} \binom{i+1}{j} \frac{[1+(1+N)x]^{(i-j+1)}}{(i-j+1)} \\
 & \left. + \frac{A_N}{[1+(1+N)x]^{\frac{2}{1+N}}} \right] - \frac{1}{x} + \frac{K}{2} x^{n+1} (1+mx)^p (1+x)^{1-N} [1+(1+N)x]^{\frac{N-1}{N+1}} \\
 & + \delta x^{n+2} (1-2ax)^p (1+x)^{1-N} [1+(1+N)x]^{\frac{N-1}{N+1}}
 \end{aligned} \tag{2.2.5}$$

Now, differentiating eq. (2.2.3) with respect to x and feeding it into eq. (2.1.34), one obtains:

$$\begin{aligned}
\frac{\kappa}{C}\rho = & -2 \left[\sum_{i=0}^p \binom{p}{i} K m^i \frac{x^{(i+n+1)}}{(i+n+1)} (i+n+2)(1+x)^{2-N} [1+(1+N)x]^{-\frac{2}{1+N}} \right. \\
& + x^{(i+n+2)} (2-N)(1+x)^{(1-N)} [1+(1+N)x]^{-\frac{2}{1+N}} \\
& - 2x^{(i+n+2)} (1+x)^{(2-N)} [1+(1+N)x]^{-\left(\frac{2}{N+1}+1\right)} \\
& + \sum_{j=0}^p \binom{p}{j} \delta(-1)^j (2a)^j \frac{x^{(j+n+1)}}{(j+n+1)} (j+n+2)(1+x)^{2-N} [1+(1+N)x]^{-\frac{2}{1+N}} \\
& + x^{(j+n+2)} (2-N)(1+x)^{(1-N)} [1+(1+N)x]^{-\frac{2}{1+N}} \\
& - 2x^{(j+n+2)} (1+x)^{(2-N)} [1+(1+N)x]^{-\left(\frac{2}{N+1}+1\right)} \\
& + (2-N)(1+x)^{1-N} - \frac{1}{2} \binom{N-1}{2} (1+x)^{1-N} [1-(N-3)x] \\
& - \sum_{i=0}^{N-4} \sum_{j=0}^{i+1} \frac{(-1)^j x}{(N+1)^{i+2}} \binom{N-1}{i+3} \binom{i+1}{j} \frac{(1+N)(i-j+1)[1+(1+N)x]^{(i-j+1)}}{(1+x)^{N-2}} \\
& + x(2-N)(1+x)^{1-N} [1+(1+N)x]^{(i-j+1)} + (1+x)^{2-N} [1+(1+N)x]^{(i-j+1)} \\
& + A_N \left[\frac{x(2-N)(1+x)^{1-N}}{[1+(1+N)x]^{\frac{2}{1+N}}} + \frac{(1+x)^{2-N}}{(1+x)^{\frac{2}{1+N}}} - \frac{2x(1+x)^{2-N}}{[1+(1+N)x]^{\frac{2}{1+N}-1}} \right] \\
& - \frac{1}{x} \left[\sum_{i=0}^p \binom{p}{i} K m^i \frac{x^{i+n+1}}{(1+x)^{N-2} [1+(1+N)x]^{\frac{2}{1+N}} (i+n+1)} \right. \\
& + \sum_{j=0}^p \binom{p}{j} \delta(-1)^j (2a)^j \frac{x^{j+n+1}}{(1+x)^{N-2} [1+(1+N)x]^{\frac{2}{1+N}} (j+n+1)} \\
& + \frac{1}{(1+x)^{N-2}} - \frac{1}{2} \binom{N-1}{2} \frac{x}{(1+x)^{N-2}} \\
& - \sum_{i=0}^{N-4} \sum_{j=0}^{i+1} \frac{(-1)^j}{(N+1)^{i+2}} \binom{N-1}{i+3} \binom{i+1}{j} \frac{x[1+(1+N)x]^{i-j+1}}{(i-j+1)(1+x)^{N-2}} \\
& \left. + A_N \frac{x}{(1+x)^{N-2} [1+(1+N)x]^{\frac{2}{1+N}}} \right] \\
& + \frac{1}{x} - \frac{K}{2} x^{n+1} (1+mx)^p (1+x)^{1-N} [1+(1+N)x]^{\frac{N-1}{N+1}}
\end{aligned} \tag{2.2.6}$$

2.3 Anisotropic charged generalization of Wyman-Adler metric $N = 2$

The astrophysical significance and the adiabatic stability of Wyman-Adler solution ($N = 2$) in modeling neutron stars was first studied by Whitman [99], Maurya and Gupta [100], they studied the behaviors of static neutral fluid spheres generated by (2.1.19) for $N \geq 1$. However, out of all integral values the case $N = 2$ generates the most relevant

EOS, for which the speed of sound $\approx \frac{1}{\sqrt{3}}$ throughout most of the star, similar to the behavior of strange quark. For $N = 2$ the solution of the Einstein-Maxwell system eq. (2.1.10)-(2.1.13) for the model charge distribution and pressure anisotropy considered in this work are then given by,

$$e^\nu = B_2(1+x)^2 \quad (2.3.1)$$

$$\begin{aligned} e^{-\lambda} = & \sum_{i=0}^p \binom{p}{i} K m^i \frac{x^{(i+n+2)}}{(i+n+1)(1+3x)^{\frac{2}{3}}} \\ & + \sum_{j=0}^p \binom{p}{j} \delta(-1)^j (2a)^j \frac{x^{(j+n+2)}}{(j+n+1)(1+3x)^{\frac{2}{3}}} \\ & + 1 + A_2 \frac{x}{(1+3x)^{\frac{2}{3}}} \end{aligned} \quad (2.3.2)$$

$$\frac{2Cq^2}{x^2} = \frac{Kx^{n+1}(1+mx)^p(1+3x)^{\frac{1}{3}}}{(1+x)} \quad (2.3.3)$$

$$\Delta = \frac{\delta x^{n+2}(1-2ax)^p(1+3x)^{\frac{1}{3}}}{(1+x)} \quad (2.3.4)$$

$$\begin{aligned} \frac{\kappa}{C} P_r = & \sum_{i=0}^p \binom{p}{i} K m^i \frac{x^{(i+n+1)}(1+5x)}{(i+n+1)(1+x)(1+3x)^{\frac{2}{3}}} \\ & + \sum_{j=0}^p \binom{p}{j} \delta(-1)^j (2a)^j \frac{x^{(j+n+1)}(1+5x)}{(j+n+1)(1+x)(1+3x)^{\frac{2}{3}}} \\ & + \frac{4}{(1+x)} + \frac{1}{2} \left[\sum_{l=0}^p \binom{p}{l} K m^l \frac{x^{(l+n+1)}(1+3x)^{\frac{1}{3}}}{(1+x)} \right] \\ & + A_2 \frac{(1+5x)}{(1+x)(1+3x)^{\frac{2}{3}}} \end{aligned} \quad (2.3.5)$$

$$\begin{aligned} \frac{\kappa}{C} P_t = & \sum_{i=0}^p \binom{p}{i} K m^i \frac{x^{(i+n+1)}(1+5x)}{(i+n+1)(1+x)(1+3x)^{\frac{2}{3}}} \\ & + \sum_{j=0}^p \binom{p}{j} \delta(-1)^j (2a)^j \frac{x^{(j+n+1)}(1+5x)}{(j+n+1)(1+x)(1+3x)^{\frac{2}{3}}} \\ & + \frac{4}{(1+x)} + \frac{1}{2} \left[\sum_{l=0}^p \binom{p}{l} K m^l \frac{x^{(l+n+1)}(1+3x)^{\frac{1}{3}}}{(1+x)} \right] \\ & + A_2 \frac{(1+5x)}{(1+x)(1+3x)^{\frac{2}{3}}} + \frac{\delta x^{n+2}(1-2ax)^p[1+3x]^{\frac{1}{3}}}{(1+x)} \end{aligned} \quad (2.3.6)$$

$$\begin{aligned}
\frac{\kappa}{C}\rho = & -2 \sum_{i=0}^p \binom{p}{i} K m^i \frac{x^{(i+n+1)} [i(3x+1) + n(3x+1) + 2(2x+1)]}{(i+n+1)(1+3x)^{\frac{5}{3}}} \\
& -2 \sum_{j=0}^p \binom{p}{j} \delta(-1)^j (2a)^j \frac{x^{(j+n+1)} [j(3x+1) + n(3x+1) + 2(2x+1)]}{(j+n+1)(1+3x)^{\frac{5}{3}}} \\
& - \sum_{i=0}^p \binom{p}{i} K m^i \frac{x^{(i+n+1)}(1+3x)}{(i+n+1)(1+3x)^{\frac{5}{3}}} - \sum_{j=0}^p \binom{p}{j} \delta(-1)^j (2a)^j \frac{x^{(j+n+1)}(1+3x)}{(j+n+1)(1+3x)^{\frac{5}{3}}} \\
& - \frac{1}{2} \sum_{l=0}^p \binom{p}{l} K m^l \frac{x^{(l+n+1)}(1+3x)^{\frac{1}{3}}}{(1+x)} - A_2 \frac{(3+5x)}{(1+3x)^{\frac{5}{3}}} \quad (2.3.7)
\end{aligned}$$

2.4 Anisotropic charged generalization for $N = 2$ and particular cases

Case I: $p = 0$ and $n = 0$

$$e^\nu = B_2(1+x)^2 \quad (2.4.1)$$

$$e^{-\lambda} = (K + \delta) \frac{x^2}{(1+3x)^{\frac{2}{3}}} + 1 + A_2 \frac{x}{(1+3x)^{\frac{2}{3}}} \quad (2.4.2)$$

$$\frac{2Cq^2}{x^2} = \frac{Kx(1+3x)^{\frac{1}{3}}}{(1+x)} \quad (2.4.3)$$

$$\Delta = \frac{\delta x^2(1+3x)^{\frac{1}{3}}}{(1+x)} \quad (2.4.4)$$

$$\begin{aligned}
\frac{\kappa}{C}P_r = & (K + \delta) \frac{x(1+5x)}{(1+x)(1+3x)^{\frac{2}{3}}} + \frac{4}{(1+x)} + \frac{1}{2} \frac{Kx(1+3x)^{\frac{1}{3}}}{(1+x)} \\
& + A_2 \frac{(1+5x)}{(1+x)(1+3x)^{\frac{2}{3}}} \quad (2.4.5)
\end{aligned}$$

$$\begin{aligned}
\frac{\kappa}{C}P_t = & (K + \delta) \frac{x(1+5x)}{(1+x)(1+3x)^{\frac{2}{3}}} + \frac{4}{(1+x)} + \frac{1}{2} \frac{Kx(1+3x)^{\frac{1}{3}}}{(1+x)} \\
& + A_2 \frac{(1+5x)}{(1+x)(1+3x)^{\frac{2}{3}}} + \frac{\delta x^2(1+3x)^{\frac{1}{3}}}{(1+x)} \quad (2.4.6)
\end{aligned}$$

$$\begin{aligned}
\frac{\kappa}{C}\rho = & \frac{-2Kx(4x+2) - Kx(1+3x)}{(1+3x)^{\frac{5}{3}}} - \frac{2\delta x(4x+2) + \delta x(1+3x)}{(1+3x)^{\frac{5}{3}}} \\
& - \frac{1}{2} \frac{Kx(1+3x)^{\frac{1}{3}}}{(1+x)} - A_2 \frac{(3+5x)}{(1+3x)^{\frac{5}{3}}} \quad (2.4.7)
\end{aligned}$$

Case II: $p = 1$ and $n = 0$

$$e^\nu = B_2(1+x)^2 \quad (2.4.8)$$

$$e^{-\lambda} = (K + \delta) \frac{x^2}{(1+3x)^{\frac{2}{3}}} + \frac{1}{2} \frac{Kmx^3}{(1+3x)^{\frac{2}{3}}} - \frac{\delta ax^3}{(1+3x)^{\frac{2}{3}}} + 1 + A_2 \frac{x}{(1+3x)^{\frac{2}{3}}} \quad (2.4.9)$$

$$\frac{2Cq^2}{x^2} = \frac{Kx(1+mx)(1+3x)^{\frac{1}{3}}}{(1+x)} \quad (2.4.10)$$

$$\Delta = \frac{\delta x^2(1-2ax)(1+3x)^{\frac{1}{3}}}{(1+x)} \quad (2.4.11)$$

$$\begin{aligned} \frac{\kappa}{C} P_r = & \frac{Kx(1+5x)}{(1+x)(1+3x)^{\frac{2}{3}}} \left(1 + \frac{1}{2}mx\right) + \frac{\delta x(1+5x)}{(1+x)(1+3x)^{\frac{2}{3}}} (1-ax) \\ & + \frac{1}{2} \left[\frac{Kx(1+3x)^{\frac{1}{3}}}{(1+x)} (1+mx) \right] + \frac{4}{(1+x)} + A_2 \frac{(1+5x)}{(1+x)(1+3x)^{\frac{2}{3}}} \end{aligned} \quad (2.4.12)$$

$$\begin{aligned} \frac{\kappa}{C} P_t = & \frac{Kx(1+5x)}{(1+x)(1+3x)^{\frac{2}{3}}} \left(1 + \frac{1}{2}mx\right) + \frac{\delta x(1+5x)}{(1+x)(1+3x)^{\frac{2}{3}}} (1-ax) \\ & + \frac{1}{2} \left[\frac{Kx(1+3x)^{\frac{1}{3}}}{(1+x)} (1+mx) \right] + \frac{4}{(1+x)} + A_2 \frac{(1+5x)}{(1+x)(1+3x)^{\frac{2}{3}}} \\ & + \frac{\delta x^2(1-2ax)(1+3x)^{\frac{1}{3}}}{(1+x)} \end{aligned} \quad (2.4.13)$$

$$\begin{aligned} \frac{\kappa}{C} \rho = & -\frac{2Kx(4x+2) + Kx(1+3x)}{(1+3x)^{\frac{5}{3}}} - \frac{2Kmx^2(2+7x) + Kmx^2(1+3x)}{2(1+3x)^{\frac{5}{3}}} \\ & - \frac{2\delta x(2+4x) + \delta x(1+3x)}{(1+3x)^{\frac{5}{3}}} + \frac{2a\delta x^2(2+7x) + \delta ax^2(1+3x)}{(1+3x)^{\frac{5}{3}}} \\ & - \frac{Kx(1+3x)^{\frac{1}{3}} + Kmx^2(1+3x)^{\frac{1}{3}}}{2(1+x)} - A_2 \frac{(3+5x)}{(1+3x)^{\frac{5}{3}}} \end{aligned} \quad (2.4.14)$$

Chapter 3

Physical Boundary Conditions

3.1 Determination of the arbitrary constant A_2

The boundary condition $P_r(R) = 0$ can be utilized to specify the constant of integration A_2 . For Wyman-Adler metric:

$$A_2 = -\sum_{i=0}^p \binom{p}{i} K m^i \frac{X^{(i+n+1)}}{(i+n+1)} - \sum_{j=0}^p \binom{p}{j} \delta (-1)^j (2a)^j \frac{X^{(j+n+1)}}{(j+n+1)} - \frac{1}{2} \sum_{l=0}^p K m^l \frac{X^{(l+n+1)}(1+3X)}{(1+5X)} - 4 \frac{(1+3X)^{\frac{2}{3}}}{(1+5X)} \quad (3.1.1)$$

where $X = CR^2$.

Case I : $p = 0$ and $n = 0$

$$A_2 = -X(K + \delta) - 4 \frac{(1+3X)^{\frac{2}{3}}}{(1+5X)} - \frac{1}{2} K X \frac{(1+3x)}{(1+5x)} \quad (3.1.2)$$

Case II : $p = 1$ and $n = 0$

$$A_2 = -KX \left(1 + \frac{mX}{2}\right) - \delta X (1 - aX) - \frac{KX}{2} (1 + mX) \frac{(1+3x)}{(1+5x)} - 4 \frac{(1+3X)^{\frac{2}{3}}}{(1+5X)} \quad (3.1.3)$$

3.2 Total charge to radius ratio Q/R

Using $X = CR^2$ in eq. (2.2.1) we obtain,

$$\frac{Q}{R} = \sqrt{\frac{K}{2} X^{n+2} \frac{(1 + mX)^p (1 + 3X)^{\frac{1}{3}}}{(1 + X)}} \quad (3.2.1)$$

3.3 Total mass to radius ratio M/R

The interior solution should match continuously with an exterior Reissner-Nordström solution given by,

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2); \quad r \geq R \quad (3.3.1)$$

This requires the continuity of e^ν , e^λ and q across the boundary $r = R$,

$$e^{\nu(R)} = e^{-\lambda(R)} = \left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right) \quad (3.3.2)$$

and $q(R) = Q$. By matching the metric coefficients obtained in eqs. (2.3.1) and (2.3.2) with the exterior Reissner-Nordström metric at the boundary and with reference to the eq. (3.2.1) one can establish the equation of compactness,

$$\frac{2M}{R} = \left(1 - e^{-\lambda(X)} + \frac{Q^2}{R^2}\right) \quad (3.3.3)$$

3.4 Total charge to mass ratio Q/M

By the use of eqs. (3.2.1) and (3.3.3) we obtain the charge to mass ratio Q/M .

3.5 Determination of the constant B_2

The constant B_2 can be specified by the boundary condition $e^{\nu(R)} = e^{-\lambda(R)}$, which gives

$$B_2 = (1 + X)^{-2} e^{-\lambda(X)} \quad (3.5.1)$$

3.6 Central and surface redshifts

From the definition of redshift,

$$z = \sqrt{1 - \frac{2M}{R} + \frac{Q^2}{R^2}} - 1 \quad (3.6.1)$$

one obtains the central and surface redshifts:

$$z_c = \sqrt{e^{-\nu(0)}} - 1 \quad (3.6.2)$$

$$z_s = \sqrt{e^{-\nu(R)}} - 1 = \frac{(1 + X)^{-1}}{\sqrt{B_2}} - 1 \quad (3.6.3)$$

Chapter 4

Construction of Physically Realistic Fluid Spheres

4.1 Pressure and density gradients for $N = 2$

A straightforward differentiation of the pressure and density eqs. (2.3.4), (2.3.5), (2.3.6) and (2.3.7) with respect to the auxiliary variable x one obtains the pressure and density gradients, respectively, for the model EOS.

$$\begin{aligned}
 \frac{\kappa}{C} \frac{dP_r}{dx} = & \sum_{i=0}^p \binom{p}{i} \frac{Km^i}{(i+n+1)} \\
 & \times \frac{x^{(i+n)} [5x^3(3i+3n+1) + 23x^2(i+n+1) + 9x(i+n) + i+n+11x+1]}{(1+x)^2(1+3x)^{\frac{5}{3}}} \\
 & + \sum_{j=0}^p \binom{p}{j} \frac{\delta(-1)^j(2a)^j}{(j+n+1)} \\
 & \times \frac{x^{(j+n)} [5x^3(3j+3n+1) + 23x^2(j+n+1) + 9x(j+n) + j+n+11x+1]}{(1+x)^2(1+3x)^{\frac{5}{3}}} \\
 & + \frac{K}{2} \sum_{l=0}^p \binom{p}{l} \frac{x^{(l+n)} [3x^2(l+n+1) + 4x(l+n) + l+n+6x+1]}{(1+x)^2} \\
 & - \frac{4}{(1+x)^2} + 2A_2 \frac{(1-5x)^2}{(1+x)^2(1+3x)^{\frac{5}{3}}} \tag{4.1.1}
 \end{aligned}$$

$$\begin{aligned}
 \frac{d\Delta}{dx} = & \frac{\delta x^{(n+1)}(n+2)(1-2ax)^{p-1}(1+3x)^{\frac{1}{3}}}{(1+x)} - \frac{2ap\delta x^{(n+2)}(1+3x)^{\frac{1}{3}}(1-2ax)^{p-1}}{(1+x)} \\
 & - \frac{\delta x^{(n+2)}(1+3x)^{\frac{1}{3}}(1-2ax)^p}{(1+x)^2} + \frac{\delta x^{(n+2)}(1-2ax)^p}{(1+x)(1+3x)^{\frac{2}{3}}} \tag{4.1.2}
 \end{aligned}$$

$$\begin{aligned}
\frac{\kappa}{C} \frac{dP_t}{dx} = & \sum_{i=0}^p \binom{p}{i} \frac{Km^i}{(i+n+1)} \\
& \times \frac{x^{(i+n)} [5x^3(3i+3n+1) + 23x^2(i+n+1) + 9x(i+n) + i+n+11x+1]}{(1+x)^2(1+3x)^{\frac{5}{3}}} \\
& + \sum_{j=0}^p \binom{p}{j} \frac{\delta(-1)^j(2a)^j}{(j+n+1)} \\
& \times \frac{x^{(j+n)} [5x^3(3j+3n+1) + 23x^2(j+n+1) + 9x(j+n) + j+n+11x+1]}{(1+x)^2(1+3x)^{\frac{5}{3}}} \\
& + \frac{K}{2} \sum_{l=0}^p \binom{p}{l} \frac{x^{(l+n)} [3x^2(l+n+1) + 4x(l+n) + l+n+6x+1]}{(1+x)^2} \\
& - \frac{4}{(1+x)^2} + 2A_2 \frac{(1-5x)^2}{(1+x)^2(1+3x)^{\frac{5}{3}}} \\
& + \frac{\delta x^{(n+1)}(n+2)(1+3x)^{\frac{1}{3}}(1-2ax)^p}{(1+x)} - \frac{2ap\delta x^{(n+2)}(1+3x)^{\frac{1}{3}}(1-2ax)^{p-1}}{(1+x)} \\
& - \frac{\delta x^{(n+2)}(1+3x)^{\frac{1}{3}}(1-2ax)^p}{(1+x)^2} + \frac{\delta x^{(n+2)}(1-2ax)^p}{(1+x)(1+3x)^{\frac{2}{3}}} \tag{4.1.3}
\end{aligned}$$

$$\begin{aligned}
\frac{\kappa}{C} \frac{d\rho}{dx} = & - \sum_{i=0}^p \binom{p}{i} \frac{Km^i}{(i+n+1)} \\
& \times \frac{x^{(i+n)} [9ix^2 + 6ix + i + 9nx^2 + 6nx + n + 3x^2 + 4x + 1]}{(1+3x)^{\frac{8}{3}}} \\
& - \sum_{i=0}^p \binom{p}{i} \frac{Km^i}{(i+n+1)} \\
& \times \left[\frac{x^{(i+n)} [36inx^2 + 24inx + 4in + 30ix^2 + 34ix + 8i + 18n^2x^2 + 12n^2x]}{(1+3x)^{\frac{8}{3}}} \right. \\
& \left. + \frac{x^{(i+n)} [2n^2 + 30nx^2 + 28nx + 6n + 8x^2 + 8x + 4]}{(1+3x)^{\frac{8}{3}}} \right] \\
& - \sum_{j=0}^p \binom{p}{j} \frac{\delta(-1)^j (2a)^j}{(j+n+1)} \\
& \times \frac{x^{(j+n)} [9jx^2 + 6jx + j + 9nx^2 + 6nx + n + 3x^2 + 4x + 1]}{(1+3x)^{\frac{8}{3}}} \\
& - \sum_{j=0}^p \binom{p}{j} \frac{\delta(-1)^j (2a)^j}{(j+n+1)} \\
& \times \left[\frac{x^{(j+n)} [36jnx^2 + 24jnx + 4jn + 30jx^2 + 34jx + 8j + 18n^2x^2 + 12n^2x]}{(1+3x)^{\frac{8}{3}}} \right. \\
& \left. + \frac{x^{(j+n)} [2n^2 + 30nx^2 + 28nx + 6n + 8x^2 + 8x + 4]}{(1+3x)^{\frac{8}{3}}} \right] \\
& - \frac{1}{2} \sum_{l=0}^p \binom{p}{l} Km^l \frac{x^{(l+n)} [x^2(3l+3n+1) + 4x(l+n+1) + l+n+1]}{(1+x)^2(1+3x)^{\frac{2}{3}}} \\
& + 10A_2 \frac{(1+x)}{(1+3x)^{\frac{8}{3}}}
\end{aligned} \tag{4.1.4}$$

4.1.1 Pressure and density gradients (particular cases)

Case I: $p = 0$ and $n = 0$

$$\begin{aligned} \frac{\kappa}{C} \frac{dP_r}{dx} = & (K + \delta) \frac{(5x^3 + 23x^2 + 11x + 1)}{(1+x)^2(1+3x)^{\frac{5}{3}}} - \frac{4}{(1+x)^2} \\ & + \frac{K(3x^2 + 6x + 1)}{2(1+x)^2} + A_2 \frac{(2 - 10x^2)}{(1+x)^2(1+3x)^{\frac{5}{3}}} \end{aligned} \quad (4.1.5)$$

$$\frac{d\Delta}{dx} = \frac{2\delta x(2x^2 + 4x + 1)}{(1+x)^2(1+3x)^{\frac{2}{3}}} \quad (4.1.6)$$

$$\begin{aligned} \frac{\kappa}{C} \frac{dP_t}{dx} = & (K + \delta) \frac{(5x^3 + 23x^2 + 11x + 1)}{(1+x)^2(1+3x)^{\frac{5}{3}}} - \frac{4}{(1+x)^2} \\ & + \frac{K(3x^2 + 6x + 1)}{2(1+x)^2} + A_2 \frac{(2 - 10x^2)}{(1+x)^2(1+3x)^{\frac{5}{3}}} \\ & + \frac{2\delta x(2x^2 + 4x + 1)}{(1+x)^2(1+3x)^{\frac{2}{3}}} \end{aligned} \quad (4.1.7)$$

$$\begin{aligned} \frac{\kappa}{C} \frac{d\rho}{dx} = & -(K + \delta) \frac{(11x^2 + 12x + 5)}{(1+3x)^{\frac{8}{3}}} - \frac{K}{2} \frac{(x^2 + 4x + 1)}{(1+x)^2(1+3x)^{\frac{2}{3}}} \\ & + 10A_2 \frac{(1+x)}{(1+3x)^{\frac{8}{3}}} \end{aligned} \quad (4.1.8)$$

Case II: $p = 1$ and $n = 0$

$$\begin{aligned} \frac{\kappa}{C} \frac{dP_r}{dx} = & (K + \delta) \frac{(5x^3 + 23x^2 + 11x + 1)}{(1+x)^2(1+3x)^{\frac{5}{3}}} - \frac{4}{(1+x)^2} \\ & + \frac{x(20x^3 + 46x^2 + 20x + 2)}{2(1+x)^2(1+3x)^{\frac{5}{3}}} (Km - 2a\delta) \\ & + \frac{K(3x^2 + 6x + 1)}{2(1+x)^2} + \frac{K}{2} \frac{mx(6x^2 + 10x + 2)}{(1+x)^2} \\ & + A_2 \frac{(2 - 10x^2)}{(1+x)^2(1+3x)^{\frac{5}{3}}} \end{aligned} \quad (4.1.9)$$

$$\frac{d\Delta}{dx} = \frac{2\delta x[-7ax^3 + x^2(2 - 12a) + x(4 - 3a) + 1]}{(1+x)^2(1+3x)^{\frac{2}{3}}} \quad (4.1.10)$$

$$\begin{aligned} \frac{\kappa}{C} \frac{dP_t}{dx} = & (K + \delta) \frac{(5x^3 + 23x^2 + 11x + 1)}{(1+x)^2(1+3x)^{\frac{5}{3}}} - \frac{4}{(1+x)^2} \\ & + \frac{x(20x^3 + 46x^2 + 20x + 2)}{2(1+x)^2(1+3x)^{\frac{5}{3}}} (Km - 2a\delta) \\ & + \frac{K}{2} \frac{(3x^2 + 6x + 1)}{(1+x)^2} + \frac{K}{2} \frac{mx(6x^2 + 10x + 2)}{(1+x)^2} \\ & + \frac{2\delta x[-7ax^3 + x^2(2 - 12a) + x(4 - 3a) + 1]}{(1+x)^2(1+3x)^{\frac{2}{3}}} \\ & + A_2 \frac{(2 - 10x^2)}{(1+x)^2(1+3x)^{\frac{5}{3}}} \end{aligned} \quad (4.1.11)$$

$$\begin{aligned} \frac{\kappa}{C} \frac{d\rho}{dx} = & (K + \delta) \frac{(11x^2 + 12x + 5)}{(1+3x)^{\frac{8}{3}}} + \left(\delta a - \frac{Km}{2} \right) \frac{x(50x^2 + 52x + 14)}{(1+3x)^{\frac{8}{3}}} \\ & - \frac{K}{2} \frac{(x^2 + 4x + 1)}{(1+x)^2(1+3x)^{\frac{2}{3}}} - \frac{K}{2} \frac{mx(4x^2 + 8x + 2)}{(1+x)^2(1+3x)^{\frac{2}{3}}} \\ & + 10A_2 \frac{(1+x)}{(1+3x)^{\frac{8}{3}}} \end{aligned} \quad (4.1.12)$$

4.2 Specifying the maximum mass and radius

For any particular set $(K, m, n, p, a, \delta, X)$ the compactness M/R and the ratio Q/R of the compact fluid sphere is obtained by the use of the eqs. (3.2.1) and (3.3.3). The maximum mass of the fluid distribution can then be calculated by using one of the following quantities: i) radius, ii) central density, iii) surface density, iv) central pressure or, v) total charge as parameter. In this section we proceed with two special cases for a prescribed stellar surface density, $\rho_s = 4.68 \times 10^{14} \text{ gcm}^{-3}$ in the following way:

4.2.1 Radius

$$R = \sqrt{\frac{X}{\kappa\rho_s}} \Omega(X) \quad (4.2.1.1)$$

where,

$$\begin{aligned}
 \Omega(X) = & -2 \sum_{i=0}^p \binom{p}{i} K m^i \frac{X^{(i+n+1)} [i(3X+1) + n(3X+1) + 2(2X+1)]}{(i+n+1)(1+3X)^{\frac{5}{3}}} \\
 & -2 \sum_{j=0}^p \binom{p}{j} \delta(-1)^j (2a)^j \frac{X^{(j+n+1)} [j(3X+1) + n(3X+1) + 2(2X+1)]}{(j+n+1)(1+3X)^{\frac{5}{3}}} \\
 & - \sum_{i=0}^p \binom{p}{i} K m^i \frac{X^{(i+n+1)}(1+3X)}{(i+n+1)(1+3X)^{\frac{5}{3}}} - \sum_{j=0}^p \binom{p}{j} \delta(-1)^j (2a)^j \frac{X^{(j+n+1)}(1+3X)}{(j+n+1)(1+3X)^{\frac{5}{3}}} \\
 & - \frac{1}{2} \sum_{l=0}^p \binom{p}{l} K m^l \frac{X^{(l+n+1)}(1+3X)^{\frac{1}{3}}}{(1+X)} - A_2 \frac{(3+5X)}{(1+3X)^{\frac{5}{3}}} \quad (4.2.1.2)
 \end{aligned}$$

Case I: $p = 0$ and $n = 0$

$$R = \sqrt{\frac{X}{\kappa \rho_s} \Gamma(X)} \quad (4.2.1.3)$$

where,

$$\begin{aligned}
 \Gamma(X) = & \frac{-2KX(4X+2) - KX(1+3X)}{(1+3X)^{\frac{5}{3}}} - \frac{2\delta X(4X+2) + \delta X(1+3X)}{(1+3X)^{\frac{5}{3}}} \\
 & - \frac{1}{2} \frac{KX(1+3X)^{\frac{1}{3}}}{(1+X)} - A_2 \frac{(3+5X)}{(1+3X)^{\frac{5}{3}}} \quad (4.2.1.4)
 \end{aligned}$$

Case II: $p = 1$ and $n = 0$

$$R = \sqrt{\frac{X}{\kappa \rho_s} \Sigma(X)} \quad (4.2.1.5)$$

where,

$$\begin{aligned}
 \Sigma(X) = & -\frac{2KX(4X+2) + KX(1+3X)}{(1+3X)^{\frac{5}{3}}} - \frac{2KmX^2(2+7X) + KmX^2(1+3X)}{2(1+3X)^{\frac{5}{3}}} \\
 & - \frac{2\delta X(2+4X) + \delta X(1+3X)}{(1+3X)^{\frac{5}{3}}} + \frac{2a\delta X^2(2+7X) + \delta aX^2(1+3X)}{(1+3X)^{\frac{5}{3}}} \\
 & - \frac{KX(1+3X)^{\frac{1}{3}} + KmX^2(1+3X)^{\frac{1}{3}}}{2(1+X)} - A_2 \frac{(3+5X)}{(1+3X)^{\frac{5}{3}}} \quad (4.2.1.6)
 \end{aligned}$$

4.2.2 Total mass

$$M = \frac{R}{2} \left(1 - e^{-\lambda(X)} + \frac{Q^2}{R^2} \right) \quad (4.2.2.1)$$

where the mass is measured in km.

Case I: $p = 0$ and $n = 0$

$$M = \frac{R}{2} \left(1 - e^{-\lambda(X)} + \frac{Q^2}{R^2} \right) \quad (4.2.2.2)$$

where,

$$e^{-\lambda(X)} = (K + \delta) \frac{X^2}{(1 + 3X)^{\frac{2}{3}}} + 1 + A_2 \frac{X}{(1 + 3X)^{\frac{2}{3}}} \quad (4.2.2.3)$$

Case II: $p = 1$ and $n = 0$

$$M = \frac{R}{2} \left(1 - e^{-\lambda(X)} + \frac{Q^2}{R^2} \right) \quad (4.2.2.4)$$

where,

$$e^{-\lambda(X)} = (K + \delta) \frac{X^2}{(1 + 3X)^{\frac{2}{3}}} + \frac{1}{2} \frac{KmX^3}{(1 + 3X)^{\frac{2}{3}}} - \frac{\delta aX^3}{(1 + 3X)^{\frac{2}{3}}} + 1 + A_2 \frac{X}{(1 + 3X)^{\frac{2}{3}}} \quad (4.2.2.5)$$

4.2.3 Total charge

Case I: $p = 0$ and $n = 0$

$$Q = R \sqrt{\frac{K}{2} X^2 (1 + 3X)^{\frac{1}{3}}} \quad (4.2.3.1)$$

Case II: $p = 1$ and $n = 0$

$$Q = R \sqrt{\frac{K}{2} X^2 (1 + mX) (1 + 3X)^{\frac{1}{3}}} \quad (4.2.3.2)$$

where the charge Q is measured in km.

4.2.4 Central density

For both Case I and Case II: $p = 0, 1$ and $n = 0$

$$\rho_c = -3A_2 \frac{X}{\kappa R^2} \quad (4.2.4.1)$$

4.2.5 Central pressure

For both Case I and Case II: $p = 0, 1$ and $n = 0$

$$P_c = \frac{X}{\kappa R^2} (4 + A_2) \quad (4.2.5.1)$$

where the central pressure is given in the unit N m^{-2} .

Chapter 5

Physical Analysis of the Models

For each choice of adjustable constant parameters the maximum mass of charged star depends on the corresponding set of maximum values of $X = X_{\max}$ upto which the pressure and density and their gradients satisfy some elementary criteria [18–20, 87, 88, 91]

- i) The solution should be free from geometric singularities, which requires, $e'' > 0$ and $e^\lambda > 0$ in the range $0 \leq r \leq R$.
- ii) To be free from physical singularities the model must have radial and tangential pressures and density positive inside the sphere, i.e., $P_r, P_t, \rho > 0$.
- iii) In order to have an equilibrium configuration the matter must be stable against the collapse of local regions. This requires that the radial pressure P_r must be a monotonically nondecreasing function of ρ ,
$$\frac{dP_r}{d\rho} \geq 0$$
- iv) The relativistic adiabatic index is given by $\Gamma = (P_r + \rho)dP_r/P_r d\rho$. The necessary condition for this exact solution to serve as a model of a relativistic star is that $\Gamma > 4/3$.
- v) The causality conditions $0 \leq dP_r/d\rho \leq 1$, $0 \leq dP_t/d\rho \leq 1$ must be satisfied.
- vi) A physically reasonable energy-momentum tensor has to obey the conditions $\rho \geq (P_r + 2P_t)$ and $(\rho + P_r + 2P_t) \geq 0$.

vii) Pressure and density should be maximum at the center and monotonically decreasing towards the boundary of the fluid sphere. Mathematically,

$$\begin{aligned} \left(\frac{dP_r}{dr}\right)_{r=0} &= 0, & \left(\frac{dP_t}{dr}\right)_{r=0} &= 0, & \left(\frac{d\rho}{dr}\right)_{r=0} &= 0, \\ \left(\frac{d^2P_r}{dr^2}\right)_{r=0} &= 0, & \left(\frac{d^2P_t}{dr^2}\right)_{r=0} &= 0, & \left(\frac{d^2\rho}{dr^2}\right)_{r=0} &= 0 \end{aligned}$$

So that,

$$\frac{dP_r}{dr} < 0, \quad \frac{dP_t}{dr} < 0, \quad \frac{d\rho}{dr} < 0, \quad 0 < r \leq R$$

viii) Additionally, the velocity of sound should be monotonically decreasing towards the surface. In this context, it is worth mentioning that for different equations of state at ultra-high densities, it is found that the speed of sound is decreasing outwards from the center of fluid sphere, i.e., [1, 98].

$$\frac{d}{dr} \left(\frac{dP_r}{d\rho} \right) < 0, \quad \frac{d}{dr} \left(\frac{dP_t}{d\rho} \right) < 0, \quad 0 \leq r \leq R$$

ix) The ratio of pressure to density, P_r/ρ , P_t/ρ , should be monotonically decreasing with the increase of r , i.e.,

$$\frac{d}{dr} \left(\frac{P_r}{\rho} \right)_{r=0} = 0, \quad \frac{d^2}{dr^2} \left(\frac{P_r}{\rho} \right)_{r=0} < 0$$

$$\frac{d}{dr} \left(\frac{P_t}{\rho} \right)_{r=0} = 0, \quad \frac{d^2}{dr^2} \left(\frac{P_t}{\rho} \right)_{r=0} < 0$$

x) Electric field intensity E , such that $E(0) = 0$, is taken to be monotonically increasing, i.e., $dE/dr > 0$ for $0 < r \leq R$.

xi) Pressure anisotropy, Δ , vanishes at the center, i.e., $\Delta(0) = 0$ and $\Delta(r = R) = \frac{\kappa}{C}(P_t(R) - P_r(R)) = \frac{\kappa}{C}P_t(R) > 0$

xii) According to Herreras cracking (or overturning) condition [101], the region for which radial speed of sound is greater than the transverse speed of sound is a potentially stable region, so the requirement for the stability of the fluid sphere $|v_t^2 - v_r^2| < 1$ is to be satisfied.

Case I : $p = 0$ and $n = 0$

a) The maximum limit of compactness parameter ($2M/R$) obtained by Adler set by the causality condition v) for the neutral isotropic fluid sphere is about 0.7, But the neutral model fails to satisfy condition viii). To generate an anisotropic charged fluid sphere we set $(m, n, p, \delta, a) = (10^4, 0, 0, 0.35, 0.6)$. For this choice the range of values, $0.780 \leq K \leq 0.996$, $0.220 \leq X \leq 0.278$. are obtained over which the fluid distribution satisfies the above mentioned inequalities. With the decrease of K , X increases.

b) The maximum value of compactness parameter is obtained $(2M/R)_{\max} = 0.54$ using eq.(3.3.3) at $K_{\min} = 0.780$ and $X_{\max} = 0.278$.

c) Corresponding to the values of $(K_{\min}, X_{\max})$ the total charge to radius ratio, and total charge to total mass ratio are found to be $Q/R = 0.240$ and $Q/M = 0.9997$ using eq. (3.2.1) and (3.3.3) For a particular choice of stellar surface density $\rho_s = 4.6794 \times 10^{14} \text{ gcm}^{-3}$ as parameter the total mass and other physical quantities are calculated by the use of eqs. (4.2.1.1), (4.2.2.2), (4.2.3.1), (4.2.4.1), (4.2.5.1) and found $M_{\max} = 1.74M_{\odot}$, $R = 10.73\text{km}$, $P_c = 79.50 \text{ MeV fm}^{-3}$, $\rho_c = 632.275 \text{ MeV fm}^{-3}$, $Q = 1.0756 \times 10^{19}\text{C}$, central and surface redshifts $z_c = 0.77$ and $z_s = 0.38$ respectively.

Case II : $p = 1$ and $n = 0$

a) To generate an anisotropic charged fluid sphere we set $(m, n, p, \delta, a) = (600, 0, 1, 0.001, 0.02)$. For this choice the range of values, $K \geq 0.014$, $0 \leq X \leq 0.141$. are obtained over which the fluid distribution satisfies the above mentioned inequalities. With the decrease of K , X increases.

b) The maximum value of compactness parameter is obtained $(2M/R)_{\max} = 0.36$ using eq. (3.3.3) at $K_{\min} = 0.014$ and $X_{\max} = 0.141$.

c) Corresponding to the values of $(K_{\min}, X_{\max})$ the total charge to radius ratio, and total charge to total mass ratio are found to be $Q/R = 0.362$ and $Q/M = 0.998$ using eq. (3.2.1) and (3.3.3) For a particular choice of stellar surface density $\rho_s = 4.6812 \times 10^{14} \text{ gcm}^{-3}$ as parameter the total mass and other physical quantities are calculated by the use of eqs. (4.2.1.6), (4.2.2.3), (4.2.3.2), (4.2.4.1), (4.2.5.1) and found $M_{\max} = 1.02M_{\odot}$, $R = 9.84$, $P_c = 38.33 \text{ MeV fm}^{-3}$, $\rho_c = 410.217 \text{ MeV fm}^{-3}$, $Q = 6.2876 \times 10^{18} \text{ C}$, central and surface redshifts $z_c = 0.229$ and $z_s = 0.0776$ respectively.

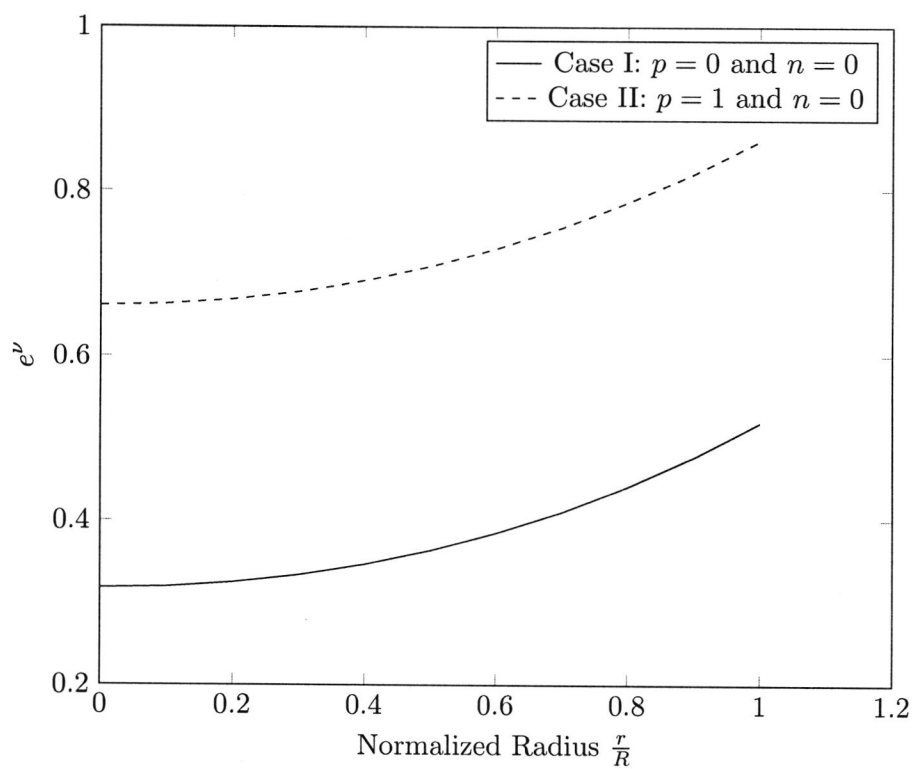
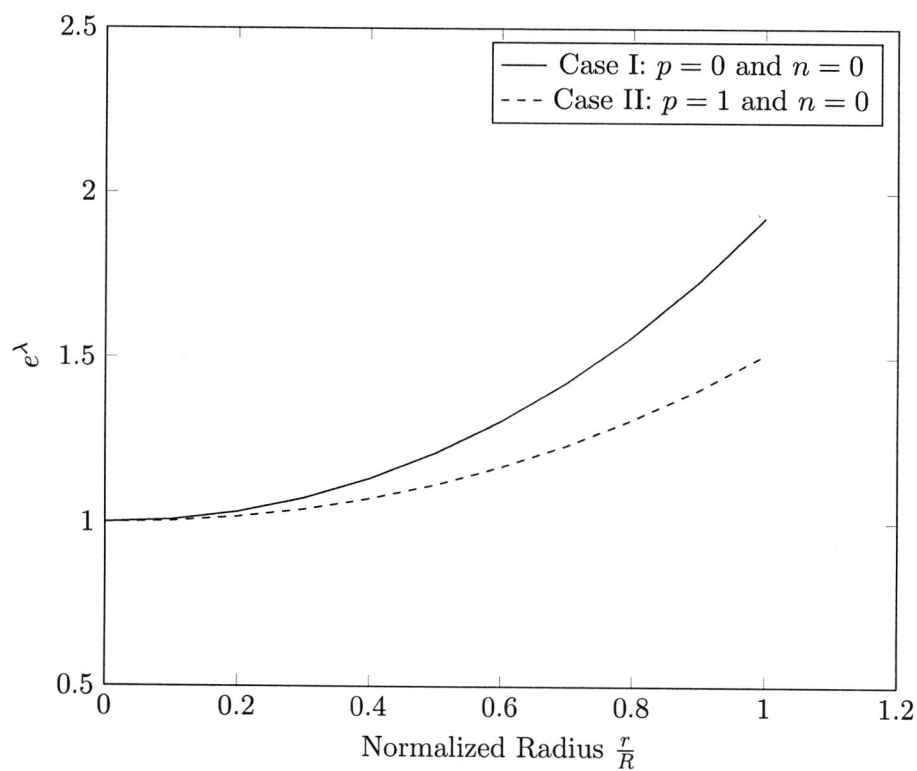
An absolute constraint of the maximally allowable mass-to-radius ratio (M/R) for electrically neutral isotropic fluid spheres was obtained by Buchdahl of the form $2M/R \leq 8/9$ (in the unit, $c = G = 1$) which states that, for a given radius, a static isotropic fluid

sphere cannot be arbitrarily massive. Böhmer and Harko [102] proved that for a compact object with charge, $Q(< M)$, there is a lower bound for the mass-radius ratio,

$$\frac{3Q^2}{2R^2} \frac{\left(1 + \frac{Q^2}{18R^2}\right)}{\left(1 + \frac{Q^2}{12R^2}\right)} \leq \frac{2M}{R}.$$

Upper bound of the mass of charged sphere was generalized by Andreásson [103] who proved that

$$\sqrt{M} \leq \frac{\sqrt{R}}{3} + \sqrt{\frac{R}{9} + \frac{Q^3}{3R}}.$$

FIGURE 5.1: e^ν vs normalized radius r/R FIGURE 5.2: e^λ vs normalized radius r/R

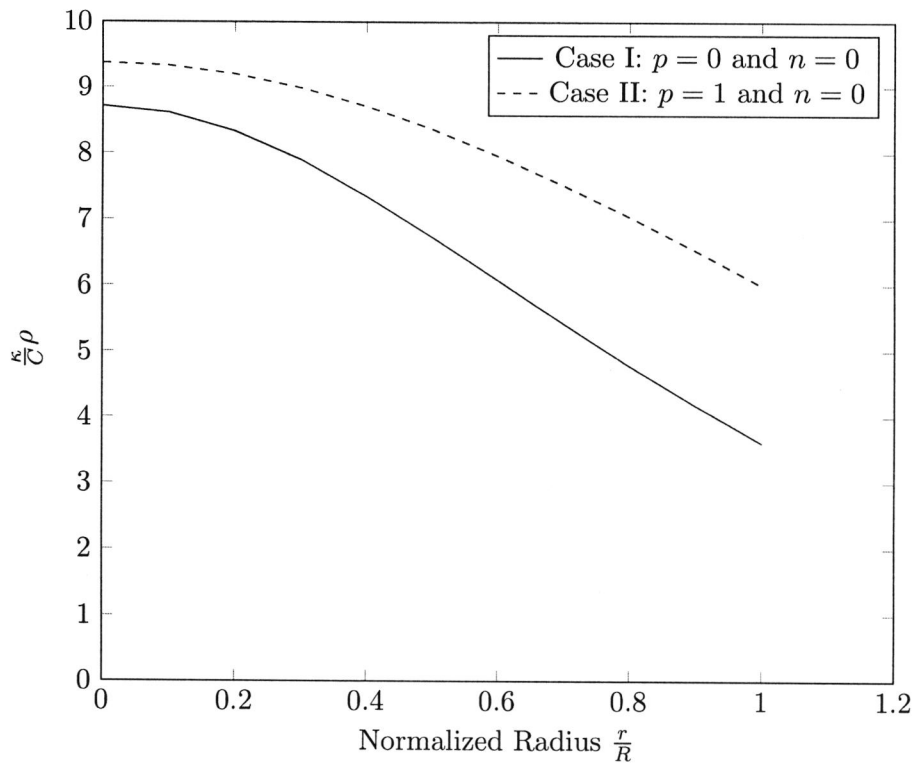


FIGURE 5.3: Energy density ρ

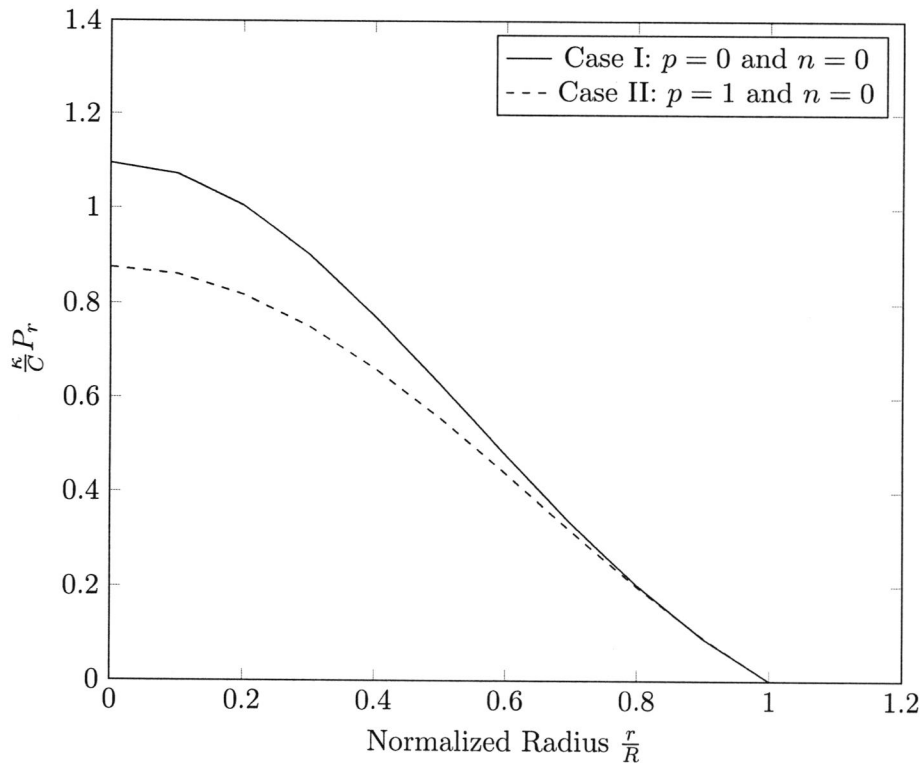


FIGURE 5.4: Radial Pressure P_r

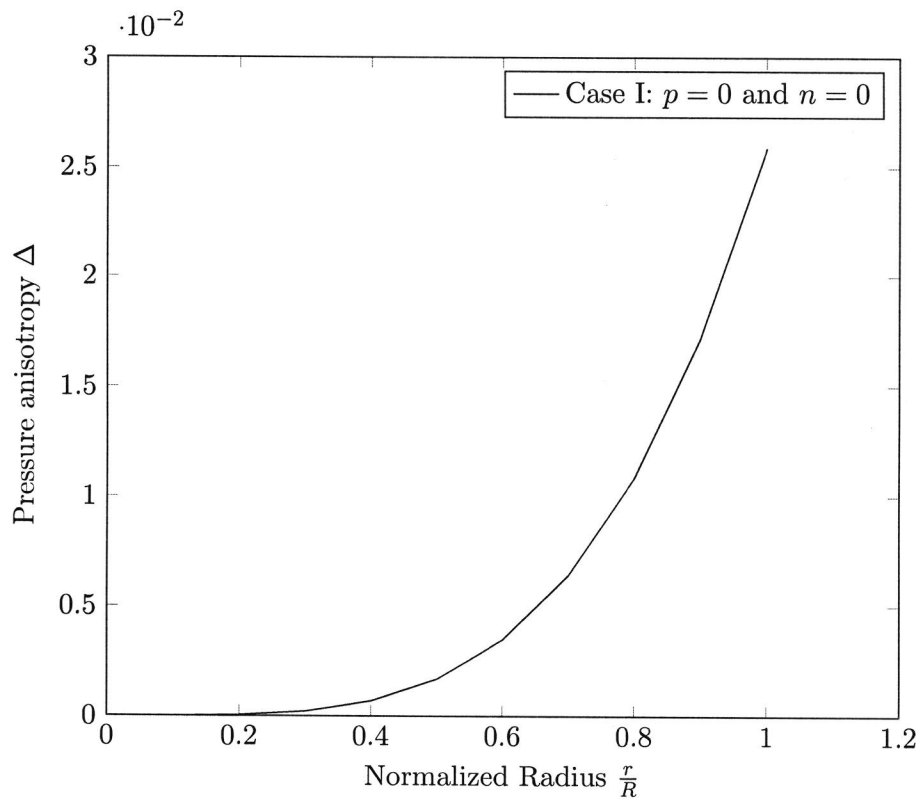


FIGURE 5.5: Pressure anisotropy Δ

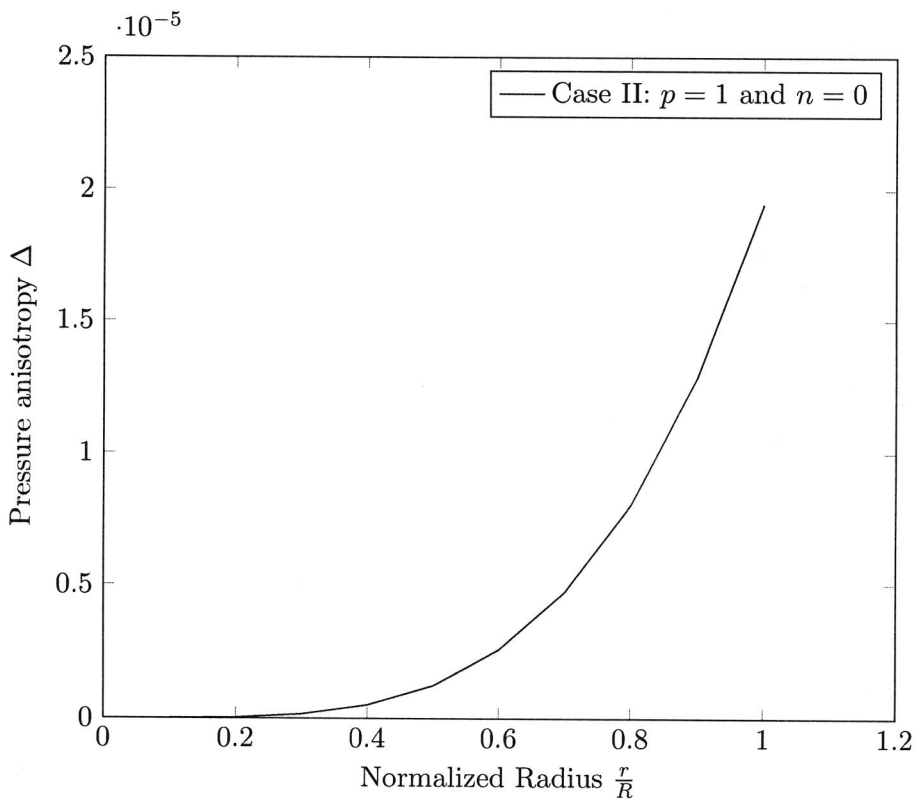


FIGURE 5.6: Pressure anisotropy Δ

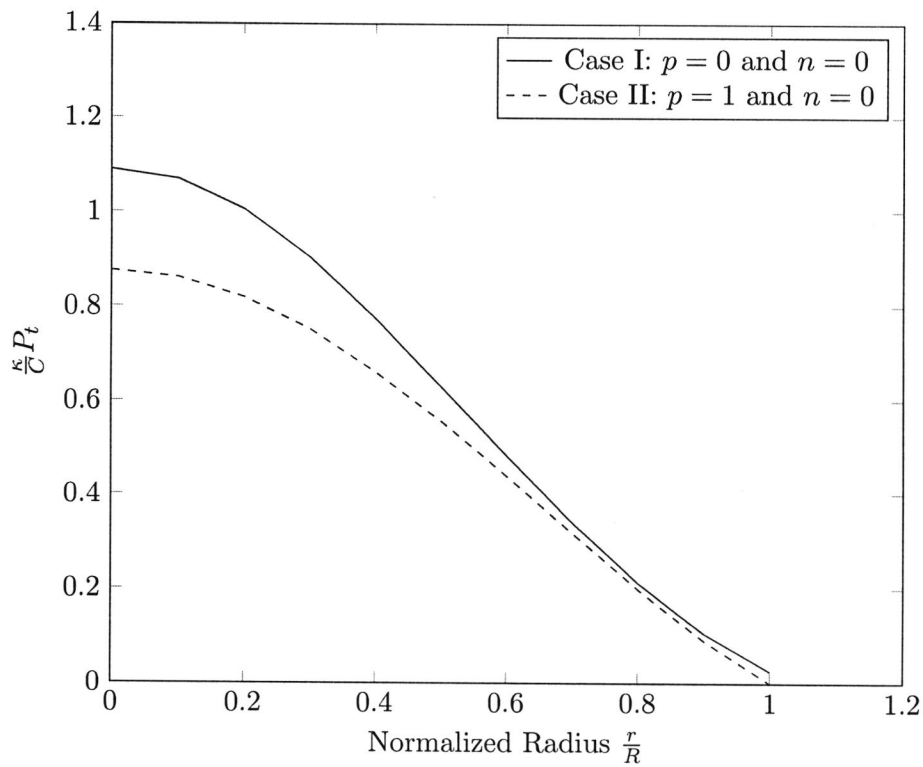


FIGURE 5.7: Tangential Pressure P_t

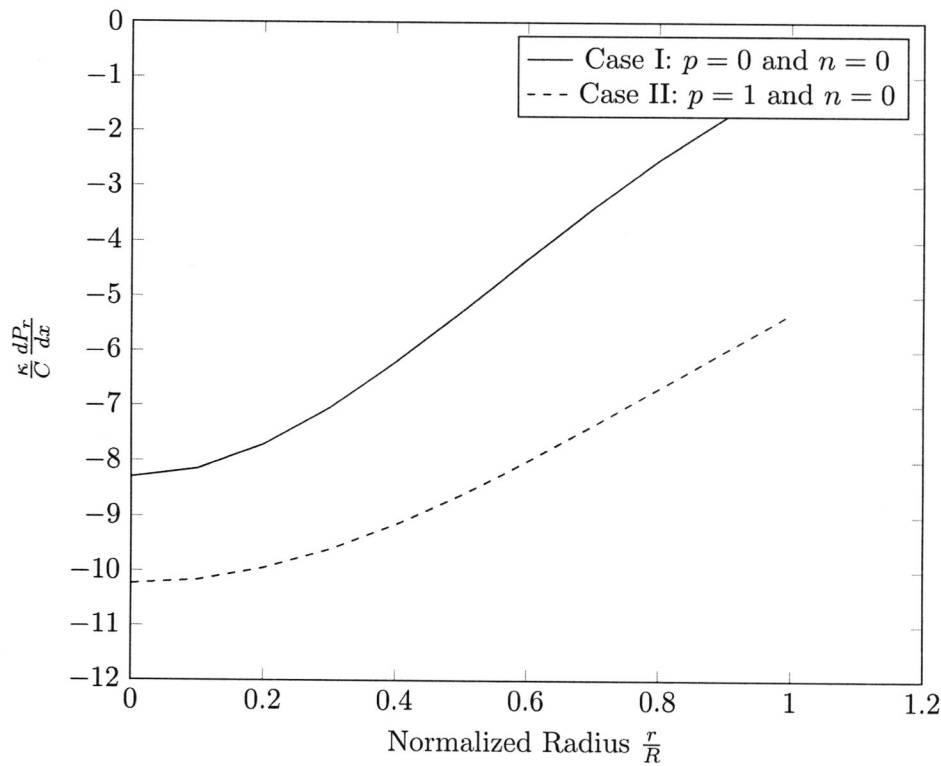


FIGURE 5.8: Gradient of Radial Pressure

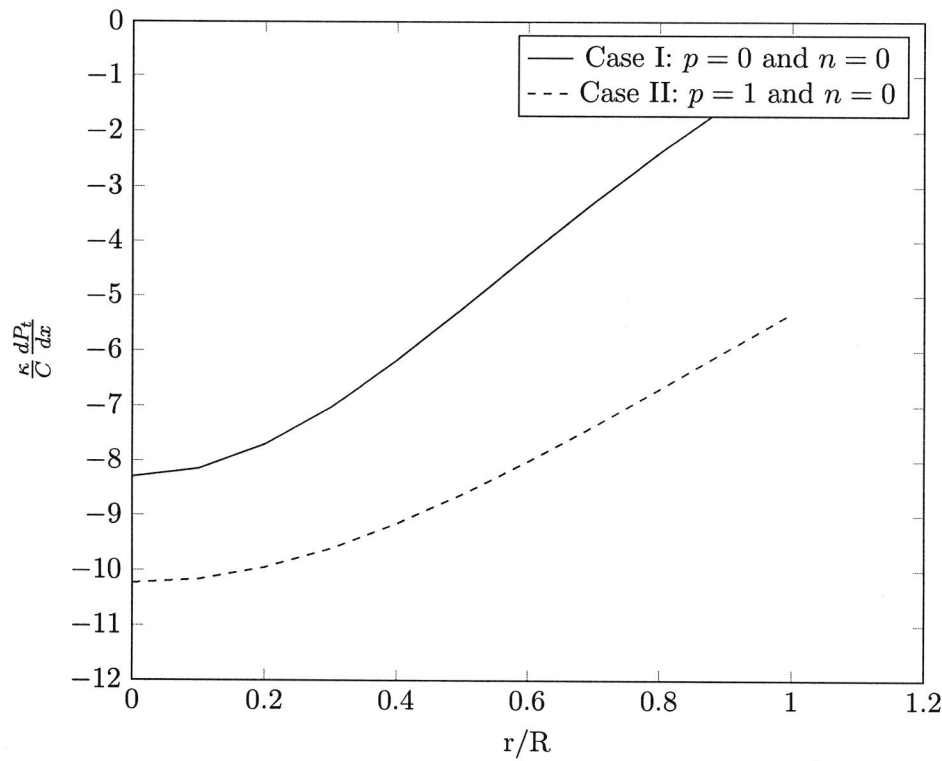


FIGURE 5.9: Gradient of Tangential Pressure

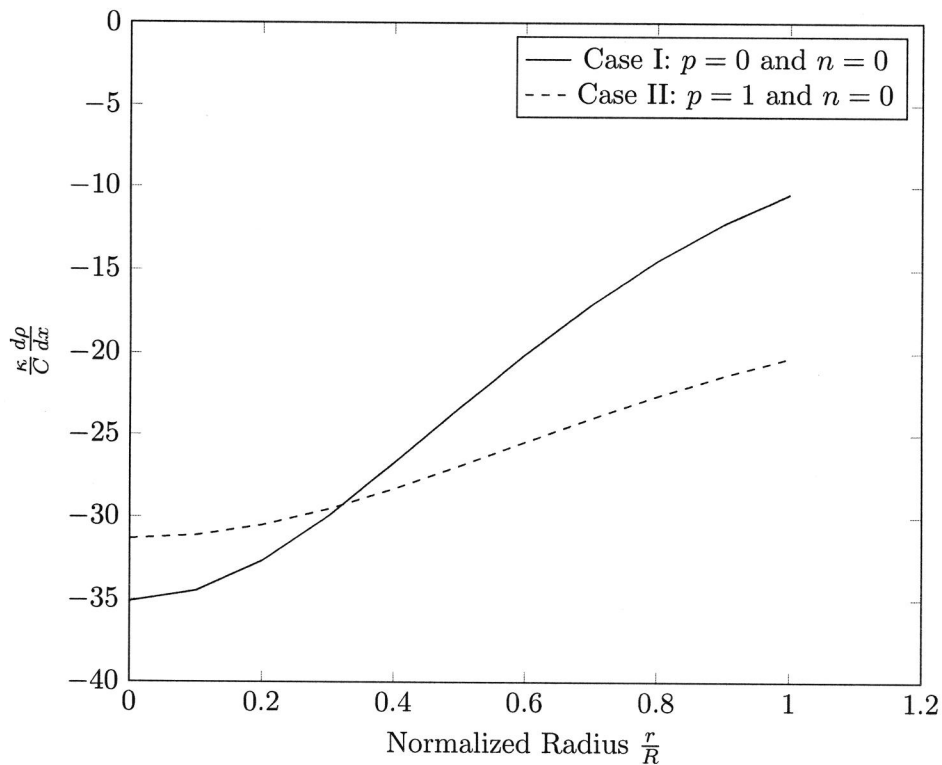


FIGURE 5.10: Gradient of Energy Density

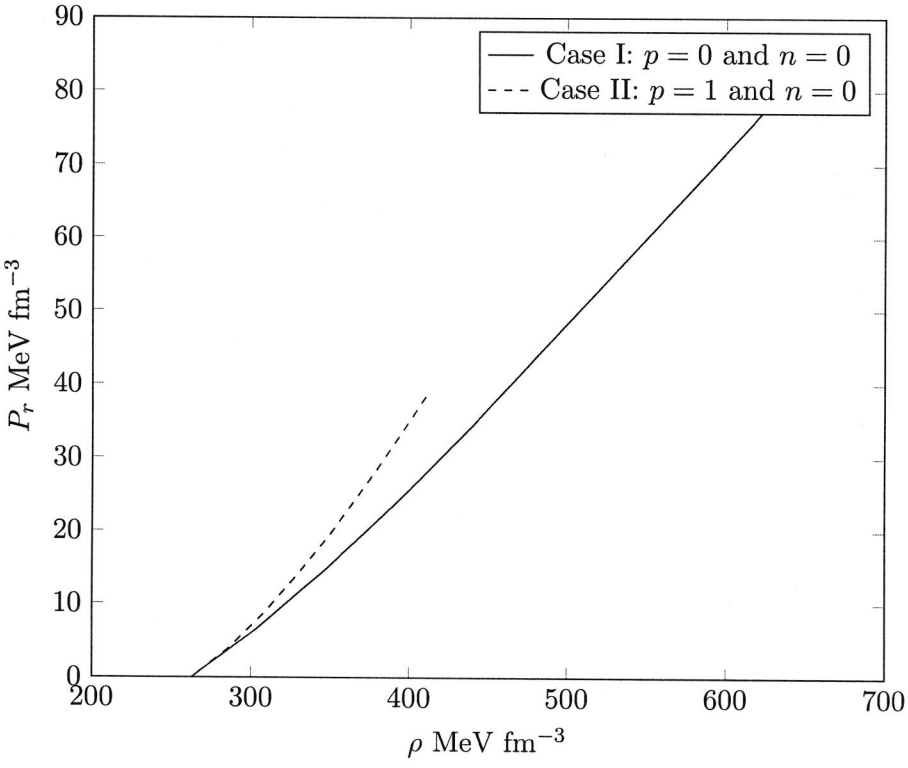


FIGURE 5.11: Radial Pressure vs density

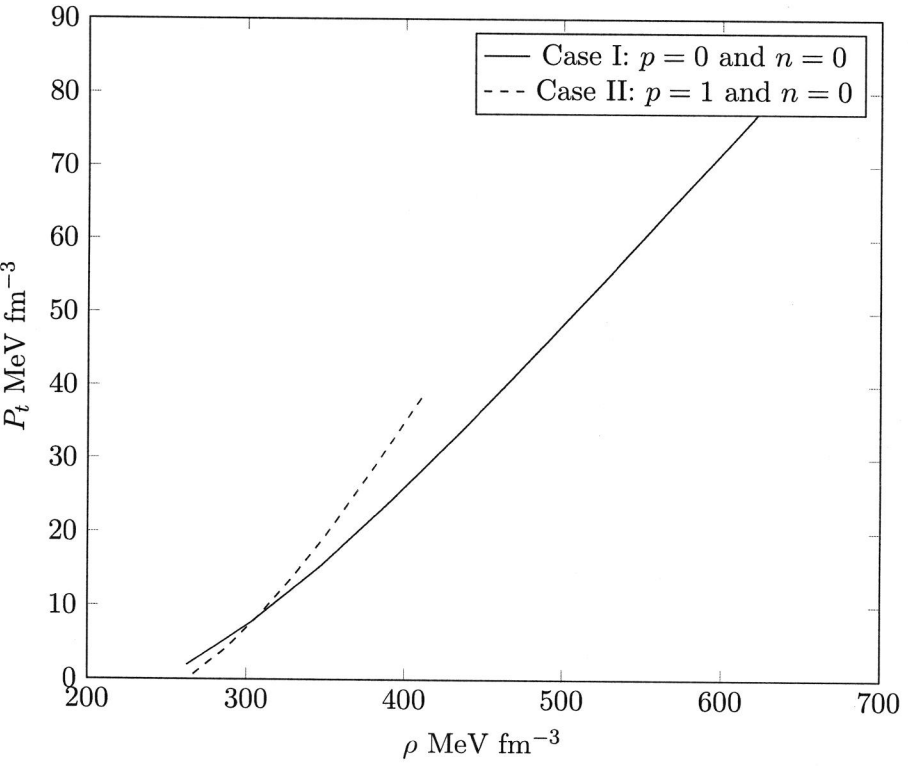


FIGURE 5.12: Tangential Pressure vs density

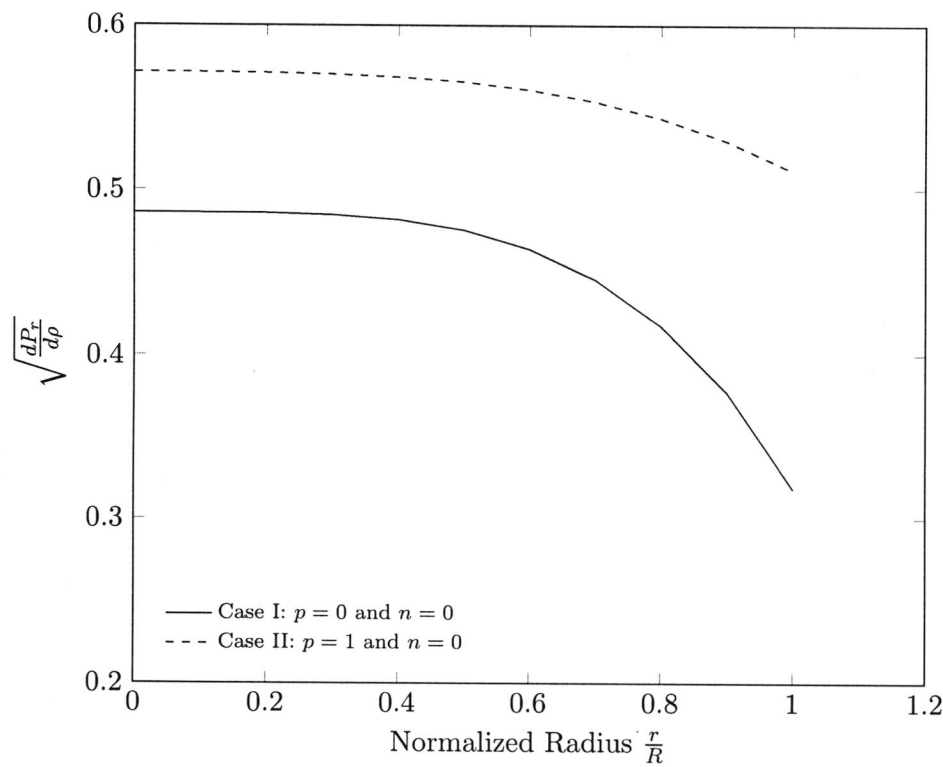


FIGURE 5.13: Speed of sound for Radial Pressure

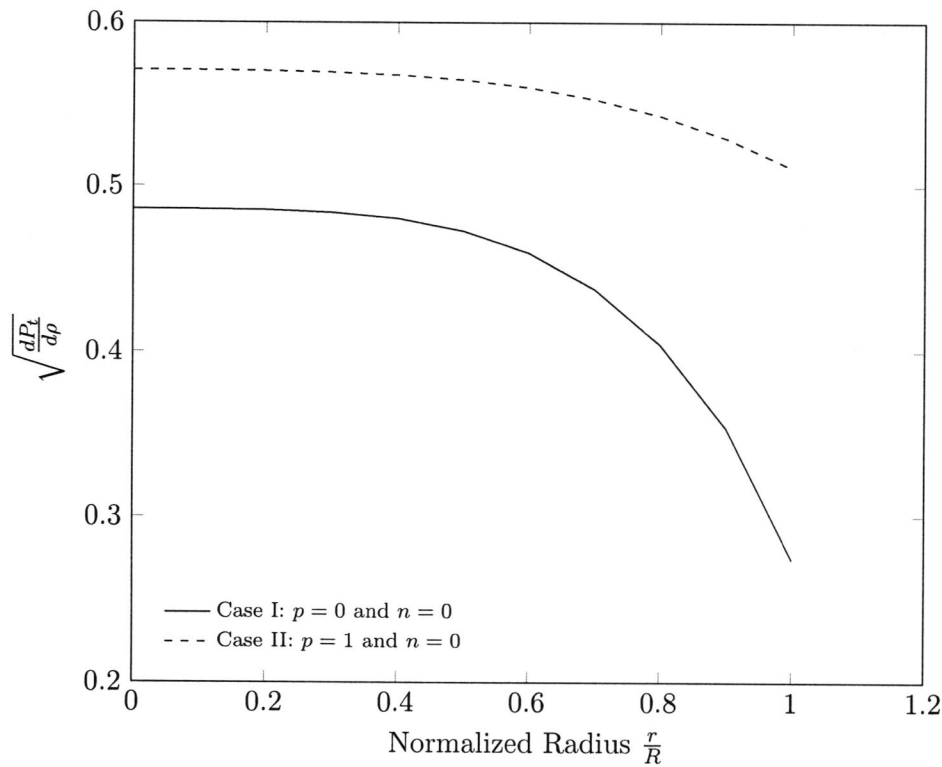


FIGURE 5.14: Speed of sound for Tangential Pressure

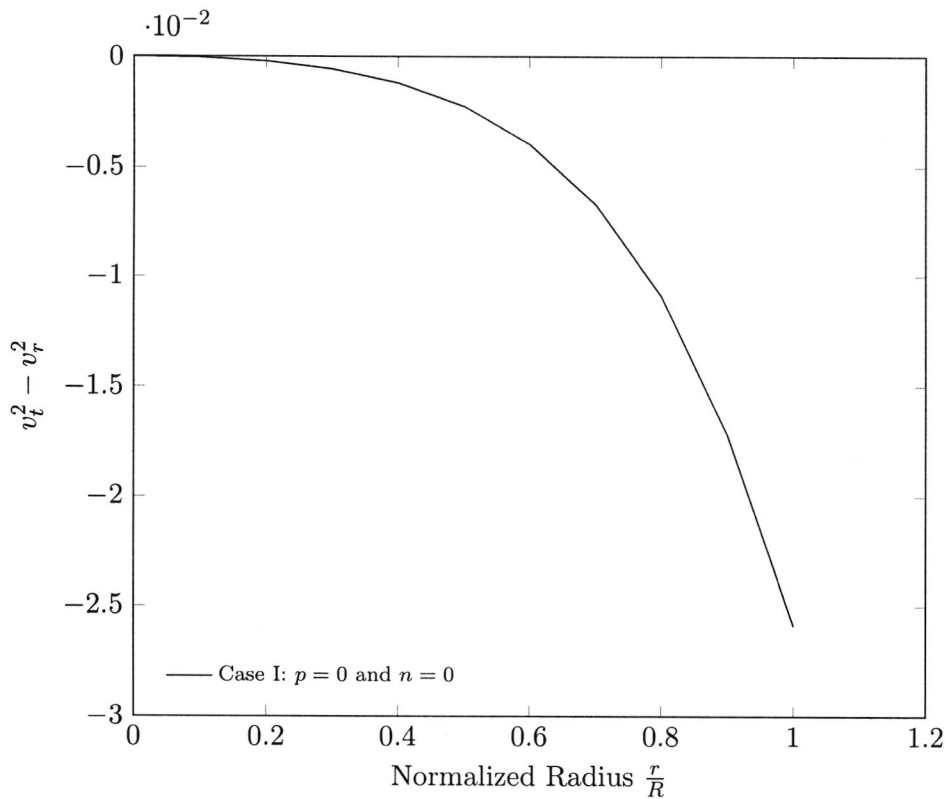


FIGURE 5.15: Requirement for stability $|v_t^2 - v_r^2| < 1$

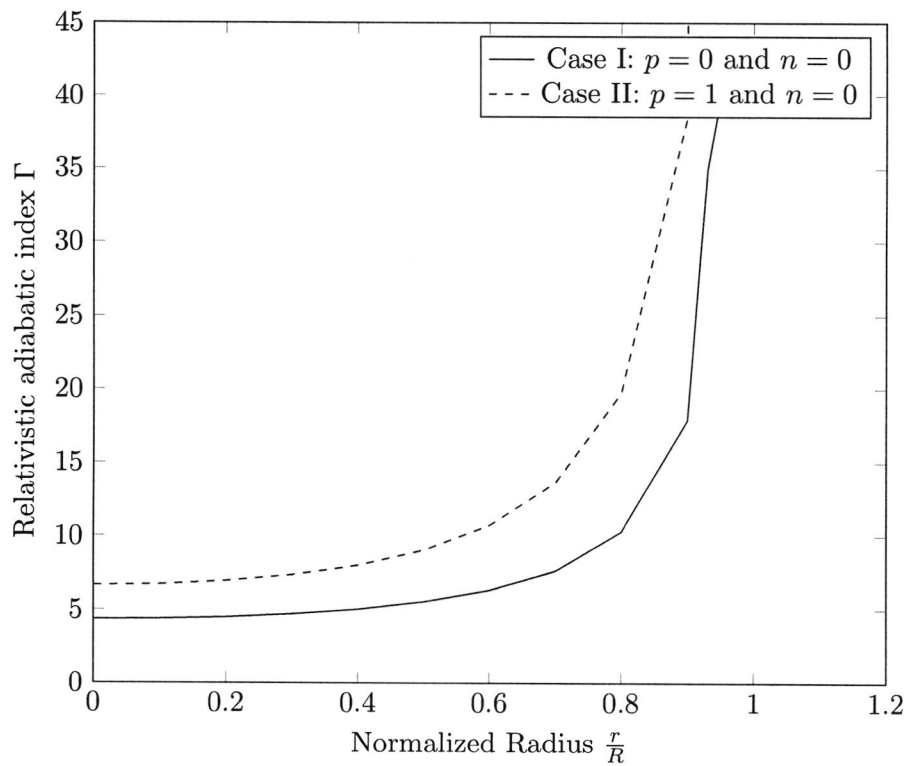


FIGURE 5.16: Relativistic adiabatic index Γ

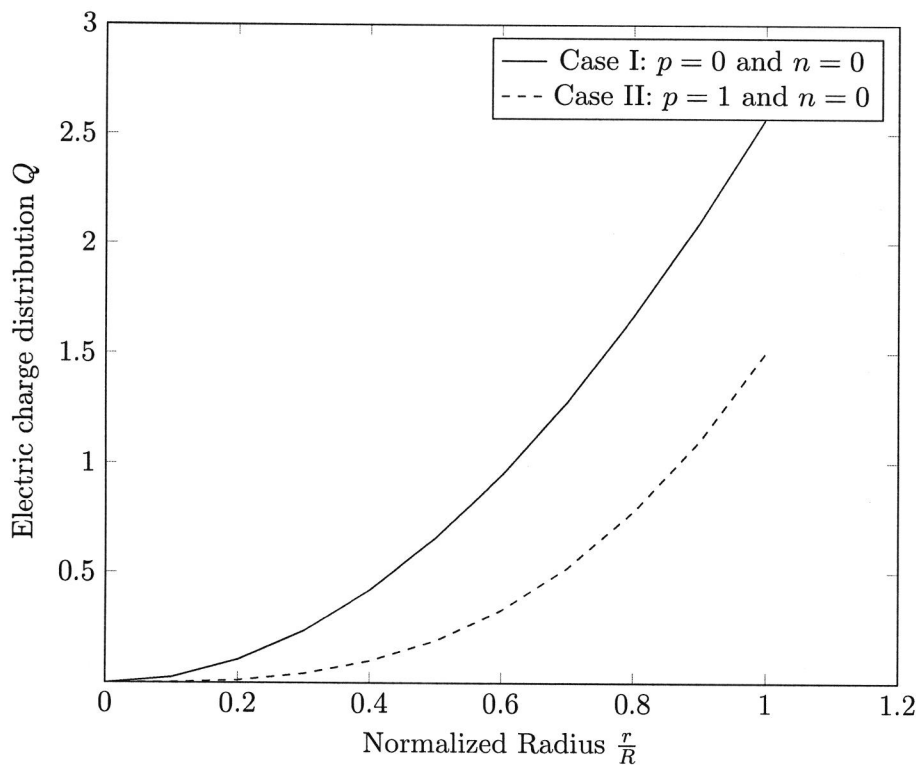


FIGURE 5.17: Electric charge distribution Q

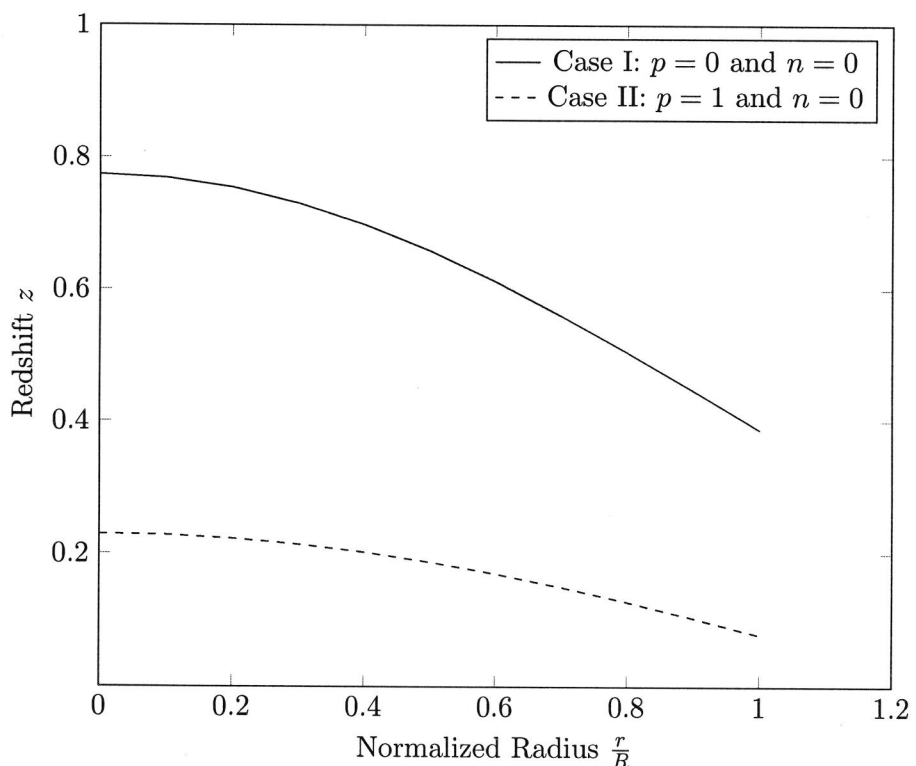


FIGURE 5.18: Redshift z

TABLE 5.1: Some values of parameters $(K_{\min}, X_{\max})$ for which well-behaved charge fluid sphere can be generated.

p	m	δ	a	$(K_{\min}, X_{\max})$	A_2	B_2	$(P_r/c^2\rho)_c$	$\sqrt{(dP_r/c^2d\rho)_c}$	$\sqrt{(dP_t/c^2d\rho)_c}$	$2M/R$	Q/R	Q/M
<i>Case I: $p = 0$ and $n = 0$</i>												
0	10^4	0.35	0.6	(0.780, 0.278)	-2.9049	0.3179	0.2270	0.4861	0.4861	0.5384	0.2403	0.9997
0	10^4	0.35	0.6	(0.990, 0.200)	-3.0832	0.4086	0.2190	0.4681	0.4681	0.4502	0.1965	0.9547
0	10^4	0.6	0.71	(0.993, 0.18)	-3.1217	0.4390	0.2187	0.4676	0.4676	0.4202	0.1774	0.9129
0	10^4	0.2	0.71	(0.996, 0.220)	-3.0531	0.3812	0.2188	0.4678	0.4678	0.4794	0.2163	0.9999
<i>Case II: $p = 1$ and $n = 0$</i>												
1	600	0.001	0.02	(0.014, 0.141)	-3.1241	0.6614	0.3264	0.5713	0.5713	0.3621	0.1532	0.9986
1	200	0.001	0.02	(0.041, 0.141)	-3.1259	0.6615	0.3235	0.5688	0.5688	0.3622	0.1533	0.9987
1	100	0.001	0.02	(0.080, 0.140)	-3.1325	0.6631	0.3193	0.5651	0.5651	0.3603	0.1523	0.9969
1	70	0.001	0.02	(0.110, 0.139)	-3.1355	0.6648	0.3161	0.5622	0.5622	0.3581	0.1499	0.9876

TABLE 5.2: Maximum mass and various physical quantities for anisotropic charged fluid sphere for a given surface density $\rho_s = 4.678 \times 10^{14} \text{gcm}^{-3}$

p	m	δ	a	$(K_{\min}, X_{\max})$	$M_{\odot}$	R (km)	$(\frac{\kappa}{C}P_r)_c$	$(\frac{\kappa}{C}P_r)_s$	$(\frac{\kappa}{C}P_t)_c$	$(\frac{\kappa}{C}P_t)_s$	$(\frac{\kappa}{C}\rho)_c$	$(\frac{\kappa}{C}\rho)_s$
<i>Case I: $p = 0$ and $n = 0$</i>												
0	10^4	0.35	0.6	(0.780, 0.278)	1.74	10.73	1.0950	0	1.0950	0.0259	8.7149	3.6159
0	10^4	0.35	0.6	(0.990, 0.200)	1.43	10.33	0.9168	0	0.9168	0.0136	9.2494	4.6564
0	10^4	0.35	0.6	(0.993, 0.180)	1.33	10.17	0.8783	0	0.8783	0.0110	9.3650	5.0190
0	10^4	0.35	0.6	(0.996, 0.220)	1.52	10.44	0.9468	0	0.9468	0.0164	9.1594	4.3283
<i>Case II: $p = 1$ and $n = 0$</i>												
1	600	0.001	0.02	(0.014, 0.141)	1.02	9.84	0.8758	0	0.8758	1.94×10^{-5}	9.3725	5.9955
1	200	0.001	0.02	(0.041, 0.141)	1.02	9.83	0.8740	0	0.8740	1.92×10^{-5}	9.3777	5.9935
1	100	0.001	0.02	(0.080, 0.140)	1.01	9.82	0.8674	0	0.8674	1.9254×10^{-5}	9.3975	6.0140

Chapter 6

Concluding Remarks

Under some *ad hoc* assumptions on one of the metric potentials together with some suitable forms of electric charge distribution and pressure anisotropy, we have studied particular simple families of relativistic charged stellar models which is obtained by solving the Einstein-Maxwell field equations for a static spherically symmetric distribution of anisotropic fluid. The analytical equation of state has been extracted from the resulting metric. The behaviors of various physical variables in the interior of the star have been investigated and found regular and well behaved throughout the fluid sphere. In figs. (5.1) and (5.2), the metric functions e^ν and e^λ have been plotted respectively which show the continuity of those quantities. In figs. (5.3), (5.4) and (5.7) the behaviors of anisotropic pressure and energy density have been shown, which are positive and monotonically decreasing functions in the interior of the star. The pressure anisotropy itself is depicted in the figs. (5.5) and (5.6) for the two cases respectively. The pressure to density ratio, for fluid sphere obtained by assigning values of the parameters is presented in figs. (5.11) and (5.12). The behaviors of the quantities of gradient of energy density, gradient of radial and tangential pressure are shown in figs. (5.8), (5.9) and (5.10) respectively. It can be also shown from the figs. (5.13) and (5.14) that the radial and tangential speeds of sound $\sqrt{dP_r/d\rho}$ and $\sqrt{dP_t/d\rho}$ always remain less than the speed of light and the condition of causality is satisfied. The stability of anisotropic fluid sphere can also be judged by $|v_t^2 - v_r^2| < 1$, which is plotted in the fig (5.15). The values of relativistic adiabatic index are plotted in fig. (5.16). For large ρ (at the center), $\Gamma \approx \frac{4}{3}$ and there is a minimum value of $\rho = \rho_s = 4B$, the surface value below which Γ becomes infinitely large. The electric charge distributions are plotted in fig. (5.17), which is zero at the center and monotonically increasing towards the pressure free interface. The redshift, monotonically decreasing from the center to surface is depicted in the fig. (5.18). In the absence of electric charge and pressure anisotropy these families of analytical relativistic stellar models reduce to wellknown Tolman IV type exact metrics and hence

the models obtained in the work may be regarded as the anisotropic charged analogues of *Generalized Tolman IV* solution which enable us to compare the results with their isotropic counterparts. The maximum mass, maximum allowable radius and the various physical parameters of charged compact fluid spheres for a given surface density are reported in tables (5.1) and (5.2). Thus it has been demonstrated that my calculation satisfies the physical requirements, for wide range of values of K , m , δ , a and x , giving us a possibility for different models of charged fluid spheres. The resulting charged fluid spheres can be utilized to construct models of physically reasonable compact charged objects such as charged *strange quark star*.

Bibliography

- [1] S. Fatema and M. H. Murad. An exact family of Einstein-Maxwell Wyman-Adler solution in general relativity. *Int. J. Theor. Phys.*, 52:2508, 2013. doi:10.1007/s10773-013-1538-y.
- [2] L. K. Patel, R. Tikekar, and M. C. Sabu. Exact interior solutions for charged fluid spheres. *Gen. Relativ. Gravit.*, 29:489, 1997. doi:10.1023/A:1018886816863.
- [3] S. S. Koppar, L. K. Patel, and T. Singh. On relativistic charged fluid spheres. *Acta Physica Hungarica*, 69:53, 1991. doi:10.1007/BF03054133.
- [4] L. K. Patel and S. S. Koppar. A charged analogue of the Vaidya-Tikekar solution. *Aust. J. Phys.*, 40:441, 1987.
- [5] L. K. Patel and N. P. Mehta. An exact model of an anisotropic relativistic sphere. *Aust. J. Phys.*, 48:635–643, 1995.
- [6] R. Sharma, S. Mukherjee, and S. D. Maharaj. General solution for a class of static charged spheres. *Gen. Relativ. Gravit.*, 33:999, 2001. doi:10.1023/A:1010272130226.
- [7] K. Komathiraj and S. D. Maharaj. Tikekar superdense stars in electric fields. *J. Math. Phys.*, 48:042501, 2007. doi:10.1063/1.2716204.
- [8] K. Komathiraj and S. D. Maharaj. Analytical models of quark stars. *Int. J. Mod. Phys. D*, 16:1803, 2011. doi:10.1142/S0218271807011103.
- [9] W. B. Bonnor. The mass of a static charged sphere. *Z. Phys.*, 160:59, 1960. doi:10.1007/BF01337478.
- [10] W. B. Bonnor. Equilibrium of a charged sphere. *Mon. Not. R. Astron. Soc.*, 29:443, 1965. doi:10.1093/mnras/129.6.443.
- [11] J. D. Bekenstein. Hydrostatic equilibrium and gravitational collapse of relativistic charged fluid balls. *Phys. Rev. D*, 4:2185, 1971. doi:10.1103/PhysRevD.4.2185.

- [12] A. Nduka. Charged fluid sphere in general relativity. *Gen. Relativ. Gravit.*, 7:493, 1976. doi:10.1007/BF00766408.
- [13] M. K. Gokhroo and A. L. Mehra. Anisotropic spheres with variable energy density in general relativity. *Gen. Relativ. Gravit.*, 26:75, 1994. doi:10.1007/BF02088210.
- [14] J. M. Lattimer and M. Prakash. Neutron star structure and the equation of state. *Astrophys. J.*, 550:426, 2001. doi:10.1086/319702.
- [15] J. M. Lattimer. The structure of strange quark matter and neutron stars. *J. Phys. G: Nucl. Part. Phys.*, 30:S479, 2004. doi:10.1088/0954-3899/30/1/056.
- [16] S. K. Maurya, Y. K. Gupta, and Pratibha. Regular and well-behaved relativistic charged superdense star models. *Int. J. Mod. Phys. D*, 20:1289–1300, 2011. doi:10.1142/S0218271811019414.
- [17] S. K. Maurya and Pratibha Gupta, Y. K. A class of regular and well behaved relativistic super-dense star models. *Int. J. Theor. Phys.*, 51:943–953, 2012. doi:10.1007/s10773-011-0968-7.
- [18] M. H. Murad and S. Fatema. A family of well behaved charge analogues of Durgapal's perfect fluid exact solution in general relativity. *Astrophys. Space Sci.*, 343:587–597, 2013. doi:10.1007/s10509-012-1277-1.
- [19] M. H. Murad and S. Fatema. A family of well behaved charge analogues of Durgapal's perfect fluid exact solution in general relativity II. *Astrophys. Space Sci.*, 344:69–78, 2013. doi:10.1007/s10509-012-1320-2.
- [20] M. H. Murad and S. Fatema. Some exact relativistic models of electrically charged *Self-bound* stars. *Int. J. Theor. Phys.*, 52:4342, 2013. doi:10.1007/s10773-013-1752-7.
- [21] S. K. Maurya and Y. K. Gupta. On charged analogues of Matese and Whitman interior solutions in general relativity. *Int. J. Theor. Phys.*, 51:1792, 2012. doi:10.1007/s10773-011-1057-7.
- [22] R. Stettner. On the stability of homogeneous, spherically symmetric, charged fluids in relativity. *Ann. Phys.*, 80:212, 1973. doi:10.1016/0003-4916(73)90325-4.
- [23] F. de Felice, Y. Yu, and J. Fang. Relativistic charged spheres. *Mon. Not. R. Astron. Soc.*, 277:L17, 1995. doi:10.1093/mnras/277.1.L17.
- [24] D. D. Dionysiou. Equilibrium of a static charged perfect fluid sphere. *Astrophys. Space Sci.*, 85:331, 1982. doi:10.1007/BF00653455.

- [25] F. Rahaman, R. Sharma, S. Ray, R. Maulick, and I. Karar. Strange stars in Krori-Barua space-time. *Eur. Phys. J. C*, 72:2071, 2012. doi:10.1140/epjc/s10052-012-2071-5.
- [26] N. Bijalwan. Charged analogues of Schwarzschild interior solution in terms of pressure. *Astrophys. Space Sci.*, 336:413, 2011. doi:10.1007/s10509-011-0780-0.
- [27] N. Bijalwan and Y. K. Gupta. Nonsingular charged analogues of Schwarzschild's interior solution. *Astrophys. Space Sci.*, 317:251, 2008. doi:10.1007/s10509-008-9887-3.
- [28] N. Bijalwan and Y. K. Gupta. Closed form Vaidya-Tikekar type charged fluid spheres with pressure. *Astrophys. Space Sci.*, 334:293, 2011. doi:10.1007/s10509-011-0735-5.
- [29] N. Pant and S. Rajasekhara. Variety of well behaved parametric classes of relativistic charged fluid spheres in general relativity. *Astrophys. Space Sci.*, 333:161, 2011. doi:10.1007/s10509-011-0607-z.
- [30] T. Kiess. Exact physical Maxwell-Einstein-Tolman-VII solution and its use in stellar models. *Astrophys. Space Sci.*, 339:329, 2012. doi:10.1007/s10509-012-1013-x.
- [31] M. Malheiro, R. Picanço, S. Ray, J. P. S. Lemos, and V. T. Zanchin. Of charged stars and charged black holes. *Int. J. Mod. Phys. D*, 13:1375, 2004. doi:10.1142/S0218271804005560.
- [32] S. Ray and B. Das. Charged static fluid spheres in general relativity. *Astrophys. Space Sci.*, 282:635, 2002. doi:10.1023/A:1021133019415.
- [33] S. Ray and B. Das. Tolman-Bayin type static charged fluid spheres in general relativity. *Mon. Not. R. Astron. Soc.*, 349:1331, 2004. doi:10.1111/j.1365-2966.2004.07602.x.
- [34] M. K. Mak, P. C. W. Fung, and T. Harko. New classes of interior solutions to Einstein-Maxwell equations in spherical symmetry. *II Nuovo Cimento B*, 111:1461, 1996. doi:10.1007/BF02741485.
- [35] M. K. Mak and T. Harko. Anisotropic relativistic stellar models. *Ann. Phys. (Leipzig)*, 11:3, 2002.
- [36] M. K. Mak and T. Harko. An exact anisotropic quark star model. *Chin. J. Astron. Astrophys.*, 2:248, 2002. doi:10.1088/1009-9271/2/3/248.

- [37] M. K. Mak and T. Harko. Anisotropic stars in general relativity. *Proc. R. Soc. Lond. A*, 459:393–408, 2003. doi:10.1098/rspa.2002.1014.
- [38] M. K. Mak and T. Harko. Quark stars admitting a one-parameter group of conformal motions. *Int. J. Mod. Phys. D*, 13:149, 2004. doi:10.1142/S0218271804004451.
- [39] F. Weber, R. Negreiros, and P. Rosenfield. *Neutron Star Interiors and the Equation of State of Superdense Matter*, volume 357 of *Astrophysics and Space Science Library*. Berlin, 2009. doi:10.1007/978-3-540-76965-1_0.
- [40] P. K. Chattopadhyay, R. Deb, and B. C. Paul. Relativistic solution for a class of static compact charged star in pseudo-spheroidal spacetime. *Int. J. Mod. Phys. D*, 21:1250071, 2012. doi:10.1142/S021827181250071X.
- [41] T. X. Zhang. Electric redshift and quasars. *Astrophys. J. Lett.*, 636:L61–L64, 2006. doi:10.1086/500255.
- [42] A. Bodmer. Collapsed nuclei. *Phys. Rev. D*, 4:1601, 1971. doi:10.1103/PhysRevD.4.1601.
- [43] N. K. Glendenning. *Compact Stars. Nuclear physics, Particle physics, and General relativity*. Springer, Astrophysics and Space Science Library, New York, 2nd edition, 2000. doi:10.1007/978-1-4684-0491-3.
- [44] I. Bombaci. *Strange quark stars: structural properties and possible signatures for their existence in Physics of Neutron Star Interiors*, volume 578 of *Lecture Notes in Physics*. Springer, Berlin, 2001. doi:10.1007/3-540-44578-1_10.
- [45] J. van. Leeuwen, editor. *Structure of quark stars*, number 291 in *Neutron Stars and Pulsars: Challenges and Opportunities after 80 Years*, 2012. Proceedings IAU Symposium, IAU. doi:10.1017/S1743921312023174.
- [46] P. Haensel, A. Y. Potekhin, and D. G. Yakovlev. *Neutron Stars 1: Equation of State and Structure*, volume 326 of *Astrophysics and Space Science Library*. Springer, Berlin, 2007.
- [47] R. P. Negreiros, F. Weber, M. Malheiro, and V. Usov. Electrically charged strange quark stars. *Phys. Rev. D*, 80:083006, 2009. doi:10.1103/PhysRevD.80.083006.
- [48] P. Haensel. Equation of state of dense matter and maximum mass of neutron stars. *EAS Publications Series*, 7:249–282, 2003. doi:10.1051/eas:2003043.
- [49] C. Alcock, E. Farhi, and A. Olinto. Strange stars. *Astrophys. J.*, 310:261, 1986. doi:10.1086/164679.

- [50] V. Canuto and J. Lodenquai. Behavior of matter at superhigh density. *Phys. Rev. D*, 11:233–244, 1975. doi:10.1103/PhysRevD.11.233.
- [51] M. Cosenza, L. Herrera, M. Esculpi, and L. Witten. Some models of anisotropic spheres in general relativity. *J. Math. Phys.*, 22:118–125, 1981. doi:10.1063/1.524742.
- [52] V. V. Usov, T. Harko, and K. S. Cheng. Structure of the electrospheres of bare strange stars. *Astrophys. J.*, 620:915, 2005. doi:10.1086/427074.
- [53] X. Y. Lai and R. X. Xu. A polytropic model of quark stars. *Astroparticle Physics*, 31:128, 2009. doi:10.1016/j.astropartphys.2008.12.007.
- [54] R. Sharma, S. Karmakar, and S. Mukherjee. Maximum mass of a class of cold compact stars. *Int. J. Mod. Phys. D*, 15:405, 2006. doi:10.1142/S0218271806008012.
- [55] R. Sharma and B. S. Ratanpal. Relativistic stellar model admitting a quadratic equation of state. *Int. J. Mod. Phys. D*, 22:1350074, 2013. doi:10.1142/S0218271813500740.
- [56] A. Kurkela, P. Romatschke, and A. Vuorinen. Cold quark matter. *Phys. Rev. D*, 81:105021, 2010. doi:10.1103/PhysRevD.81.105021.
- [57] V. V. Usov. Electric fields at the quark surface of strange stars in the color-flavor locked phase. *Phys. Rev. D*, 70:067301, 2004. doi:10.1103/PhysRevD.70.067301.
- [58] R. P. Negreiros, I. M. Mishustin, S. Schramm, and F. Weber. Properties of bare strange stars associated with surface electric fields. *Phys. Rev. D*, 82:103010, 2010. doi:10.1103/PhysRevD.82.103010.
- [59] S. Ray, A. L. Espíndola, M. Malheiro, J. P. S. Lemos, and V. T. Zanchin. Electrically charged compact stars and formation of charged black holes. *Phys. Rev. D*, 68:084004, 2003. doi:10.1103/PhysRevD.68.084004.
- [60] F. Weber, M. Meixner, R. P. Negreiros, and M. Malheiro. Ultra-dense neutron star matter, strange quark stars, and the nuclear equation of state. *Int. J. Mod. Phys. E*, 16:1165, 2007. doi:10.1142/S0218301307006599.
- [61] R. Tikekar and K. Jotania. Relativistic superdense star models of pseudo spheroidal spacetime. *Int. J. Mod. Phys. D*, 14:1037, 2005. doi:10.1142/S021827180500722X.
- [62] M. Kalam, A. A. Usmani, F. Rahaman, S. M. Hossein, I. Karar, and R. Sharma. A relativistic models for strange quark stars. *Int. J. Theor. Phys.*, 52:3319, 2013. doi:10.1007/s10773-013-1629-9.

- [63] S. Thirukkanesh and S. D. Maharaj. Charged relativistic spheres with generalized potentials. *Math. Meth. Appl. Sci.*, 32:684, 2009. doi:10.1002/mma.1060.
- [64] S. Faruqi and N. Pant. Well-behaved relativistic charged super-dense star models. *Astrophys. Space Sci.*, 341:485–490, 2012. doi:10.1007/s10509-012-1132-4.
- [65] R. Sharma S. D. Maharaj S. Karmakar, S. Mukherjee. The role of pressure anisotropy on the maximum mass of cold compact stars. *Gen. Relativ. Gravit.*, 21:637–644, 1989. doi:10.1007/s12043-007-0088-3.
- [66] T. Singh, G. P. Singh, and A. M. Helmi. New solutions for charged anisotropic fluid spheres in general relativity. *II Nuovo Cimento B*, 110:387–393, 1995. doi:10.1007/BF02741446.
- [67] S. D. Maharaj, J. M. Sunzu, and S. Ray. Some simple models for quark stars. *Eur. Phys. J. Plus*, 129, 2014. doi:10.1140/epjp/i2014-14003-9.
- [68] S. K. Maurya and Y. K. Gupta. A family of well behaved charge analogues of a well behaved neutral solution in general relativity. *Astrophys. Space Sci.*, 332: 481–490, 2011. doi:10.1007/s10509-010-0541-5.
- [69] S. K. Maurya and Y. K. Gupta. On a family of well behaved perfect fluid balls as astrophysical objects in general relativity. *Astrophys. Space Sci.*, 334:145, 2011. doi:10.1007/s10509-011-0705-y.
- [70] S. K. Maurya and Y. K. Gupta. Extremization of mass of charged superdense star models describe by Durgapal type space-time metric. *Astrophys. Space Sci.*, 334: 301, 2011. doi:10.1007/s10509-011-0736-4.
- [71] L. Herrera, A. D. Prisco, J. Ospino, and E. Fuenmayor. Conformally flat anisotropic spheres in general relativity. *J. Math. Phys.*, 42:2129–2143, 2001. doi:10.1063/1.1364503.
- [72] L. Herrera, J. Ospino, and A. D. Prisco. All static spherically symmetric anisotropic solutions of Einstein’s equations. *Phys. Rev. D*, 77:027502, 2008. doi:10.1103/PhysRevD.77.027502.
- [73] K. Schwarzschild. “Golden Oldie”: On the gravitational field of a mass point according to Einstein’s theory. *Gen. Rel. Grav.*, 35:951, 2003. doi:10.1023/A:1022971926521.
- [74] R. C. Tolman. Static solutions of Einstein’s field equations for spheres of fluid. *Phys. Rev.*, 55:364, 1939. doi:10.1103/PhysRev.55.364.

- [75] J. R. Oppenheimer and G. M. Volkoff. On massive neutron cores. *Phys. Rev.*, 55:374, 1939. doi:10.1103/PhysRev.55.374.
- [76] P. C. Vaidya and R. J. Tikekar. Vaidya-Tikekar type superdense star admitting conformal motion in presence of quintessence field. *Astrophys. Astron.*, 3:325, 1982. doi:10.1140/epjc/s10052-015-3340-x.
- [77] M. Wyman. Radially symmetric distributions of matter. *Phys. Rev.*, 75:1930, 1949. doi:10.1103/PhysRev.75.1930.
- [78] M. P. Korkina. Static configuration with an ultrarelativistic equation of state at the center. *Sov. Phys. J.*, 24:468, 1981. doi:10.1007/BF00898413.
- [79] M. C. Durgapal. A class of new exact solutions in general relativity. *J. Phys. A: Math. Gen.*, 15:2637, 1982. doi:10.1088/0305-4470/15/8/039.
- [80] M. Ishak, L. Chamandy, N. Neary, and K. Lake. Exact solutions with w modes. 64:024005, 2001. doi:10.1103/PhysRevD.64.024005.
- [81] K. Lake. All static spherically symmetric perfect-fluid solutions of Einstein's equations. *Phys. Rev. D*, 67:104015, 2003. doi:10.1103/PhysRevD.67.104015.
- [82] R. J. Adler. A fluid sphere in general relativity. *J. Math. Phys.*, 15:727, 1974. doi:10.1063/1.1666717.
- [83] H. Heintzmann. New exact static solutions of Einstein's field equations. *Z. Physik*, 228:489, 1969. doi:10.1007/BF01558346.
- [84] R. L. Bowers and E. P. T. Liang. Anisotropic spheres in general relativity. *Astrophys. J.*, 188:657–665, 1974. doi:10.1086/152760.
- [85] S. D. Maharaj and R. Maartens. Anisotropic spheres with uniform energy density in general relativity. *Gen. Relativ. Gravit.*, 21:899–905, 1989. doi:10.1007/s10509-007-9409-8.
- [86] M. S. R. Delgaty and K. Lake. Physical acceptability of isolated, static, spherically symmetric, perfect fluid solutions of Einstein's equations. *Comput. Phys. Commun.*, 115:395, 1998. doi:10.1016/S0010-4655(98)00130-1.
- [87] M. H. Murad. A new well behaved class of charge analogue of Adler's relativistic exact solution. *Astrophys. Space Sci.*, 343:187–194, 2013. doi:10.1007/s10509-012-1258-4.
- [88] M. H. Murad and S. Fatema. Some new Wyman-Adler type static relativistic charged anisotropic fluid spheres compatible to self-bound stellar modeling. 2014. arXiv.org:1408.5126.

- [89] M. H. Murad and S. Fatema. Anisotropic charged stellar models in *Generalized Tolman IV* spacetime. *Eur. Phys. J. Plus*, 130:3, 2015. doi:10.1140/epjp/i2015-15003-y.
- [90] A. Giuliani and T. Rothman. Absolute stability limit for relativistic charged spheres. *Gen. Rel. Grav.*, 40:1427–1447, 2008. doi:10.1007/s10714-007-0539-7.
- [91] M. H. Murad and S. Fatema. Some static relativistic compact charged fluid spheres in general relativity. *Astrophys. Space Sci.*, 350:293, 2014. doi:10.1007/s10509-013-1722-9.
- [92] R. N. Mehta, N. Pant, D. Mahto, and J. S. Jha. A well-behaved class of charged analogue of Durgapal solution. *Astrophys. Space Sci.*, 343:653–660, 2013. doi:10.1007/s10509-012-1289-x.
- [93] D. N. Pant and N. Pant. Variety of well behaved parametric classes of relativistic charged fluid spheres in general relativity. *J. Math. Phys.*, 34:2440, 1993. doi:10.1063/1.530129.
- [94] D. N. Pant and A. Sah. Massive fluid spheres in general relativity. *J. Math. Phys.*, 20:2537, 1979. doi:10.1063/1.524059.
- [95] D. N. Pant. Relativistic modeling of a superdense star containing a charged perfect fluid. *Astrophys. Space Sci.*, 215:97, 1994. doi:10.1007/BF00627463.
- [96] N. Pant, N. Pradhan, and M. H. Murad. A family of exact solutions of einstein-maxwell field equations in isotropic coordinates: An application to optimization of quark star mass. *Astrophys. Space Sci.*, 352:135–141, 2014. doi:10.1007/s10509-014-1904-0.
- [97] N. Pant and P. S. Negi. Variety of well behaved exact solutions of Einstein-Maxwell field equations strange quark stars, neutron stars and pulsars. *Astrophys. Space Sci.*, 338:163–169, 2012. doi:10.1007/s10509-011-0919-z.
- [98] A. H. M. M. Rahman and M. H. Murad. Some electrically charged relativistic stellar models in general relativity. *Astrophys. Space Sci.*, 351:255–265, 2014. doi:10.1007/s10509-014-1823-0.
- [99] P.G. Whitman and R.C. Burch. Charged spheres in general relativity. *Phys. Rev. D.*, 24:311, 1981. doi:10.1103/PhysRevD.24.2049.
- [100] S. K. Maurya and Y. K. Gupta. A family of anisotropic super-dense star models using a space-time describing charged perfect fluid distributions. *Phys. Scr.*, 86:025009, 2012. doi:10.1088/0031-8949/86/02/025009.

- [101] L. Herrera. Cracking of self-gravitating compact objects. *Physics Letters A*, 165:3: 206–210, 1992. doi:10.1016/0375-9601(92)90036-L.
- [102] C. G. Böhmer and T. Harko. Bounds on the basic physical parameters for anisotropic compact general relativistic objects. *Class. Quantum Grav.*, 23:6479, 2006. doi:10.1088/0264-9381/23/22/023.
- [103] H. Andréasson. Sharp bounds on the critical stability radius for relativistic charged spheres. *Commun. Math. Phys.*, 288:715, 2009. doi:10.1007/s00220-008-0690-3.