

Modelling Option Prices Using Neural Networks

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A thesis submitted to the Department of Computer Science and Engineering
in partial fulfillment of the requirements for the degree of
B.Sc. in Computer Science

Department of Computer Science and Engineering
Brac University
Summer 2019

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Declaration

It is hereby declared that

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2. The thesis does not contain material previously published or written by a third party, except where this is appropriately cited through full and accurate referencing.
3. The thesis does not contain material which has been accepted, or submitted, for any other degree or diploma at a university or other institution.
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Abstract

In this research, modelling of the European option prices of S&P 500 index options was carried out using Multi-layer Perceptron Neural Networks. The goal was to train the neural networks using historical data to accurately determine option prices, given the index price, strike price and time to expiry as inputs. There is no hard and fast formula for pricing options, with the exception of the Black Scholes model, which is only a theoretical model and often under-performs in practical applications. Therefore, developing a model for pricing real options is of great importance, and Neural Networks have the potential to be vital vehicles to that end. That is what motivated this study. Different results with respect to accuracy are achieved by partitioning the data according to moneyness of options, with the Neural Network performing exceptionally for in-the-money options, but poorly for out-of-the-money options. This suggest that in a volatile market the neural network outperforms the Black Scholes model for in-the-money options, however the Black Scholes model is still better for at-the-money options.

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Nomenclature

The next list describes several symbols & abbreviation that will be later used within the body of the document

BP Back Propagation

BS Black-Scholes

BSM Black-Scholes-Merton

CCP Central Counter Party

ETD Exchange-Traded-Derivative

FDI Foreign Direct Investment

FX Foreign Exchange

MLP Multi-Layer Perceptron

MNN Modular Neural Network

OPM Option Pricing Model

OTC Over-The-Counter

RBF Radial Basis Function

ReLU Rectified Linear Unit

SCG Scaled Conjugate Gradient

Chapter 1

Introduction

1.1 Derivatives : A Financial Perspective

According to financial terms, a derivative is an agreement between at least one side where the worth depends on a settled fundamental monetary resource (like a security) or set of monetary resources (like an index). Bonds, commodities, currencies, interest rates, market indexes, and stocks incorporates as some common primary assets. The derivatives market has achieved the most astounding development of all as of late and has turned into a noteworthy supporter of stability in the money related world. As of late, derivatives markets have developed by immense edge in rising economies and have provided the economic rewards and losses faced by financial specialists in developing nations. In such derivative markets North America ruled for quite a long time. As expressed by Hassan and Rahman[24], the developing markets in Asia-Pacific district has surpassed North America as the world's biggest derivatives market in the midst of expanding interest for futures and options contracts in the area's quickly developing economies. As indicated by Washington-based Futures Industry Association (FIA), derivatives contracts represented 38 percent of the worldwide aggregate in the Asia-Pacific region while 33 percent in North America during the first half of 2010. The derivatives markets have included quality, assortment and in particular better risk management in such regions

Derivatives are secondary assets whose worth is exclusively based (inferred) on the estimation of the underlying assets. Derivatives without any link to underlying asset is considered useless. Some common derivatives are futures contracts, warrants, forward contracts, options and swaps. Futures contract, for instance, is a derivative since its worth is influenced by the value of its fundamental asset. A stock option, likewise, is a derivative because its value is "derived" from the underlying stock. A derivative's value can be "derived" from a security, but doesn't mean that an investor of a derivative owns a security.

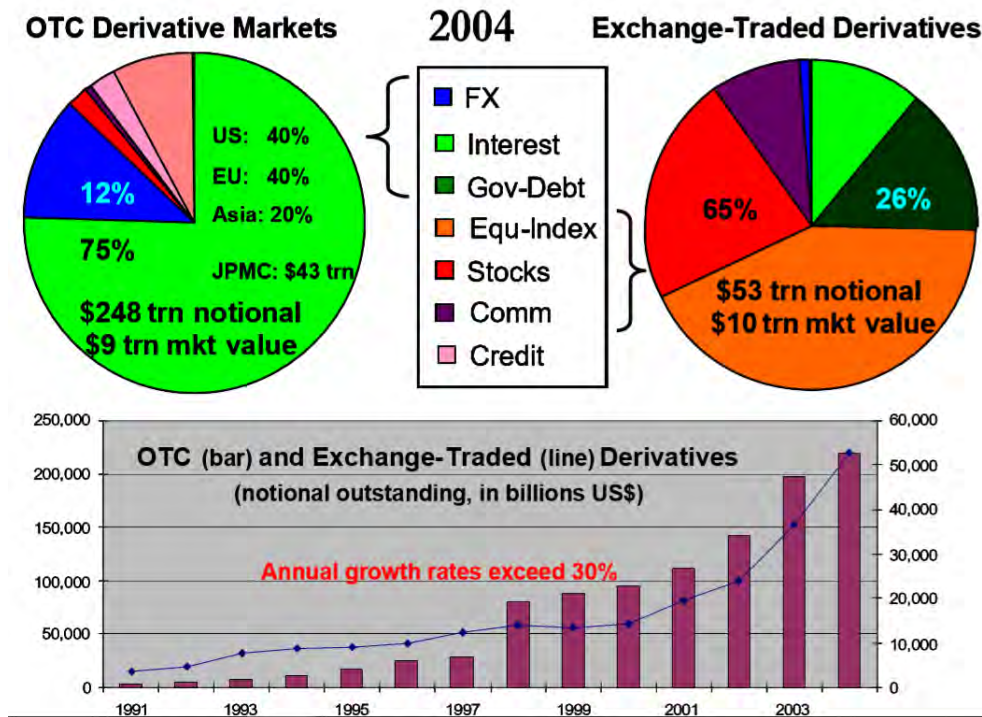


Figure 1.1: Global Derivatives Market (2004) [19]

1.2 Types of Derivatives Product

Derivative products can be classified into two - "lock" and "option". As stated by Fratzscher[19] lock products (e.g. swaps, futures, or forwards) investors are bound to the settled upon terms over the life of the agreement. However option products (e.g. interest rate swaps) offer investors the right, however not the commitment, to the contract with agreed terms. The investment philosophy of such derivative products can be bound by the gain-loss equation, that is investors can either use derivatives as a means to reduce risk through hedging or gain rewards through speculation. Derivatives used for hedging is a means of transferring the risks associated with the primary security's price from one party to another.

Chapter 2

Derivatives Contracts

Contracts are of three types: **Options, Swaps and Futures/Forward contracts**
- with contrasts in each type.

2.1 Options

Options are agreements that allows an investor to sell or buy a security without having the commitment to do it. When there is a large shift in share prices and the scenario becomes favourable for the derivatives' buyer, they bank on such cases without purchasing the primary asset in the first place.

The most common option strategies are:

- Long Call - Buyer believes the price of security will augment and purchase the right to own the underlying asset. For a long call investor profit occurs when the market price of the security exceeds the strike price by the charge of the premium paid for the call.
- Long Put - Buyer believes the price of security will curtail and purchase the right to sell the underlying asset. For a long put investor profit occurs when the strike price of the security exceeds the market price by the charge of the premium paid for the put.
- Short Call - Buyer will sell the call believing the price of security will fall. When a call is sold, the counter-party (the long call) of exercising the call as the power of exercising is transferred from the call writer to call buyer. For a short call writer the profit is restricted only to the premium received, that is only if the buyer of the call does not exercise the call.
- Short Put - Buyer will sell a put option believing the the price of security will augment. For a short put writer the profit is restricted only to the premium received, that is only if the buyer of the call does not exercise the call.

2.2 Swaps

Swaps are subsidiaries where counter-parties trade money streams or different factors related with various speculations. Commonly a swap happens in light of the fact that one gathering has a similar bit of leeway, such as acquiring assets under factor loan fees, while another individual can be borrowing more unreservedly at fixed rates. The easiest variety of a swap is classified "plain vanilla" ; however there are numerous sorts, including:

- Interest Rate Swaps - A fixed rate loan is exchanged between two person for a floating rate loan. However when a trader has floating rate liabilities in his fixed rate loan, that trader could swap his fixed rate with another person for a floating rate in order to equal the liabilities. Option strategies can be used for interest rate swaps however in case of swaption investors are not bound to do the swap but just has the mere right to do so.
- Currency Swaps - Loan payment and capital in one currency is exchanged between two parties for payment and capital in another currency.
- Commodity Swaps - Depending on the value of a principal commodity, a contract is made between two parties to trade their cash flows.

2.3 Forward & Future contracts

For a specific price in the future, two sides with forward and future contracts can either purchase or sell an asset. Utilizing the spot or most current price these agreements are generally composed. The distinction between the spot cost at the hour of conveyance and the forward or future cost determines the buyer's benefit or loss. These agreements are regularly used for hedging and speculation. Futures are institutionalized contracts that exchange on centralized exchange system while non-standard forwards are exchanged over the counter through an intermediary seller network.

Chapter 3

Derivative Products in Asian Markets

3.1 State of the Asian Derivative Markets

The powerlessness of the capital markets crosswise over Asia centering the bank finance and unregulated Over-The-Counter (OTC) subsidiary markets without supporting instruments for organizations was prevalent in the Asian Crisis. The **five** principle subsidiary items exchanged in Asian markets as expressed by Fratzscher [19] in his discoveries are as follows:

- Outside trade items for monetary forms are exchanged in Tokyo, Singapore, and Hong Kong for the most part in OTC derivatives, and there are offshore centers chiefly in Singapore for minor and non-convertible currencies. The joined Asian FX markets are huge with turnover representing 33% of overall markets.
- Loan cost subsidiaries in Asia are little and represent under 5% of world markets with a declining pattern in OTC markets and a relocation towards Exchange-Traded-Derivative (ETD) markets. The two overwhelming areas that are exchanging mostly JPY and US\$ swaps (OTC) and futures (ETD) are Tokyo and Singapore. Local fixed income derivative markets have as of late been created and hasn't grown much.
- Equity derivatives have seen the quickest development, regularly multiplying every few years. The most generally exchanged items with huge interest of institutional speculators and critical foreign participation was index futures. Over 44% of overall ETD value turnover is right now occurring on Asian trades. This is due to an amazingly high turnover ratios of multiple times outstanding stock of futures and multiple times extraordinary stock for options, when contrasted with a normal worldwide turnover of 25.
- Product subsidiaries have a long history, particularly in China, where the Soybean futures contract at the Dalian Commodity Exchange is the third biggest

	Australia	China	Hong Kong	India	Indonesia	Japan	Korea	Malaysia	Philippines	Singapore	Thailand
Index											
Futures	✓	✗	✓	✓	✓	✓	✓	✓	✗	✓	✗
Options	✓	✗	✓	✓	✗	✓	✓	✓	✗	✓	✗
Options on futures	✓	✗	✗	✗	✗	✓	✗	✗	✗	✓	✗
Stock											
Futures	✓	✗	✓	✓	✗	✗	✗	✗	✗	✓	✗
Options	✓	✗	✓	✓	✗	✓	✓	✗	✗	✗	✗
Currency											
Futures	✓	✗	✗	✗	✓	✓	✓	✗	✗	✗	✗
Options	✗	✗	✗	✗	✗	✗	✓	✗	✗	✗	✗
Interest rate											
Futures	✓	✗	✓	✓	✗	✓	✓	✓	✗	✓	✗
Options on futures	✓	✗	✗	✗	✗	✓	✗	✗	✗	✓	✗
Bonds											
Futures	✓	✗	✓	✗	✗	✓	✓	✓	✗	✓	✗
Options on futures	✓	✗	✗	✗	✗	✓	✓	✗	✗	✓	✗
Commodities											
Futures	✓	✓	✗	✗	✓	✓	✓	✓	✗	✓	✗
Options on futures	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
# of products traded	12	1	6	5	3	10	9	5	0	9	0

Figure 3.1: Derivative Products and Turnover across Asia [19]

subordinates contract crosswise over Asia (among the world's 20 biggest subordinates contracts). What's more, wheat, rubber, gold, and oil futures are huge and are for the most part exchanged on Chinese or Japanese item trades. However subsidiaries represent under 10% of turnover on the trades.

- Credit subsidiaries are among the quickest developing items, particularly credit default swaps that record for half of the OTC market. It is assessed that 10% of the overall \$6 turn credit subordinate market is situated in Asia, generally in Tokyo and Hong Kong. Such unregulated markets have made regulators worried as they are less straightforward and exceptionally utilized.

3.2 Characteristic of Derivative Instruments

A feature of subordinates instruments is that they are intended to **transfer risk**, while equity or fixed income securities are intended to be an unequivocal case on the money streams created by the exchange of a budgetary resource in a specific purview. Investors exchanging in subordinates markets structure have assumptions regarding hidden resource costs and are better put to deal with the risks related with price changes. Numerous specialists have progressively taken a worldwide perspective on resource enhancement and risk management, and have looked to make exposures to resource classes that are not promptly accessible in the market. This trademark makes subsidiaries increasingly consistent to exchanging, making OTC subordinates markets global than nationalistic equity markets. The pace of improvement of financial developments and new subordinates instruments through advances in data innovation, has encouraged the monetary mediators to ceaselessly present and grasp new derivatives instruments.

	Australia	China	Hong Kong	India	Indonesia	Japan	Korea	Malaysia	Philippines	Singapore	Thailand
Liquidity											
Fixed income benchmarks	✓	○	○	✓	○	✓	✓	✓	✗	✓	✓
Fixed income liquidity	✓	○	✓	○	✗	✓	✓	○	○	✓	○
Equity market liquidity	✓	✓	✓	✓	○	✓	✓	✓	✗	✓	○
Regulation											
Derivatives law	✓	✗	✓	✓	✗	○	○	○	✗	○	○
Securities lending	✓	○	✓	✓	○	✓	✓	✗	✗	✓	○
Accounting standards	✓	○	✓	○	✓	○	○	✓	✗	✓	○
Exchanges											
Clearing and settlement CCP	✓	✗	✓	✓	✗	○	✓	○	✗	✓	○
ISDA netting opinion	✓	✗	✓	✓	✓	✓	✓	✓	✓	✓	✓
Demutualized exchange	✓	✗	✓	✓	✗	○	✓	○	✗	✓	○
Taxes											
Tax harmonization	✓	✗	○	✓	✓	○	✓	○	✗	✓	✗
Transaction costs and IT	✓	○	✓	✓	✗	✓	✓	✓	○	✓	○
Investor base and NBFI	✓	○	✓	✓	✗	○	✓	○	✗	✓	○
# best practice components	12	1	10	10	3	7	10	5	1	11	2

Notes: ✓ denotes best practice ; ○ denotes progress on existing deficiencies ; and ✗ denotes major problems.

Figure 3.2: Derivatives Market Infrastructure Across Asia [19]

3.3 Driving Factors for Derivative Markets in Asia

As per Hassan and Rahman [24], the primary driving components for subsidiary markets in different economies of Asia have been the monstrous advantages from **risk sharing** in progressively complete capital markets, corporate interest for hedging apparatuses, institutional requirements for upgraded liquidity in a situation of huge cross-outskirt streams and exchange coordination, just as curtailing exchange costs in electronic exchanging. At present, the greater part of the subsidiary markets in Asia offer numerous items for various client necessities.

There is a solid relationship between current subsidiaries items in Asia (Figure 3.1) and the fundamental subsidiaries framework (Figure 3.2). Fratzscher [19] states that so as to create subsidiary instruments, nations need to set up various prerequisites that identify with market liquidity (counting fixed pay benchmarks), strong book-keeping and administrative gauges (counting subsidiaries law), present day framework at trades (focal counter-party), and an assessment domain that creates equal opportunity for every investors. At the point when Figure 3.1 is contrasted with Figure 3.2 it creates the impression that Australia, Hong Kong, India, Japan, Korea, and Singapore have as of now generally settled prescribed procedures in their fundamental foundation, though China, Indonesia, Malaysia, the Philippines, and Thailand have still real impediments to survive, albeit noteworthy advancement is being made in every one of these nations to improve the current budgetary framework.

Chapter 4

Importance of Derivatives Market in Bangladesh

The estimation of subsidiary securities is evident for a financing economies. The capital market of Bangladesh has as of late seen unfortunate market shift in stock prices and the ongoing intermittent falls at Dhaka Stock Exchange (DSE) has shown the absence of information and apparatuses were required to battle this disturbing market sector instability.

Hassan and Rahman [24] states that one of the factors behind such high unpredictability of our capital market is the absence of hedging apparatus that could secure the speculators, individuals and firms. Throughout the previous two decades, developing markets to be specific Korea, Malaysia, Brazil and India have creatively and sensibly utilized a lively subsidiary market for risk moderation and furthermore yielded generous advantages as far as market extension and financial development. In perspective on the positive result in comparable markets of the world, the effect of subsidiaries market in Bangladesh ought to saliently affect the nation's capital market and the general economy.

4.1 Methods of Introducing Derivatives Market in Bangladesh

The following set of recommendations will address both the preparatory evaluation need and also relevant building blocks of creating an efficient derivative exchange:

- As a matter of importance, the Security and Exchange Commission should create a board of trustees for setting up extensive report on the performance of trade exchanged subordinates showcased in Bangladesh.
- The idea and use of subsidiaries is still very ambiguous to stakeholders, for example, regulators, monetary foundations and import/export substances. Such

constrained comprehension of subordinates exchanging with respect to competitors can turn into a prevention to the improvement of the market. Subsequently, the underlying procedure of making a subordinate market ought to include teaching stakeholders on the different subsidiaries accessible in world-wide markets and furthermore find out their prerequisite of subsidiary instruments. Such extensive market study can be an intuitive strategy for sharing thoughts, concerns and proposals for going ahead.

- The valuing of subsidiaries items and the management of risk models require merchants/vendors, regulators and end-clients to have adequate instructive preparation in monetary and quantitative strategies utilized for resource estimating. Along these lines, it is fundamental that experts attend specific courses in monetary administration and subsidiaries.
- Subsidiaries, presented both by means of trades and OTC have been acknowledged fundamentally as a successful instrument for hedging. They guarantee profundity and liquidity to the capital markets and henceforth support exchanging. In perspective on late disturbance in the equity markets, it is suggested that more straightforward subsidiaries items be propelled first, pursued by progressively complex items as our business sectors advance. Staged presentation of subsidiaries, with index futures(momentary) trailed by options on index and individual resources ought to be in mid-term horizon. When the trade garnered expertise by exchanging index futures, commodity futures ought to likewise be presented.
- When the advisory board has effectively finished its investigation and gave an administrative solution, the emphasis should move on redesigning the framework to suit subsidiary exchanging. A particular sub-group of the advisory group containing mathematicians, investigators and subsidiary trade specialists ought to be accountable for setting up all guidelines relating to exchanging, clearing, settlement, margin upkeep and enrollment qualification criteria.
- So as to keep up the effectiveness and straightforwardness of subsidiary security trade, clearing and settlement of subsidiaries items ought to be executed by a focal counter-party with several multi-item close-out netting arrangements.

In outline, subsidiaries give risk management devices just as elective speculation chances to investors. Developing economies in which index futures and options have been presented have encountered huge gains in both securities exchange capitalization and exchanging volume. There is no denying the way that our current institutional set up and administrative structure probably won't be sufficient for advanced instruments like subordinates. In any case, the ongoing calamitous fall of our securities exchange, quickly declining FDI and shortage of venture openings in a value driven economy, is shouting out for an inventive and adaptable financial item for hedging and market extension. The significance of the improvement of institutional foundation and human capital can't and ought not be disregarded; in any case, if the capital market of Bangladesh must be in the focused level like other rising economies of India, China, Malaysia or Singapore, the procedure of dynamic policy making cannot make up with the pace of the counter-parts in progressive

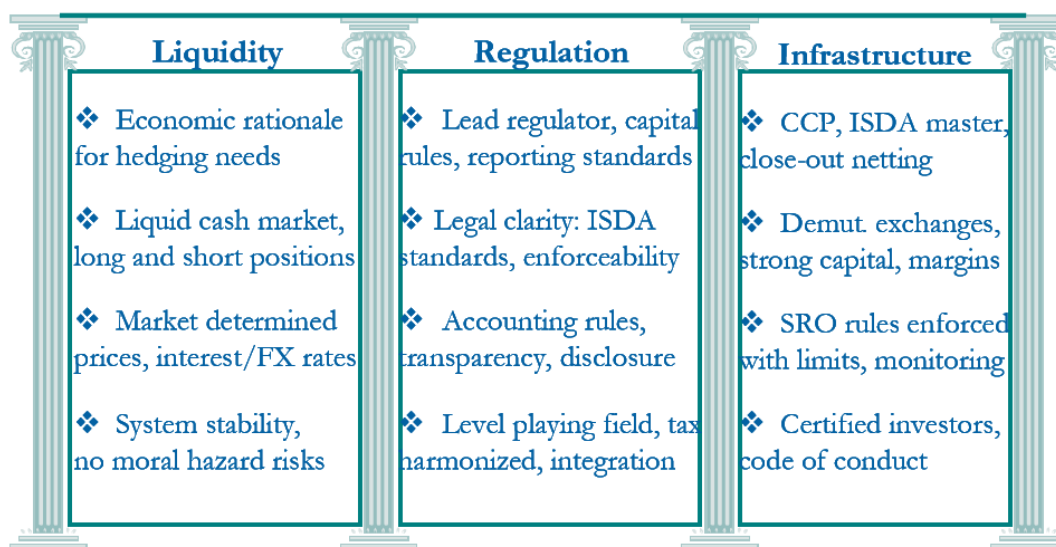


Figure 4.1: Three Pillars for Sound Derivative Market

budgetary markets. Subsequently, the policy makers and regulators must start a procedure for presenting subsidiary securities in Bangladesh.

Chapter 5

Options: A Detailed Analysis

Options are financial instruments that are derivatives or based on underlying securities such as stocks. An options contract offers the buyer the opportunity to buy or sell — depending on the type of contract the buyer holds with the underlying asset. There is no fixed rule for the carrier of an asset to sell or buy their financial instrument.

- The holder of call options can buy the asset at an agreed price within a time range.
- The holder of put options can sell the asset at an agreed price within a time range.

Pros of Call & Put Options :

- A buyer of call option can buy their assets at a lower price than the market's when the price of stock is augmenting.
- The owner of a put option can earn profit if they sell their securities at the strike price while the strike price is more than the market price.
- The writer of an option earn a premium fee for selling the option to the option buyer.

Cons of Call & Put Options :

- If the market is failing, the seller of put option might be required to purchase the security at an increased strike price than paid in market.
- The writer of call option might be forced to purchase shares at an augmented price if the price of stock keeps rising while they face boundless risk.
- The buyers of an option are required to pay a premium price to the option writer.

Every option has a particular date of expiration and every option investor needs to abide by this time duration for exercising their option. The price mentioned for an option is called its strike price or exercise price. The purchase and selling of options usually occur through online site and retailers.

Options, a type of adaptable investment product involves a trade in two sides involving a seller and a buyer . Every call option involves a buoyant buyer and a bearish seller, while on the other hand put options involve bearish buyer and a buoyant seller. An option contract comprises of 100 shares of the underlying security and a premium fee is paid for each contract. For instance, an option with a premium price of 45 cents per contract, would cost \$45 ($\$0.45 \times 100 = \45) when bought.

There are two factors that determine the price of the premium - the strike price and the expiration date. The strike price is the price agreed on selling or buying the security till the date of expiration whereas the expiration date sets the limit of the date for exercising the option.

There are several reasons why traders and investors will sell and buy options. For instance speculation of options grants a trader a position of advantage in holding an asset at a lesser cost than purchasing the shares of the asset. Options are particularly used by investors to curtail the exposure of risk in their finance portfolio. Option holder can also earn profit by purchasing call options or by selling the options and earning a premium.

Unlike European Options, American options can be exercised at any time period before its date of expiration whereas European Options can only be exercised on the date of expiration. Here exercising means the option owner can use his right to sell or buy the underlying security.

5.1 Profits and Risks of Buying Call Options

The bearer of call options does not have any commitment to purchase the asset if the bearer does not want to. The rise and fall of the underlying stock does not have any impact.

Buyers of call options expect the price of share will augment more than the option's strike price before the option gets expired. If the stock price augments above the strike price, the option holder will buy the stock at its strike price by exercising the option and sell the stock immediately at the current market price in order to earn a profit.

The profit earned per contract earned through this trade is the share price of the market minus the strike price and the expense/s required to pay for the option - this could be the premium price and/or the brokerage commission required to place the order/s. However the total profit earned by the holder would be the profit earned per contract multiplied by the number of contracts bought multiplied by the number of shares per contract. On the other hand if the underlying asset's price does not

augment above the strike price by the date of expiration, the option will expire worthless as the bearer will not exercise it. This results in a loss(risk) for the bearer of the option as a premium had to be paid for the call option.

Risk : The only risk to the buyer of call option is restricted to price of the premium required to be paid.

5.2 Profits and Risks of Selling Call Options

To sell a call option is to write a contract. The writer of the call option will be receiving a premium fee from the buyer of the option. For an option writer the highest profit earned in the trade is the premium received in selling the call option. A call option seller is bearish in nature, that is the investor believes the underlying security's price will curtail below the strike price of the option or be close to the strike price while the option is active. If the share price of the market is at or below the strike price during the life of the option then the option will not be exercised by the buyer of the option as it will deem worthless. This is a gain for the option seller as the writer will not be incurring any loss.

On the other hand if the share price of the market exceeds the strike price by the date of expiration then the option writer must sell the shares to the option buyer at the lower strike price. The option writer could either sell the shares from their portfolio holdings or needs to purchase the stock at the current market price in order to sell to the buyer of the call option. This results in a loss for the option writer where the level of loss depends on the share's cost to transact the option order plus any commission of the brokerage order but minus any premium received through the option contract.

To sum up, the risk faced by the writers of a call option is far more than the exposure of risk to the buyers of call.

Risk : Option writer's level of risk is boundless because the price of the stock could continue to augment which would create significant losses.

5.3 Profits and Risks of Buying Put Options

A put option buyer believes that the market price of stock will curtail below the strike price before the option expires. They earn profit when the price of the stock price falls below the strike price. Thus put option investors exercise their option when they observe current market price to be below the strike price. They collect their profit by selling their shares at the strike price when such scenario occurs. Investors can replace their shares by purchasing them in the open market.

The profit through such trade is the strike price minus the present price of stock in the market and the expense/s required to pay for the option - this could be

the premium price and/or the brokerage commission required to place the order/s. However the total profit earned by the holder would be the profit earned per contract multiplied by the number of contracts bought multiplied by the number of shares per contract. As the stock price curtails the value of holding a put option augments and vice versa.

Risk : The only risk to the buyer of put option is restricted to price of the premium required to be paid.

5.4 Profits and Risks of Selling Put Options

Writing a contract means to sell put options. A writer of put option has the ambition to see the security's price to stay same or augment over the period of the option's life - which is being bullish. A put option buyer has the privilege of making the option writer purchase shares of the underlying security at the exercise price by the date of expiration. The put option will become worthless if the primary stock's price becomes above the strike price by the expiration date. Then the writer will be earning his maximum profit which is the premium minus any brokerage commission since the option will not be exercised by the buyer.

On the other hand if the value of the stock market curtails beneath the strike price then the put option buyer will be inclined to exercise it prompting the writer to buy shares of the primary security at the strike price. The put option buyer will then sell his share at the strike price getting him a profit as the strike price is more than the market value of the stock.

For the writer of put option, he can either sell the shares and have a loss or keep hold of the shares with the expectation that price of the stock will increase above the price of purchase. Therefore the loss incurred will be the higher strike price minus the lower market price minus the premium received plus any brokerage commission required to be paid.

Risk : Depending on the appreciation value of the shares, a put writer can face significant loss if the price of the market falls below the strike price where by the date of expiration would be required to purchase the shares at the strike price.

Chapter 6

Black-Scholes Model

6.1 The Black-Scholes Model

According to M. Morelli et. al [18] and J. Yao et. al [17], the Black Scholes model, otherwise called the Black-Scholes-Merton (BSM) model, is a model featuring financial instruments with a variation of price over time, for example, stocks that can, in addition to other things, be utilized to decide the price of an European call option. BSM was the first widely used model for option pricing. The model assumes that the price of heavily traded assets follow a geometric Brownian motion with constant drift and volatility. When applied to a stock option, the model incorporates :

- the constant price variation of the stock
- the time value of money
- the option's strike price
- the time to the option's expiry
- expected interest rates
- expected volatility

The assumptions underlying the original Black-Scholes model are as follows:

- The model considers the option to be European and can be exercised only at the date of expiration.
- During the life of the option no dividends are paid.
- Markets are not considered volatile.
- No transaction costs (e.g. brokerage commission) are considered in purchasing the option.

- The underlying asset's risk-free rate is considered familiar and constant.
- The profit/loss of the primary security are thought to be dispersed.

As Anders, Korn and Schmitt [16] states that asset prices follow a geometric Brownian motion; mean returns and volatilities are constant over time; interest rates are both constant over time and equal for all maturities; trading occurs continuously on frictionless markets and no arbitrage opportunities exist. Based on the above determinations, Black and Scholes presented the European call option that was written on a non-dividend-paying stock:

$$C_{BS}(t) = SN(d_1) - X_e^{-r(T-t)} + N(d_2) \quad (6.1)$$

$$d_1 = \frac{\ln S/X + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}} \quad (6.2)$$

$$d_2 = d_1 - \sigma\sqrt{T - t} \quad (6.3)$$

where

$S \equiv$ price of the underlying asset

$X \equiv$ strike price of the option

$\sigma \equiv$ volatility of the continuously compounded asset returns

$r \equiv$ continuously compounded interest rate

$T - t \equiv$ time to maturity of the option contract

and $N(x)$ is the cumulative distribution function of the standard normal distribution.

General conclusion of Black-Scholes model as stated by P.Ritchken [7]:

- For pricing at-the-money options, the BSM model is considered superior especially when expiration time surpasses 2 months.
- In the cases of deep in-the-money and out-of-the-money options, there is a major difference between prices of the market and the price of the model.
- Options with a validity of less than one month are often wrongly quoted.
- Options with exceedingly low and exceedingly high volatility stocks are often not properly priced.

6.2 Black Scholes Model's Limitations

As expressed already, the BSM model is just used to value European options and doesn't consider that U.S. options could be used before the termination date. Besides, the model accept profits and risk free rates are consistent, yet this may not be valid in all actuality. The model expect volatility stays consistent over the option's life, which isn't the situation since volatility changes with the degree of free market activity. The model additionally accept that there are no exchange costs or

expenses; that interest rate is consistent for all developments and that there are no risk free exchange openings. These suppositions can prompt costs that stray from actual price where these components are available.

Black and Scholes introduced in 1973 their milestone OPM(Option Pricing Model). Andreou, Charalambous and Martzoukos [20] states that despite the fact that BS and its variants are considered as the most prominent achievements in financial theory in the last three decades, empirical research has shown that the formula suffers from systematic biases. The BS bias stems from the fact that the model has been developed under a set of simplified assumptions such as geometric Brownian motion of stock price movements, constant variance of the underlying returns, continuous trading on the underlying asset, constant interest rates, etc.

Yao, Li and Tan [17] states that in spite of the fact that BSM model outperforms other option pricing models aside from the cases of deep-in-the-money and deep-out-of-the-money options, the way the market's data contradicts the model's presumption makes it untrustworthy.

Violations may be found in the following situations :

- According to Peters [14], fractal may be a better use for finding volatility instead of random walk description. Akgiray [8] has also declined the assumption of constant variance of stocks.
- American option overwhelms the genuine markets. Individuals need more decisions when the market changes, for example, when it is close to an opportunity to pay profits.
- In cases of stocks dividends are regular practice.

6.3 Fine-tuning Black Scholes Model

Several practices have been made to alter the BSM model to restrict the constraints it takes under assumption. Some of the following practices have been mentioned below :

- Pure jump model as found by Cox and Ross [2]
- Mixed diffusion jump model, found by Merton [3], based on continuous constraints
- Square root constant elasticity of variance diffusion model as found by Cox and Ross [2]
- Displaced diffusion model as found by Rubinstein [5]
- Compound option diffusion model, found by Geske [4], based on the constant volatility assumption

- Merton's [1] stochastic interest rate extensions
- Stochastic volatility models as found by Hull and White [6]
- Regime switching model, found by Naik [11]
- Implied binomial tree as found by Deman and Kani [12] and Rubinstein [15]

The above practices make a precise and pragmatic option pricing model taking minor considerations.

Chapter 7

Literature Review

7.1 Neural Networks

Neural networks are an entirely adaptable class of measurable models which has been enlivened by the investigations of the brain and the sensory system. It can be portrayed as a system of arcs/connections and nodes/neurons and is a gathering of interconnected handling components organized in progressive layers . These layers can be organized progressively, and the **1st layer is known as the input layer, the last layer is known as the output layer, and the inside layers are known as the center or hidden layers.** Feed-forward systems guide contributions to yields with signal streaming one way from the input layer to the output layer.

A numerical weight is related with every neurons that impacts the input cell on an output cell. Reinforcement is indicated by positive weights; inhibition is indicated by negative weights. With supervised learning, through a training procedure, connection weights are found out by the network, as models from a preparation set are introduced more than once to the network.

Fixed through continuous or discrete values each processing element has an activation level. The activation level of a node in an artificial neural network is the output generated by the activation function. Malliaris and Salchenberger [10] states that if the neuron is in the input layer, its activation level is determined in response to the input signals it receives from the environment. For cells in the middle or output layers, the activation level is computed as a function of the activation levels on the cells connected to it and the associated connection weights. This function is called the transfer function or activation function which sets the output behavior of each node, or “neuron” in an artificial neural network and may be a linear discriminant function, e.g, a positive signal is output if the value of this function exceeds a threshold level, and zero otherwise. It may also be a continuous, non-decreasing function.

Robert R. Trippi and Efraim Turban [9] stated, NNs are reforming practically every part of finance. NN are utilized worldwide by financial firms to handle troublesome

undertakings including instinctive judgment or requiring the recognition of data pattern, which evade customary analytic strategies. Numerous onlookers accept NNs will inevitably beat even the best brokers and financial specialists. NNs are as of now being utilized in the markets of securities, to estimate the economy and to examine credit hazard.

7.2 Neural Networks Applied to Option Pricing

Neural network model has been generally connected and examined in the option pricing model in order to stay away from the deficiency of the customary parametric models. Huchison [13] was the first to utilize RBF(Radial Basis Function) and BP(Back Propagation) neural network models for evaluating European options. His examination showed that the model is superior to the Black-Scholes model. Huchison [13] proposed a non-parametric strategy for evaluating the value of derivative using NN. Huchison [13] stated that although his approach might not be a substitute for the more conventional arbitrage-based pricing formulas, network pricing equations might be progressively precise and computationally increasingly effective when the primary assets are volatile and obscure, or when the pricing formula cannot be tackled analytically.

The forecast of option prices of Nikkei 225 index futures was conducted by Jingtao Yao et al. [17] through the use of back propagation neural networks. Various outcomes as far as precision were accomplished by gathering the information differently. The outcomes state that a NN incorporated option pricing model outflanks the conventional BSM model for volatile markets. Jingtao Yao et al. [17] used data partition based on moneyness in the NN model and stated that investors who prefer high risk - high return can opt for NN model outcomes.

Marco J. Morelli et al. [18] used neural network algorithms for finding option price and tuned it for the the nonlinear aspect of financial derivatives. He used two types of NN, namely - multi-layer perceptions and radial basis functions, and compared their performances. American and European options were taken into account as well as the Greek letters in case of hedging. His research showed an outcome of fruitful prediction of option prices and Greek Letters with high exactness and good computational time.

A non-parametric modular NN was designed by Gradojevic et al. [21] to find the value of S&P-500 European call options. The model depends on time to maturity and moneyness of the options. The model has proven out-of-sample pricing performance when differentiated with parametric and non-parametric models. Gradojevic et al.'s [21] study suggested that modularity ameliorates the generalization functions of normal feedforward NN option pricing models.

In order to ameliorate the prediction of the price of derivatives of securities, Yi-Hsien Wang [22] used a hybrid asymmetric volatility system into artificial NN option pricing model. His method can be curtailed to the stochastic and non-linearity of the error term sequence and also get the asymmetric volatility value. Yi-Hsien Wang's

[22] model showed that Grey-GJR–GARCH volatility gives better predictability than other volatility methods.

Based on the BSM model, a hybrid wavelet NN model was suggested by Zhang et al. [23] where hybrid forecasting models amalgamated with hybrid Wavelet NN and genetic algorithm was built. According to his model the options were categorized based on moneyness and the weighted implied volatility rates were considered as inputs to the NN. He ran his model on the Hong Kong derivative market and found that hybrid models like his are better than the general BSM model and other NN models.

A NN-based demand model was built by S. Shakya et al. [27] using evolutionary algorithms to optimize pricing policies. The pros of such an approach are the flexibility of modelling a range of different demand scenarios found in products and services and the versatility of deciphering intricate models. S. Shakya et al.’s [27] research output showed consistency, adaptability and high accuracy of pricing policy when compared with other models.

Mitra [26] in his research showed that BSM formula has methodical biases as market prices differed from the output of the BSM equation. In some studies the biases were corrected using a rectification mechanism in the input data. Mitra [26] used ANN to ameliorate the option pricing prediction where input parameters were modified using appropriate multipliers where the value of the multipliers were found using known data that reduces error in valuation.

The performance of Artificial NN was differentiated with modified BSM model based on option pricing and hedging by Fei Chen and Charles Sutcliffe [25]. The ANNs were trained using high-frequency data to find option prices. Fei Chen’s and Charles Sutcliffe’s [25] research stated that hybrid ANN is better than modified BSM model and the standard ANN in finding put and call option’s prices. Apart from finding option prices, their research also stated that ANNs trained on real hedge ratios far outperforms the pricing models and modified BSM models.

7.3 Novelty

The novelty in our study comes from a combination of aspects. We took a Multi-Layer Perception approach with two hidden layers in our model, as opposed to only one used in a majority of studies. The benefit of two hidden layers is improved performance for high frequency data, as was the case in our study. We partitioned the data into separate models based on their moneyness, which allowed the network to learn each class of option prices separately. Implied volatility and risk free interest rate were not included as inputs in our models. In the case of interest rate, it was discarded as the value remains largely constant throughout the year, and adding a constant as an input would hamper the performance of the network. As for implied volatility, since we trained our model on sequential data ordered by transaction time, it allowed our network to account for time dependencies and hence capture the implied volatility in the hidden layers. This alleviates one of the major pain

points of the traditional Black Scholes Model - determining the implied volatility. The sequential approach in our training also means that our model can potentially be applied on the trading floor, using a real-time 'sliding-window' application.

Chapter 8

Dataset Collection and Preparation

8.1 Raw Data Collection

The data used in our study are one year of historical data of S&P 500 Index Options (SPX) traded in the Chicago Board Options Exchange (CBOE), ranging from 2nd January 2018 to 31st December 2018. Due to the unavailability of free option price datasets with our required features, we resorted to purchasing data from IVolatility - a service specializing in options data.

The raw dataset consisted of 2,674,746 rows of data, each row representing a single option contract, with the following columns: symbol, exchange, date, adjusted close, option symbol, expiration, strike, call/put, style, ask, bid, volume, open interest, unadjusted.

symbol	exchange	date	adjusted close	option symbol	expiration	strike	call/put	style	ask	bid	volume	open interest	unadjusted
SPX	CBOE	04/03/2018	2614.45	SPXW 180413P02755000	04/13/2018	2755	P	E	143.3	139.5	153	429	2612.317
SPX	CBOE	04/03/2018	2614.45	SPXW 180413C02760000	04/13/2018	2760	C	E	0.75	0.5	42	1505	2612.317
SPX	CBOE	04/03/2018	2614.45	SPXW 180413P02760000	04/13/2018	2760	P	E	148.2	144.3	53	496	2612.317
SPX	CBOE	04/03/2018	2614.45	SPXW 180413C02765000	04/13/2018	2765	C	E	0.65	0.4	313	2163	2612.317
SPX	CBOE	04/03/2018	2614.45	SPXW 180413P02765000	04/13/2018	2765	P	E	153.1	149.2	1	589	2612.317
SPX	CBOE	04/03/2018	2614.45	SPXW 180413C02770000	04/13/2018	2770	C	E	0.55	0.35	47	341	2612.317
SPX	CBOE	04/03/2018	2614.45	SPXW 180413P02770000	04/13/2018	2770	P	E	158	154.1	4037	4696	2612.317
SPX	CBOE	04/03/2018	2614.45	SPXW 180413C02775000	04/13/2018	2775	C	E	0.45	0.25	8	984	2612.317
SPX	CBOE	04/03/2018	2614.45	SPXW 180413P02775000	04/13/2018	2775	P	E	162.9	159	8	711	2612.317
SPX	CBOE	04/03/2018	2614.45	SPXW 180413C02780000	04/13/2018	2780	C	E	0.35	0.2	41	7080	2612.317

Table 8.1: Raw Data

8.2 Data Pre-processing and Cleaning

The raw dataset did not include traded prices of the option contracts, therefore to substitute for that, we computed the mean of ask and bid and set that as the

substitute for option price (price). We then reformatted the two date variables (date and expiration) and used them to compute the time to expiration in days (duration) of each option. Furthermore, we added a column for the moneyness of each option, computed as the ratio of index price (adjusted close) and strike price (strike).

```
def alterData(df):
    df['price'] = (df['ask']+df['bid'])/2
    df['date'] = df['date'].apply(dateFormat)
    df['expiration'] = df['expiration'].apply(dateFormat)
    df['duration'] = df['expiration'] - df['date']
    df['duration'] = df['duration'].apply(findDays)
    df['moneyness'] = df['adjusted close'] / df['strike']
    return df
```

Listing 1: Data Preparation

To clean the dataset, we dropped all entries representing put options, since our objective was to work with call options only. We also dropped entries with duration less than 10 days and price less than \$5, as such entries were found to be unrealistic trading scenarios which would only confuse our neural network. We then dropped all the unnecessary columns of data, obtaining a final dataset of 957,564 data points with columns arranged as follows: Trading day (date), Time to Expiry (duration), Adjusted Index Close Price (close), Strike Price (strike) and Option Price (price).

```
def dropData(df):
    df = df.dropna(how = 'all')
    df.drop(df[df['call/put'] == 'P'].index, inplace = True)
    df.drop(df[df['style'] != 'E'].index, inplace = True)
    df.drop(df[df['duration'] < 10].index, inplace = True)
    df.drop(df[df['price'] < 5].index, inplace = True)
    df.drop(columns = ['symbol', 'exchange', 'option symbol',
                      'style', 'call/put', 'ask', 'bid', 'expiration', 'volume',
                      'open interest', 'unadjusted'], inplace = True)
    return df
```

Listing 2: Data Cleaning

Figure 8.1 is the pairwise scatter plot of our final cleaned dataset. It can be seen that there is a linear relation between strike and price, but other pairs of features are primarily nonlinear with price. This supports our hypothesis that a neural network should be able a good learner of option prices, as opposed to more traditional methods like linear regression.

We used moneyness to split the data into different datasets in order to obtain a narrower range of data for more efficient training. The moneyness is defined as the ratio of index price (S) and strike price (X). Options are labelled as in-the-money, at-the-money and out-of-the-money with regards to the moneyness. A call option

date	duration	close	strike	price
02/01/2018	10	2695.81	1200	1492.2
02/01/2018	10	2695.81	1300	1392.3
02/01/2018	10	2695.81	1400	1292.35
02/01/2018	10	2695.81	1500	1192.4
02/01/2018	10	2695.81	1600	1092.45
02/01/2018	10	2695.81	1650	1042.5
02/01/2018	10	2695.81	1700	992.55
02/01/2018	10	2695.81	1750	942.6
02/01/2018	10	2695.81	1800	892.55
02/01/2018	10	2695.81	1850	842.6

Table 8.2: Final Data

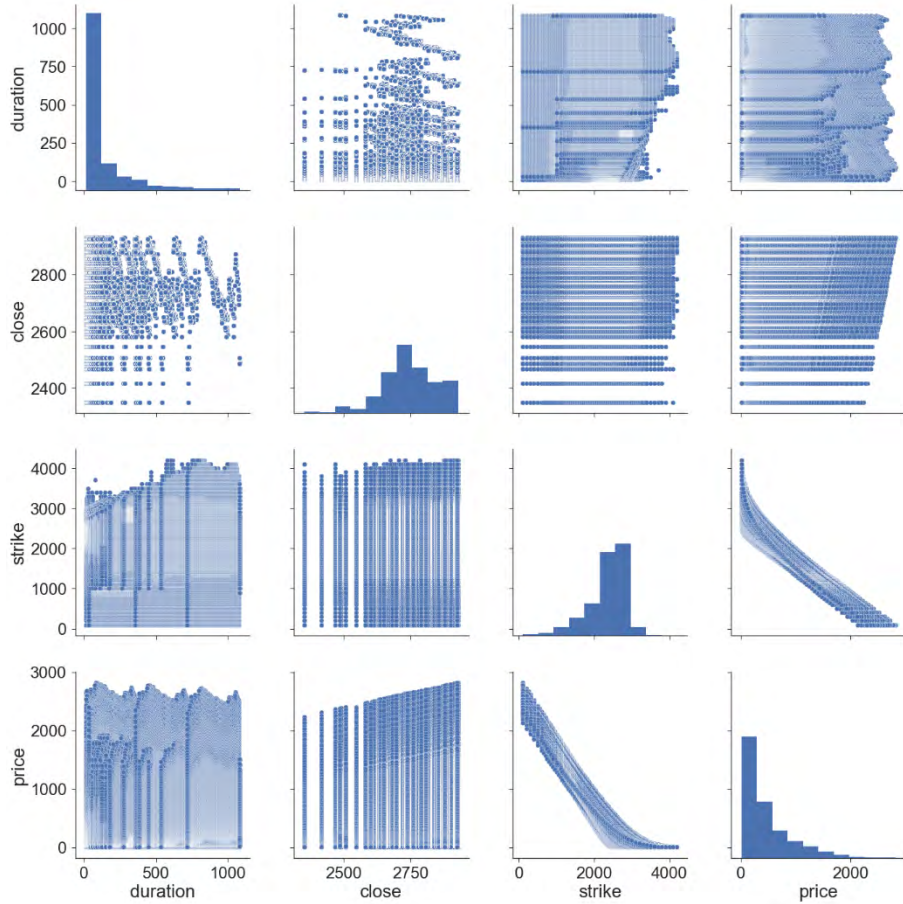


Figure 8.1: Pairwise Scatter Plot

is in-the-money when $S > X$, at-the-money when $S = X$ and out-of-the-money when $S < X$. In day-to-day real life trading, at-the-money options are hard to come across. Therefore, we label the options with $S \approx X$ as at-the-money. We use α to signify the

tolerance of $S \approx X$ to define the at-the-money dataset. Table 8.3 shows the number of entries in each of our dataset based on the choice of α . To balance the number of datapoints in each dataset, α has been set at 0.02.

α	OUT ($S/X \leq 1 - \alpha$)	AT ($1 - \alpha < S/X \leq 1 + \alpha$)	IN ($S/X > 1 + \alpha$)
0.02	119611	123398	714555
0.03	94995	177915	684654
0.04	75445	226794	655325
0.05	60258	270439	626867
0.06	48457	310373	598736

Table 8.3: Number of entries in each dataset based on choice of α

As a final pre-processing step before moving onto training, the input variables close, strike and duration were normalized on a scale between -1 to 1. However, we kept our target variable (price) unscaled. This was done to avoid having to revert the output after forecasting, which could have led to loss of information. Our data was partitioned into training, validation and testing sets following a partitioning rule of 70%, 20% and 10% respectively.

```

#-----Data Scaling-----#
max_abs_scaler = preprocessing.MaxAbsScaler()
df[['duration', 'close', 'strike']] = max_abs_scaler.fit_transform
    (df[['duration', 'close', 'strike']])

#-----Train/Validate/Test Split-----#
n = len(df.index)

n_train = (int)(0.7 * n)
train = df[0 : n_train]
x_train = train[['duration', 'close', 'strike']].values
y_train = train['price'].values

n_val = (int)(0.9 * n)
val = df[n_train+1 : n_val]
x_val = val[['duration', 'close', 'strike']].values
y_val = val['price'].values

test = df[n_val+1 : n]
x_test = test[['duration', 'close', 'strike']].values
y_test = test['price'].values

```

Listing 3: Preprocessing

The total number of points of the data sets we explored, and corresponding train-validation-test split for our neural network models can be found in Table 8.4. Dataset spx18c refers to the whole data we have; while spx18c-in, spx18c-at, and spx18c-out refers to the in-the-money, at-the-money, and out-of-the-money datasets respectively.

Name	All	Train	Validation
spx18c	957564	670294	191512
spx18c-in	714555	500188	142910
spx18c-at	123398	86378	24679
spx18c-out	119611	83727	23921

Table 8.4: Number of entries in each dataset

Chapter 9

Neural Network Model

9.1 Multi-Layer Perceptron

Morelli et al. [18] states that neural networks (NN) are known as universal approximators, since they are able to capture and model a variety of unknown relationships, once a sufficiently large sample of training data $(\vec{x}_t, \hat{y}_t)$ is provided. Looking at this from a mathematical perspective, the modelling task can be put in the form of minimizing the following loss function:

$$H(\hat{f}) \equiv \sum_{t=1}^T (\|\hat{y}_t - \hat{f}(\vec{x}_t)\|^2 + \lambda \|\nabla f(\vec{x}_t)\|^2) \quad (9.1)$$

Here ∇ is the gradient operator and λ is a parameter representing the smoothness of the required solution. This is reduced to minimize the distance between a given set of points $\hat{y}_t$ and the answer of the unknown function $\hat{f}(\vec{x}_t)$.

Morelli et al. [18] further states that the chosen Multi-Layer Perceptron (MLP) is a single layer back-propagation network which uses the scaled conjugate gradient (SCG) technique to update the parameters of the network. This network can be summarized as follows:

$$\hat{f}(\vec{x}_t) = h \sum_{i=1}^k c_i h(\beta_{0i} + \vec{\beta}_{1i} \vec{x}_t) + c_0 \quad (9.2)$$

Where $(\vec{x}_t)$ are the input variables of the data, $h(x)$ is a smooth, monotonic, increasing function, known as the activation function, which for our case is the Rectified Linear Unit (ReLU) function: $f(x) = x^+ = \max(0, x)$, c and β_i are the parameters tuned by the SCG technique to minimize the error and k is the number of hidden layer nodes or neurons. The Loss function used in our study is Mean Absolute Percentage Error represented by:

$$M = \frac{1}{n} \sum_{t=1}^n \left| \frac{A_t - F_t}{A_t} \right| \quad (9.3)$$

Morelli et al. [18] also states that a typical backpropagation neural network usually consists of one input layer, one or more hidden layers and one output layer. The individual nodes starting from the input layer to the output layer are connected in a feedforward structure. The weights of each connection are initially given random values. The error between the predicted output value and the actual expected value is calculated and backpropagated through the network and is used to update the weights in the next iteration. This is a supervised learning procedure that aims to minimize the error between the actual and the predicted output values. Figure 9.1 shows a two-hidden-layer neural network which is a generic representation of the models used in our research.

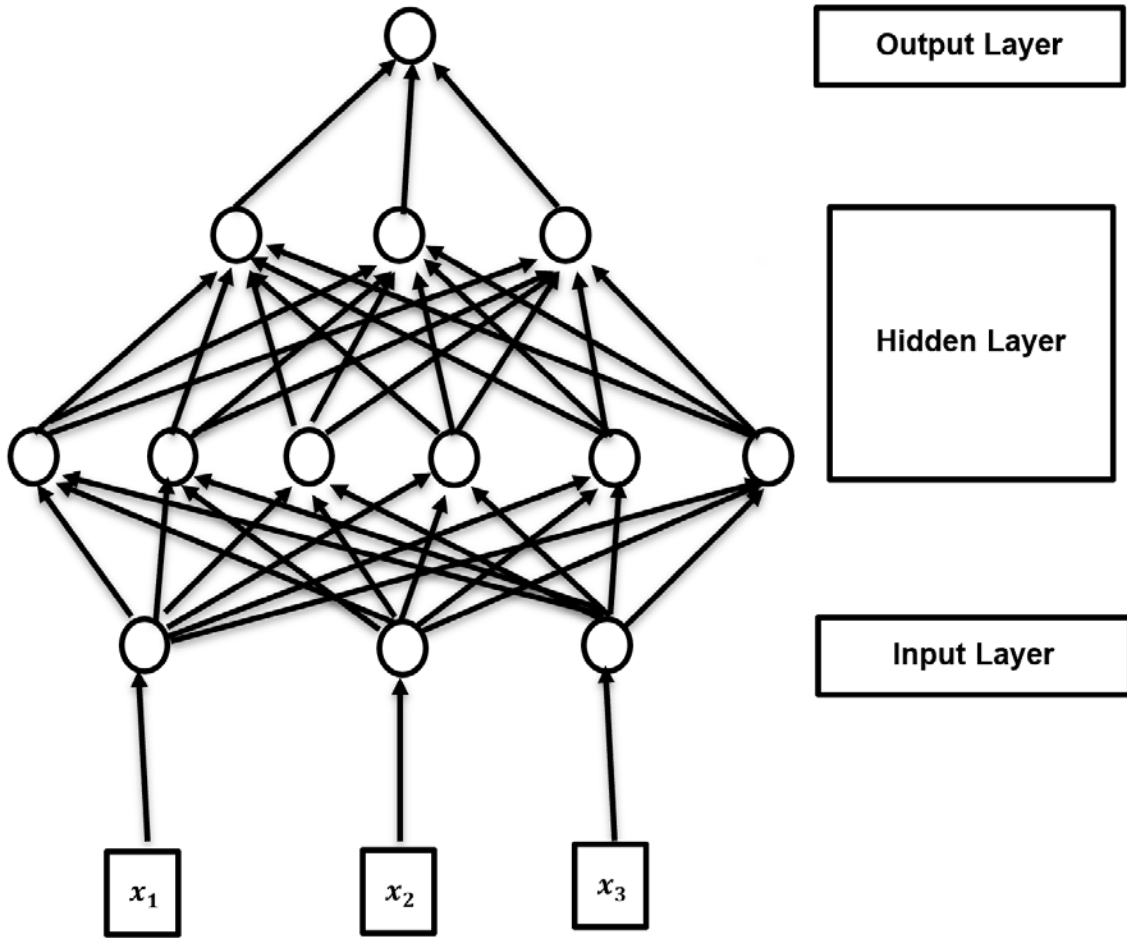


Figure 9.1: A two-hidden-layer feed-forward neural network

Provided sufficient data, a neural network can in theory model any kind of data pattern given that it is trained sufficiently, with sufficient data. When using a neural network as a forecasting tool, the first step involves training the network. During training, the neural network will iteratively learn the nature of the data based on our given inputs and output. For example, we have a large dataset consisting of four columns - x_1 , x_2 , x_3 and y . The neural network may be trained on the data patterns (x_1, x_2, x_3, y) , where the inputs are x_1 , x_2 and x_3 while the output is y . For this training, the hypothesis is that y depends on x_1 , x_2 and x_3 . Provided a sufficient variety of data tuples, a neural network model representing the relationship between the inputs and the output can be constructed, and this model can be used to forecast

the value of y based on the values of x_1 , x_2 and x_3 .

For our MLP NN model, various architectures have been experimented over a combination of nodes for the two hidden layers and we tried to obtain the model resulting in minimum training error and testing error. With 3 nodes as input and 1 node as output, different combinations of hidden layer nodes were implemented for each of our datasets.

The model is designed as follows:

```
#---Parameters of Each Model---#
h1 = # number of nodes in the first hidden layer
h2 = # number of nodes in the second hidden layer
batch = # batch size
epoch = # number of epochs
#-----#

model = Sequential()
model.add(Dense(h1, input_dim = 3, activation = 'relu'))
model.add(Dense(h2, activation = 'relu'))
model.add(Dense(1))

model.compile(loss = 'mape', optimizer = 'rmsprop',
              metrics = ['accuracy'])

model.fit(x_train, y_train, batch_size = batch, epochs = 20,
          validation_data = (x_val, y_val), verbose = 2)
model.save("model.h5")
```

Listing 4: MLP Model

Chapter 10

Results and Discussion

10.1 Experiment 1 – Entire Dataset

For the model dealing with the entire dataset, we determined an architecture of 6 neurons in the first hidden layer and 3 neurons in the second hidden layer gave the best results.

```
Train on 670294 samples, validate on 191512 samples
Epoch 1/20
- 28s - loss: 89.2151 - acc: 1.9394e-04 - val_loss: 87.9951 - val_acc: 2.0364e-04
Epoch 2/20
- 26s - loss: 88.5195 - acc: 1.8947e-04 - val_loss: 87.7194 - val_acc: 2.0364e-04
Epoch 3/20
- 21s - loss: 87.7909 - acc: 1.9991e-04 - val_loss: 85.5561 - val_acc: 1.9320e-04
Epoch 4/20
- 17s - loss: 72.1097 - acc: 3.8789e-04 - val_loss: 50.1820 - val_acc: 5.4305e-04
Epoch 5/20
- 18s - loss: 21.6856 - acc: 8.3545e-04 - val_loss: 15.8366 - val_acc: 9.1378e-04
Epoch 6/20
- 27s - loss: 10.1727 - acc: 0.0012 - val_loss: 14.4170 - val_acc: 9.6600e-04
Epoch 7/20
- 26s - loss: 8.5142 - acc: 0.0016 - val_loss: 12.5081 - val_acc: 0.0011
Epoch 8/20
- 26s - loss: 7.6472 - acc: 0.0021 - val_loss: 12.0299 - val_acc: 0.0013
Epoch 9/20
- 25s - loss: 7.3880 - acc: 0.0025 - val_loss: 11.4663 - val_acc: 0.0016
Epoch 10/20
- 17s - loss: 7.3337 - acc: 0.0027 - val_loss: 11.7046 - val_acc: 0.0014
Epoch 11/20
- 17s - loss: 7.3105 - acc: 0.0026 - val_loss: 12.3324 - val_acc: 0.0014
Epoch 12/20
- 17s - loss: 7.2954 - acc: 0.0027 - val_loss: 12.3797 - val_acc: 0.0017
Epoch 13/20
- 17s - loss: 7.2800 - acc: 0.0028 - val_loss: 12.2552 - val_acc: 0.0018
Epoch 14/20
- 17s - loss: 7.2697 - acc: 0.0028 - val_loss: 12.4381 - val_acc: 0.0019
Epoch 15/20
- 17s - loss: 7.2691 - acc: 0.0027 - val_loss: 11.5828 - val_acc: 0.0020
Epoch 16/20
- 21s - loss: 7.2597 - acc: 0.0028 - val_loss: 11.9105 - val_acc: 0.0019
Epoch 17/20
- 26s - loss: 7.2540 - acc: 0.0028 - val_loss: 11.8564 - val_acc: 0.0015
Epoch 18/20
- 26s - loss: 7.2488 - acc: 0.0029 - val_loss: 11.2167 - val_acc: 0.0020
Epoch 19/20
- 27s - loss: 7.2473 - acc: 0.0028 - val_loss: 11.7047 - val_acc: 0.0018
Epoch 20/20
- 27s - loss: 7.2416 - acc: 0.0028 - val_loss: 11.6428 - val_acc: 0.0021
```

Figure 10.1: Training on all options

The model had a training error of around 10% and a testing error of almost 19%. This large difference in error was likely due to an uneven distribution of options of varying moneyness in the training and testing sets.

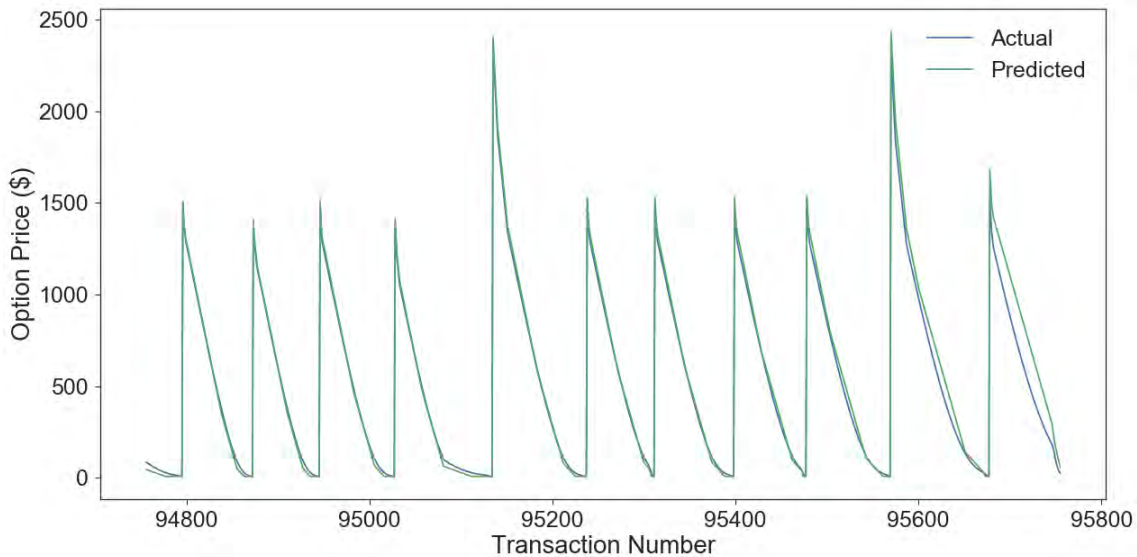


Figure 10.2: Last 1000 transactions of all options

10.2 Experiment 2 – In-the-money Options

The in-the-money model behaved very similarly to the entire dataset, but performed significantly better. This was likely due to the majority of options in our core dataset being in-the-money. Similar to the previous model, we determined 6 neurons in the first hidden layer and 3 neurons in the second hidden layer as the optimal architecture.

The model converged with a training error of about 1.5% and a testing error of just under 3%. This came out to be the best performer among all our models.

10.3 Experiment 3 – At-the-money Options

For at-the-money options, a configuration of 6 neurons in the first hidden layer and 4 neurons in the second hidden layer resulted in the barely acceptable, but best possible model.

With a training error of almost 15% and a testing error of around 20%, this model was surely underwhelming.

Train on 500188 samples, validate on 142910 samples

Epoch 1/20
- 23s - loss: 1.5788 - acc: 0.0035 - val_loss: 2.7886 - val_acc: 0.0022

Epoch 2/20
- 23s - loss: 1.5738 - acc: 0.0035 - val_loss: 2.6751 - val_acc: 0.0027

Epoch 3/20
- 23s - loss: 1.5681 - acc: 0.0035 - val_loss: 2.6362 - val_acc: 0.0027

Epoch 4/20
- 23s - loss: 1.5633 - acc: 0.0036 - val_loss: 2.6354 - val_acc: 0.0029

Epoch 5/20
- 24s - loss: 1.5585 - acc: 0.0036 - val_loss: 2.7590 - val_acc: 0.0026

Epoch 6/20
- 23s - loss: 1.5543 - acc: 0.0036 - val_loss: 3.1454 - val_acc: 0.0015

Epoch 7/20
- 23s - loss: 1.5501 - acc: 0.0037 - val_loss: 2.8536 - val_acc: 0.0022

Epoch 8/20
- 23s - loss: 1.5459 - acc: 0.0037 - val_loss: 2.7621 - val_acc: 0.0021

Epoch 9/20
- 23s - loss: 1.5422 - acc: 0.0036 - val_loss: 2.7961 - val_acc: 0.0026

Epoch 10/20
- 23s - loss: 1.5388 - acc: 0.0037 - val_loss: 2.6445 - val_acc: 0.0030

Epoch 11/20
- 23s - loss: 1.5357 - acc: 0.0036 - val_loss: 2.5654 - val_acc: 0.0030

Epoch 12/20
- 23s - loss: 1.5325 - acc: 0.0037 - val_loss: 3.0461 - val_acc: 0.0016

Epoch 13/20
- 23s - loss: 1.5298 - acc: 0.0038 - val_loss: 2.7294 - val_acc: 0.0024

Epoch 14/20
- 23s - loss: 1.5278 - acc: 0.0037 - val_loss: 3.0741 - val_acc: 0.0016

Epoch 15/20
- 23s - loss: 1.5250 - acc: 0.0038 - val_loss: 2.6998 - val_acc: 0.0024

Epoch 16/20
- 23s - loss: 1.5227 - acc: 0.0038 - val_loss: 2.7512 - val_acc: 0.0026

Epoch 17/20
- 23s - loss: 1.5212 - acc: 0.0039 - val_loss: 2.8426 - val_acc: 0.0020

Epoch 18/20
- 23s - loss: 1.5189 - acc: 0.0037 - val_loss: 3.2843 - val_acc: 0.0013

Epoch 19/20
- 20s - loss: 1.5167 - acc: 0.0038 - val_loss: 2.9381 - val_acc: 0.0022

Epoch 20/20
- 16s - loss: 1.5155 - acc: 0.0040 - val_loss: 2.8153 - val_acc: 0.0021

Figure 10.3: Training on in-the-money options

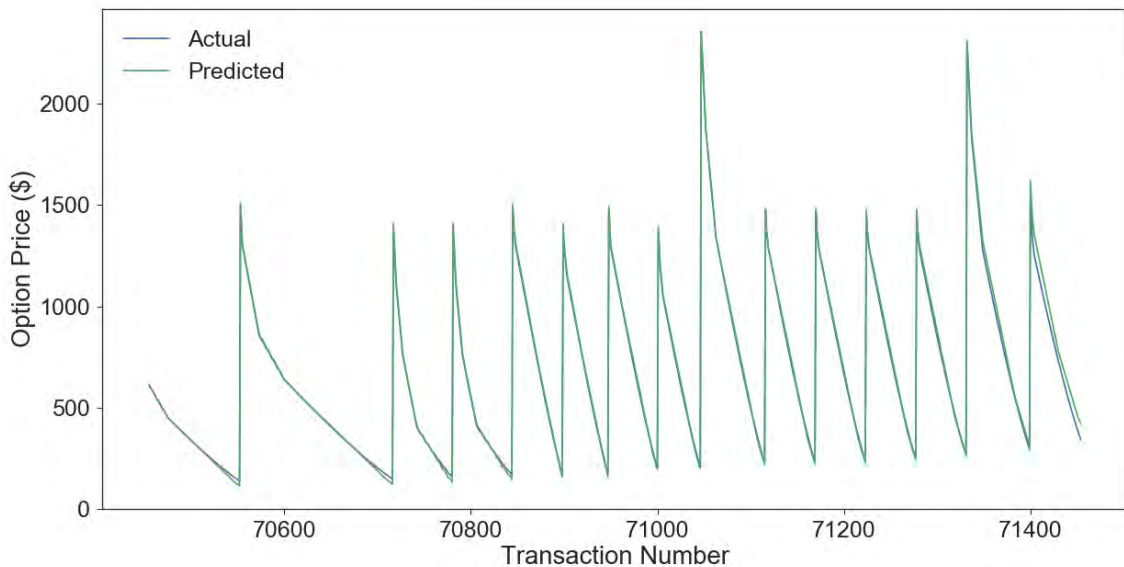


Figure 10.4: Last 1000 transactions of in-the-money options

```

Train on 86378 samples, validate on 24679 samples
Epoch 1/50
- 7s - loss: 13.8825 - acc: 0.0029 - val_loss: 25.3622 - val_acc: 0.0014
Epoch 2/50
- 7s - loss: 13.8398 - acc: 0.0032 - val_loss: 29.8048 - val_acc: 4.4572e-04
Epoch 3/50
- 7s - loss: 13.8487 - acc: 0.0030 - val_loss: 25.1381 - val_acc: 0.0023
Epoch 4/50
- 7s - loss: 13.7810 - acc: 0.0031 - val_loss: 28.5597 - val_acc: 8.1041e-04
Epoch 5/50
- 7s - loss: 13.7120 - acc: 0.0031 - val_loss: 23.4543 - val_acc: 0.0023
Epoch 6/50
- 9s - loss: 13.6163 - acc: 0.0032 - val_loss: 31.2943 - val_acc: 2.8364e-04
Epoch 7/50
- 11s - loss: 13.5640 - acc: 0.0031 - val_loss: 25.6115 - val_acc: 0.0016
Epoch 8/50
- 11s - loss: 13.4868 - acc: 0.0031 - val_loss: 23.6374 - val_acc: 0.0019
Epoch 9/50
- 12s - loss: 13.4949 - acc: 0.0029 - val_loss: 24.2336 - val_acc: 0.0022
Epoch 10/50
- 11s - loss: 13.4325 - acc: 0.0031 - val_loss: 22.9644 - val_acc: 0.0018
.....
Epoch 41/50
- 11s - loss: 13.0907 - acc: 0.0036 - val_loss: 25.4139 - val_acc: 0.0018
Epoch 42/50
- 11s - loss: 13.0648 - acc: 0.0033 - val_loss: 24.7348 - val_acc: 0.0015
Epoch 43/50
- 10s - loss: 13.0723 - acc: 0.0033 - val_loss: 28.4503 - val_acc: 7.6989e-04
Epoch 44/50
- 10s - loss: 13.0457 - acc: 0.0039 - val_loss: 23.4972 - val_acc: 0.0022
Epoch 45/50
- 8s - loss: 13.0578 - acc: 0.0033 - val_loss: 22.4328 - val_acc: 0.0016
Epoch 46/50
- 7s - loss: 13.0875 - acc: 0.0036 - val_loss: 21.1826 - val_acc: 0.0016
Epoch 47/50
- 7s - loss: 13.0659 - acc: 0.0033 - val_loss: 24.5385 - val_acc: 0.0022
Epoch 48/50
- 7s - loss: 13.0554 - acc: 0.0032 - val_loss: 24.4187 - val_acc: 0.0020
Epoch 49/50
- 7s - loss: 13.0446 - acc: 0.0034 - val_loss: 23.1650 - val_acc: 0.0020
Epoch 50/50
- 6s - loss: 13.0321 - acc: 0.0036 - val_loss: 30.6879 - val_acc: 5.2676e-04

```

Figure 10.5: Training on at-the-money options

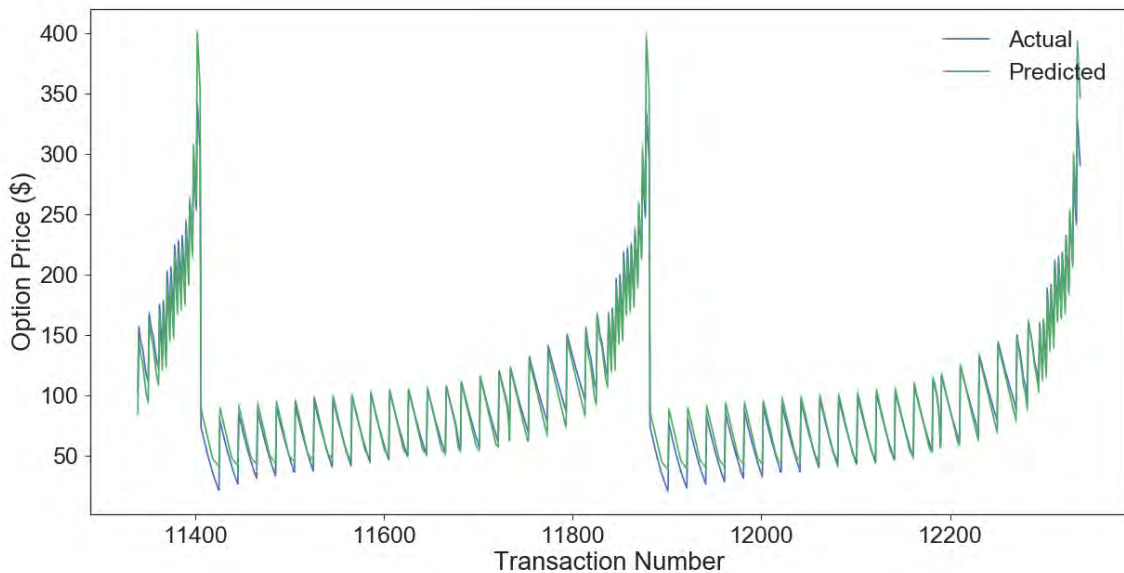


Figure 10.6: Last 1000 transactions of at-the-money options

10.4 Experiment 4 – Out-of-the-money Options

The neural network was by far the least successful with out-of-the-money options. The best performing configuration was determined to be the same as that of experiment 3 - 6 neurons in the first hidden layer and 4 neurons in the second hidden layer.

```
Train on 83727 samples, validate on 23921 samples
Epoch 1/50
- 5s - loss: 27.6251 - acc: 0.0052 - val_loss: 41.2807 - val_acc: 0.0031
Epoch 2/50
- 5s - loss: 27.5867 - acc: 0.0051 - val_loss: 38.7516 - val_acc: 0.0028
Epoch 3/50
- 5s - loss: 27.5319 - acc: 0.0056 - val_loss: 41.4580 - val_acc: 0.0028
Epoch 4/50
- 5s - loss: 27.5181 - acc: 0.0057 - val_loss: 37.3754 - val_acc: 0.0032
Epoch 5/50
- 5s - loss: 27.4584 - acc: 0.0058 - val_loss: 41.2820 - val_acc: 0.0028
Epoch 6/50
- 5s - loss: 27.4084 - acc: 0.0058 - val_loss: 39.3387 - val_acc: 0.0029
Epoch 7/50
- 5s - loss: 27.3958 - acc: 0.0058 - val_loss: 34.8586 - val_acc: 0.0031
Epoch 8/50
- 5s - loss: 27.3299 - acc: 0.0060 - val_loss: 38.7229 - val_acc: 0.0032
Epoch 9/50
- 5s - loss: 27.2990 - acc: 0.0062 - val_loss: 35.7915 - val_acc: 0.0033
Epoch 10/50
- 6s - loss: 27.2298 - acc: 0.0061 - val_loss: 40.5698 - val_acc: 0.0029
.....
Epoch 41/50
- 12s - loss: 26.4677 - acc: 0.0047 - val_loss: 38.3237 - val_acc: 0.0031
Epoch 42/50
- 12s - loss: 26.4617 - acc: 0.0049 - val_loss: 40.1344 - val_acc: 0.0029
Epoch 43/50
- 12s - loss: 26.4278 - acc: 0.0046 - val_loss: 35.5456 - val_acc: 0.0035
Epoch 44/50
- 12s - loss: 26.4349 - acc: 0.0047 - val_loss: 36.6960 - val_acc: 0.0033
Epoch 45/50
- 12s - loss: 26.4224 - acc: 0.0048 - val_loss: 38.0592 - val_acc: 0.0033
Epoch 46/50
- 12s - loss: 26.4433 - acc: 0.0050 - val_loss: 36.1101 - val_acc: 0.0028
Epoch 47/50
- 12s - loss: 26.4288 - acc: 0.0047 - val_loss: 36.6483 - val_acc: 0.0031
Epoch 48/50
- 12s - loss: 26.4294 - acc: 0.0049 - val_loss: 38.7553 - val_acc: 0.0031
Epoch 49/50
- 12s - loss: 26.3994 - acc: 0.0047 - val_loss: 38.6996 - val_acc: 0.0029
Epoch 50/50
- 12s - loss: 26.4052 - acc: 0.0051 - val_loss: 38.0338 - val_acc: 0.0029
```

Figure 10.7: Training on out-of-the-money options

The model barely managed to converge, with a training error of around 26% and a testing error of almost 40%.

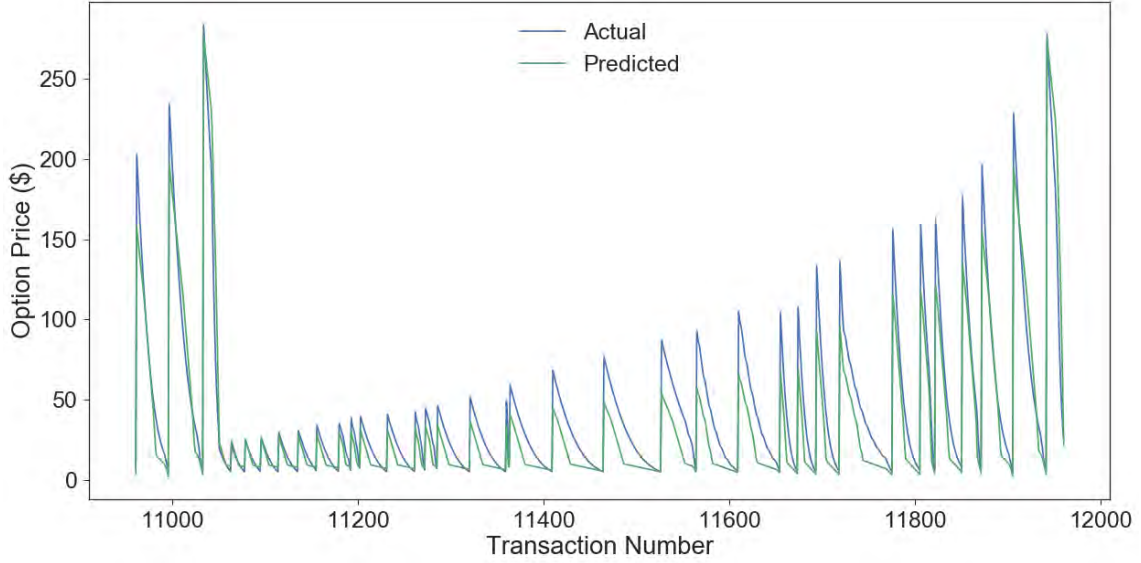


Figure 10.8: Last 1000 transactions of out-of-the-money options

10.5 Discussion

Dataset	Model	Training Error %	Testing Error %
spx18c	3-6-3-1	10.53	18.90
spx18c-in	3-6-3-1	1.48	2.79
spx18c-at	3-6-4-1	14.78	21.83
spx18c-out	3-6-4-1	26.13	38.95

Table 10.1: Neural Network Results for different models

From Table 10.1, it is evident that our neural network model performs exceptionally on in-the-money options, whereas it performs poorest on out-of-the-money options. On the other hand, the neural network is able to price the collection of options with mixed moneyness with an acceptable degree of error, but that may be primarily due to the larger portion of our dataset being in-the-money.

According to Ritchken [7], the Black-Scholes model is generally known to misprice in-the-money and out-of-the-money options. Our neural network, on one hand, prices in-the-money options exceptionally well; but on the other hand, grossly misprices out-of-the-money options. Clearly the neural network cannot completely replace the Black-Scholes model, but they can both be complementary to each other, as one makes up for the lacking of the other to a certain degree.

In the Black-Scholes Model, implied volatility is calculated on the basis of historical data such as the standard deviation of daily asset returns for a given period. This estimation of implied volatility is one of the biggest challenges of the Black-Scholes Model. In our model, the implied volatility was not used as an input. This was

based on the assumption that the features of the daily assets were fed into the network and therefore the daily returns, the core factor of implied volatility, could easily be captured and modelled by the neural network. We found that partitioning the data arbitrarily caused the neural network to fail to converge, which confirmed our assumption that the implied volatility is indeed time-dependent, and sequential partitioning of the data is crucial to modeling option prices. Therefore, our data was partitioned in transaction order, which resulted in the implied volatility being encapsulated by the neural network in its black box (hidden layers).

Chapter 11

Conclusion

Our result demonstrate that the MLP Neural Network methodology is a suitable alternative to the Black-Scholes Model for estimating in-the-money option prices. However, it can not replace the Black-Scholes model entirely, as the latter is superior for at-the-money options. Neither stands out when it comes to out-of-the-money options, which means there is still a need for further study into other potential models. As it stands now, the Black-Scholes Model used alongside a Neural Network has great potential, although in an ideal market, the Black-Scholes Model should be prioritized as the first choice. Conversely, the Neural Network model has the potential of being applied to American options, where the traditional Black-Scholes Model cannot be implemented. Moreover, the neural network model is more appropriate for highly volatile markets where the Black-Scholes' assumption of stable volatility is violated. Partitioning the data according to different moneyness ranges helps the Neural Network build a more robust model. Also, since maintaining the transaction sequence of the data results in the neural network successfully capturing the implied volatility, it can be implemented using a 'sliding window' format, where on every trading day, n days of historical data is used to generate a model to forecast m days of option prices. This implementation could very well make it's way to the trading floor in the future.

For further development of our model we will be looking for more sophisticated algorithms for the optimization of the Multi-layer Perceptron architecture. Since there are several limitations that restrict the use of neural network model for estimation such as determining the appropriate number of nodes for the hidden layer, the model's training thus becomes computationally intensive while its results are bound by the selection of learning parameters, activation function and the composition of dataset. This creates room for further experimentation to build even more robust models.

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