Axion Monodromy Inflation and the Late Time Universe

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Declaration

It is hereby declared that

1. The thesis submitted is our own original work while completing degree at BRAC University.
2. The thesis does not contain material previously published or written by a third party, except where this is appropriately cited through full and accurate referencing.
3. The thesis does not contain material which has been accepted, or submitted, for any other degree or diploma at a university or other institution.
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Abstract

In this paper we analyse the viability of a solution for early and late time evolution of the universe. The standard model of cosmology, the ΛCDM model, contains subtle inconsistencies with the nature of the Cosmic Microwave Background radiation. To alleviate these problems cosmic inflation of some form is required to explain early universe evolution. In this paper, we focus on a large-field inflationary model called axion monodromy inflation (AMI). Large-field models require a UV-complete to fully explain the high-energy physical phenomena. We opt to use superstring theory as the UV-complete theory. This means that we must construct a de Sitter minimum for the universe at late time from superstring theory, for which we choose the Kachru Kallosh Linde Trivedi (KKLT) mechanism. We analyse the consequences of taking these two mechanisms together and conclude that a naïve integration of the two models is not consistent, leading to an η-problem in the axion monodromy model.

Keywords: Early Universe Cosmology; Cosmic Inflation; String Theory; KKLT; Axion Monodromy Inflation; Eta Problem

Dedication

Dedicated to Shariq,

who has become an unwitting student of Quantum Gravity but will still listen to my unending ramblings.

- Saif Ar Rasul

For Ma and Baba,

often harangued but indefatigably supportive.

- Sitima Moeen

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The support of these wonderful people can scarcely be ranked, and was pivotal to the existence of this work.

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Conventions

All expressions in this work make use of natural units where

$$c = 1,\ \hbar = 1,\ M_{\rm Pl}^2 = 1/8\pi G = 1. \tag{0.1}$$

Where units are reintroduced for analysis, it will be stated explicitly.

Index Conventions

Throughout this work, unless stated otherwise in a particular section, the Greek letter indices such as μ,ν,ρ,σ will run over Minkowski spacetime directions $0,1,2,3$ and lowercase Latin letter indices, i,j,k etc. will run over purely spatial directions $1,2,3$. Uppercase Latin letters $M,N,..$ runs over all dimensions in the theory up to $D=26$. When decomposed, the Greek indices run over the Minkowski spacetime and, $m,n,..$ runs over the extra spacetime directions from 4 to D.

Metric Conventions

In general, the metric for Minkowski space is taken in the (+,-,-,-) convention.

Moduli Conventions

The axio-dilaton, S, will be complexified as

$$S = e^{-\phi} + iC_0 \tag{0.2}$$

and the singular Kähler modulus is

$$T = \tau + i\rho \tag{0.3}$$

where τ is the volume modulus.

Chapter 1

Introduction

Our ultimate goal as cosmologists is to come up with a theory that explains the whole universe for all of cosmic time. The universe, being quite large, requires a theory of Physics at the largest scales. It happens to be the case that at this scale, the major interaction between bodies is gravitation.

General relativity, our best effort at describing gravitation, is remarkably precise. According to Pogge [1], the time dilation calculated for a satellite orbiting the Earth results in a discrepancy of $38\mu s$ a day compared to a clock on Earth. Without taking into account this seemingly insignificant deviation, the global GPS array would fail to give accurate positional measurements within 2 minutes and introduce an error of 10 km per day. Using general relativity, we have put forward what is currently considered the standard model of cosmology, the ΛCDM model, which combines our classical understanding of the universe using the so-called FLRW metric with the addition of the recently discovered dark matter and dark energy. The success of this model is evident in its numerous agreements with experiment, such as the detection of the Cosmic Microwave Background (CMB), a remarkably homogeneous isotropic background of radiation. The temperature spectrum of the CMB was found to be an accurate match to the Planck spectrum of a black body which is difficult to account for in an expanding universe without a hot stage [2]. This along with further results in Nucleosynthesis, which predicted the abundance of light elements in the universe to great accuracy, made the Big Bang a very successful model of cosmology.

However the standard model of cosmology starts to exhibit subtle problems once we take a closer look. While detection of the CMB radiation was pivotal in solidifying standard cosmology as we know it, upon further inspection several aspects of the CMB turn out to be inexplicable using only standard cosmology. Issues start to arise that cannot be explained by a classical "Big Bang" scenario. To address these issues, Alan Guth proposed the theory of Cosmic Inflation in 1981 [3], by hypothesising a period of accelerating expansion right before the Big Bang as presented in standard cosmology. However inflation is not a theory per se, it is a framework for building theories. The task remains to produce a concrete model of inflation that both solves the issues plaguing standard cosmology while also matching with current day observations such as from BICEP2 and the Planck satellite.

There is an extra challenge to understanding the early universe. In Physics, a successful theory breaking down at a certain regime is usually the result of unknown and not-yet-understood Physics becoming relevant in that regime. While general relativity is wildly successful, it breaks down at the center of black holes and at the beginning of the universe. This is because in both these regimes we shrink down below the Planck length and exceed energies past the Planck mass - and in precisely this combination of conditions our understanding of Physics breaks down. General relativity approaches the realm of quantum field theory, and a reconciliation of these two theories has not yet been made, at least not one that is generally agreed upon. So if we are to understand the universe at early times, it seems we need a quantum theory of gravity, usually called a quantum gravity. The most mathematically complete, calculable theory so far has been string theory, and it will be our high-energy theory of choice in this paper. Since the time of Guth, many models of inflation have been put forward. Many models exist that employ string theory, as can be seen in Baumann and McAllister [4]. We shall focus on a specific model of inflation called axion monodromy inflation (AMI). There are a few reasons why it is an interesting candidate model.

1. It is still viable within experimental bounds set by the 2018 Planck data [5].
2. Axion monodromy, as a large-field model, may be confirmed by near-future measurements of B-modes of the CMB anisotropy field.
3. Remnant axions after inflation may also partially or fully explain the abundance of dark matter in the universe.

If we are to put stock in string theory being the fundamental theory of the universe, it concerns us to see if we can recover our familiar universe at "late time" (now). This is especially relevant since the modern formulation of string theory, superstring theory, is only consistent in 10 dimensions and also predicts the existence of new particles which come from the extra symmetry known as supersymmetry. To fix the problem of having too many dimensions we compactify the extra dimensions - curl them up into a compact space with a very small radius to the point of undetectability - to produce the apparent 4D universe like the one we observe. The problem is that this usually produces a universe with a negative cosmological constant, Λ, known as an anti-de Sitter space. However, all observations point to the fact that our universe has a very small but positive cosmological constant, evidenced from the accelerating expansion, which corresponds to a de Sitter space. Part of the problem here is that supersymmetric vacuums seem to naturally end up being anti-de Sitter, and since supersymmetry is unobserved in the first place, one way is to add something to the theory that breaks the supersymmetry but also "lifts" the vacuum to a de Sitter vacuum. A seminal paper on string cosmology proposed a mechanism to do just that. The KKLT mechanism [6], in which the eponymous authors Kachru, Kallosh, Linde and Trivedi put forward a procedure for constructing a de Sitter vacuum from string theory computations.

Our paper seeks to assess the viability of axion monodromy inflation when applied to the universe together with the KKLT mechanism for producing the late-time de Sitter vacuum. We look at the constraints on the parameters of the inflation model as well as the evaluate the effect this may have on the η-problem, an issue that can arise in inflationary theory preventing it from being successful.

Structurally, the paper is broken down as follows. In chapter 2, we give a brief review of standard cosmology before the advent of inflation, along with evidence behind and motivation for the widespread acceptance of the theory and lastly an explanation of the problems that arise in standard cosmology. Then in chapter 3 we explain the core idea of cosmic inflation, how it addresses each problem in turn, walk through the calculations for a classical field theory that would give rise to inflation in a slow-roll model, discuss the parameters to keep track of inflation and end on a brief phenomenological discussion of the stages of inflation. Chapter 4 is a review of the parts of superstring theory required to discuss AMI and KKLT. Then in Chapter 5 we start by talking about the problems with natural inflation, a more naïve model of inflation involving axions, and how we resolve them with the AMI model. We follow this up with some of our calculations on inflationary constraints using the 2018 Planck data [5]. In chapter 6 we explain the KKLT mechanism and then discuss the behaviour of AMI when modified by the imposition of constraints from KKLT. Lastly, we discuss future research directions and end with some concluding remarks.

Chapter 2

Standard Cosmology

The best validated description of the evolution of the universe so far is the Λ-CDM model which maps the different stages of the universe as different eras and models expansion for each one. To do this the expansion of an isotropic, homogeneous universe is related to its energy content via the Friedmann equations and the change in expansion is mapped for each type of energy. It is observed that for all known entities the expansion is decelerating but the expansion of the present-day universe is accelerating. This is accounted for by the introduction of a cosmological constant into the general relativistic relationship between energy and geometry.

The following sections detail the understanding of the evolution of the universe offered by standard cosmology, up to the concordance (ΛCDM) model. The following sections are based on discussions in Zyla et al. [2] and Baumann [7].

2.1 Radii

The **Friedmann-Lemaître-Robertson-Walker metric**, which was used to model the evolution of the stages of the universe, relies on the assumption of an isotropic and homogeneous universe. In co-moving coordinates, it is given as

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right). \tag{2.1}$$

where k is the curvature constant. The **scale factor**, $a(t)$ denotes the relative size of the hypersurface of three dimensional space in a particular timeslice.

The **Hubble parameter** denotes rate of expansion, $H = \frac{\dot{a}}{a}$. These can then be used to calculate the horizons which determine the observable regions of spacetime.

The (co-moving) **particle horizon** (CPH), χ_{ph}, is the maximum co-moving distance light can travel from the origin of the universe (which is set to $t = 0 \Rightarrow \tau = 0 \Rightarrow a = 0$) up until a time t. It is defined by

$$\chi_{\mathrm{ph}} = \tau(t) - \tau(0) = \int_0^t \frac{dt}{a(t)} = \int_0^a d\ln a \frac{1}{aH}. \tag{2.2}$$

It is therefore the farthest point of causal contact. The quantity $d\tau = dt/a$ is defined as conformal time, and can be thought of as time that gets shorter as the universe expands. It is the time as experienced by a light ray at that point.

The (co-moving) **Hubble radius** (CHR), $(aH)^{-1}$, is the distance at which objects are receding from an observer at the speed of light. This means this is the farthest possible distance from which objects can communicate with an observer at this time [7]. It should be noted, this is different to particle horizon which is the maximum distance at which objects could have ever communicated. The relationship between these horizons is governed by the evolution of $a(t)$, such that it depends on the energy content of the universe and which type of energy is dominating at any point. In standard cosmology when only the characteristics of ordinary matter is considered, $\chi_{ph} \sim (aH)^{-1}$

2.2 The Friedmann Equations

The next task would be to determine how this expansion may have progressed. This is done by using the general relativistic precept that the geometry of spacetime will be determined by what is contained within it. To begin we use the **Einstein field equations**,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}. \tag{2.3}$$

$T\mu\nu$ is the **stress-energy tensor**, determined by the distribution of matter-energy. It is conventional to assume the universe is a perfect fluid with

$$T_{\mu\nu} = (\rho + P) u_\mu u_\nu - P g_{\mu\nu}. \tag{2.4}$$

where u^μ is the velocity vector for the fluid in co-moving coordinates. Plugging this into equation (2.3) along with equation (2.1), we get the **Friedmann equations** which relates rate of expansion of spacetime to the composition of its energy content. They are

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \tag{2.5}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P). \tag{2.6}$$

Applying the law of mass-energy conservation, $\nabla_\mu T_{\mu\nu} = 0$, to this we get the continuity equation [2],

$$\dot{\rho} = -3\left(\frac{\dot{a}}{a}\right)(\rho + P). \tag{2.7}$$

Different forms of energy have different density to pressure relationships that can be introduced to equation (2.6). For this we define the equation of state parameter $\omega \equiv \frac{P}{\rho}$, and integrate equation (2.7) to give,

$$\rho \propto a^{-3(1+\omega)}. \tag{2.8}$$

The ω for each type of matter are given in Baumann [7]. Matter is defined as anything that has $\omega = 0$, which gives $\rho \sim a^{-3}$. This is like saying density scales with inverse of volume. Radiation (or relativistic matter) is defined as having $\omega = \frac{1}{3}$ which gives $\rho \sim a^{-4}$. This means not only is the number density of particles spread out by expansion but so is their energy. For both of these ω obeys the strong energy condition $\omega \geq 0$ which is simply the assumption that energy density is always positive. This is an assumption from equations (2.3), Einstein's equations, to prevent gravity from becoming repulsive. However, dark energy has been measured to have $\omega \approx -1$ which produces the relationship $\rho \propto a^0$. This implies that the energy content increases as spacetime grows, and can be expressed in Einstein's equation (2.3) by the addition of the cosmological constant, Λ[1].

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{2.9}$$

[1]In Quantum Field Theory, there is a prediction for energy inherent to empty spacetime given by the ground state energy of the vacuum. This could theoretically motivate a contribution to the energy content as new spacetime is created. But the value predicted by QFT deviates from astronomical observations by about 10^{120}.

This parameter need not, however, be constant and could be dynamically evolving.

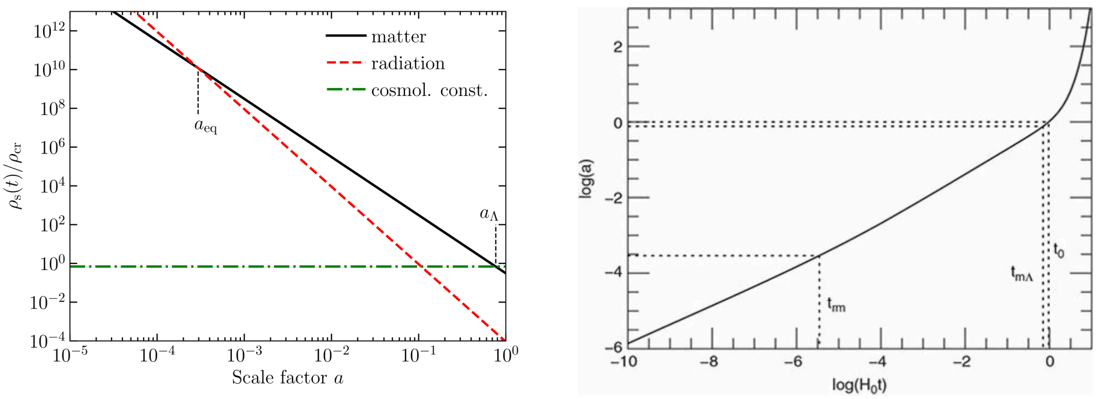
With the addition of the cosmological constant the Friedmann equations take the form [2]

$$\begin{aligned} H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}, \\ \frac{\ddot{a}}{a} &= \frac{\Lambda}{3} - \frac{4\pi G}{3}(\rho + 3p). \end{aligned} \tag{2.10}$$

The second equation makes obvious how a positive cosmological constant adds to accelerated expansion. Using equation (2.8) and ω's to find the time evolution of the scale factor and mapping it we get

$$a(t) = \begin{cases} t^{\frac{2}{3}(1+\omega)}, & \text{if } \omega \geq 0 \\ \exp Ht, & \text{if } \omega = -1 \end{cases}. \tag{2.11}$$

The results are used to plot the following graphs.



(a) ρ vs. a

(b) a vs. t

Figure 2.1: Logarithmic plots showing evolution of a determined by ρ. , Reproduced from Dodelson [8] and Ryden [9].

The different functions of density come to dominate the change of a at different times, t, thus leading to the second graph with different "eras" of expansion. The first is radiation-dominated, the second is matter-dominated, and the current is the cosmological constant(?)-dominated era. Current measurements of red-shift also suggest that the expansion of the universe is accelerating [7]. As such the need for cosmological models to corroborate a positive Λ term in the metric is significant.

This complete trajectory for the evolution of the universe is known as the

ΛCDM model [8] and, along with Big Bang Nucleosynthesis, was extremely effective at explaining the universe as it is observed today. However there were stark gaps in its explanatory power, particularly to do with the isotropy observed in the CMB.

2.3 Outstanding Problems in Standard Cosmology

2.3.1 Horizon and Flatness problem

The solutions to the Friedmann equations give a result for the age of the universe which causes a significant problem. As stated the CMB is extremely homogeneous, with a temperature distribution that is uniform to an order of $\Delta T/T \approx 10^{-5}$ [2]. But there are points in the CMB which are outside of each others' particle horizons when calculated from the time of the Big Bang singularity to the formation of the background. This would suggest thermalisation between what would have been causally disconnected regions of spacetime. Fluctuations grow with expansion rather than shrink, so the deviations in temperature must have been even smaller in the past. Because patches of spacetime that should be outside each other's particle horizon are communicating (thermalising), this is known as the **horizon problem**. This is illustrated in figure (2.2).

A further problem is observed in [2] by manipulating the Friedmann equation in the following way:

$$H^2 = \frac{1}{3}\rho(a) - \frac{k}{a^2} \tag{2.12}$$

$$\Rightarrow 1 = \frac{1}{3H^2}\rho(a) - \frac{k}{(aH)^2} \tag{2.13}$$

$$\Rightarrow 1 - \frac{\rho(a)}{\rho_{\text{crit}}} = -\frac{k}{(aH)^2} \tag{2.14}$$

$$\Rightarrow 1 - \Omega(a) = -\frac{k}{(aH)^2}. \tag{2.15}$$

Here $\Omega(a)$ is the curvature parameter, which is determined by the ratio of the actual energy density to the critical energy density, $\rho_{\text{crit}} := 3H^2$. A value of $\Omega = 1$ would correspond to perfect flatness. One can see from equation (2.15) that any deviation of $\Omega(a)$ from 1, *i.e.* perfect flatness, will grow with the co-moving Hubble radius. An increasing co-moving Hubble radius cor-

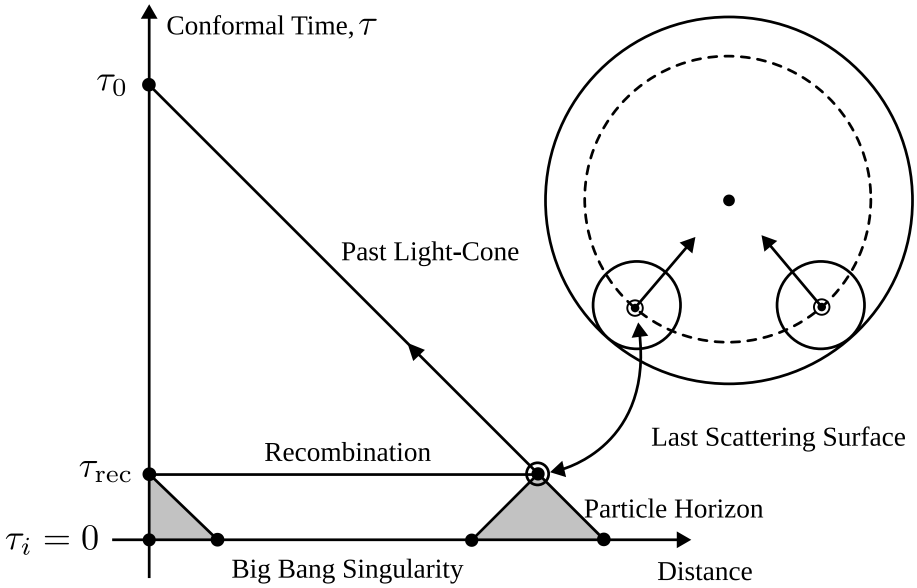


Figure 2.2: A conformal diagram of the past light cone in standard Big Bang cosmology. This is a reproduction of figure 8 of Baumann [10]

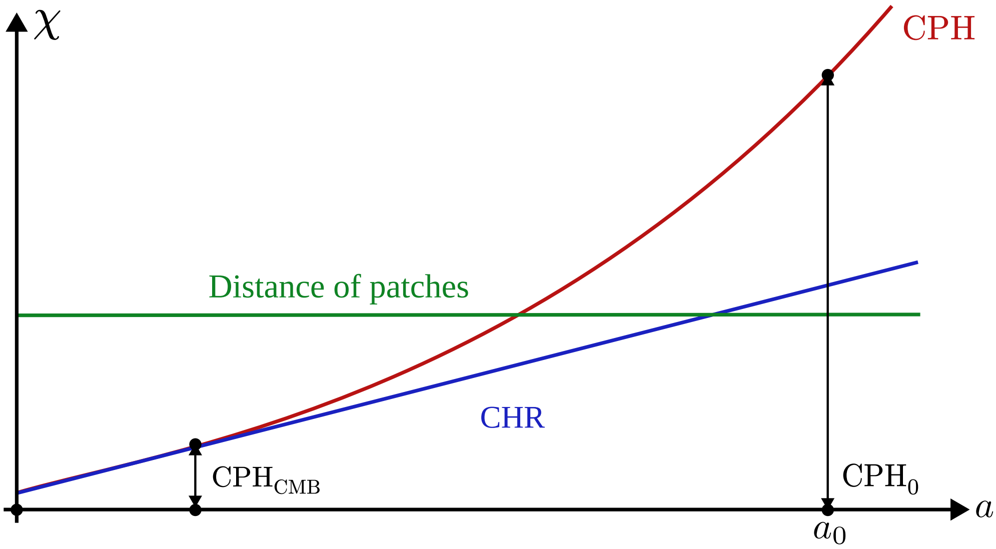


Figure 2.3: Plot of the various co-moving distance versus scale factor *a* in standard Big Bang cosmology.

responds to a universe that deviates away from flatness quickly. The current measurement from WMAP data for $1-\Omega$ is at 0.03 [11]. Since this value diverges during expansion this requires the value at time of the point of greatest density, i.e. the beginning of expansion, to be as low as 10^{-61}, an incredibly microscopic deviation from flatness which would nonetheless be essential to existence of structure in the universe. Any large deviation would have caused the universe to collapse early during expansion or expand too rapidly for anything to form [7].

The Hubble parameter is positive for an expanding universe, and the Big Bang model requires the Hubble radius to increase monotonically with time. This means if we travel back in time the early universe should be in far less communication with itself but nonetheless produce a world that is so perfectly calibrated that it gives rise to a flat expansive universe with large-scale structures. The dramatic flatness of the early universe cannot be predicted by standard cosmology, but must instead be assumed in the initial conditions. Likewise, the striking large-scale homogeneity of the universe is not explained or predicted by the model, but instead must simply be assumed.

2.3.2 Magnetic Monopoles

Inflation was originally conceived to address the monopole problem. Both Maxwell's theory of electromagnetism as well as Grand Unified Theories (GUTs) predict the existence of magnetic monopoles. According to GUT theories, topological phase-changes that should have happened near the beginning of the universe should have produced large quantities of magnetic monopoles. This becomes a problem in the hot Big Bang model, when one calculates the number of monopoles produced in events, such as the electroweak symmetry breaking. One finds that they would be the dominant matter in the universe [9]. This is contrary to the fact that no monopole has ever been observed. The monopoles would affect the curvature of the universe and in turn the Hubble parameter, galaxy formation, etc. making them unwanted relics.

However magnetic monopoles have not yet been observed. We will not focus on the monopole problem in this paper, in favour of giving more time to the horizon and flatness problems.

* * *

These problems are not strictly inconsistent with the concordance model, but they require gross ad-hoc assumptions, *i.e.* fine-tuning of parameters, about the beginning of the universe which is something we would like to avoid. Inflation naturally produces the very particular initial conditions that are presupposed in the Big Bang.

Chapter 3

Cosmic Inflation

Cosmic Inflation is a period of exponential expansion of space in the universe hypothesised to have occurred shortly after the $t = 0$ singularity. In this period, the co-moving Hubble radius decreases, thereby addressing the **horizon** and **flatness problems**, which plague standard cosmology. The stretching and subsequent freezing of quantum fluctuations could also explain the formation of energy density variations and thus of large structure in the universe. For reference, this section is based on the works of Baumann and McAllister [4], Baumann [10], Di Marco, Orazi, and Pradisi [12] and Vaudrevange [13].

3.1 Slow-Roll Inflation

Inflation can be achieved in multiple ways. Slow-roll inflation is a simple variant of inflation that postulates the existence of a single scalar field, ϕ, whose energy contribution dominates over a local patch of spacetime. The potential for this scalar field has a very shallow gradient and the initial field amplitude is displaced with respect to the global minimum. The field very slowly rolls towards the minimum while in the flat region, during which time the potential term dominates the kinetic term causing exponential expansion. At some point corresponding to a critical value of the inflaton field, ϕ_{end}, the kinetic term regains dominance and inflation ends. This initiates a period called **reheating**, during which, through coupling to standard model particles, the inflaton releases its stored energy, “heating up” the universe.

3.2 Addressing the Outstanding Problems with Standard Cosmology

Since both the horizon and flatness problem can be thought of as stemming from the monotonically increasing co-moving Hubble radius, $(aH)^{-1}$, the inflationary solution is to propose a theoretical framework where the co-moving Hubble radius decreases for a sufficiently long period of time to account for observations of homogeneity and flatness.

3.2.1 Addressing the Horizon Problem

The co-moving Hubble radius is always positive and therefore always adds to the co-moving particle horizon via the integral given by equation (2.2). However, if the co-moving Hubble radius shrinks for a period of time, the co-moving Hubble radius could have been larger in the past and contributed more to the co-moving particle horizon. This is illustrated in figure (3.1). For a certain size of the co-moving particle horizon, then, we could have had a much larger co-moving Hubble radius during the creation of the CMB. Thus, given inflation lasts for long enough, the co-moving Hubble radius could be large enough to encompass the two distinct patches of the universe, at least at the time that the CMB would have been formed.

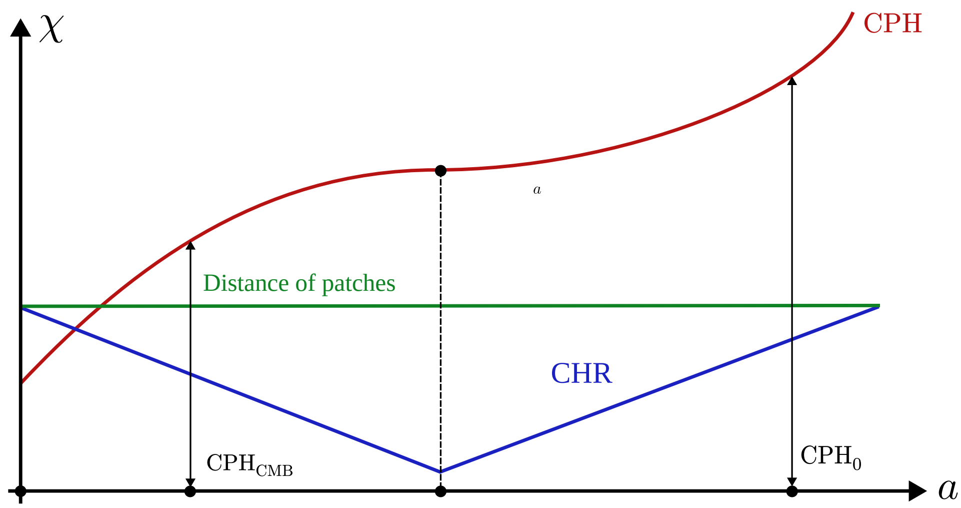


Figure 3.1: Plot of the various co-moving distance versus scale factor a with the addition of inflationary period.

We can also visualise this using the idea of **conformal diagrams of space-time**. In the standard Big Bang cosmology $a(\tau = 0) = 0$ (in conformal time), so the universe seems to start at $\tau = 0$ and as can be seen in figure (2.2), two distant parts of space are causally disconnected and yet appear to be in thermal equilibrium.

Inflation has the effect of changing $a(\tau)$ to $-\frac{1}{H_\tau}$ which pushes $a = 0$ to $\tau = -\infty$, that is, the infinite past (again in terms of conformal time) and that means that we have "much more time" than we thought before the apparent Big Bang for the past light cones to trace back and intersect, thereby allowing two now-disconnected regions to have communicated some time in the past, as can be seen from figure (3.2).

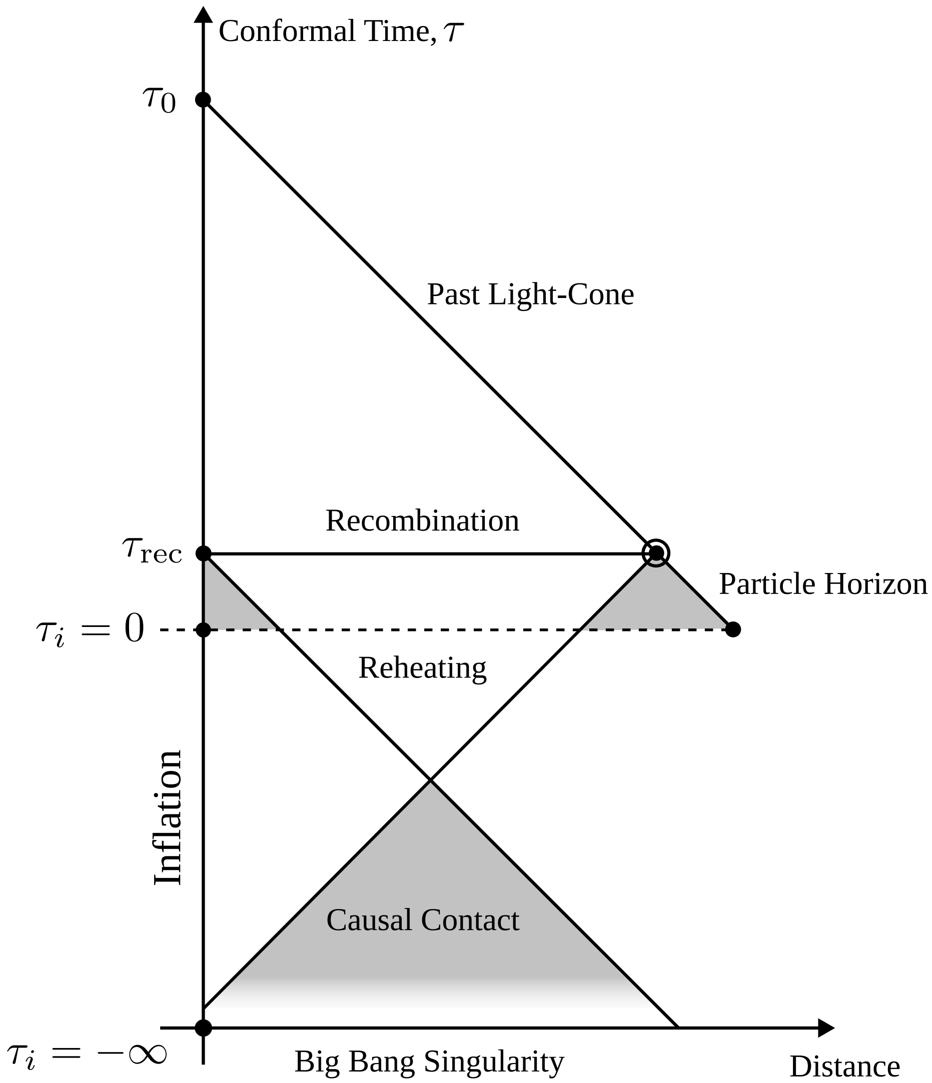


Figure 3.2: A conformal diagram of the past light cone with the addition of an inflationary period. This is a reproduction of figure 9 of Baumann [10]

Of course the analysis holds in regular time, as is necessary to be relevant.

Conformal time only serves to make the conclusion more obvious by transforming the curved light paths into straight $45°$ lines, making the diagram easier to read.

3.2.2 Addressing the Flatness Problem

If the co-moving Hubble radius were to decrease rather than increase, $\Omega = 1$, which used to be an solution of equation (2.15) where $\Omega = 1$ becomes an attractive point instead. This means that we expect the universe to be driven towards flatness during inflation rather than away from it, which means with enough inflation we can solve the fine-tuning of flatness in the current observable universe.

3.2.3 Addressing the Monopole Problem

As there would be a fixed number of monopoles formed after the topological phase change in the early universe, inflation can explain the lack of observations of magnetic monopoles by scaling up the universe to such a magnitude that the density of magnetic monopoles falls under ~ 1 per volume-of-our-observable-universe. In such a scenario, it is likely that we would not observe a single magnetic monopole.

3.3 Classical Dynamics of Slow-Roll Inflation

In this section we will look at the mathematics underlying the principle of slow-roll inflation.

Firstly, let us start off with the stipulation that we require a period of accelerated expansion. This corresponds to a period of decreasing co-moving Hubble radius, as can be seen from

$$\ddot{a} > 0 \Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{aH}\right) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\dot{a}}\right) = -\frac{\ddot{a}}{\dot{a}^2} < 0. \tag{3.1}$$

In this class of models, a scalar field, the so-called **inflaton**, ϕ, is used to generate the required period of accelerated expansion. Let us deduce the criteria under which this will occur:

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3P) \tag{3.2}$$

$$\ddot{a} > 0 \Rightarrow P < -\frac{1}{3}\rho \tag{3.3}$$

In other words, our scalar field must exert negative pressure, since $\rho > 0$ always.

We introduce the Einstein-Hilbert action minimally coupled to ϕ [13],

$$S = \int \mathrm{d}^4x\sqrt{|g|}\left(\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right). \tag{3.4}$$

If we assume the ϕ is homogeneous in space we can omit the spatial part of the derivative to arrive at [10]

$$S = \int \mathrm{d}^4x\sqrt{|g|}\left(\frac{1}{2}R + \frac{1}{2}\dot{\phi}^2 - V(\phi)\right). \tag{3.5}$$

There are two sets of equations-of-motion, one obtained from varying the action with respect to the metric $g_{\mu\nu}$, which give the Einstein Field Equations,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}(\phi). \tag{3.6}$$

Inputting the FLRW metric in equation (3.6) leads to the equation

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) - \frac{k}{a^2}, \tag{3.7}$$

where equation (3.7) is the Friedmann equation for ϕ.

One may identify the expression on the RHS of equation (3.7) as the energy of the fluid filling space,

$$\rho_\phi := \frac{1}{2}\dot{\phi}^2 + V(\phi). \tag{3.8}$$

The other equation-of-motion comes from varying the action with respect to the scalar field ϕ, that being [13]

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V(\phi) = 0 \tag{3.9}$$

Implicitly differentiating equation (3.7) and substituting $\ddot{\phi}$ from equation (3.9) allows us to establish the equation

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2 + \frac{k}{a^2}, \tag{3.10}$$

from which we can infer

$$\Rightarrow \dot{H} + H^2 = -\frac{1}{6}\left(\rho_\phi + 3\left\{\frac{1}{2}\dot{\phi}^2 - V(\phi)\right\}\right) \tag{3.11}$$

$$\Rightarrow P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{3.12}$$

Using equation (3.8) and (3.12) one may compute the equation-of-state parameter, ω_ϕ,

$$\omega_\phi = \frac{P_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}. \tag{3.13}$$

We can rephrase equation (3.3) to

$$P_\phi < -\frac{1}{3}\rho_\phi \Rightarrow \omega_\phi < -\frac{1}{3}. \tag{3.14}$$

Thus equation (3.13) tells us that

$$\frac{1}{2}\dot{\phi}^2 \ll V(\phi) \tag{3.15}$$

to ensure the universe inflates. In other words, the kinetic energy of the field, ϕ, must be (much) less than its potential energy.

3.3.1 Slow-Roll Conditions

We would like to define some parameters that would allow us to track the progress of inflation.

First let us form a parameter to tell us whether the universe is inflating *at a certain point in time* (or equivalently, scale factor). From the definition of H we can deduce that

$$H^2 + \dot{H} = \frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = \frac{\ddot{a}}{a} \tag{3.16}$$

$$\Rightarrow \frac{\ddot{a}}{a} = H^2\left(1 + \frac{\dot{H}}{H^2}\right) := H^2(1 - \epsilon_H). \tag{3.17}$$

So if we define accelerated expansion as $\frac{\ddot{a}}{a} > 0$ then we get our **first slow-roll condition** [13]

$$0 < \epsilon_H < 1; \quad \epsilon_H := -\frac{\dot{H}}{H^2}. \tag{3.18}$$

If ϵ_H is within the given interval, the universe is currently inflating.

In addition to keeping track of the fact that inflation is active at any moment, we also need to keep track of the overall length of the inflationary period.

Suppose that we are currently inflating. This means, as we have established, that the kinetic energy of the inflaton field is much smaller than its potential energy. In order for the inflaton field to remain in this regime, the change in velocity of the field must be small, *i.e.* $\ddot{\phi}$ must be very small, so that the kinetic energy stays small. We formalise this by the requirement that we can neglect the $\ddot{\phi}$ term compared to the $-3H\dot{\phi}$, in other words the **the second slow-roll condition** is

$$|\ddot{\phi}| \ll \left|-3H\dot{\phi}\right|. \tag{3.19}$$

This condition can be encapsulated in a second slow-roll parameter:

$$\eta_H \ll 1; \quad \eta_H := -\frac{\ddot{\phi}}{H\dot{\phi}} = -\frac{1}{2}\frac{\ddot{H}}{\dot{H}H} \tag{3.20}$$

The value of η does not affect inflation directly, but larger values mean that ϵ will grow out of range too quickly for inflation to match observations, as indicated by the following relationship.

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}N} = 2\epsilon(\eta - \epsilon) \tag{3.21}$$

where N is known as the number of **e-folds** (until the end) of inflation. A situation in which η grows too quickly is known as an $\boldsymbol{\eta}$**-problem**.

The interpretation of N can be seen more clearly from the following computation.

$$dN = -Hdt \tag{3.22}$$

$$\Rightarrow N(t) = \int_t^{t_{\text{end}}} \mathrm{d}t'\, H(t') = \int_t^{t_{\text{end}}} \mathrm{d}t'\, \frac{\frac{\mathrm{d}a}{\mathrm{d}t}}{a} = \int_a^{a_{\text{end}}} \frac{\mathrm{d}a}{a} = \int_a^{a_{\text{end}}} \mathrm{d}(\ln a) \tag{3.23}$$

$$\Rightarrow N_{\text{total}} = \int_{a_{\text{start}}}^{a_{\text{end}}} \mathrm{d}(\ln a) = \ln\left(\frac{a_{\text{end}}}{a_{\text{start}}}\right) \tag{3.24}$$

$$\Rightarrow e^{N_{\text{total}}} = \frac{a_{\text{end}}}{a_{\text{start}}}. \tag{3.25}$$

In words, N captures how many times the universe will magnify by a factor of e until the end of inflation. It is used as a standard method of measuring the exponential growth of the universe. Current models put $40 \lesssim N_{\text{total}} \lesssim 60$ [10], in order for inflation to last long enough to expand the universe to a sufficient degree to account for observations of flatness and homogeneity.

3.3.2 Flatness of the Potential

If we utilise the constraint that kinetic energy must be much lower than potential energy from equation (3.15) and that the acceleration of the inflaton field, $\ddot{\phi}$, must be negligible from equation (3.19), equation (3.7) becomes

$$H^2 = \frac{1}{3}V(\phi). \tag{3.26}$$

and the other equation-of-motion of ϕ becomes

$$\begin{aligned} 3H\dot{\phi} + \partial_\phi V(\phi) &= 0 && (3.27)\\ \Rightarrow \dot{\phi} &= -\frac{1}{3H}\partial_\phi V(\phi). && (3.28) \end{aligned}$$

We can translate the slow-roll parameters to be in terms of $V(\phi)$. First let us use equation (3.28) to find $\dot{H}$, by setting

$$\begin{aligned} \dot{H} &\approx -\frac{1}{2}\dot{\phi}^2 \quad \text{(ignoring curvature)} && (3.29)\\ &= -\frac{1}{2}\frac{1}{9H^2}(\partial_\phi V(\phi))^2. && (3.30) \end{aligned}$$

Using (3.18), (3.20) and (3.30), we obtain the following **potential slow-roll parameters** (where M_{Pl} has been reinstated) [13]:

$$\begin{aligned} \epsilon_V &:= \frac{M_{\text{Pl}}^2}{2}\left(\frac{(\partial_\phi V(\phi))}{V}\right)^2 \approx \epsilon_H, && (3.31)\\ \eta_V &:= \frac{M_{\text{Pl}}^2 \partial_\phi^2 V(\phi)}{V} \approx \epsilon_H + \eta_H. && (3.32) \end{aligned}$$

In principle, the slow-roll parameters in terms of potential $V(\phi)$ and in terms of the Hubble parameter H are equivalent *only* when $\phi(t)$ has a trajectory approaching an **attractor solution**[1]. In practice, this seems to hold in the cases we are concerned with [12].

As such we conclude from these new potential slow-roll parameters, which contain the *slope* and *curvature* of the inflaton potential respectively, that since the slow-roll parameters must both be small for inflation to occur, this means that the potential, $V(\phi)$, must be close to flat in Planck units.

[1]An attractor solution is one in which the system veers towards the slow-roll regime, $\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$, over time.

3.3.3 Exponential Expansion

Lastly, let us show that the setup above does indeed produce a period of exponential expansion, as advertised.

First note that from equation (3.10) we can infer that:

$$\dot{H} \propto \partial_\phi V(\phi) \approx 0 \Rightarrow H = \text{constant} \Rightarrow H^2 = \text{constant}. \tag{3.33}$$

If we also use equation (3.17) we find that

$$\frac{\ddot{a}}{a} = H^2(1 - \epsilon_H) \approx H^2, \tag{3.34}$$

as long as $\epsilon_H \ll 1$, which should by definition be satisfied during inflation. From here we can deduce that

$$\frac{\ddot{a}}{a} \approx H^2 \tag{3.35}$$

$$\Rightarrow \ddot{a} = H^2 a \tag{3.36}$$

$$\Rightarrow a(t) = a_0 \exp H(t - t_0). \tag{3.37}$$

And thus we find that given the conditions we derived, we get an exponential growth in the scale factor $a(t)$, in other words, inflation. Note that this means that inflation corresponds to de Sitter expansion.

3.3.4 Phenomenological Synopsis of Slow-roll Inflation

Given the analysis above, a short, qualitative synopsis of inflation may be in order.

Before inflation, the universe is filled with a homogeneous scalar field that exerts negative pressure by virtue of having a potential energy much higher than its kinetic energy. This negative pressure acts as a source for the exponential expansion of spacetime. Like all systems, the inflaton tends to approach the minimum of its potential, but the inflaton potential is very shallow, so this rolling is very slow. The lower bound on the time it takes is calibrated to observations of flatness and homogeneity of the universe. At some point, the inflaton exits the region of the potential that is approximately flat, and the curvature becomes significant. Since the kinetic energy is proportional to the curvature, as can be seen from equation (3.28), the kinetic energy increases. Curving down also means that the potential energy decreases. Both of these factors break the condition that the potential

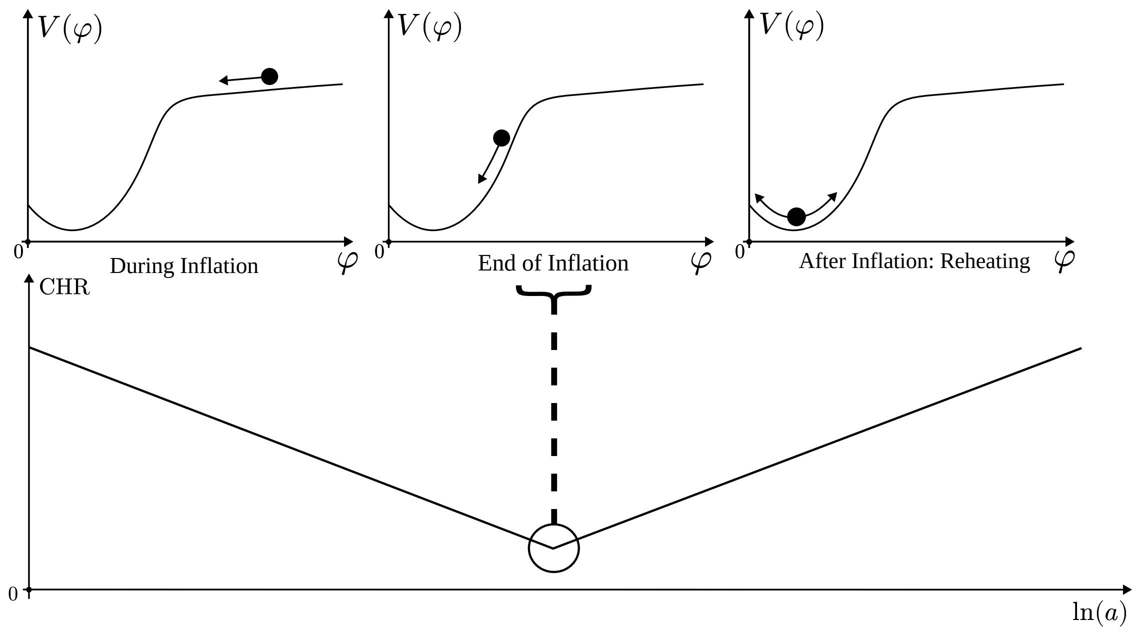


Figure 3.3: A heuristic diagram depicting the evolution of the inflaton over its potential and the co-moving Hubble radius during inflation.

energy of the inflaton field dominates the kinetic energy and so inflation ends. After this a period called **reheating** occurs where the inflaton loses its energy to radiation, which heats up the universe, preparing a hot, dense state which we observe as the beginning of the apparent hot Big Bang. After this, standard cosmology continues as established.

3.4 Inflationary Observables

Quantum fluctuations occur in the metric during inflation, denoted $\delta g_{\mu\nu}$, and in the matter fields, denoted $\delta T_{\mu\nu}$. These fluctuations can be decomposed into scalar (S), vector (V), and tensor (T) components in what is known as the **SVT decomposition**. In practice the vector component is negligible for the purposes of inflation [12], so we will focus on the scalar and tensor components. The following are some quantities relevant to characterising these quantum fluctuations.

3.4.1 Power Spectra

A **power spectrum**, in the context of inflation, is a measure of the squared amplitudes of metric perturbations during inflation . Perturbations are affected by choice of gauge, but this can be circumvented with clever choice of **gauge-invariant perturbations**. One such choice is the **co-moving cur-**

vature perturbation, $\mathcal{R}$, from which we define the **scalar power spectrum** Δ_s^2,

$$\Delta_s^2 := \Delta_{\mathcal{R}}^2 = \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k). \tag{3.38}$$

where $P_{\mathcal{R}}(\boldsymbol{k})$ is defined using the 2-point correlator between two modes of $\mathcal{R}$,

$$\langle \mathcal{R}_{\boldsymbol{k}} \mathcal{R}_{\boldsymbol{k}'} \rangle = (2\pi)^3 \delta(\boldsymbol{k} + \boldsymbol{k}') P_{\mathcal{R}}(k). \tag{3.39}$$

According to Baumann and McAllister [4], in the slow-roll approximation the scalar power spectrum boils down to

$$\Delta_s^2(k) = \frac{1}{24\pi^2} \frac{V(\phi)}{M_{\text{Pl}}^4} \frac{1}{\epsilon_V}\bigg|_{k=aH} = \frac{1}{12\pi^2} \frac{1}{M_{\text{Pl}}^6} \frac{V^3}{V'^2}\bigg|_{k=aH}. \tag{3.40}$$

The **tensor power spectrum** can be defined similarly if we account for the two polarisations of the tensor perturbation, h_{ij}, and so define

$$\Delta_t^2 := 2\Delta_h^2 = 2\frac{k^3}{2\pi^2} P_h(k), \tag{3.41}$$

where $P_h(k)$ is defined analogously to equation (3.39).

If we take the ratio of these the power spectra we obtain a quantity known as the **tensor-to-scalar ratio**, r,

$$r := \frac{\Delta_t^2(\boldsymbol{k})}{\Delta_s^2(\boldsymbol{k})} = 16\epsilon_V. \tag{3.42}$$

The tensor-to-scalar ratio can be shown to follow the **Lyth bound**,

$$\frac{\Delta\phi}{M_{\text{Pl}}} = \mathcal{O}(1) \times \sqrt{\frac{r}{0.01}}. \tag{3.43}$$

3.4.2 Spectral Tilt

In order to track the running of the scalar and tensor perturbations over co-moving wavenumber we can introduce the **scalar spectral index** and the **tensor spectral index** (or alternatively **spectral tilt**),

$$n_s - 1 \equiv \frac{\mathrm{d}\ln\Delta_s^2}{\mathrm{d}\ln k} = 2\eta_V - 6\epsilon_V; \quad n_t{}^{2} \equiv \frac{\mathrm{d}\ln\Delta_t^2}{\mathrm{d}\ln k} = -2\epsilon_V \tag{3.44}$$

[2] n_t is defined differently for historical reasons.

3.4.3 Scalar Amplitude

Using the scalar power spectrum, Δ_s^2, and the scalar spectral tilt, n_s, we can define the **scalar amplitude**, A_s (proportional to the **density fluctuations**). From Baumann and McAllister [4]

$$\Delta_S^2(k) = A_s \left(\frac{k}{k_\star}\right)^{n_s - 1} \tag{3.45}$$

where $k_\star$ is known as the **pivot scale**, which is used as a reference scale to standardise comparisons of theoretical and experimental values. We shall use a pivot scale of $k = 0.002$ Mpc^{-1} to match the 2018 Planck data [5].

Chapter 4

Superstring Theory

The model of inflation under consideration in this paper, axion monodromy inflation (AMI) is a large-field model ($\Delta\phi \geq M_{\text{Pl}}$), meaning that the perturbations produced during inflation and hence the energy scale must be very high, about $\delta g, \delta T \gtrsim O(M_{\text{Pl}})$, as indicated by the Lyth bound, (3.43), on the tensor-to-scalar ratio, r.

This allows us to make contact with recent and near-future observations. Observation efforts like BICEP2 and Planck are trying to ascertain increasingly accurate values for the magnitude of the tensor-to-scalar ratio. For large-field models the Lyth bound, (3.43), tells us that the tensor-to-scalar ratio, $r > 0.01$. These would mean large tensor perturbation amplitude, on the order of the Planck scale. In other words, they would correspond to detectable gravitational waves. These are encoded in the divergence-free modes of the CMB anisotropy vector field, known as its **B-mode**. Detecting these B-modes would constitute conclusive evidence of large-field inflation.

Given a large-field model, the inflaton will interact with Planck-scale physics. This will introduce quantum corrections from the heavier modes in the UV complete theory (see equation (5.1)). To protect the theory a symmetry needs to be present all the way through the journey across the energy scale. This means that we will approach a regime where the EFT requires input from the UV-complete theory, in other words, a theory of quantum gravity (QG). Axion inflation uses string theory as its candidate UV-complete theory, and so we shall establish a small subset of String Theory required to discuss both AMI as well as the KKLT mechanism.

4.1 Supersymmetry

For consistency reasons, string theory requires a symmetry called **supersymmetry**. Specifically, without supersymmetry string theory cannot produce fermions. As such, we first establish the formalism of supersymmetry. This section is based on the works of Krippendorf, Quevedo, and Schlotterer [14], Wess and Bagger [15] and Muller-kirsten and Wiedemann [16].

For a treatment of spinor representations and spinor indices see Krippendorf, Quevedo, and Schlotterer [14].

Symmetries come in two broad classes:

1. **Spacetime Symmetries** These act on spacetime coordinates of fields explicitly.

   $$x^{\mu} \longrightarrow x'^{\mu}(x^{\nu})$$

   Notably the Poincaré group fall under this category.

2. **Internal Symmetries** These symmetries connect different fields together by transforming them into each other, usually by rotating states in an abstract space:

   $$\Phi^{a}(x) \longrightarrow M^{a}{}_{b}\Phi^{b}(x)$$

   The standard model gauge group, $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)$ is an internal symmetry group.

The spacetime symmetries of the Standard Model are given by the Poincaré group, $\mathbb{R}^{1,3} \ltimes SO(1,3)$ which has the following Lie algebra:

$$\left[P^{\mu}, P^{\nu}\right] = 0, \tag{4.1}$$

$$\left[M^{\mu\nu}, P^{\mu}\right] = i(P^{\mu}\eta^{\nu\sigma} - P^{\nu}\eta^{\mu\sigma}), \tag{4.2}$$

$$\left[M^{\mu\nu}, M^{\rho\sigma}\right] = i(M^{\mu\sigma}\eta^{\nu\rho} + M^{\nu\rho}\eta^{\mu\sigma} - M^{\mu\rho}\eta^{\nu\sigma} - M^{\nu\sigma}\eta^{\mu\rho}). \tag{4.3}$$

where P^{μ} are the spacetime translation generators and $M^{\mu\nu}$ are the Lorentz group generators for rotation and boosts, given by

$$\text{Rotations, } J_i = \frac{1}{2}\varepsilon_{ijk}M^{jk}; \quad \text{Boosts, } K_i = M^{0i} \tag{4.4}$$

There is a no-go theorem by Coleman and Mandula [16] that states that, under certain assumptions, the symmetry group of a QFT with a scattering matrix S is constrained to be of the form *Poincaré* $\otimes$ *Internal*. Each generator corresponds to a continuous symmetry and thus a corresponding conserved charge. The P^μ generators correspond to linear momenta, J_i correspond to angular momenta, K_i correspond to centre-of-mass energy. The Coleman-Mandula theorem tells us that these are the only possible space-time charges, any more and the scattering angle would be overconstrained. However, Golfand and Likhtmann found a loophole in the theorem. One of the assumptions was that the generators must be bosonic, that is, carry only tensor indices. These operators are related to each other via a commutator, inside a Lie algebra. One may evade the theorem by adding spinorial generators, which would anticommute. These spinorial generators carry spinor charges and act as the generators for supersymmetry[1]. Adding anticommuting elements means that we must extend the Lie algebra of the Poincaré group to a **Lie superalgebra**.

4.1.1 The Poincaré Lie Superalgebra

The Poincaré Lie algebra is extended to the Poincaré Lie superalgebra (for a precise definition see Muller-kirsten and Wiedemann [16] or Wess and Bagger [15]) with the following commutators and anticommutators,

$$\left[Q_\alpha, M^{\mu\nu}\right] = (\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta, \tag{4.5}$$
$$\left[Q_\alpha, P^\mu\right] = \left[\bar{Q}^{\dot\alpha}, P^\mu\right] = 0, \tag{4.6}$$
$$\left\{Q_\alpha, Q_\beta\right\} = 0, \tag{4.7}$$
$$\left\{Q_\alpha, \bar{Q}_{\dot\beta}\right\} = 2(\sigma^\mu)_{\alpha\dot\beta} P_\mu. \tag{4.8}$$

The commutators and anticommutators for Q_α and $\bar{Q}_{\dot\beta}$ can be deduced from index-matching arguments, up to constant prefactors [14]. The indices α and β here run over $1,..,4$ as they are Majorana spinor indices. The matrices $(\sigma^\mu)^{\dot\alpha}{}_{\dot\beta}$ are the Pauli matrices and the matrices $(\sigma^{\mu\nu})_\alpha{}^\beta$ form a representation of the Lorentz algebra, and are given by

$$\begin{aligned}
(\sigma^{\mu\nu})_\alpha{}^\beta &:= \frac{i}{4}\left(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu\right)_\alpha{}^\beta, \\
(\bar\sigma^{\mu\nu})^{\dot\alpha}{}_{\dot\beta} &:= \frac{i}{4}\left(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu\right)^{\dot\alpha}{}_{\dot\beta}, \\
(\bar\sigma^\mu)^{\dot\alpha\alpha} &:= \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}\left(\sigma^\mu\right)_{\beta\dot\beta}.
\end{aligned} \tag{4.9}$$

[1]Later in 1975 Haag, Lopuszanski and Sohnius proved a generalised form of the Coleman-Mandula theorem that forbade any further fermionic generators that transform with higher dimensional representations of the Lorentz group from being included. As such, supersymmetry seems to be the *only* extension we can make that makes sense.

4.1.2 Action of Supersymmetry Operators

Supersymmetry proposes a symmetry between bosons and fermions. A short justification for interpreting Q_α and $\bar{Q}^{\dot{\alpha}}$ in this manner is outlined below.

Consider massless states, $p^\mu = (E, 0, 0, E)$. Evaluating the anticommutator gives us

$$\left\{Q_\alpha, \bar{Q}_{\dot{\beta}}\right\} = 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu = 2E(\sigma^0 + \sigma^3)_{\alpha\dot{\beta}} = 4E \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_{\alpha\dot{\beta}} \tag{4.10}$$

which means that

$$\{Q_1, \bar{Q}_{\dot{1}}\} = 4E; \quad \{Q_2, \bar{Q}_{\dot{2}}\} = 0. \tag{4.11}$$

We can use Q_1 and $\bar{Q}_1$ to construct ladder operators,

$$a := \frac{Q_1}{2\sqrt{E}}, \quad a^\dagger := \frac{\bar{Q}_1}{2\sqrt{E}}, \tag{4.12}$$

which inherit the anticommutation relations

$$\{a, a^\dagger\} = 1, \quad \{a, a\} = \{a^\dagger, a^\dagger\} = 0. \tag{4.13}$$

Now if we apply the J^3 operator on a state $|p, \lambda\rangle$ we get

$$J^3(a|p,\lambda\rangle) = \left(aJ^3 - \left[a, J^3\right]\right)|p^\mu, \lambda\rangle = \left(aJ^3 - \frac{a}{2}\right)|p^\mu, \lambda\rangle = \left(\lambda - \frac{1}{2}\right)(a|p^\mu, \lambda\rangle), \tag{4.14}$$

where we have used the result that $\left[a, J^3\right] = \frac{1}{2}a$ [14]. The calculation for $a^\dagger$ is similar and results in

$$J^3(a^\dagger|p^\mu, \lambda\rangle) = \left(\lambda + \frac{1}{2}\right)(a^\dagger|p^\mu, \lambda\rangle), \tag{4.15}$$

which uses $\left[J, a^\dagger\right] = -\frac{1}{2}a^\dagger$. So a removes half a unit of spin and $a^\dagger$ adds half a unit of spin. This means we find new particles in the spectrum which are called **superpartners** of the standard model particles, specifically we obtain a boson for every fermion and vice versa.

The case for massive particles is more complicated, but the action on spin is the same. A walk-through of the calculation can be found in Muller-kirsten and Wiedemann [16].

4.1.3 Superspace

An interesting way to view Minkowski space is to consider it as the coset space obtained when one takes the quotient of the Poincaré group by the Lorentz group. Elements of the Poincaré group are parametrised by $\{a^\mu, \omega^{\mu\nu}\}$ and the Lorentz group is parametrised by $\{\omega^{\mu\nu}\}$, so after we send $\omega^{\mu\nu} \to 0$ we are left with a^μ, which is isomorphic to the set of position vectors in Minkowski space.

The general element of the super-Poincaré group is [14]

$$g = \exp\left[i(\omega^{\mu\nu}M_{\mu\nu} + a^\mu P_\mu + \epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}})\right], \tag{4.16}$$

where ϵ is a Grassmann number (for a full treatment see Nakahara [17]). If we do the same thing with the super-Poincaré group, whose parameters are $\{a^\mu, \epsilon^\alpha, \bar{\epsilon}_{\dot{\alpha}}, \omega^{\mu\nu}\}$ we obtain a space with coordinates $(x^\mu, \theta_\alpha, \bar{\theta}^{\dot{\alpha}})$; this is called **superspace**. The additional coordinates, θ and $\bar{\theta}$, use Grassmann numbers, and are sometimes called "Grassmann directions".

4.1.4 Superfields

A **superfield**, $S\left(x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}\right)$, is a field in superspace is given by [14]

$$\begin{aligned} S\left(x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}\right) =& \phi(x) + \theta^\alpha \psi_\alpha(x) + \bar{\theta}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}(x) + \theta^\alpha\theta_\alpha M(x) + \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} N(x) \\ &+ \theta^\alpha \left(\sigma^\mu\right)_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} V_\mu(x) + \theta^\alpha\theta_\alpha\bar{\theta}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}}(x) \\ &+ \theta^\alpha\bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\rho_\alpha(x) + \theta^\alpha\theta_\alpha\bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} D(x). \end{aligned} \tag{4.17}$$

While S is the most general field we can write down, it is not an irreducible representation of the supersymmetric algebra. Smaller superfields can be obtained by imposing certain constraints. First define the supercovariant derivatives as

$$D_\alpha = \partial_\alpha + i(\sigma^\mu)_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu, \tag{4.18}$$

$$\bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i\theta^\beta(\sigma^\mu)_{\beta\dot{\alpha}}\partial_\mu. \tag{4.19}$$

1. We obtain a **chiral superfield** when we set $D_\alpha S = 0$. We denote these by Φ.
2. Similarly we obtain an **antichiral superfield** when we set $\bar{D}_{\dot{\alpha}} S = 0$. We denote these by $\bar{\Phi}$.

3. Lastly we obtain a **vector superfield** if we set $S = S^\dagger$. We denote these by V.

Out of these, of particular interest to us is the chiral superfield. Applying the constraint $D_\alpha \Phi = 0$ we obtain the following form for the superfield [14]

$$\Phi(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}) = \varphi(x) + \sqrt{2}\theta^\alpha \psi_\alpha(x) + \theta^\alpha\theta_\alpha F(x) + i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu\varphi(x) - \frac{i}{\sqrt{2}}(\theta^\alpha\theta_\alpha)\partial_\mu\psi^\beta(x)(\sigma^\mu)_{\beta\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} - \frac{1}{4}\theta^\alpha\theta_\alpha\bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu\partial^\mu\varphi(x). \tag{4.20}$$

4.1.5 $\mathcal{N} = 1$ Supergravity and the F-term Potential

In standard QFT when we synthesise viable Lagrangians we have to ensure that the action is Poincaré invariant. This amounts to requiring that the variation of the Lagrangian, $\delta\mathcal{L}$ in response to a general Poincaré transformation be a total derivative term, such that it vanishes upon integration.

Similarly, if we want to construct Lagrangians that also respect supersymmetry, we require that the Lagrangian we write in terms of superfields produce an action that is invariant under super-Poincaré transformations. It turns out that there are two types of terms for which this is true,

1. The general superfield S from (4.17) has a term called a D-term (the term with the coefficient D). Under a super-Poincaré transformation it transforms as $\delta D = \frac{i}{2}\partial_\mu(\epsilon\sigma^\mu\bar{\lambda} - \rho\sigma^\mu\bar{\epsilon})$, a total derivative. This is a good candidate for building a supersymmetric theory.

2. The other interesting term is from (4.20) which is called the F-term (similarly, the term with coefficient F). This term also has a variation that is a total derivative, $\delta F = i\sqrt{2}\,\overline{\epsilon\sigma}^\mu\partial_\mu\psi$

We shall focus on the chiral superfield as it is relevant to our work. The most general renormalizable Lagrangian one can write using Φ is [14]

$$\mathcal{L}_\Phi = K(\Phi, \Phi^\dagger)\Big|_D + \left(W(\Phi)\Big|_F + \text{h.c.}\right). \tag{4.21}$$

where $\Big|_D$ means picking out the D-term and $\Big|_F$ means picking out the F-term. The "h.c." means we add the Hermitian conjugates of the previous term.

Here $K(\Phi, \Phi^\dagger)$ is known as the **Kähler potential**. It is a real function of Φ and $\Phi^\dagger$, making it a vector superfield. In addition to that, the coordinates of the chiral superfield form a Kähler manifold. For a detailed discussion on Kähler manifolds see Appendix A of Baumann and McAllister [4].

$W(\Phi)$ is known as the **superpotential**, a holomorphic function of chiral superfield Φ, which makes it a chiral superfield as well.

We can restrict the form of $K(\Phi, \Phi^\dagger)$ and $W(\Phi)$ by dimensional arguments. The overall Lagrangian must have a mass dimension of 4, that is, $[\mathcal{L}] = 4$. For scalar fields, $[\Phi] = 1$, and for fermions $[\psi] = \frac{3}{2}$. If we use the chiral superfield expansion in (4.20) we can also deduce that $[\theta] = -\frac{1}{2}$ and in turn $[F] = 2$, from the dimension of Φ. This restricts the dimension of $K(\Phi, \Phi^\dagger)$ and $W(\Phi)$ in the following way: [14]

$$K_D \subseteq \mathcal{L}_\Phi \Rightarrow [K_D] \leq 4, \tag{4.22}$$

$$W_F \subseteq \mathcal{L}_\Phi \Rightarrow [W_F] \leq 4, \tag{4.23}$$

$$K = \cdots + (\theta\theta)(\bar{\theta}\bar{\theta})K_D \Rightarrow [K] \leq 3, \tag{4.24}$$

$$W = \cdots + (\theta\theta)W_F \Rightarrow [W] \leq 2. \tag{4.25}$$

Also since K is real it can only contain $\Phi^\dagger\Phi$ but not $\Phi + \Phi^\dagger$ or $\Phi\Phi$. This leaves us with the most general form for $K(\Phi, \Phi^\dagger)$ and $W(\Phi)$ is [15]

$$K(\Phi, \Phi^\dagger) = \Phi^\dagger\Phi; \quad W(\Phi) = \alpha + \lambda\Phi + \frac{m}{2}\Phi^2 + \frac{g}{3}\Phi^3. \tag{4.26}$$

With this, we can write the minimal chiral superfield Lagrangian as [15]

$$L_\Phi = \int d^2\theta d^2\bar{\theta}\Phi^\dagger\Phi + \int d^2\theta\, \alpha + \lambda\Phi + \frac{m}{2}\Phi^2 + \frac{g}{3}\Phi^3 + \text{h.c.} \tag{4.27}$$

$$= \int d^2\theta \left[-\frac{1}{8}\bar{D}\bar{D}\Phi^\dagger\Phi + \alpha + \lambda\Phi + \frac{m}{2}\Phi^2 + \frac{g}{3}\Phi^3 \right] + \text{h.c.} \tag{4.28}$$

where the second line is known as the component form of the superfield. $\bar{D}$ is the supercovariant derivative defined earlier, with an implicit sum over the field components.

We also need the supergravity Lagrangian, which we write in analogy with the Einstein-Hilbert Lagrangian as [15]

$$L_{SG} = -\frac{6}{\kappa^2}\int d^2\Theta\mathcal{E}R + \text{h.c.} \tag{4.29}$$

Here $\kappa^2 = 8\pi G$, $\mathcal{E}$ is the chiral superspace density, generalising the $e = \sqrt{-g}$, the Vielbein determinant[2], in the Einstein-Hilbert Lagrangian and R is the superspace scalar curvature, generalising the Ricci scalar. They both have

[2]The Vielbein formalism, a generalisation of the *Vierbein* or *tetrad* formalism is a procedure for simplifying certain calculations by decomposing the metric into a pair of vector fields called Vielbein fields, $e_a{}^\mu$. For more information see chapter 2 of Nastase [18].

expansions in Θ that convert L_{SG} into the following expression [15]:

$$L_{\text{SG}} = -\frac{1}{2}e\mathcal{R} - \frac{1}{3}e\bar{M}M + \frac{1}{3}eb^a b_a + \frac{1}{2}e\varepsilon^{\alpha\beta\gamma\delta}(\bar{\psi}_\alpha\bar{\sigma}_\beta\tilde{\mathcal{D}}_\gamma\psi_\delta - \psi_\alpha\bar{\sigma}_\beta\tilde{\mathcal{D}}_\gamma\bar{\psi}_\delta), \tag{4.30}$$

where $(e^\mu{}_a, \psi^\mu{}_\alpha, M, b_a)$ is known as the supergravity multiplet (See Wess and Bagger [15] for a full treatment) and forms the components of Φ that are remain applying the gauge.

In order to take L_Φ to curved space we add (4.29) and we make a coordinate transform from $\theta \to \Theta, \mathrm{d}^2\theta \to \mathrm{d}^2\Theta\ 2\mathcal{E}$ and lastly $\bar{D}\bar{D} \to (\bar{\mathcal{D}}\bar{\mathcal{D}} - 8R)$, where $\mathcal{D}$ is the gauge covariant version (accounting for curvature) of the supercovariant derivative defined in (4.19). This leaves us with [15], [19]

$$\begin{aligned}
L &= L_{\text{SG}} + L_\Phi \\
&= \int \mathrm{d}\Theta 2\mathcal{E}\left[-3R - \frac{1}{8}(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8R)\Phi^\dagger\Phi - \frac{1}{8}(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8R)[c\Phi + \bar{c}\Phi^\dagger]\right. \\
&\qquad\qquad\qquad\qquad \left. + \alpha + \lambda\Phi + \frac{m}{2}\Phi^2 + \frac{g}{3}\right] \qquad (4.31)\\
&:= \int \mathrm{d}\Theta 2\mathcal{E}\left[-\frac{1}{8}(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8R)\Omega(\Phi,\Phi^\dagger) + W(\Phi)\right]. \qquad (4.32)
\end{aligned}$$

This is the minimal chiral superfield Lagrangian with matter-coupling for $\mathcal{N} = 1$ **supergravity**. A supergravity is the gauge theory obtained when we restrict the Lagrangian to be invariant under local SUSY transformations rather than global SUSY transformations. If we substitute Φ and $\Phi^\dagger$ in (4.32) we obtain a lengthy component expansion. The derivation of this expansion is out of scope here (it can be found in Wess and Bagger [15]) but we will need the scalar potential that arises when the auxiliary fields are integrated out of the Lagrangian, known as the **F-term potential**, which explicitly is [14], [15], [19]

$$V_F = \exp(K)\left[K^{i\bar{j}}D_i W D_{\bar{j}}\overline{W} - 3|W|^2\right]. \tag{4.33}$$

Here K is the **Kähler potential**, which is a type of potential that can be defined on a Kähler manifold. Our compact space is a Calabi-Yau manifold, which is a type of Kähler manifold. The Kähler potential stores information about the nature of our compact space. $D_a := \partial_a + (\partial_a K)$ is the Kähler covariant derivative. $g^{a\bar{b}} := (\partial_a\partial_{\bar{b}}K)^{-1}$ is the Kähler metric, the metric on the Kähler manifold, with a and b being indices that go over all the moduli (more on this in section (4.2.6)). Finally, W is the GVW superpotential.

The result (4.33) will be significant when we cover the KKLT mechanism for constructing a de Sitter vacuum.

4.2 String Theory

String theory posits that on a fundamental level what we think of as particles are not zero-dimensional points but instead are actually one-dimensional strings. Here, strings are the most fundamental dynamical objects and they trace out a worldsheet instead of a worldline. These sections are based on the works of Tong [20], Baumann and McAllister [4], Johnson [21].

The action for a relativistic string can be written in analogue with the following reparametrisation invariant action of a point particle:

$$S_{\text{particle}} = -m \int d\tau \sqrt{-\dot{X}^{\mu}\dot{X}^{\nu}\eta_{\mu\nu}}. \tag{4.34}$$

Here the action is simply proportional to the length of the worldline and so we produce a string action proportional to the area of the worldsheet. Points on the worldsheet are parametrised by spacelike coordinate, σ and timelike coordinate, τ which are mapped to spacetime coordinates by functions $X^M(\sigma,\tau)$, where $M = 0,1,\ldots,D-1$.

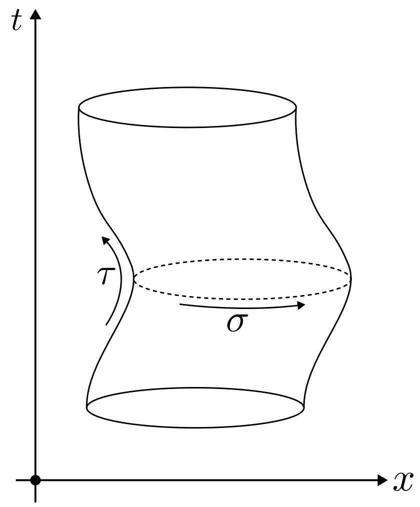


Figure 4.1: A string worldsheet in the target spacetime showing the parametrisation on the sheet. Reproduced from Tong [20]

An induced metric can be defined on this worldsheet as

$$\gamma_{ab} = \frac{\partial X^M}{\partial \sigma^a}\frac{\partial X^N}{\partial \sigma^b}\eta_{\mu\nu} = \begin{pmatrix} \dot{X}^2 & \dot{X}\cdot X' \\ \dot{X}\cdot X' & X'^2 \end{pmatrix}, \tag{4.35}$$

where $a,b = 0,1$ runs over the two worldsheet coordinates and $\dot{X} = \frac{\partial X}{\partial \tau}$, $X' = \frac{\partial X}{\partial \sigma}$. This is the pull-back of the Minkowski metric on the curved worldsheet. The infinitesimal area element is then given by $\sqrt{-\det(\gamma_{ab})}$. Thus the action can be written as

$$S = -T \int d\tau d\sigma \sqrt{(\dot{X} \cdot X')^2 - (\dot{X})^2 (X')^2}. \tag{4.36}$$

where T is a constant of unit M/L and has the interpretation of tension of the string.[3] This is known as the **Nambu-Goto action**. But a more convenient and equivalent form is the **Polyakov action**, given by

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-g} g^{ab} \partial_a X^M \partial_b X^N \eta_{MN}, \tag{4.37}$$

where, $g \equiv \det g_{ab}$ and g_{ab} is the **worldsheet metric** and, from its equation of motion, is the same as γ_{ab} up to a conformal factor. The action (4.37) essentially couples D scalar fields X to a 2-dimensional gravity via the metric, g_{ab}, which is now a dynamical field. The Polyakov action has **Weyl invariance**[4] (in addition to reparametrisation invariance and Poincaré symmetry), which means g_{ab} can be transformed to the Minkowski metric, η_{ab}, i.e. the conformal gauge. This simplifies the action to

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \partial^a X^M \partial_a X_M. \tag{4.38}$$

This is an action of D free fields with $SO(1, D-1)$ global Lorentz symmetry.

4.2.1 Quantisation

A detailed analysis of the quantisation can be found in Tong [20]. The resultant expressions are reproduced here to make relevant observations. Consider a closed string, such that $X^M(\sigma + 2\pi, \tau) = X^M(\sigma, \tau)$. When the canonical (bosonic) commutation relationships are imposed on the solutions of the action given above the string can be quantised to produce states

$$\tilde{\alpha}^i_{-n} \alpha^j_{-n} |0, p\rangle \text{ [5]} \tag{4.39}$$

[3] $T = \frac{1}{2\pi\alpha'}$ where $[\alpha'] = 2$. Thus there is a natural length scale which can be associated to the string, $\alpha' = l_s^2$. l_s is known as the string length.

[4] The property that X^M is invariant under the transformation $g_{ab} \to \Omega^2(\sigma) g_{ab}$. This is a conformal (angle-preserving) transformation.

in the Fock space, where the two oscillators come from the left and right sector with each state having mass

$$M^2 = \frac{4}{\alpha'}\left(N - \frac{D-2}{24}\right). \tag{4.40}$$

N is the level of the states. The ground state $(N = 0)$ is a tachyon[6] and the first excited states $\tilde{\alpha}^i_{-1}\alpha^j_{-1}|0,p\rangle$ have mass $M^2 = \frac{4}{\alpha'}(1 - \frac{D-2}{24})$. In order to retain the original Lorentz symmetry the first excited states must be massless[7], which is only possible if $\mathbf{D = 26}$. A similar argument requires 26 dimensions for the Lorentz symmetry of quantised open strings. The above massless states transform under $SO(24)$ which decomposes into three irreducible representations to each of which can be assigned a massless gauge field. These are a symmetric tensor G_{MN} describing a particle of spin-2 *i.e.* the graviton [19]; an anti-symmetric tensor, B_{MN} called the **Kalb-Ramond field**; a trace or scalar field, Φ known as the **dilaton** [7].

4.2.2 Branes

The similar process of quantisation for an open string is not detailed here, other than to introduce where strings end. For open strings, where $\sigma = [0,\pi]$, the determination of the equations of motion requires boundary conditions at the endpoints of the string. This is the Dirichlet boundary condition,

$$\delta X^M = 0 \quad \text{at } \sigma = 0, \pi, \tag{4.41}$$

which confines the X^M coordinate to a specific point. The other option is the Neumann boundary condition,

$$\partial_\sigma X^M = 0 \quad \text{at } \sigma = 0, \pi, \tag{4.42}$$

which allows the end point to move freely along the σ direction. A combination of the two types of boundaries for different coordinates, say

$$\partial_\sigma X^I = 0 \quad \text{for } I = 0, 1, ..., p, \tag{4.43}$$

$$X^J = c^J \quad \text{for } J = p+1, ..., D \tag{4.44}$$

[6]Particle of negative square mass, as when located at the unstable maximum of the potential

[7]The $N = 1$ states number $(D-2)^2$, of which there is no representation possible for $SO(D-1)$ symmetry. Massive states always produce an appropriate representation, so these are assumed to be massless.

define a $(p+1)$ dimensional hypersurface the string ends on which breaks the spacetime Lorentz symmetry into

$$SO(1,D-1) \to SO(1,p) \times SO(D-p-1). \tag{4.45}$$

This surface, known as a **D-brane**, (or **Dp-brane**) is itself a dynamical fundamental object in string theory.

The principle of constructing the action as higher dimensional analogues of relativistic particles can be extended to Dp-branes. Considering the worldvolume of the brane along with the presence of a background gauge field, $A^a(X)$ gives us the **Dirac-Born-Infeld action**,

$$S = -T_p \int \sqrt{-\det(\gamma_{ab} + 2\pi\alpha' F_{ab})}, \tag{4.46}$$

where γ_{ab} is once again defined as in equation (4.35) but with a,b running over the worldvolume coordinates on the brane, ξ. F_{ab} is the field strength of the constant gauge field and T_p is the tension of the brane. [20] If the background is inhabited by the massless closed string modes, G_{MN}, B_{MN} and Φ, then the action changes to

$$S = -T_p \int d^{p+1}\xi \, e^{-\Phi} \sqrt{-\det(\gamma_{ab} + 2\pi\alpha' F_{ab} + B_{ab})}, \tag{4.47}$$

where the metric pull-back changes to

$$\gamma_{ab} = \frac{\partial X^M}{\partial \xi^a} \frac{\partial X^N}{\partial \xi^b} G_{MN} \tag{4.48}$$

and G_{MN} describes the curved spacetime. B_{ab} is the pullback of the anti-symmetric field tensor,

$$B_{ab} = \frac{\partial X^M}{\partial \xi^a} \frac{\partial X^N}{\partial \xi^b} B_{MN}. \tag{4.49}$$

.

4.2.3 Perturbative Corrections

String actions can have non-perturbative corrections and two types of perturbative corrections. These are detailed below.

1. α' **expansion** The action for a string in a background profile with the

massless fields, G_{MN}, B_{MN} and Φ, is given by

$$S_\sigma = -\frac{1}{4\pi\alpha'}\int \sqrt{-g}\left([g^{ab}G_{MN}(X)+\epsilon^{ab}B_{MN}(X)]\partial_a X^M \partial_b X^N + \alpha'\Phi(X)R(g)\right), \tag{4.50}$$

where $R(g)$ is the Ricci scalar for g_{ab}. The background fields can be expanded about a given point X_0^M, to give terms such as

$$g^{ab}\,\partial_P G_{MN}(X_0)\,\delta X^P\,\partial_a \delta X^M\,\partial_b \delta X^N .$$

These terms describe an interacting field theory from which the equations of motion of the background field can be determined. If the gradients and curvatures of the fields are small in units of α', the interaction is perturbative and the expansion is known as the **σ-model** or **α' expansion**. [4] Alternatively, the equations of motion can be found from an action (given below) that describes directly the interactions of the background fields. Such an action is constructed as an effective field theory. The full action at the $M_s \equiv (\alpha')^{-1/2}$ energy scale from which the massive modes have been integrated out leaves only the effective action [20]

$$S_{\mathrm{B}} = \frac{1}{2\kappa_D^2}\int \mathrm{d}^D X\sqrt{-G}e^{-2\Phi}\left(R+4(\partial\Phi)^2-\frac{1}{2}|H_3|^2-\frac{2(D-26)}{3\alpha'}+O(\alpha')\right). \tag{4.51}$$

Here, $H_{(3)} = dB_{(2)}$ is the field-strength of B_{MN} and κ_D is a coupling constant. Higher order terms in α' contain higher-derivative operators in the fields. For example, the first α' correction to the bosonic string at $O(\alpha')$ is proportional to $R^2+\ldots$.

2. **String loop corrections** The scattering of strings can be represented in diagrams like figure (4.2) [8], with higher order loop diagrams appearing as worldsheets of higher genus. The S-matrices for the diagrams are calculated by the legs extended to infinity and assigned arbitrary momentum. A conformal map brings the legs to a finite distance with a vertex operator for each leg thus making the different loop diagrams surfaces of different topologies. In the path-integral formalism, for given initial and final states the integral prescribes summing over all

[8]These diagrams are for closed string in particular.

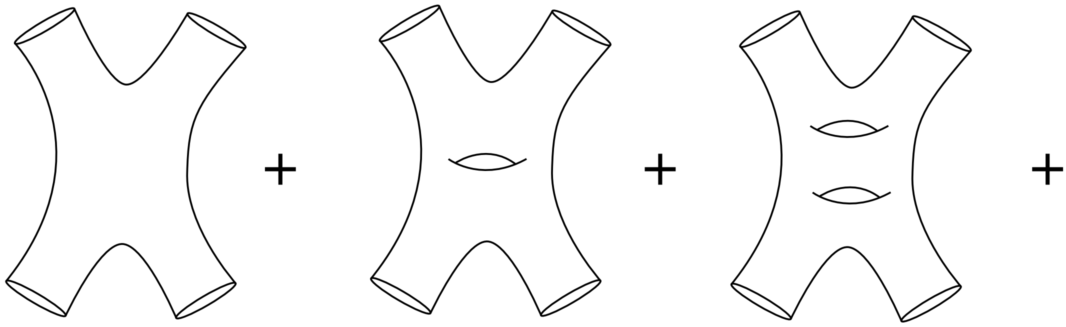


Figure 4.2: Scattering diagrams of closed strings at different loop numbers

metrics and all topologies[20],

$$\sum e^{-S_{string}} = \sum_{\text{topologies}} e^{-\lambda\chi} \int \mathcal{D}X\mathcal{D}g e^{-S_{\text{Polyakov}}}, \tag{4.52}$$

where χ is the Euler characteristic of the worldsheet given by

$$\chi = \frac{1}{4\pi} \int d^2\sigma\sqrt{g}R\ . \tag{4.53}$$

χ is a topological constant of the worldsheet and R is the Ricci scalar. The coupling for each diagram and defined as the **string coupling**, $g_s = e^{\lambda}$.

For a constant dilaton field $\Phi(X) = \lambda$ the last term in the action (4.50) becomes

$$S_{\text{dilaton}} = \lambda\chi \tag{4.54}$$

just as it appears in equation (4.52) so that the string coupling g_s is determined by the constant mode of the dilaton which is taken to be,

$$\Phi_0 = \lim_{X\to\infty} \Phi. \tag{4.55}$$

where X is the coordinate function as in (4.50). Therefore the string coupling is given by,

$$g_s = e^{\Phi_0} \tag{4.56}$$

4.2.4 Superstrings and Types of String Theory

The addition of fermions to the theory adds additional spinor terms to the action. In the simplest case of $\mathcal{N} = (1,1)$,

$$S = S_{\text{Polyakov}} + S_{\text{Fermion}} \tag{4.57}$$
$$= -\frac{1}{4\pi\alpha'} \int d^2\sigma \left(\partial^a X^M \partial_a X_M - i\bar{\psi}^M \gamma^a \partial_a \psi_M\right) \tag{4.58}$$

where γ^a are two-dimensional Dirac matrices and ψ^M are Dirac spinors that also transform as vectors under Lorentz transformation. This means that the Lorentz invariance requirement puts different constraints on the group so that only $D = 10$ dimensions are required. The spectrum of states for S_{Fermion} cannot be determined without defining the periodicity conditions for the string. If the fermions are periodic *i.e.* $\psi^M_\pm(\sigma+\pi) = \psi^M_\pm$, then they are in the **Ramond** sector. If anti-periodic *i.e.* $\psi^M_\pm(\sigma+\pi) = -\psi^M_\pm$, they are in the **Neveu-Schwarz** sector. Each independent component of ψ, *i.e.* the left-moving and right-moving components, could each take either periodicity leading to four sectors, NS-NS, R-R, R-NS and NS-R.

In practice, at the relevant energy scales, systems are described by constructing a 10 dimensional low-energy effective action of the interactions of massless states of the superstring, which are known as **Supergravity theories.** The massless modes in the bosonic quantisation, G_{MN}, B_{MN} and Φ, arise in the NS-NS sector for all types of string theory.[9] The states in the other sectors are determined by which type of string theory is being considered. Type II string theory is discussed here as the type in which the relevant model for AMI is constructed. In order to build a consistent theory the operators of the theory are subjected to what is known as a Gliozzi–Scherk–Olive (GSO) projection. If the NS-R and R-NS sectors are projected identically, so as to have the same spectra, it produces type IIB string theory. If the projections are opposite, it produces type IIA string theory.

The R-R sector comprises, for each of the types, the following fields[21] :

$$\begin{aligned} \text{type IIA}: &\quad C_{(1)}, C_{(3)}, C_{(5)}, C_{(7)}, \\ \text{type IIB}: &\quad C_{(0)}, C_{(2)}, C_{(4)}, C_{(6)}, C_{(8)}. \end{aligned}$$

Here, $C_{(p)}$ is a p-form making the first two objects in each category Hodge dual to the final two. A Dp-brane couples to a $(p+1)$-form i.e. $C_{(p+1)}$ which

[9] Type I string theory excludes the Kalb-Ramond field on the grounds that it is not invariant under worldsheet parity.

has a field strength given by $F_{(p+2)}$, a $(p+2)$-form. Since an n- and $(10-n)$-form are dual to each other, a p and $(6-p)$ source the same field, electrically and magnetically. It is shown in Johnson [21] that D-branes carry gauge symmetry charges under the R-R fields (unlike strings) and necessarily couples to the appropriate fields. As such, each theory has a specific set of D-branes. In type-IIA, there are Dp-branes of $p = 0,2,4,6,8$. In type-IIB, there are Dp-branes of $p = 1,3,5,7,9$. The field strengths of particular interest in the our potential, those defined in Type IIB, are shown below:

$$\begin{aligned}
&F^{(1)} = dC^{(0)}, F^{(3)} = dC^{(2)}, F^{(5)} = dC^{(4)};\ H^{(3)} = dB^{(2)},\\
&\tilde{F}^{(1)} = F^{(1)},\ \tilde{F}^{(3)} = F^{(3)} - C^{(0)} \wedge H^{(3)},\\
&\tilde{F}^{(5)} = F^{(5)} - \frac{1}{2}C^{(2)} \wedge H^{(3)} + \frac{1}{2}B^{(2)} \wedge F^{(3)}.
\end{aligned}$$

Another object of significance is the NS5-brane. A closed string couples electrically to B_{MN}. The Hodge dual of this field is given by $dB_{(6)} = \star dB_{(2)}$ and the five-dimensional object which couples to it (as shown below) is an object with an NS-NS charge known as the NS5-brane [21]. It is the magnetic dual of the 10D closed string. The $B_{(6)}$ field has an electric coupling to it of the form

$$\nu_5 \int_{\mathcal{M}} d^6\xi B_{(6)}, \tag{4.59}$$

where $\mathcal{M}$ is the worldvolume of the NS5-brane.

4.2.5 Flux Compactification

Supergravity inherits the ten dimensions needed for consistency in supersymmetric string theories. This proves a big phenomenological issue, as we observe the universe as a 4D spacetime. The solution to this issue is a process known as **compactification**. We imagine that the dimensions beyond the four that we observe are curled up or *compactified* such that their characteristic length is below the threshold of observations. More rigorously, this means we are postulating that the full 10D space can be decomposed as follows [4]:

$$\mathcal{M}_{10} = \mathcal{M}_4 \times Y_6. \tag{4.60}$$

where Y_6 is a compact 6D manifold. The best understood configurations take Y_6 to be a 3 complex dimensional Calabi-Yau manifold. These are preferred because the world around us seems to have no supersymmetry, or at least broken supersymmetry. Calabi-Yau orientifold compactifications break some supersymmetries and in type IIB case that we deal with, we are left with $\mathcal{N}=1$ supersymmetry.

The metric should in turn decompose to the following form in a vacuum [4]:

$$G_{MN}\mathrm{d}X^M\mathrm{d}X^N=\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu+g_{mn}\mathrm{d}y^m\mathrm{d}y^n. \tag{4.61}$$

Here G_{MN} must follow be a solution to the vacuum Einstein equations and so the space must be Ricci flat, $R_{\mu\nu}=R_{mn}=0$. [10]

The 4D action that comes from a compactification is computed by a **dimensional reduction**. We start with the 10D geometry determined by [4]

$$G_{MN}\mathrm{d}X^M\mathrm{d}X^N=e^{-6u(x)}g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu+e^{2u(x)}\hat{g}_{mn}\,\mathrm{d}y^m\,\mathrm{d}y^n. \tag{4.63}$$

where $\hat{g}$ is a reference metric defined by $\int_{Y_6}\mathrm{d}^6y\sqrt{\hat{g}}:=\mathcal{V}$. The function $e^{u(x)}$ is called a **breathing mode**, it captures the change in the size of Y_6 over the 4D spacetime.

Focusing just on the Einstein-Hilbert part of the 10D action[11], we have

$$S_{\mathrm{EH}}^{(10)}=\frac{1}{2\kappa^2}\int\mathrm{d}^{10}X\sqrt{-G}e^{-2\Phi}R_{10}. \tag{4.64}$$

where R_{10} is the Ricci scalar constructed from G_{MN} . We would like to express R_{10} in terms of R_4 and $\hat{R}_6$, the Ricci scalars constructed from $g_{\mu\nu}$ and $\hat{g}_{mn}$, respectively. Re-expressing (4.64) in terms of the 4D and 6D Ricci scalars, we obtain [4]

$$S_{\mathrm{EH}}^{(10)}=\frac{1}{2\kappa^2}\int\mathrm{d}^4x\sqrt{-g}\int_{Y_6}\mathrm{d}^6y\sqrt{\hat{g}}e^{-2\Phi}\left(R_4+e^{-8u}\hat{R}_6+12\partial_\mu u\partial^\mu u\right). \tag{4.65}$$

If we integrate out the integral in d^6y and if the string coupling, $g_s=e^\Phi$ is constant over the compact space then we can recover the 4D Einstein-Hilbert

[10] In theories that have sources of stress-energy such as D-branes and background fields, the metric is modified. The so-called **warped metric** is given by

$$G_{MN}\mathrm{d}X^M\mathrm{d}X^N=e^{2A(y)}g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu+e^{-2A(y)}g_{mn}\,\mathrm{d}y^m\,\mathrm{d}y^n, \tag{4.62}$$

where $A(y)$ is known as the **warping factor**.

[11] Note that the 10D action before compactification is supersymmetric but the solutions of the equations of motion of the supergravity need not preserve it. Nevertheless, they usually are made to preserve it because it makes the calculations easier.

term as [4]

$$S_{\mathrm{EH}}^{(4)}=\frac{M_{\mathrm{pl}}^2}{2}\int \mathrm{d}^4x\sqrt{-g}R_4;\quad M_{\mathrm{Pl}}^2:=\frac{\mathcal{V}}{g_s^2\kappa^2}. \tag{4.66}$$

The double derivative term in (4.65) is a canonical kinetic term for a 4D scalar field. As such we can interpret that, after dimensional reduction, we obtain a massless scalar field called a **modulus field**, because its origin is a spacetime dependent deformation of the original 10D solution. For Calabi-Yau solutions the breathing mode corresponds to one of the Kähler moduli.

More general Kaluza-Klein reductions incorporate more complicated actions with p-form gauge fields and generalisations of the breathing mode $u(x)$, but the principle is the same: We factorise the 10D space into a product of non-compact 4D spacetime and a compact 6D manifold, usually a Calabi-Yau threefold, and the result is that deformation parameters show up as 4D spacetime scalar fields called moduli fields.

4.2.6 Moduli

Coming back to the inflationary point-of-view, our UV-complete theory of choice is string theory. As we saw in the previous section, compactifying the 10D supergravity of a string theory produces 4D scalar fields called moduli fields.

Moduli in Calabi-Yaus

We will briefly discuss the moduli that arise in Calabi-Yau compactifications. For an in-depth explanation of the mathematical origins of moduli arise we refer the reader to Appendix A of Baumann and McAllister [4].

The geometric moduli correspond with to deformations of the metric g_{mn}. To maintain Ricci flatness as required by the CY-condition that is

$$R_{mn}(g+\delta g)=0. \tag{4.67}$$

This gives us two kinds of moduli. These arises due to deformations of the Kähler form and the holomorphic complex structure form.

1. the deformations of the Kähler form[4]

$$\delta J\equiv i\delta g_{i\bar{j}}\,\mathrm{d}z^i\wedge\mathrm{d}\bar{z}^{\bar{j}}, \tag{4.68}$$

where $z^i,\bar{z}^{\bar{j}};i,\bar{j}=1,2,3$ are the complex coordinates on the Calabi-Yau

Y_6. The **Kähler moduli** are given by [4]

$$J = t^I(x)\omega_I. \tag{4.69}$$

Here $t^I(x)$ are 4D scalar fields, the Kähler moduli, and ω_I are a set of $h^{1,1}$ harmonic $(1,1)$-forms that form a basis for the **Dolbeault cohomology group** $H^{1,1}_{\bar\partial}(Y_6,\mathbb{C})$ where $I = 1,\dots,h^{1,1}$. An explanation of the Dolbeault cohomology groups and **Hodge numbers** $h^{i,j}$ can be found in Baumann and McAllister [4].

2. the complex structure deformations [4],

$$\delta g_{\bar i \bar j} = c\zeta^A(x)\,(\chi_A)_{kl\bar i}\bar\Omega^{kl}{}_{\bar j}, \tag{4.70}$$

define the **complex structure moduli**, ζ^A. Here the χ^A are a basis of $(1,2)$-forms for the $H^{1,2}_{\bar\partial}(Y_6,\mathbb{C})$ with $A = 1,\dots,h^{1,2}$. Ω is the unique $(3,0)$-form on Y_6.

Additional scalar fields can be obtained by expanding the NS-NS and R-R potentials in the bases of harmonic forms, resulting in [4]

$$\begin{aligned} B_{(2)} &= B_2(x) + b^I(x)\omega_I, \\ C_{(2)} &= C_2(x) + c^I(x)\omega_I, \\ C_{(4)} &= \vartheta^I(x)\tilde\omega_I. \end{aligned} \tag{4.71}$$

where $\tilde\omega_I$; $I = 1,\dots,h^{1,1}$ form a basis for $H^{2,2}$ and $B_2(x)$ and $C_2(x)$ are the 4D versions of the $B_{(2)}$ and $C_{(2)}$.

The proper Kähler coordinates of the moduli space are determined by assembling the invariant scalars into the bosonic components of chiral multiplets. First we have the complex axio-dilaton,

$$S = e^{-\phi} + iC_0, \tag{4.72}$$

then we have the complex structure moduli, ζ_a, which are fine unaltered. The scalars arising from the 2-forms combine to give the complex combination

$$G_\alpha \equiv c_\alpha - Sb_\alpha, \tag{4.73}$$

and lastly we define the last complexication,

$$T_i \equiv \tau_i + i\theta_i \tag{4.74}$$

where τ_i are the 4-cycle volumes.

Overall we are left with $h^{1,1}_+$ number of T_i's, $h^{1,1}_-$ number of G^α's, the axio-dilation S and the $h^{1,2}_+$ complex structure moduli ζ^a as our moduli.

So far we have discussed moduli in 4D $\mathcal{N}=2$ supersymmetry. To make contact with $\mathcal{N}=1$ supersymmetry. This can be achieved by adding **orientifold planes** which are fixed points under a certain involution operator, $\mathcal{O}$, called the **orientifold action** that reverse the orientation of the string worldsheet. The most important ones are of the form,

$$\mathcal{O} = (-1)^{F_L}\Omega_{ws}\sigma \tag{4.75}$$

where F_L is the worldsheet fermion number, Ω_{ws} is the worldsheet reversal operator and σ is a geometric involution operator that flips the $(3,0)$-form Ω. Under $\mathcal{O}$ the cohomology group $H^{1,1}$ can be decomposed as,

$$H^{1,1} = H^{1,1}_+ \oplus H^{1,1}_- \tag{4.76}$$

where the factors include the 2-forms that are even and odd under the involution operator, respectively. These leaves us with the moduli [4]

$$\begin{aligned} B_2 &= b^\alpha(x)\omega_\alpha \\ C_2 &= c^\alpha(x)\omega_\alpha \\ C_4 &= \vartheta^i(x)\tilde{\omega}_i \end{aligned} \tag{4.77}$$

where $\alpha = 1,\ldots,h^{1,1}_-$ runs over the odd eigenspace.

Moduli in Inflation

These moduli fields gain mass by interacting with sources of stress-energy like D-branes and quantised fluxes, in a process called **moduli stabilisation**.

Integrating out heavy scalar fields, the result of moduli stabilisation, in the effective theory causes further corrections to the inflationary potential and thereby exacerbates the η-problem. The existence of other light scalars is also problematic for our theory because they would absorb energy during inflation and subsequently ruin predictions about the abundance of light elements from Big Bang nucleosynthesis in addition to mediating unobserved fifth forces by virtue of having only gravitational coupling to observable particles. So we assume for simplicity that the inflaton is the only light moduli field and all other moduli-fields barring the inflaton are heavy, such that their mass, $m \gg H$ [10], which means that after moduli stabilisation we would have a single scalar field, ϕ. This is also in line with what is required for KKLT to be successful.

4.2.7 Axions

The moduli in (4.77) give rise to particles called **axions**, arising in string theory from the integration of p-form gauge fields over p-cycles of a compact space. Integrating out moduli give us the following axions,

$$b_I = \frac{1}{\alpha'} \int_{\Sigma_2^I} B_{(2)}, \quad c_I = \frac{1}{\alpha'} \int_{\Sigma_2^I} C_{(2)}, \quad \vartheta_I = \frac{1}{(\alpha')^2} \int_{\Sigma_4^I} C_{(4)}, \tag{4.78}$$

We call the set of all these particles axions, $a = \{b_I, c_I, \vartheta_I, C_0, b_U, c_U\}$. Orientifolding by O from (4.75) projects out some of the axion, leaving

$$a = \{C_0, b_\alpha, c_\alpha, \vartheta_I \,\Big|\, \alpha = 1, \ldots, h_-^{1,1}, I = 1, \ldots, h_+^{1,1}\}. \tag{4.79}$$

Axions enjoy a continuous shift symmetry at all orders in perturbation theory. Baumann and McAllister [4] illustrate this using the b axion. If we expand the field B_{MN} around a fixed point $X_{(0)}$ we obtain [4]

$$B_{MN}(X) = B_{MN}\left(X_{(0)}\right) + X^P \partial_P B_{MN}\left(X_{(0)}\right) + O(X^2) \tag{4.80}$$

The terms after $B_{MN}\left(X_{(0)}\right)$ in the series have at least one derivative on B_{MN} which erases any shifts in the b axion. If the first term, which is a total derivative, vanishes, the axion enjoys a continuous shift symmetry.

However, the total derivative will be non-vanishing for a non-trivial cycle, known as a **worldsheet instanton** Σ_2, this contributes a term as follows[4]:

$$S_{\text{inst}} = \exp\left(-\frac{1}{2\pi\alpha'} \int_{\Sigma_2} (J + iB_2)\right) \propto \exp\left(-i\frac{b}{2\pi}\right). \tag{4.81}$$

where J is the Kähler form. This is a correction that is non-perturbative in α'. Thus, the continuous shift symmetry is spontaneously broken, resulting in a discrete shift symmetry,

$$a \to a + (2\pi)^2. \tag{4.82}$$

Other non-perturbative corrections, known as Euclidean D-branes, wrapping Σ_2 can cause periodic shift-symmetry breaking terms for the c axion. The symmetry may also be explicitly broken by adding D-branes to the compactification setup.

The discrete shift symmetry forces the Lagrangian to be of the form [4]

$$\mathcal{L}(a) = -\frac{1}{2} f^2 (\partial a)^2 - \Lambda^4 \left[1 - \cos\left(\frac{a}{2\pi}\right)\right] + \cdots , \tag{4.83}$$

where Λ is a scale constant that is determined by the dynamics of the theory and f is known as the **axion decay constant**. We may convert the axion to its **canonically normalised form**,

$$\phi := af, \tag{4.84}$$

in which case the periodicity becomes $(2\pi)^2 f$. The decay constant can be derived from the effective Kähler potential after dimensional reduction.

Chapter 5

Axion Inflation

The potential constructed in models of inflation are effective potentials where the symmetries of light objects, in this case the axions and light modes of D-branes, are used to write a Lagrangian containing all allowed operators suppressed by the scale of quantum gravitational effects, i.e. $\Lambda = M_{\rm Pl}$. The inflaton field minimally coupled to gravity has the following Lagrangian [7]:

$$S_{\rm eff}[\phi] = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + \mathcal{L}_l[\phi] + \sum_i c_i \frac{\mathcal{O}_i[\phi]}{\Lambda^{\delta_i - 4}}\right]. \tag{5.1}$$

The effects of the massive fields are encoded in the higher dimensional operators, $\mathcal{O}_i$. In general this introduces corrections to the mass of the inflaton through the series of **Planck-suppressed operators** which become more significant as we get closer to the Planck-scale (and beyond). For example, scalar masses (like Higgs in the standard model) receives corrections proportional to the cut-off of the theory,

$$\Delta m^2 \sim \Lambda^2. \tag{5.2}$$

These changes generate corrections to the inflationary potential, correspondingly causing $\Delta\eta > 1$ which causes an η-problem, and thereby ruins the flatness of the potential. Specifically we would have

$$\Delta\eta_V = \frac{\Delta m_\phi^2}{3H^2} \geq 1. \tag{5.3}$$

These contributions can be mitigated by the use of symmetries. Supersymmetry reduces the mass corrections to the order of H but that still leaves $\eta \sim 1$. An approximate shift symmetry however, suppresses quantum corrections by the parameter that measures the weak breaking of the symmetry [4]. In axion inflation models, these higher order corrections are prohibited by the shift symmetry of the axion. The non-perturbative effects such as instantons etc. which can give the axion a potential, break this symmetry weakly, allowing the problem to be skirted [7]. This makes them attractive candidates for the inflaton field.

5.1 Natural Inflation and its Issues

The discrete shift symmetry discussed in section (4.2.7) results in the periodic potential for the axion:

$$V(\phi) = \frac{\Lambda^4}{2}\left[1 - \cos\left(\frac{\phi}{f}\right)\right]. \tag{5.4}$$

from equation (4.83) such that $\phi \equiv af$. This is a possible potential for natural inflation [4].

However this model has limitations which are discussed in Pajer and Peloso [22]. Contradictions with experimental data arise in relation to the quantity f. The axion decay constant determines the strength of the shift-symmetric coupling and therefore determines the energy scale up to which the effective field theory (EFT) construction of the potential holds. With M_{Pl} as the UV cut-off scale,

$$\frac{f^2}{M_{Pl}^2} \approx \frac{\alpha'^2}{L^4} \ll 1 \tag{5.5}$$

where L is the characteristic length scale of the compactification. [4] The inequality comes from the requirement that the compactification (KK) scale should be well below the string scale ($M_{KK} << M_s$), therefore

$$\frac{L_s^4}{L_{KK}^4} = \frac{M_{KK}^4}{M_s^4} << 1. \tag{5.6}$$

Therefore the $f/M_{\mathrm{Pl}} \ll 1$ bound is well-established for construction.

Measurements for observational parameters, namely n_s and r however, causes complications. If we set the value of f for natural inflation at $N = 60$ and demand sufficient flatness, WMAP and Planck data gives $f = 3.7M_{\mathrm{Pl}}$ and $f = 10M_{\mathrm{Pl}}$ respectively [22]. Therefore, for the region in which natural inflation is defined i.e. $f < M_{\mathrm{Pl}}$, n_s is found to be too small for observational bounds. Thus it seems that we are looking for a field with a super-Planckian displacement whilst being confined to $f < M_{\rm Pl}$.

5.2 Axion Monodromy Inflation

The discussions above motivate cases where the inflaton field is able to move over a large range. We require in our model that the large field excursions, $\Delta\phi \gg M_{\rm Pl}$, must be "kinematically allowed", i.e., the scalar field space must allow amplitudes on the order of $M_{\rm Pl}$. The flatness of the inflaton potential must be left intact dynamically over this large field excursion. As previously discussed, axions enjoy a shift symmetry when outside the influence of symmetry-breaking stress-energy sources, such as branes. The unmodified axion potential has a sub-Planckian period leading to a maximal field excursion that is sub-Planckian, and so the axion only enables small-field inflation. Adding a brane to the picture allows the potential to become aperiodic by introducing a term that is linear in the axion field. This allows unbounded field excursions and thus large-field inflation. The potential of the symmetry-broken field may look as follows [4]:

$$V(\phi) = \mu^3\phi + \Lambda^4 \cos\left(\frac{\phi}{2\pi f}\right); \quad \mu^3 := \frac{1}{f}\frac{\rho}{(2\pi)^6 g_{\rm s}\,(\alpha')^2}. \tag{5.7}$$

This is called a **monodromy**, a phenomenon in which every circuit of the naïve configuration space does *not* bring the axion potential to the same point.

The potential can be derived as in McAllister, Silverstein, and Westphal [23] by considering a D5-brane wrapping a 2-cycle, threaded by the $B_{(2)}$ field. The 2-cycle is the same as the one B is integrated over in equation (4.78). An anti-D5-brane must be present at a suitably distant region in space wrapping a homologous 2-cycle to ensure the cancellation of the charge[1] per the tadpole condition. Integrating the action for the D5-brane over the 2-cycle produces an action with four-dimensions containing the axion. From equation (4.47), the DBI action with $F_{ab} = 0$ is given by

$$S_{\rm D5} = \frac{1}{(2\pi)^5\,(\alpha')^3\,g_{\rm s}} \int_{\mathcal{M}_4\times\Sigma_2} {\rm d}^6\sigma \sqrt{-\det\left(G_{ab} + B_{ab}\right)}, \tag{5.8}$$

where the 2-cycle Σ_2 is of size $l\sqrt{\alpha'}$ making ${\rm Vol}(\Sigma_2) \equiv \int d^2\sigma = l^2\alpha'$. Taking the

[1]The compactification of a D5-brane over a 2-cycle leaves a charge coupling to a 3-form flux. The charge would be dependent on b which would fix the value of b by Gauss' law.[4]

integral over Σ_2, the four-dimensional potential for b is

$$V(b) = \frac{\rho}{(2\pi)^6 (\alpha')^2 g_s} \sqrt{(2\pi)^2 \ell^4 + b^2}, \tag{5.9}$$

where ρ is the contribution from warping. When the vev of the axion is large, i.e. $b \gg \ell^2$ the potential is approximately linear.

$$V(b) \propto b \tag{5.10}$$

The kinetic term for b comes from the $H = dB$ term in the effective action, equation (4.51), for 10 dimensions, which looks like

$$S_{10\mathrm{D}} \supset -\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-G_{10}} e^{-2\phi} H_{MNP} H^{MNP} \tag{5.11}$$

$$\supset -\frac{1}{4\kappa_{10}^2} \int d^4x \sqrt{-g_4} \int d^6x \sqrt{-g^6} e^{-2\phi} g^{\mu\nu} \left(\partial_\mu b_I\right) \left(\partial^\mu b_J\right) \omega^I_{mm'}\, \omega^J_{nn'} g^{mm'} g^{nn'}. \tag{5.12}$$

After compactification,

$$S = \frac{1}{2} \int d^4x \sqrt{g_4} \gamma^{IJ} g^{\mu\nu} \partial_\mu b_I \partial_\nu b_J, \quad \gamma^{IJ} = \frac{1}{6(2\pi)^7 g_s^2 \alpha'^4} \int \omega_I \wedge \star\omega_J \tag{5.13}$$

$$\equiv -\int \frac{1}{2} \sqrt{g_4} \sum_I f_I^2 \left(\partial_\mu b_I\right)^2 \equiv -\frac{1}{2} \int \left(\partial_\mu \phi_I\right) \left(\partial^\mu \phi^I\right) \tag{5.14}$$

wherein the γ_{IJ} metric is diagonalised to find a kinetic term for each axion. Therefore, for the canonical normalisation $\phi = bf$, f is determined to be

$$f^2 = \frac{\mathcal{V}^{1/3}}{2 g_s^4 (\pi)^7 (\alpha')}. \tag{5.15}$$

The full Lagrangian can then be written as

$$\mathcal{L} = -\frac{1}{2} (\partial\phi)^2 - \mu^3 \phi, \quad \text{with} \quad \mu^3 \equiv \frac{1}{f} \frac{\rho}{(2\pi)^6 g_s (\alpha')^2}. \tag{5.16}$$

The potential here introduces a monodromy to the b_i axions and analogously NS5-branes introduce a monodromy for the c_i axions [23].

The effect of instantons then introduces periodic corrections to the potential as discussed before. They can be suppressed exponentially by the size of the cycle wrapped by the worldsheet making the periodic term sub-leading [23].

The final potential looks as in equation (5.7).[2]

5.3 Calculation for Parameters involved in Inflation

5.3.1 Inflationary Parameters for Axion Monodromy Inflation

In this section we will calculate the value of the scalar spectral tilt, n_s, the tensor-to-scalar ratio, r, and the scalar power spectrum Δ_s^2.

First of all, we must compute the slow-roll parameters, ϵ_V and η_V. In order to do this we need a potential $V(\phi)$, which we will take to be the axion monodromy potential to leading order. This means that

$$V(\phi) = \mu^3 \phi. \tag{5.17}$$

Evaluating equations (3.31) and (3.32), we can obtain the expression for the potential slow-roll parameters, giving us

$$V'(\phi) = \mu^3; \quad V''(\phi) = 0, \tag{5.18}$$

$$\Rightarrow \epsilon_V = \frac{M_{\text{Pl}}^2}{2\phi^2}; \quad \eta_V = 0. \tag{5.19}$$

From this information we can calculate n_s and r given by (3.44) and (3.42), that being

$$\begin{aligned} n_s &= 1 + 2\eta_V^\star - 6\epsilon_V^\star, \\ r &= 16\epsilon_V^\star. \end{aligned} \tag{5.20}$$

where the $\star$ means that we must evaluate them at horizon exit (roughly the start of inflation). To calculate these we need to find the value of the field ϕ at horizon exit, that is, we need $\phi_\star$. To calculate $\phi_\star$ we will need to work backwards from the constraint that N, the number of e-folds, has to be 40 - 60 and that inflation ends when $\epsilon_V \approx 1$, which will give us ϕ_{end}. For

[2]Terms proportional to $\phi \cos(\frac{\phi}{2\pi f})$ are suppressed for being sub-leading.

concreteness, we will take $\epsilon_V(\phi_{\text{end}}) = 1$ as the cutoff. Setting

$$\begin{aligned} \frac{M_{\text{Pl}}^2}{2\phi_{\text{end}}^2} &= 1 \\ \Rightarrow \phi_{\text{end}} &= \frac{1}{\sqrt{2}} M_{\text{Pl}}. \end{aligned} \tag{5.21}$$

This will allow us to evaluate $\phi_\star$ using the expression for the number of e-folds in slow-roll inflation from Baumann and McAllister [4], which tells us that

$$\begin{aligned} N(\phi) &= \int_{\phi_{\text{end}}}^{\phi} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_V} M_{\text{Pl}}} \\ \Rightarrow N_{\text{total}} = N(\phi_\star) &= \int_{\frac{1}{\sqrt{2}}M_{\text{Pl}}}^{\phi_\star} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_V} M_{\text{Pl}}} = \int_{\frac{1}{\sqrt{2}}M_{\text{Pl}}}^{\phi_\star} \frac{\mathrm{d}\phi}{\sqrt{\frac{M_{\text{P}}^2}{\phi^2}} M_{\text{Pl}}} = \int_{\frac{1}{\sqrt{2}}M_{\text{Pl}}}^{\phi_\star} \frac{\phi}{M_{\text{Pl}}^2}\, \mathrm{d}\phi \\ &= -\frac{1}{4} + \frac{(\phi_\star)^2}{2M_{\text{Pl}}^2}. \end{aligned} \tag{5.22}$$

Substituting in the conservative estimate of $N_{\text{total}} = 60$, we obtain that

$$60 = -\frac{1}{4} + \frac{(\phi_\star)^2}{2M_{\text{Pl}}^2} \Rightarrow \phi_\star \approx 10.9772 M_{\text{Pl}}. \tag{5.23}$$

Now let us calculate $\epsilon_V^\star$ and $\eta_V^\star$:

$$\begin{aligned} \epsilon_V^\star = \epsilon_V(\phi_\star) = \frac{M_{\text{Pl}}^2}{2\phi_\star} &= \frac{M_{\text{Pl}}^2}{2 \times 10.9772 M_{\text{Pl}}} \approx 4.15486 \times 10^{-3}, \\ \eta_V^\star &= \eta_V(\phi_\star) = 0. \end{aligned} \tag{5.24}$$

With this we can finally obtain the tensor-to-scalar ratio, r, and the scalar spectral tilt, n_s:

$$\begin{aligned} n_s = 1 + 2\eta_V^\star - 6\epsilon_V^\star &= 1 + 2 \times 0 - 6 \times 4.15486 \times 10^{-3} \approx 0.975071, \\ r &= 16 \times \epsilon_V^\star \approx 6.64778 \times 10^{-2}. \end{aligned} \tag{5.25}$$

For comparison the most precise bounds from the Planck 2018 data [5] states that

$$n_s = 0.9649 \pm 0.0042 \text{ (68\% CL, Planck TT,TE,EE + Low E + Lensing)} \tag{5.26}$$

$$r < 0.056 \text{ (68\% CL, Planck TT + Low E + Lensing + BK15)} \tag{5.27}$$

5.3.2 Calculating μ from Planck Data

We shall now use (3.40) in order to calculate the value of μ from equation (5.17).

Firstly the Planck 2018 [5] data's value of A_s gathered from combining the Planck TT, TE, EE power spectrum data, low multipole E polarisation data and gravitational lensing data is

$$10^{10}A_s \approx \exp 3.044 \tag{5.28}$$

$$\Rightarrow A_s \approx \frac{\exp 3.044}{10^{10}} \tag{5.29}$$

$$\Rightarrow A_s \approx 2.099 \times 10^{-9}. \tag{5.30}$$

We can use this to derive the value of μ from equation (5.17), which says that

$$A_s = \Delta_s^2(k) = \frac{1}{12\pi^2}\frac{1}{M_{\text{Pl}}^6}\frac{V^3}{V'^2}\bigg|_{k=k_\star} \tag{5.31}$$

$$\Rightarrow A_s = \frac{1}{12\pi^2}\frac{1}{M_{\text{Pl}}^6}\frac{(\mu^3\phi_\star)^3}{(\mu^3)^2} = \frac{\mu^3\phi_\star^3}{12M_{\text{Pl}}^6\pi^2} \tag{5.32}$$

$$A_s|_{60\text{ e-folds}} \approx 2.099 \times 10^{-9} \tag{5.33}$$

$$\Rightarrow \frac{\mu^3\,(10.9772M_{\text{Pl}})^3}{12M_{\text{Pl}}^2\pi^2} \approx 2.099 \times 10^{-9} \text{ (Using equation (5.23))} \tag{5.34}$$

$$\Rightarrow \frac{1.323 \times 10^3\mu^3}{12M_{\text{Pl}}^3\pi^2} \approx 2.099 \times 10^{-9} \tag{5.35}$$

$$\Rightarrow \frac{110.228\mu^3}{M_{\text{Pl}}^3} \approx 2.099 \times 10^{-9} \tag{5.36}$$

$$\Rightarrow \frac{110.228\mu^3}{M_{\text{Pl}}^3} \approx 2.099 \times 10^{-9} \tag{5.37}$$

$$\Rightarrow \left(\frac{\mu}{M_{\text{Pl}}}\right)^3 \approx 1.904 \times 10^{-11} \tag{5.38}$$

$$\Rightarrow \frac{\mu}{M_{\text{Pl}}} \approx \sqrt[3]{1.904 \times 10^{-11}} \tag{5.39}$$

$$\Rightarrow \frac{\mu}{M_{\text{Pl}}} \approx 2.670 \times 10^{-4} \tag{5.40}$$

$$\Rightarrow \mu \approx 2.670 \times 10^{-4}M_{\text{Pl}}. \tag{5.41}$$

5.3.3 Constraints on string parameters

Now, for the benefit of calculations, the second term is reintroduced below to calculate η at $\phi_\star \sim 11M_{\rm Pl}$.

$$
\begin{aligned}
\eta =& M_{\rm Pl}^2 \frac{V''}{V} \\
=& M_{\rm Pl}^2 \, \frac{\Lambda^4 (2\pi f)^2 \cos\left(\frac{\phi}{2\pi f}\right)}{\mu^3 \phi - \Lambda^4 \cos\left(\frac{\phi}{2\pi f}\right)} \\
\approx& M_{\rm Pl}^2 \, \frac{\Lambda^4 (2\pi f)^2}{11\mu^3 M_{\rm Pl} - \Lambda^4} \\
=& M_{\rm Pl} \frac{\Lambda^4 (2\pi f)^2}{11\mu^3 - \frac{\Lambda^4}{M_{\rm Pl}}} \\
\approx& M_{\rm Pl} \frac{\Lambda^4 (2\pi f)^2}{11\mu^3}
\end{aligned}
$$

This puts the constraint on Λ to be

$$
\Lambda^4 \leq \frac{11\mu^3}{M_{\rm Pl}} (2\pi f)^2 \tag{5.42}
$$

This establishes that the periodic effect must be largely subdominant to the monodromy term, justifying our choice of leading order potential.

Chapter 6

The KKLT Mechanism

6.1 Overview of the Mechanism

The current accelerated expansion of the universe points to the fact that our universe occupies a de Sitter vacuum state, characterised by constant positive scalar curvature and small positive cosmological constant. Frustratingly, mathematical construction of such vacua proves difficult, with flat or anti-de Sitter spaces being much easier to construct. The Kachru Kallosh Linde Trivedi (KKLT) mechanism is a proposal for constructing a de Sitter vacuum[1]. As such, the use of the KKLT mechanism in connecting inflation to the late-time universe is of particular interest in this paper.

The KKLT mechanism applies a series of steps from the starting point in F-theory to arrive at a de Sitter vacuum. The following has been summarised from a review of KKLT by Houmes [26] (although notably we use a different notational convention). For more detail one may look at the original paper, Kachru et al. [6]. The steps to construct the de Sitter minimum are:

1. **Starting Point and Enforcing the Tadpole Condition** The mechanism starts from a theory called **F-theory**, which is a 12D super-theory to certain variants of string theory. In F-theory there is a higher dimensional analogue of Gauss's law for fluxes called the **tadpole condition**[2], *i.e.* that flux lines must end somewhere in a compact space. It is

[1] Other proposals include the Large Volume Scenario (LVS) [24], perturbative moduli stabilisation and dS [25].

[2] The word "tadpole" here refers to Feynman diagrams with at least one line that starts and ends on the same vertex. They look like tadpoles, hence the name.

given by [26]

$$\frac{\chi(X)}{24} = N_{D3} + \frac{1}{2\kappa^2 T_3}\int_X H_{(3)} \wedge F_{(3)}. \tag{6.1}$$

where χ is the Euler characteristic of the compact space X in F-theory, N_{D3} is the number of D3-branes minus the number of anti-D3-branes, κ is the 10D Newton constant as in section (4.1.5), T_3 is the D3-brane tension and H_3 and F_3 are the 3-form fluxes. The reason for N_{D3} entering the equation is interesting. D3-branes and anti-D3-branes produce flux lines analogously to how electric charges produce electric flux lines. Since the tadpole condition tells us flux lines must end somewhere, the satisfaction of this condition depends on the number of D3-brane and anti-D3-brane. In a non-compact space like ours, electric charges can have flux lines that extend to infinity, but in a compact space like X, they must end somewhere finite. The interpretation of the LHS term counts the negative D3-brane charge coming from the O3 planes and the induced D3 charge on D7 branes, while the terms on the right-hand side count the net D3 charge from transverse branes and fluxes in the CY manifold [6].

2. **Transferring the Tadpole Condition to Type-IIB Theory** The tadpole condition is then transferred to type-IIB string theory by taking a certain limit of F-theory that involves compactifying over a 2-torus [27]. According to Houmes [26], transferring the tadpole condition this way allows one to make arguments in the type-IIB theory without needing to get into explicit detail.

3. **Stabilising All but One Modulus** The mechanism then continues in the $\mathcal{N} = 1$ supergravity (low energy limit) of type-IIB theory. It uses the general supergravity potential given by equation (4.33), which we repeat here:

$$V = e^K \left(\sum_{a,b} g^{a\bar{b}} D_a W \overline{D_b W} - 3|W|^2 \right). \tag{6.2}$$

The Kähler potential is given by

$$K = -3\ln(T+\bar{T}) - \ln(S+\bar{S}) - \ln\left(-i\int_M \Omega \wedge \bar{\Omega}\right). \tag{6.3}$$

where we have converted the definition from Houmes [26] into our own notational convention. $T = \tau + i\rho$, with τ being the volume modulus. $S = s + iC_0 = e^{-\phi} + iC_0$ is the axio-dilaton[3]. Ω is the unique holomorphic (3,0)-form on the compact space M of the type-IIB theory.

The leading order GVW superpotential W_0 is

$$W_0 = \int_M G_{(3)} \wedge \Omega. \tag{6.4}$$

where $G_{(3)}$ is a composite 3-form flux given by $G_{(3)} = F_{(3)} - SH_{(3)}$.

If we substitute (6.3) and (6.4) we find that [26]

$$g^{T\overline{T}} D_T W \overline{D_T W} = 3|W|^2. \tag{6.5}$$

So the term with indices $(a,b) = (T,T)$ in equation (6.2) cancels the $-3|W|^2$ term. This is known as the **no-scale cancellation**. This means the potential contains all moduli except the volume modulus. There exists a freedom in the choice for the fluxes $F_{(3)}$ and $H_{(3)}$. KKLT [26] chooses $F_{(3)}, H_{(3)} \in H^3(M,\mathbb{Z})$, the 3rd de Rham Cohomology group [4], such that their integrals give integral values, fulfilling a version of Dirac quantisation. This results in a specific effective potential for all moduli except the volume modulus, and so they are now stabilised.

4. **Stabilising the Volume Modulus** One unstabilised modulus remains: the volume modulus, τ. Before stabilisation a remark is made to justify this step. The previous moduli have received masses of order $\sim \frac{\alpha'}{R^3}$. The volume modulus on the other hand follows $\tau \propto R^4$ (it is the volume of a 4-cycle). For small R, this means we can treat all other moduli as large in mass compared to the volume modulus. As such, we can take the low energy limit and treat τ as the only dynamical variable and treat the other moduli as fixed. This separation of scale is crucial to the successful implementation of the KKLT mechanism.

   With that established, we stabilise τ by considering correction terms to the F-term potential which include T, so that an effective potential can be generated for it. While noting that other kinds of corrections may exist, Houmes [26] considers corrections to the superpotential of the form

$$W_{\text{correction}} = Ae^{-aT} \Rightarrow W = W_0 + Ae^{-aT}. \tag{6.6}$$

[3] C_0 is the zero-form defined in equation (4.59)

We can then find W_0 by applying the supersymmetric minimum condition, $D_T W = 0$, giving us [26]

$$W_0 = -Ae^{-aT}\left(1+\frac{2}{3}a\tau_\star\right) = -Ae^{-a\tau_\star}e^{ia\rho_\star}\left(1+\frac{2}{3}a\tau_\star\right), \tag{6.7}$$

where the $\star$ denotes quantities evaluated at the minimum of the potential. Using equations (6.2) and (6.6) generates the potential [26]

$$V = \frac{aAe^{-a\tau}}{2\tau^2}\left(\frac{1}{3}\tau Aae^{-a\tau} + W_0 + Ae^{-a\tau}\right). \tag{6.8}$$

The minimum of this potential can then be computed, giving [26]

$$V_{\text{min,AdS}} = -\frac{a^2A^2e^{-2a\tau_\star}e^{2ia\rho_\star}}{6\tau_\star}. \tag{6.9}$$

[4]

At this point plotting V as a function of τ shows an Anti-de Sitter minimum,

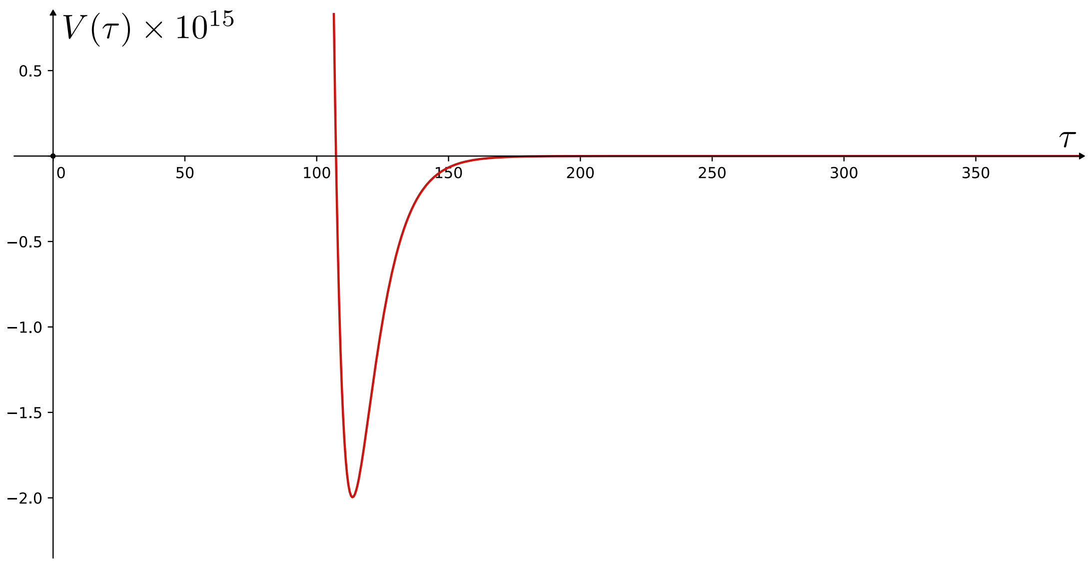


Figure 6.1: Plot of V vs. τ, with $a = 0.1, A = 1$ and $W_0 = -10^{-4}$, a reproduction of figure (3.1) of Houmes [26]

5. **Anti-D3-brane Uplifting** The next step is the one that actually constructs the de Sitter minimum from the established anti-de Sitter minimum, called anti-D3-brane uplifting. As its name suggests, this step involves adding a number of anti-D3-branes to the setup. Additional flux is also added in order to satisfy the tadpole condition. These anti-D3-branes each contribute a term proportional to $\frac{1}{\tau^3}$ so overall we ob-

tain a term of the form $\frac{D}{\tau^3}$ where D is a parameter that depends on the number of anti-D3-branes added.

At this point it is important to mention the concept of **warping**. Much like the warping of spacetime by sources of stress-energy, the compactified manifold M is warped by the presence of flux or D-branes. A crucial part of this step is that the warping caused by the anti-D3-branes added will dampen their contribution to the potential so that it cannot significantly distort the shape of the potential and ruin the setup so far.

We assume that this term simply adds onto the previous potential which gives us the following potential [26]:

$$V = \frac{aAe^{-a\tau}}{2\tau^2}\left(\frac{1}{3}\tau Aae^{-a\tau} + W_0 + Ae^{-a\tau}\right) + \frac{D}{\tau^3}. \tag{6.10}$$

Since D is discretely tunable and we assume exponential warping, we should be able to add a small but positive contribution, $D \geq -V_{\text{min,AdS}}$ (Keep in mind $V_{\text{min,AdS}} < 0$). The trade-off to this setup is that the minimum is no longer global as can be seen from the following plot of V against τ which means that the stability of the vacuum against vacuum

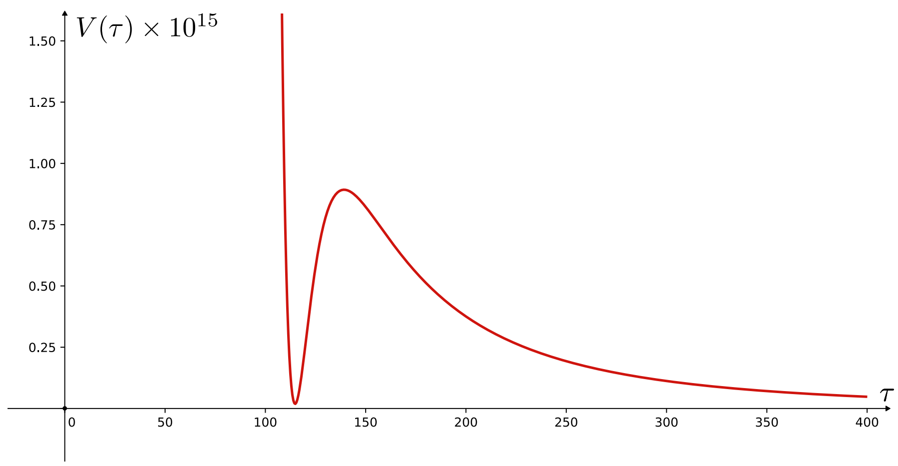


Figure 6.2: Plot of V vs. τ, with $a = 0.1, A = 1, W_0 = -10^{-4}$ and $D = 3\times10^{-9}$, a reproduction of figure (3.2) of Houmes [26]

decay to its true (Minkowski) minimum is in order. Houmes [26] analyses the vacuum using the thin-wall approximation and concludes that the “meta-stable vacuum can be effectively be considered stable” by which the author means that the vacuum decay time is larger than the expected age of the universe but smaller than the Poincaré recurrence time[5] (which has been argued is required for consistency).

With this, the de Sitter vacuum has been constructed and is discretely tunable to, for example, match the observed cosmological constant.

6.2 Computing the Cosmological Constant Given a Scalar Field Potential

The cosmological constant can be computed from the scalar field sector of the Lagrangian of the low-energy theory compactified to d dimensions. We follow the derivation laid out in [28]. The Lagrangian is given by

$$e^{-1}\mathcal{L} = M_{\text{Pl}}^{d-2}\mathcal{R}_n - \frac{1}{2}G_{ij}(\phi)\partial\phi^i\partial\phi^j - V(\phi), \tag{6.11}$$

where $e = \sqrt{-g}$ is the Vielbein determinant and $G_{ij}(\phi)$ is the metric on the scalar manifold.

We can set $G_{ij} = \delta_{ij}$ at a certain point in the potential, $\phi_\star$. This corresponds to a critical point of the potential, that is,

$$\partial_i V\Big|_{\phi_\star} = 0. \tag{6.12}$$

Finding the critical points of the scalar potential involves solving a set of non-trivial coupled algebraic equations. But once we find $\phi_\star$ we can compute $V(\phi_\star)$ which determines the vacuum energy of the solution. This is because the equations of motion are,

$$\begin{aligned} R_{\mu\nu} &= \frac{1}{M_{\text{Pl}}^{d-2}}\left(\frac{1}{2}G_{ij}\partial_\mu\phi^i\partial_\nu\phi^j + \frac{2}{(d-2)}g_{\mu\nu}V(\phi)\right), \\ \partial_\mu&\left(eg^{\mu\nu}G_{ki}\partial_\nu\phi^i\right) - \frac{1}{2}e\partial_k\left(G_{ij}\right)g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j - e\partial_k V = 0, \end{aligned} \tag{6.13}$$

[5]The Poincaré recurrence time is the average time over which the universe repeats itself, statistically speaking. It is argued that no causal Physics can take place at time scales longer than this period.

which combine to give[6]

$$R_{\mu\nu} = \frac{1}{M_{\text{Pl}}^{d-2}} \frac{2}{(d-2)} g_{\mu\nu} V(\phi_\star) \tag{6.14}$$

at $\phi = \phi_\star$.

Equation (6.14) is the vacuum Einstein equation ($T_{\mu\nu} = 0$), in its trace-reversed form,

$$R_{\mu\nu} = \frac{2}{d-2} \Lambda g_{\mu\nu}. \tag{6.15}$$

Thus we can infer that

$$\Lambda = \frac{1}{M_{\text{Pl}}^{d-2}} V(\phi_\star). \tag{6.16}$$

In other words, Λ is determined by the vacuum expectation value of the effective scalar potential at the critical point $\phi_\star$.

In the case of KKLT our potential, V, would be given by equation (6.10).

6.3 Combining KKLT and Axion Monodromy Inflation

The previous analysis details how the KKLT formalism is used to derive the cosmological constant that appears to us in late-time. However, if this process of compactification is used to explain phenomenology in the AMI model, we must also account for the fact that the F-term potential (6.2), incorporates the effect of the inflaton field on the moduli. This causes a conflict. The stabilisation of the moduli in the KKLT mechanism includes the use of non-perturbative corrections. Since these are not precluded by the axion shift symmetry, the correction here can break the symmetry to the extent that higher dimensional corrections become relevant. In particular for the b axion case, the Kähler potential mixes terms for the Kähler modulus T and candidate axion b. This contribution appears as [23]

$$K = -3\ln(\tau - \gamma\bar{\phi}\phi), \tag{6.17}$$

[6] There seems to be a typo in van Reit's work here. The numerator of the coefficient of the Λ term should be 2 in order to agree with the trace reversed Einstein equation. Equation (2.6) and (2.7) of Riet and Zoccarato [28] do not seem to agree with each other, but this cannot be confirmed without their version of the Einstein equation, which is not mentioned in the paper, so it is possible they have an extra $\frac{1}{2}$ in their Einstein equation. We choose to add an extra factor of 2 here since it at best redefines the scale of Λ.

where γ is a constant dependent on the dilaton. The physical volume of the compactification is therefore dependent on the combined term, $(\tau+\gamma b^2) \sim \mathcal{V}^{-\frac{2}{3}}$. The argument in (6.17) is invariant and so τ and b can run in a flat direction. The F-term potential becomes

$$V_F(\tau,\phi) = e^K \left[K^{i\bar{j}} D_i W D_j \overline{W} - 3|W|^2\right] \tag{6.18}$$

$$= \frac{V(\tau,\phi)}{(\tau-\gamma\bar{\phi}\phi)^3}, \tag{6.19}$$

where $V(\tau,\phi)$ contains the terms in the square bracket above it. The volume modulus enters the potential in the e^K pre-factor. This encodes the contributions the potential receives from the energy sources in the theory such as branes and fluxes. Moduli stabilisation is achieved by the non-perturbative correction to the superpotential, equation (6.6). This is a function of T in particular and stabilises only T and not the invariant volume, $\mathcal{V}$. Thus the potential becomes dependent on both $T+\bar{T}+\gamma b^2$ and T, and stabilising T inadvertently stabilises the axion. The axion loses its free direction, gaining a mass term, and the global symmetry does not remain [4]. This problem does not occur for c axions since it does not appear in the Kähler potential.

For a stabilised τ sitting at its minima, a change in $\phi = fb$ would have an effect on the potential since $\mathcal{V}$ relates the two. This origin of the mass of the axion is illustrated in Kachru et al. [29] for the $D3/\overline{D3}$ case[7] by taking an expansion about the de Sitter minimum ($V(\phi_0) = V_0 > 0$) of the potential, assumed to be at $\phi = 0$, we find that

$$V(\phi) = \frac{V_0}{(\tau-\gamma\phi^2)^3} \tag{6.20}$$

$$= \frac{V_0}{\tau^3}\left(1+3\frac{\phi^2}{\tau-\gamma\phi^2}+\ldots\right) \tag{6.21}$$

$$= V_0\left(1+\varphi\bar{\varphi}+\ldots\right). \tag{6.22}$$

The canonically normalised inflaton is now considered to be $\varphi = \phi\sqrt{\frac{3}{\tau}}$[8]. At this stage the V_0 is considered of the order of the Hubble parameter making the inflaton mass, $m_\phi \sim 3H$ leading to corrections of the form (5.3). Note that perturbations cannot be prevented by the shift symmetry at this stage since it has been broken by the moduli stabilisation.

However, this analysis is not accurate for AMI since the expansion in ϕ would

[7] The small-field model for which the KKLT formalism was created

[8] Since the kinetic term in the Lagrangian is modified to $-K_{\phi\bar{\phi}}\partial_\mu\phi\partial^\mu\bar{\phi}$

have to include the higher order terms in (6.22) which cannot be suppressed for $\phi > M_{\rm Pl}$. A series expansion about this minimum is plainly divergent and cannot be defined. At any rate, for this potential $V'' \neq m_\phi$ so the corrections to η are not as before.

Explicitly, if the potential is expressed as

$$V_{\rm eff}(\phi) = V(\phi) f(\phi). \tag{6.23}$$

the slow-roll parameter η is as follows:

$$\begin{aligned} \eta &= \frac{V''(\phi)}{V(\phi)} + \frac{V'(\phi)f'(\phi)}{V(\phi)f(\phi)} + \frac{f''(\phi)}{f(\phi)} \\ &\equiv \eta + \Delta\eta_1 + \Delta\eta_2 \end{aligned} \tag{6.24}$$

Now, assuming η already satisfies slow-roll condition[9], the corrections to η need to be $\ll 1$.

$$\Delta\eta_1 \simeq \sqrt{\epsilon}\frac{f'(\phi)}{f(\phi)}, \quad \Delta\eta_2 = \frac{f''(\phi)}{f(\phi)} \tag{6.25}$$

Taking $f(\phi) = \frac{1}{(1-\alpha\phi^2)^3}$ from equation (6.19) the corrections turn out to be

$$\Delta\eta_2 = \frac{f''(\phi)}{f(\phi)} = \frac{48\alpha^2\phi^2}{(1-\alpha\phi^2)^2} \simeq \frac{48}{\phi^2/M_{\rm Pl}^2}, \tag{6.26}$$

$$\Delta\eta_1 = \sqrt{\epsilon}\frac{f'(\phi)}{f(\phi)} = \sqrt{\epsilon}\,\frac{6\phi M_{\rm Pl}}{1-\alpha\phi^2} < 1. \tag{6.27}$$

The Plank mass factor has been reintroduced in the last terms. In this model $\phi_\star \sim 11 M_{pl}$ so corrections to η will be $\delta\eta \sim 0.4$.

However the more aggressive problem would come from the reintroduction of the Plank-suppressed terms in equation (5.1), which are no longer prevented by symmetry. These will necessarily introduce large corrections since the inflaton is of the energy scale of the gravitational coupling. The single field inflationary treatment will also breakdown.

Lastly, as KKLT introduces an eta-problem for AMI, the late time picture obtained via KKLT is not compatible with AMI. The main problem, as men-

[9] The potential terms other than e^K are assumed to be minimally affected by the axion, since $W \neq W(b)$

tioned previously, is using non-perturbative effects to stabilise the volume modulus. One way out is to consider perturbative corrections, like that of Cicoli et al. [30], to stabilise the volume modulus. The authors of [25] used perturbative corrections to stabilise the volume modulus and solve the η problem in brane-antibrane inflation.

6.4 Future Research Directions

While this review presents the difficulties associated with phenomenological agreement for AMI, there are certain directions of further analysis that suggest promise for making it more tenable.

The application of alternative methods of moduli stabilisation, such as that of perturbative corrections to the Kähler potential hold promise for a solution without an η-problem for the b axion. In Cicoli et al. [25], a method is describing for this process in the brane-antibrane inflation case, where the perturbations in τ give rise to α' and string-loop corrections. We employed perturbative stabilisation in our upcoming work [31] and got rid of the eta-problem. However, the linear potential is in tension with the experimental constraints.

It is further noted that the AMI potential could necessarily be refined by a mechanism called **flattening**. Heavy modes Ψ, affect the potential for the axion, which were cut off by Λ in the effective theory. For large ϕ, and a coupling between Ψ and ϕ, integrating out Ψ has the effect of reducing power of ϕ in the potential $V(\phi)$, *i.e.* the potential becoming flatter [4]. We found that the potential is modified to [31]

$$V(\phi_b) = \mu^{16/5}\phi_b^{4/5}, \tag{6.28}$$

where

$$\phi_b \sim \left[\frac{M_P^2 (g_s M)^{1/2}}{\mathcal{V}}\right]^{1/2} b^{5/4}. \tag{6.29}$$

This affects the predictions for inflationary parameters, such as n_s and r and brings them far better within observational bounds. The spectral tilt and scalar to tensor ratio that we get for this potential

$$n_s = 0.97770, \quad r = 0.05246. \tag{6.30}$$

The scalar to tensor ratio r is within experimental bounds but n_s appears to have a slight tension with experiments. The most precise bounds on the Planck 2018 data [5] states that

$$n_s = 0.9649 \pm 0.0042 \text{ (68\% CL, Planck TT,TE,EE + Low E + Lensing)} \tag{6.31}$$

$$r < 0.056 \text{ (68\% CL, Planck TT + Low E + Lensing + BK15)} \tag{6.32}$$

These flattened potentials are being studied to see if a non-zero parameter space can be achieved via axion monodromy inflation. This puts up a very strong challenge for power law potentials and experiments seem to prefer potentials mostly of the form [5, 25]

$$V(\phi) = C_0 \left(1 - \frac{C_1}{\phi^n}\right) \qquad \text{or} \qquad V(\phi) = C_2 \left(1 - C_3\, e^{-m\phi}\right). \tag{6.33}$$

Chapter 7

Conclusion

In this paper we assessed the viability of a theory that incorporates explanations for both the early and late universe. Given the issues with the older standard model of cosmology, we motivated the necessity for introducing cosmic inflation. With some preliminaries on supersymmetry and string theory, we introduce our candidate model of inflation, axion monodromy inflation, to serve as our theory for early universe evolution. We chose the KKLT mechanism as our explanation of the late universe, being the premier method to constructing a de Sitter vacuum from string theory. With all this established, we look at the viability of axion monodromy inflation in a universe in which moduli stabilisation has already been accomplished as per the KKLT procedure. We find that the disruption of the shift-symmetry of the axion by the stabilised volume modulus leads to an η-problem that prevents inflation from lasting long enough to account for the homogeneity and flatness of the universe. Given that as we established, inflation is required to explain discrepancies in the standard model of cosmology, this implies that *the universe's evolution cannot be adequately explained by taking the KKLT mechanism and axion monodromy inflation and integrating them naïvely*. That being said, modifications to the method of stabilising moduli and adding corrections to the potential from massive integrated-out states, may allow us to have a viable theory of both late time and early time universe respecting experimental bounds. This is being explored further in an upcoming work [31].

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