

OPTIMAL ORDER QUANTITY INTEGRATING OPERATIONS AND CARBON EMISSION COSTS IN PRODUCTION DELIVERY SUPPLY CHAIN

By

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Declaration

It is hereby declared that,

1. The thesis submitted is my own original work while completing degree at BRAC University.
2. The thesis does not contain material previously published or written by a third party, except where this is appropriately cited through full and accurate referencing.
3. The thesis does not contain material which has been accepted, or submitted, for any other degree or diploma at a university or other institution.
4. I have acknowledged all main sources of help.

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Dedication

I dedicate this thesis to my beloved parents and my dear wife for their continuous support, love, and inspiration throughout this journey. Their unwavering belief in my capabilities served as the greatest motivational force.

I am deeply thankful to my supervisor Dr. Md. Abdul Hoque, Professor, BRAC Business School (BBS), BRAC University for providing his patience and valuable guidance throughout the research process. His thoughtful guidance combined with ongoing support played a vital role in developing this research and advancing my academic development.

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List of Acronyms

SCM	Supply Chain Management
JIT	Just-In-Time
EOQ	Economic Order Quantity
MOQ	Minimum Order Quantity
EPQ	Economic Production Quantity
ESG	Environmental, Social, and Governance
GSCM	Green Supply Chain Management
CSCs	Circular Supply Chains
SEOQ	Sustainable Economic Order Quantity
CO ₂	Carbon Dioxide
SO ₂	Sulfur Dioxide
AI	Artificial Intelligence
IoT	Internet of Things
LSS	Lean Six Sigma
VCA	Value Chain Analysis
MOO	Multi Objective Optimization
EVs	Electric Vehicles
PCF	Product Carbon Footprint
EU-ETS	European Union Emissions Trading System
SCOR	Supply Chain Operations Reference
PCF	Product Carbon Footprint
BDT	Bangladeshi Taka

Glossary

Terms	Definitions
Supply chain optimization	Supply chain optimization is the process of increasing supply chain efficacy to achieve lower operational expenses while reaching maximum operational outcomes.
Lean manufacturing	The production methodology reduces waste while creating maximum value through ongoing alignment between actual market needs and production operations.
Just-in-time (JIT)	This inventory strategy provides products when needed while minimizing inventory storage expenses and reducing the risk of items becoming outdated.
Economic order quantity (EOQ)	The model finds the best order size for a retailer to achieve the minimum annual expenses of ordering and inventory holding.
Economic production quantity (EPQ)	The model finds the best order size for a manufacturer to achieve the minimum annual expenses of setting up machines for processing a product and inventory holding during production and selling of the product.
Minimum order quantity (MOQ) contract	A supplier-retailer contract commonly used to specify the possible lowest number of products supplied by a supplier to meet customers' demand as accurately as possible.
Sustainable EOQ (SEOQ)	An advanced EOQ model that includes environmental measures in calculating emission taxes in the inventory management system of deciding EOQ.
Lean six sigma (LSS)	Lean and six sigma approaches merge into a methodology that enhances a supply chain system through productivity improvement and waste elimination.
Circular supply chains (CSCs)	Supply chains focusing on efficient utilization of resources and sustainable packaging for better product recovery by shifting away from traditional linear operations to cyclic resource-based operational development.
Green supply chain management (GSCM):	Supply chains operations management requiring sustainable practices to lower environmental destruction.

Environmental sustainability	Environmental operational practices in building long-term natural system sustainability in order to minimize ecologically harmful consequences.
Carbon emissions	CO ₂ gas emissions from various sources of human activities contributing to atmospheric changes that drive to a destructive climate change.
Product carbon footprint (PCF)	A model that assesses carbon emission throughout the entire lifecycle of a product from manufacturing to end use and disposal.
AI route optimization	An artificial intelligence-based technology that helps to optimize delivery routes in order minimize both the fuel use and the carbon emissions.
Electric vehicles (EVs)	An artificial intelligence-based technology that helps to optimize delivery routes and minimize both the fuel use and carbon emissions.
Multi-objective optimization (MOO)	An optimization technique that serves in developing a supply chain and logistics modeling by optimizing various objectives in order to strike balances between the economic performance and the environmental stewardship as well as social responsibility.
Simulation-optimization	An artificial data-based optimization technique to addresses sophisticated supply chain challenges in real-life settings, where other optimization techniques fail to achieve an expected result.
Advanced planning and scheduling (APS)	A technology that uses distributed simulation methods to optimize supply chain planning and execution approaches.

Abstract

This thesis develops a mathematical model that incorporate both the operational and the carbon emission costs to find the optimal order quantity (Q) in a production-delivery supply chain. In this model the order quantity (Q) is set equal to α (a real positive multiplier) times the demand (D). We derive a formula by applying the calculus method of differentiation for determining the optimal order quantity (Q). For an illustrated numerical example problem, the optimal α value is found to be as 0.1290, along with the optimal order quantity (Q) of 12,900 units and the minimum annual total cost of 970,057.23 BDT (total operational cost of 886,453.31 BDT plus carbon emission costs of 83,603.92 BDT). A sensitivity analysis shows that the optimal order quantity and the total supply chain cost including operational and carbon emission costs respond to adjustment of the demand, production rate and other operational parameters. This model has the capability of helping organizations in making eco-friendly business choices leading to better sustainable practices. The result highlights the importance of integrating environmental attentions with operational efficiency in supply chain management. This research presents an actionable recommendation to optimize the order quantity (Q) by minimizing the total cost of managing operations together with the reduction in carbon emission, which in turn measures for creating effective and environmentally responsible supply chain operations. The research findings enhance both theoretical understanding of sustainable supply chain optimization and its practical applications.

Keywords: Optimal order quantity (Q); Carbon emission cost; Operational cost; Sustainable supply chain; Integrated cost emission framework

Chapter 1 Introduction

1.1 Research Context and Background

Optimizing supply chains is vital for businesses to improve operational efficiency, reduce costs, and maintain a competitive advantage in today's highly interrelated global markets. Advanced technologies such as intelligent automation and AI together with real-time data analysis boost organizational operational efficiency and decision-making processes to enable faster responses to the newly evolving market requirements (Chowdhury et al., 2024; (Muralidhar et al., 2024). Supply chain optimization helps to create sustainable outcomes due to its capability of allowing organizations to strengthen their environmental performance in the global economy by eliminating waste (Verma, 2024).

The modern economy requires optimized supply chain approaches since its operations depend on sophisticated interconnected systems. Businesses depend on global supply networks that provide them with material access together with production facilities and customer delivery capability. The benefits of global supply chain networks lie with three major challenges that include intensifying risks together with higher financial costs alongside adverse environmental impact. Organizations must focus on supply chain optimization because it establishes a strategic foundation that improves their business viability alongside sustainability practices.

Organizations achieve optimal supply chain success through the adoption of lean manufacturing principles. Lean manufacturing systems help businesses eliminate waste and enhance process operations to generate customer value. Companies those maintain production aligning with real-time market needs eliminate surplus inventory by minimizing delivery times while decreasing production expenses. JIT practices under lean manufacturing enhance efficiency by providing required materials at specified delivery times which reduces storage cost and prevents product obsolescence.

Due to the increasing scope of research in this field this supply chain can be organized well. Research facilitates advanced technology-based supply chain infrastructure along with data analytics to boost operational performance alongside building resilience capabilities. By using predictive analytics for demand forecasting, organizations can control inventory levels easily, thereby mitigating supply constraints and efficiently managing demand variations.

Effective supply chain management is vital for fostering sustainable business growth and achieving market leadership within the modern diversified economic systems. Advanced technologies and effective practices enable companies to boost efficiency by lowering costs while enhancing their maintenance of environmental standards. Efficient optimization strategies are essential for long-term success because supply chains will become increasingly complex in carrying out operations.

The necessity of environmental sustainability-based logistics operations continues to grow as an essential factor for business survival and climate change resolution. The adoption of advanced technologies comprising AI and big data platforms as well as the IoT, has enabled companies to maximize their eco-friendly logistics operations resulting in lowered carbon emissions and better energy effectiveness (Nicoletti & Appolloni, 2024; Leogrande, 2024). The reduction of substantial greenhouse gas emissions from the logistics sector requires sustainable practices like eco-friendly packaging together with energy-efficient warehousing (Nagy & Szentesi, 2024).

The three components of logistics operations known as transportation, warehousing and packaging, emit substantial greenhouse gas. Trucks, ships and airplanes among other vehicles use fossil fuels to transport goods over long distances and collectively share major emission levels of the global market. The process of warehousing and packaging uses power-intensive storage sites alongside non-recycling packaging products raises the total adverse environmental impact.

Organizations are integrating advanced technology alongside innovative practices to boost the sustainability performance of their logistics systems. AI route optimization technology enhances sustainability by establishing optimum delivery ways that minimizes fuel usage along with emission outputs. EVs for deliveries through the final distribution phase create more sustainable operations because they generate no carbon emissions as operations are performed with renewable power.

Thus, the process of developing sustainable logistics systems faces multiple obstacles during transition. First the high expense barriers often prevent companies from implementing advanced technologies and they face complicated environmental rule sets to be practiced. The successful removal of these barriers needs enterprises work closely with governments while involving supply chain partners at every level. Governments should provide financial support and set precise guidelines and drive innovation, and businesses must integrate ESG principles, invest in green technologies and work together to develop best operational practices.

The necessity of developing sustainable supply chain practices requires combating with climate changing environment and build stronger enterprise competitiveness with increased resilience levels. Sustainable supply chain adoption by companies creates both the environmental benefit and the regulatory compliance alongside enhancing organizational reputation. Nowadays, the global marketplace demands sustainable practices more and more requiring advanced technologies and eco-friendly strategies for continued growth. We have included such a sustainable research work in this thesis.

1.2 Research Problem

Organizations struggle to decrease their supply chain carbon emissions while maintaining cost efficiency at reasonable levels. Balancing between these two extremes is crucial because regulatory bodies alongside consumers require sustainable practices. However, achieving this equilibrium has become challenging. Economic efficiency in CO₂ reduction programs works

best through optimizing the path between emission-cost payments and unpredictable environmental damage expenses (Tahvonen et al., 1994). The unpredictable environmental consequences make it harder to find economical solution by reducing carbon emission appropriately. In the context of logistics, a bi-objective lot-sizing model exposes that minimization of carbon emission can be costly compared to maintaining cost-optimal results, specifically when previous investments make sunk costs (Turkensteen & Heuvel, 2023).

Governments in different regions maintain distinct regulations and energy costs and technological boundaries resulting in challenging cost-emission balance (Manne & Richels, 1995; Li et al., 2017). Organizations need to handle an intricate combination of external elements that strongly affect their ability to find operational cost-emission balance effectively.

Today's inventory and logistics optimization technologies contain multiple setbacks that make them ineffective at reducing expenses and environmental impact. Failing to consider product expiration dates together with business economic patterns results in stock overage or depletion which raises expenses through speeded-up deliveries and product waste (Popova et al., 2018).

The use of advanced technologies like machine learning combined with swarm intelligence boosts efficiency although this comes at the expense of major capital investments together with organizations needing to adapt their structures (Chaitanya Palanki, 2023; Kumar et al., 2013; Madas et al., 2024). The potential use of advanced technologies faces such ongoing implementation obstacles.

Organizations need to establish strong risk management systems since geopolitical conflicts along with natural disasters and pandemic outbreaks affect their supply chains (Anaba et al., 2024). Current supply chain frameworks struggle with the operational cost optimization along with carbon emission management of integrated methods, which results in increased operational risks. To achieve both the economic results and the eco-friendly environmental targets in SCM, operations management requires an advanced adaptive modeling approach of

this problem and its solution in order to sustain in this situation. We intended to work in this direction.

1.3 Research Objectives

The main objective of this research is to develop a mathematical model that will integrate operational and carbon emission costs in a production-delivery supply chain. The main objective of this study is to find the optimal α value ($\alpha > 0$) such that the optimal order quantity (Q), $Q = \alpha D$ that minimizes the cost of operations and carbon emission for processing of a product and its delivery, while maintaining demand and production constraints. Further, this study explores how economic choices impact environmental sustainability in carrying out supply chain operations. In this regard we have done the following:

- Develop a mathematical model combining operational costs with carbon emission costs in production and delivery supply chain.
- Identify the optimal order quantity (Q) that minimizes both operational costs and carbon emission costs while meeting demand and production constraints.
- Perform sensitivity analysis to ensure the optimal value functions correctly when varying demand, production rate and others operational parameters.
- Perform an evaluation to determine the economic-environmental balances involved in supply chain decisions.
- Extend the existing inventory and logistics optimization research through the incorporation of cost-effective carbon emission assessment models.
- Recommendations are provided to organizations for optimizing order quantities effectively for sustainable SCM.

1.4 Significance of the Study

This study delivers significant advancements within the field that focuses on sustainable supply chain practices. The study develops a mathematical model connecting both the operational and the carbon emission costs and an organized mathematical solution procedure to optimize production-delivery supply chain order quantities. The manufacturing and logistics companies can apply this solution process to tackle carbon emissions problem significantly. There is a growing focus on transitioning to green logistics solutions because of logistics sector is a major contributor to greenhouse gas emissions. These efforts highlight the adoption of environmentally friendly transportation system, energy-efficient warehousing, and as well as environmentally responsible packaging approaches (Nagy & Szentesi, 2024).

This analysis specifically investigates operational efficiency in combination with carbon emissions to enhance knowledge about sustainable practices within supply chains. The research will aid policy-makers together with industry leaders by showing the necessity for operational sustainability integration to develop an environment-positive supply chain framework.

This research delivers an optimal solution process for practical benefits that align specifically with industries which make carbon reduction essential to their operational activities. Also, it provides industrial organizations with specific process-oriented methods that help them cut costs while reducing their environmental impact through the optimized order quantity management.

Decision-makers can find practical tools along with empirical evidence via implementation of our mathematical optimization procedure which they may apply to face inherent challenges. The research-based practical insights will grant industries tools to execute sustainable practices both economically and sustainably for their journey to reach reduced carbon emission targets.

1.5 Scope and Boundaries

Scope:

The main purpose of this research is to assess order quantity (Q) optimization within supply chain process that focuses on production-delivery operations. The research creates a mathematical solution which finds optimal order quantities (Q) between operational cost reduction and carbon emission control for manufacturing and distribution operations dedicated to environmental sustainability.

The research examines different parameters including demand, production rates, etc., and carbon emission coefficients which stem from distinct logistical activities. The developed mathematical model offers decision-makers a framework to find optimal supply chain plans which minimize costs and enhance sustainability across supply chain operations.

Boundaries:

The boundaries of this thesis are established by certain assumptions along with specific areas of focus. The model operates with deterministic demand patterns although real supply chains actually encounter unforeseeable market shifts and operational disruptions. The research is concentrating on the operational and carbon emissions expenditures, but is overlooking social, and ethical aspect. Although these technologies (artificial and machine learning), are mentioned, the research doesn't study the problems with regard to the implementation of these technologies at scale. Also, the research does not consider regulatory conditions or even discuss disruption factors such as changes in material costs or geopolitical uncertainties. Future research must develop the model's scope to support its application across various supply chain configurations.

1.6 Thesis Organization

The research is organized over five chapters contributing to supply chain order quantity optimization focusing on integrated cost management of business operations and carbon emissions.

Chapter 1: Introduction

The first chapter sets the research margins through supply chain optimization analysis combined with environmental sustainability interests. The section defines the research borders through its explanation of the main problem statement and research goals and demonstrates its significance along with the scope and boundaries.

Chapter 2: Literature Review

This chapter analyzes the existing works related to supply chain optimization together with inventory management structures (EOQ, MOQ, and EPQ) and environmental issues as well as integrated cost-emission frameworks. This section reveals key missing information in previous studies that make it essential to conduct this present investigation.

Chapter 3: Model Formulation and Solution Methodology

The section outlines the mathematical model development steps which include proposed assumptions and notations followed by total cost function definition and then optimization framework alongside sensitivity evaluation.

Chapter 4: Results, Discussion and Sensitivity Analysis

The study presents parameter identification, optimizing results, sensitivity analysis and illustrates the trade-offs between operational and carbon emission costs before drawing practical and theoretical conclusions.

Chapter 5: Conclusion and Recommendations

The conclusion includes crucial study results alongside contributions to sustainable supply chain optimization, actionable recommendations for sustainable supply chains with the specified research limitations for future directions.

References

The references section provides a comprehensive list of all the sources cited throughout the thesis, formatted in a standardized APA 7th edition style.

Appendix A

The Appendix A contain computational python coding sections both the optimization model and the sensitivity analysis.

Chapter 2 Literature Review

2.1 Introduction to Supply Chain Optimization

2.1.1 Overview of traditional and modern approaches

Supply chain optimization has progressed from the basic traditional methods to the modern sophisticated approaches which solve complex real-life supply chain challenges. The development benefits from advanced operations research techniques together with technological advancements. The framework of integrated mathematical modeling combined with simulation-optimization has demonstrated effective management of supply chain determinants and stochastic components (Pourhejazy & Kwon, 2016). These methods produce efficient risk management outcomes while optimizing profit and lead time performance and supporting decision-making across inventory allocation and order management systems (Aqlan & Lam, 2016). The methods act as a powerful solution base to address various supply chain problems arises in planning and design. The field adopts sophisticated optimization approaches to solve problems via operational execution across diverse supply chain areas (Geunes et al., 2007).

Technological progress and market fluctuations drive the transition from traditional SCM to its modern version. Traditional SCM featured linear hierarchical structures which limited communication between parties while utilizing manual processes that created inefficient and information-alternating systems (Hasan & Habib, 2023; Hao et al., 2002). Modern SCM systems unite next-generation information technologies which incorporate both IoT and blockchain features. Modern SCM utilizes these information technology tools to optimize resource distribution while improving processing speed and enhanced supply chain transparency (Wang, 2020; Hasan & Habib, 2023). The digitalization revolution has created enhanced inventory management while speeding up response capabilities and generating better market trend predictions (Wang, 2020).

Growing supply chain data volumes require modern management practices as an essential tool to handle large-scale global supply chain operations. Modern SCM practices have become crucial for effective management of growing supply chain data as the amount of data expands exponentially (Mitroo, 2019).

2.1.2 Advanced optimization techniques for efficiency and effectiveness

Supply chain optimization techniques using advanced methods have been continuing to draw extensive attention in the recent years. The implementation of integrated mathematical modeling and simulation-optimization frameworks has proven beneficial in attaining supply chain network optimization (Pourhejazy & Kwon, 2016). System performance achieves better results via utilizing these techniques in facility location and transport and inventory management.

The incorporation of Advanced Planning and Scheduling functions with distributed simulation represents a new supply chain development. A distributed simulation method implementing Advanced Planning and Scheduling procedures, presents an innovative technology for supply chain optimization collaboration throughout the semiconductor manufacturing industry (Lendermann et al., 2001). Through this approach organizations can optimize their planning processes and execution methods quickly while implementing simulation findings without difficulty.

Supply chain sustainability, efficiency, and effectiveness depend mainly on effective communication and seamless goods and information transmission together with appropriate partner selection and environmental considerations (Bhagwat, 2018). LSS has become a lead method to apply in this environment. The combination of LSS methodologies has become a widely adopted method to optimize supply chain systems (Madhani, 2020).

Strategic decisions heavily depend on the successful measurement of both efficiency and effectiveness even though these metrics are hard to assess. Organizations functioning adapting with the modern supply chains must wisely manage the trade-offs between these dimensions in order to attain optimal performance results.

2.2 Inventory Management and Optimization Models

2.2.1 EOQ cost minimization, limitations and environmental impact

The EOQ model works to achieve cost-efficient inventory management by finding the optimal order quantity. Researchers across various periods conduct multiple investigations to enhance this cost minimization method. A practical approach to apply EOQ models for retail cycle stock inventories focused on simple implementation for small companies (Bassin, 1990). The adapted EOQ method gives smaller businesses a practical way to maximize inventory levels and minimize costs. The EOQ model expanded through research which merged unit-load and material-handling elements that included transportation expenses and physical limitation restrictions on volume and weight along with storage space capacity (Tanchoco et al., 1980) to enhance model suitability for difficult logistical industries.

Despite being extensively deployed the conventional EOQ model, suffers from essential drawbacks when evaluating sustainability aspects during its utilization. The traditional EOQ model cannot address typical industry complexities that include mandatory minimum order levels together with imperfect product quality standards as well as CO₂ emission reduction obligations (Alnahhal et al., 2024). As the demand of sustainable supply chain has been increasing, sustainable EOQ models need to be analyzed including uncertain factors, lead times along with multiple transportation options as well as carbon emission costs (Lee et al., 2017).

Sustainable inventory management implementation necessitates the analysis of waste management and preservation technology elements. Total inventory costs can decrease together with environmental sustainability through waste management implementation

alongside carbon-sensitive demand monitoring and preservation technology incorporation (Kumar et al., 2024). The EOQ model applied to agriculture benefits sustainability when it incorporates variables that measure fuel consumption and logistics costs. Farms' performance improvement with sustainability benefits becomes possible through EOQ model adaptation via fuel consumption analysis, and vehicle load ratio management and transportation cost evaluation between internal and external providers (Gao et al., 2019). Research evidence shows that modern supply chains need EOQ models to incorporate environmental sustainability features because this increases their effectiveness in today's competitive market dynamics.

Recent research shows growing interest in incorporating environmental aspects into EOQ models to promote sustainability issues into inventory management practices (Arslan & Turkay, 2013). The traditional EOQ systems focus on attaining reduced costs despite overlooking environmental effect evaluations. However, the research suggests modifications of supply chain models to add social values and environmental standards for better sustainability evaluations (Arslan & Turkay, 2013).

To meet sustainability goals researchers have developed new models that manage substandard products and observed environmental rules such as carbon emission control and carbon tax policies (Raiya & Mittal, 2022; Kazemi et al., 2018). Under different carbon regulations, researchers developed sound ordering approaches combining profit maximization while minimizing environmental impact (Yang et al., 2019). The models demonstrate their environmental usefulness through livestock slaughter optimization which aligns breeding practices with sustainability goals and reduces CO₂ emissions while meeting operational constraints (Nobil et al., 2023).

SEOQ frameworks have emerged to incorporate emission taxes and environmental measures specifically for manufacturing operations (Baraskar et al., 2024). The frameworks incorporate different operational complexities such as imprecise environmental conditions together with

product failures and inventory operations' need for carbon reduction (Chaudhary et al., 2023). The research focuses on slow-moving deterioration as well as waste disposal methods, and flexible demand patterns in addition to trade credit and carbon pollution factors (S. Kumar et al., 2024).

The EOQ model evolved to deal with environmental issues demonstrates a fundamental solution to the urgent requirement of sustainable inventory practices which merges economic performance with environmental stewardship.

2.2.2 MOQ role in supplier contracts, cost and sustainability

The use of MOQ contracts received extensive academic attention in supply-chain-based supplier-retailer relationships. MOQ contracts serve as effective coordination tools for supply chains when properly deployed for improved operational performance. The simplicity and efficiency of MOQ contracts make them ideal choices according to recent research which has proven their effectiveness in reducing contract complexity-associated cognitive burdens (Tuncel et al., 2022). Through the implementation of advance-order discounts MOQ contracts enable better supply chain coordination and enhanced operational efficiency (Chintapalli et al., 2017). MOQ constraints can also be integrated into constancy contracts to decrease storage costs for retailers (Hellion et al., 2015). The selling format selection of suppliers in online retail platforms depends on MOQ contracts because they opt for marketplace distribution when MOQ thresholds are low but prefer reselling under high MOQ conditions (Chen et al., 2023). Manufacturing flexibility stands as a key factor for resolving production difficulties in settings with high mix and low volume output while operating under current market dynamics. Customer flexibility which represents unimportance of specific product attributes or delivery timelines expands the possible solutions for supply-demand matching (Zhang & Tseng, 2009). Research shows that manufacturing flexibility depends on four main factors: disruption risks, digitalization, sustainability and supplier integration (Ivanov et al., 2018).

The implementation of sustainable initiatives across value chains encounters several obstacles because suppliers do not fully commit, and lack expertise regarding sustainable practices (Dwivedi et al., 2021). However, the fundamental role of flexibility enables manufacturers to enhance their operational efficiency and system durability as well as reach sustainable supply chain operations.

A growing body of research delves into the multifaceted impact of MOQ on costs and sustainability within supply chains. MOQ contracts enhance supply chain coordination and performance outcomes by uniting with advance-order discount programs and buyback options (Chintapalli et al., 2017; Chan et al., 2018). Studies in recent times demonstrate how Minimum Order Quantities (MOQ) functions as a coordination tool between manufacturing and retail operations, when deploying fast-response supply methods using clean technologies to produce mutual advantages for producers and retailers. Through MOQ implementation manufacturers and retailers can create joint advantages when using quick response methods supported by cleaner technology (Chan et al., 2018). Multi-echelon systems require improved ordering policies for optimal cost optimization because MOQ requirements increase inventory management complexity (Shen et al., 2019). The EOQ model requires modifications through MOQ analysis to handle multiple needs such as supply chain disruptions and sustainability considerations (Alnahhal et al., 2024). Studies reveal that investments in sustainability practices decrease supply chain risks especially in emerging markets where they boost reactive risk management strategies (Gouda & Saranga, 2018). The study explores the relationship between supply chain management and sustainable business practices, representing their significant improvement effect on performance metrics.

2.2.3 EPQ batch production, efficiency and sustainability

Traditional EPQ research models fail to acknowledge several essential elements which affect inventory management and production strategies. New research initiatives attempt to resolve

the identified model limitations. The research combined operation routing and EPQ determinations to reduce manufacturing expenses in adaptable environments (Egbelu, 1996). The EPQ model now incorporates partial backordering alongside quality discount options for batches that revealed quick sale of imperfect items that can minimize overall costs (Cunha et al., 2018). The multi-stage EPQ model which establishes manufacturing cycle time relative to lot size showed potential economic benefits than standard methods (Szendrovits, 1975). The analysis of net present value evaluated the EPQ model to derive an exact formula for cash flow present value alongside finding the optimal batch size (Disney et al., 2013). These research studies demonstrate the necessity of including routing flexibility and imperfect quality together with multi-stage production, and the time value of money in EPQ models for enhancing decision-making and cost optimization in batch production systems.

Studies in current times focus on integrating product life cycle elements together with environmental factors within sustainable EPQ frameworks. The EPQ modeling framework benefits from studies, which analyze how to include carbon emissions generated during production and storage together with product recycling elements (Karim & Nakade, 2022). Research models now exist to optimize production quantity and costs for perishable goods by accommodating Weibull-distributed deterioration and variable demands and defective items (Rabiu & Majahar Ali, 2024). The inclusion of greenhouse gas and wastewater emission costs in expanded EPQ models now offers manufacturing decision-support to executives working in pulp and paper execution (Yee et al., 2023). Sustainability education enters into engineering education through project-based learning methods including European Project Semester that unites students to learn sustainable development (Duarte et al., 2020). This research provides valuable insights via this literature review for expanding the scientific knowledge regarding sustainable production and inventory management.

2.3 Environmental Concerns in Supply Chains

2.3.1 Carbon emissions in supply chains

Current research on carbon emissions in supply chains identifies operational choices and supply chain collaboration as crucial strategies to decrease environmental impact. Research demonstrates that sustainable supply chain approaches decrease emissions and supplier ESG training enhances these results (Karaman et al., 2024). Standard operational models show that businesses can manage carbon emissions in their decisions by modifying operations while collaborating with their supply chain networks without affecting costs (Benjaafar et al., 2012). Game theory analysis examines manufacturing-retail relationship that integrates variables from carbon trade to consumer preferences accordingly (Tian et al., 2024). Researchers have investigated how social planners perform better with carbon footprint distribution combined with contract development to stimulate members of supply chains toward emission cutbacks (Lee & Choi, 2021). The research results offer important knowledge pointing to help supply chain managers achieve equilibrium between economic performance and environmental sustainability.

2.3.2 Environmental impact of transportation and logistics

The environmental impacts of transportation systems interlock with inventory practices where multiple considerations create difficult trade-offs. Centralized warehousing produces major economic and environmental advantages though transporting goods across longer distances results in higher costs (Matthews & Hendrickson, 2002). The complete life-cycle assessment of transportation systems shows non-operational elements produce larger total emissions than tailpipe operations regardless of energy inputs during the life cycle period (Chester & Horvath, 2009). Innovation within E-logistics practices that cover supply chain assets and distribution management and delivery implementation, create extensive ecological consequences for both the supply and the value chain operations (Sarkis, 2012). Integrated manufacturers-retailers

mathematical optimization policy utilizing MOQ contract, minimizes delivery costs and decrease carbon emissions when moving products from sources to destinations (Hoque & Babu, 2024). By properly matching transportation methods with inventory decisions, organizations can benefit from transportation as a substitute for inventory stock and gain better control of their speed-versus-cost trade-off. Joint optimization of these decisions produces substantial cost reductions (Constable & Whybark, 1978). The results show that suppliers should adopt a complete system to measure environmental effects from transport operations and inventory control throughout supply chain networks.

2.3.3 Circular supply chain and sustainable practices

The implementation of CSCs leads supply chain systems beyond linear patterns to develop cyclic resource-based operations (Bimpizas-Pinis et al., 2022). The eight essential processes that define CSCs include plan, source, make, deliver, use, return, recover and enable (Vegter et al., 2020). Resource efficiency and sustainable packaging together with product recovery represent key priorities within these eight core processes (Vegter et al., 2020). CSCs development depends on improved connection and unity between supply chain entities as well as novel stakeholder participation and market sector collaboration (Bimpizas-Pinis et al., 2022; González-Sánchez et al., 2020). The development of CSCs requires four key elements which include intensified relationships, adapted logistics and organizational structures, disruptive technologies and a supportive environment (González-Sánchez et al., 2020). Rapid evolution of CSCs requires further clarification between CSCs and other sustainability-related fields as well as standard definitions and conceptualizations (Montag, 2023). Investigators need to advance the CSCs framework and investigate its distinctive aspects (Montag, 2023).

2.4 Integrated Cost Emission Models in Supply Chains

2.4.1 Cost emission trade-offs and sustainable inventory management

The field of SCM research now centers its attention on how costs relate to environmental impact especially when discussing carbon emissions. At present researchers work on developing optimization models that handle economic factors and environmental outcomes with a main emphasis on carbon emission reduction (Yang et al., 2016; Ramudhin et al., 2010). Supply chain optimization models evaluate transportation expenses alongside greenhouse gas output levels under carbon regulatory systems (Yang et al., 2016; Paksoy & Özceylan, 2014). Simulation techniques have emerged as a solution to eliminate constraints in analytical optimization which can lead to reduced costs and emissions (Keramydas et al., 2017). Researchers examine supply chain design approaches that combine carbon market analysis through which decision-makers can assess variations of costs and carbon offsets resulting from different regulatory frameworks (Ramudhin et al., 2010). The studies show managers with tools to enhance their understanding of supply chain optimization through economic and environmental factor analysis which supports sustainable supply network design (Paksoy & Özceylan, 2014; Ramudhin et al., 2010).

2.4.2 Green logistics and carbon footprint considerations

Green logistics practices represent a critical component of sustainable business expansion as well as environmental preservation. The practices combine essential aspects from green transportation with warehousing operations and management activities along with waste management initiatives (Vienažindienė et al., 2021). Organizations adopting green logistics strengthen resources and technology systems while building better customer relationships (Brădescu, 2014). AI currently employs foundation models to advance logistics sustainability by streamlining delivery pathways and minimizing product waste and optimizing warehouse structures (Nicoletti & Appolloni, 2024). Research findings demonstrate how green logistics

operations enhance supply chain sustainability while showing a more indirect connection to actual green practices (Roy & Mohanty, 2024). Green logistics operates as a mechanism which maintains operational efficiency and reduces costs and enhances corporate image (Brădescu, 2014).

2.4.3 Balancing costs and sustainability in supply chains

Accurate monitoring of carbon emissions throughout supply chains is essential for sustainable supply chain management. The measurement of emissions makes use of both input-output models along with standardized assessment procedures (Cordero, 2013). The SCOR framework establishes the foundation which enables researchers to develop the PCF model for sustainability evaluation (He et al., 2019). The VCA system which operates on the blockchain provides protected and transparent reporting for carbon emissions (Heiss et al., 2024). The integration of research-based carbon footprint management into supply chains occurs through a combination of regulatory needs along with consumer preferences and environmental sustainability benefits (Aivazidou et al., 2013).

Research shows that operational adjustments alongside partnership-based decisions minimize carbon emissions while creating negligible added costs (Benjaafar et al., 2012). The analysis performed by strategic decision models reveals that when social cost rates increase, operations become less expensive and carbon emissions decrease (Tseng & Hung, 2014). Linear programming models with multiple objectives allow organizations to optimize logistics expenses against carbon management results (Ramudhin et al., 2010). The supply chain coordination benefits from emissions trading systems EU-ETS which leads to optimized inventory costs together with reduced carbon footprint (Jaber et al., 2013).

2.5 Research Gap and Justification

2.5.1 Challenges in multi-objective optimization (MOO) and sustainability

MOO represents a common framework in sustainable supply chain and logistics modeling yet struggles to handle all sustainability dimensions (Jayarathna et al., 2021). Economic evaluation and environment considerations frequently receive attention in research yet social aspects remain insufficiently studied (Chen & Andresen, 2014; Jayarathna et al., 2021). The Bees Algorithm and other swarm-based algorithms form part of a variety of MOO approaches, which is used by researchers to optimize supply chain configurations that minimize cost and lead time (Mastrocinque et al., 2013). Research has started to merge complete sustainability pillars - economic, environmental and social elements within supply chain optimization models (Chen & Andresen, 2014). Through the application of fuzzy logic researchers analyze qualitative variables while employing genetic algorithms to resolve complex optimization problems in supply chain configuration (Nepal et al., 2011). Sustainable supply chain optimization requires improved sophisticated approaches and hybrid meta-heuristic methods (Jayarathna et al., 2021).

2.5.2 Integrated cost emission models in supply chain decision making

Researchers have showed rapid growth in the integration of cost and emission factors into SCM models. Operational adjustments show substantial emission reduction without significant cost growth (Benjaafar et al., 2012). The electricity sector faces a dual challenge between lowering operational expenses and reducing emissions particularly with regard to thermal power generation of SO₂ emissions (Chattopadhyay, 1995). Both plug-in EVs and renewable power supply hold promise for reducing transportation industry costs while lowering emissions in smart grid structures (Saber & Venayagamoorthy, 2011).

Data shows an "energy efficiency gap" because the market fails to implement cost-effective energy-saving technologies according to the EPA benefit-cost analysis of vehicle greenhouse

gas regulations (Helfand & Dorsey-Palmateer, 2015). Different research studies demonstrate how organizations need to use emission factors in their operational decision-making systems across industries.

2.5.3 Emerging research trends in green supply chain optimization

Research on Green Supply Chain Management shows rising awareness about sustainability issues throughout supply chains. Research streams in GSCM are outlined through an established framework which describes nine boundaries with five resource flow components (Sarkis, 2012). The meta-analysis driven on 362 papers highlights that Multi-Criteria Decision-Making methods and firm-level studies are the most predominant methods in Sustainable Supply Chain Management study, reflecting their major role in shaping the field (Khan et al., 2021). Researchers developed a conceptual framework which includes three environmental dimensions together with 21 categories and 64 green practices for GSCM analysis (Herrmann et al., 2021). Research reveals that successful execution of sustainable global supply chain initiatives needs three critical components which include collaboration between partners, top management endorsements and environmental policy implementation (Giuffrida & Mangiaracina, 2020). Future GSCM research needs advanced modeling techniques alongside comprehensive frameworks and studies of different geographical settings and technologies according to these published papers.

2.6 Thesis Contribution

Research gaps in previous studies:

Our literature review shows that past research has incorporated the EOQ model with sustainability considerations, but often in a very limited scope. Also, the studies addressed on narrow aspects of sustainability without considering wider environmental impacts and also integrating dynamic variations in operational parameters. Moreover, existing researched

models have not been developed by incorporating both the operational and environmental costs.

Contribution of this study:

This study fills up this research gaps by developing a mathematical model that integrates both the operational and the carbon emission costs in the EOQ and EPQ models. This model optimizes the total cost function by using the equation $Q = \alpha D$ ($\alpha > 0$, D-annual demand), which pursues an optimal order quantity (Q). Through this method we develop an advanced optimization technique of the operational costs together with the carbon emission cost.

By calculating the optimal order quantity (Q) that will minimize both the operational and the carbon emission costs, this model will deliver a holistic approach to supply chain optimization. The (Q) value will enable the organizations in implementing sustainable inventory practices that reduce both the operational costs and the environmental impact. This study will highlight the value of adaptive strategies, empowering professionals to make informed decisions that maximize profit potential while obtaining sustainable outcomes in the competitive markets.

This thesis will contribute to the academic community by proposing a detailed analysis of the incorporation of economic and environmental factors. The research will also provide industry professional with practical approaches to align operational supply chains with sustainability initiatives. By demonstrating the complex relationship between cost decrease efforts and carbon emissions management within sustainable operational contexts, this study will show a significant advancement in supply chain optimization in a production-delivery context.

In addition, sensitivity analysis was conducted to evaluate the model's robustness across varying operational parameters, such as demand, production rate, setup cost, ordering and holding costs of a manufacturer and a retailer. The analyses highlight how changes in these parameter values affect the optimal order quantity and the total supply chain cost including

operational and carbon emission costs. Through various scenario evaluations the sensitivity analyses deliver actionable insights which may help organizations in building responsive supply chain strategies while maintaining resilience.

Lastly, this research will link the gap between traditional cost optimization models and the wider environmental sustainability, proposing valuable insights and practical solutions for advanced GSCM.

Chapter 3 Model Formulation and Solution Methodology

3.1 Introduction to the Mathematical Model

This section outlines a detailed mathematical formulation of the problem of delivering a product after production at a source to a destination with minimum integrated cost of operations and carbon emission. Our intention is to find the optimal order quantity within this production-delivery supply chain. The main purpose of this model is to combine operational cost reduction with the minimum carbon emission cost in order to support supply chain operations by ensuring effective delivery to meet demand effectively.

The modern supply chain requires methods to achieve economic efficiency and environmental sustainability because of its complex nature. The integrated model deals with the production-delivery supply chain defining parameters to determine total cost through its integration of production rates along with demand volumes, setup cost, transportation cost, ordering cost, holding cost and carbon emission cost.

The model establishes a structured approach for determining the optimal order quantity (Q) by using a deterministic method connecting the identified parameters. The mathematical expression for the optimal order quantity depends crucially on the optimization coefficient (α) that establishes the optimal ratio between order quantities and market demand.

The mathematical model functions as an essential resource for SCM professionals who utilize quantitative methods to measure operational improvements alongside lowering carbon effects on supply chain processes. The subsequent sections will present detailed equations and analyses demonstrating the development of total cost functions alongside optimization conditions which lead to applicable sustainability insights for SCM.

3.2 Assumptions and Notations Used in the Model

The mathematical model development requires specific assumptions and notations for the optimization of the production-delivery supply chain order quantity and subsequent effective analysis. We develop a designed framework to evaluate the relationships among variables and the operational dynamics within the model. We develop the mathematical model underpinning the following assumptions and notations.

Assumptions:

- 1) All parameter values are deterministic constants over a time period.
- 2) The model and the analyses deal with the production of a single product and its delivery.
- 3) Optimal order quantity/lot (Q) is obtained based on the optimal α via the formula $Q = \alpha D$ ($\alpha > 0$).
- 4) A specific reorder point is maintained without allowing deliberate shortages.
- 5) The replenishment of a lot is supposed to be instantaneous when an order is placed.
- 6) Uniform constant inventory holding cost on all units held.
- 7) Transportation and carbon emissions cost coefficients are deterministic constant over all lots.
- 8) Manufacturer's setup cost and retailer's ordering cost remain constant over a year.
- 9) Due to low carbon emission during order process execution, emission factors such as electricity used for computers, printers, etc., are negligible.

Notations:

- D = Annual demand rate of a product
- P = Annual production rate at a manufacturer
- Q = Quantity per lot, a decision variable
- α = A coefficient relating the demand and the order quantity by $Q = \alpha D$ ($\alpha > 0$), a decision variable
- S_m = Setup cost per setup of a production process for processing Q units at a manufacturer
- T = Transportation cost for transferring a lot from a manufacturer to a retailer
- A_r = Retailer's ordering cost for placing an order to the manufacturer
- h_m = Holding cost per unit per year at a manufacturer
- h_r = Holding cost per unit per year at a retailer

- e_t = Carbon emission coefficient, amount of carbon emitted per lot-shipment (kg CO₂/shipment)
- e_{hm} = Carbon emission coefficient for holding a unit of inventory for a year at a manufacturer (kg CO₂/unit/year), released via electricity uses for each unit inventory storage at a manufacturing site
- e_{hr} = Carbon emission coefficient for holding a unit of inventory for a year at a retailer (kg CO₂/unit/year), released via electricity usage for each unit inventory storage at the retailer.
- e_p = Carbon emission coefficient during production of a lot at a manufacturer (kg CO₂/lot)
- λ = Carbon emission cost per kg of CO₂ released (Nordhaus, 2017)
- d_t = Total distance traversed per shipment (miles)
- EF_t = CO₂ emission coefficient for diesel driven medium- and heavy-duty truck freight transportation (kg CO₂/short ton-mile – amount of CO₂ emitted by transporting one short ton (907.18 kg) of cargo for one mile), (U.S. Environmental Protection Agency [EPA], 2025)
- u_s = Total number of units carried per shipment
- w_u = Weight per unit item (short ton/unit)
- W = Total weight of a shipment (in short tons), calculated as $W = u_s \cdot w_u$
- E_{sm} = Electricity consumed for storage a unit of inventory at a manufacturer (kWh/unit)
- EF_e = CO₂ emission coefficient for usage of electricity (kg CO₂/kWh) (Lee & Lee, 2021)
- E_{sr} = Electricity consumed for storing one unit of inventory at a retailer (kWh/unit)
- E_c = Average electricity consumption per setup (kWh/setup)
- N_{gc} = Average natural gas consumption per setup (m³/setup)
- EF_{ng} = CO₂ emission coefficient for natural gas (kg CO₂/m³) (EPA, 2025)
- D_{ic} = Average diesel consumption per setup (liters/setup)
- EF_{di} = CO₂ emission coefficient for diesel (kg CO₂/liter) (EPA, 2025)
- W_{st} = Average waste generated per setup (short tons/setup)
- EF_{st} = CO₂ emission coefficient for waste treatment (kg CO₂ /short ton) (Mohsen et al., 2019)
- TC = Total Cost (the operational and the carbon emissions costs)

3.3 Development of the Total Cost Function

The total cost function consists of operational and carbon emission costs which combine into a single summation. Multiple components create the total cost which includes:

- Manufacturer annual setup cost: $\left(\frac{D}{Q}\right) \cdot S_m$

The manufacturer setup cost is incurred when each time an order of Q units is placed by a retailer. The annual total demand (D) is satisfied by D/Q number of orders per year. The setup cost per order is denoted by S_m , making the annual total setup cost is $\frac{D}{Q} \cdot S_m$.

- Manufacturer annual holding cost: $\left(\frac{D \cdot Q}{2P}\right) \cdot h_m$

This term denotes the annual cost incurred for holding inventory at the manufacturer facility during processing of a product. The term $\frac{D \cdot Q}{2P}$ represents the average work-in-process (WIP) inventory level per year, where h_m is the manufacturer inventory holding cost per unit per year.

- Annual transportation cost: $\left(\frac{D}{Q}\right) \cdot T$

This term represents the transportation cost incurred per year. Number of orders placed per year is D/Q . The transportation cost T is incurred when a shipment is completed and as a result the annual transportation cost is $\frac{D}{Q} \cdot T$

- Retailer annual ordering cost: $\left(\frac{D}{Q}\right) \cdot A_r$

The retailer ordering cost is incurred for each replenishment cycle. Order is placed for a lot of Q units, hence the annual number of orders is D/Q , with every order resulting an ordering cost of A_r and the annual ordering cost is $\frac{D}{Q} \cdot A_r$

- Retailer annual holding cost: $\left(\frac{Q}{2}\right) \cdot h_r$

This term represents the average retailer annual inventory holding cost for keeping inventory in their stock for meeting demand. The cost of holding a unit of inventory for a year at the retailer site is h_r , which results to a total annual inventory holding cost is $\frac{Q}{2} \cdot h_r$

- Annual carbon emission cost for $\frac{D}{Q}$ number of shipments: $\left(\frac{D}{Q}\right) \cdot e_t \cdot \lambda$

Total environmental impacts of supply chain operations include carbon emission generated through transportation services. The carbon emission for each transportation lot is e_t (in kg), while the yearly number of shipments equals D/Q . So, an annual carbon emission cost totals to $\frac{D}{Q} \cdot e_t \cdot \lambda$, where the parameter λ serves as the cost coefficient per unit (kg) carbon emission.

Transportation carbon emission coefficient per shipment e_t (in kg) is calculated as follows:

$$e_t = (d_t \cdot W \cdot EF_t) = \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\}, \text{ where, } W = (u_s \cdot w_u)$$

In this function, d_t stands for the complete transportation distance (in miles) traversed in transporting (diesel driven medium-and heavy-duty truck) a lot of products from a manufacturer to a retailer. The variable W denotes the total shipment weight (in short ton) obtained through total units u_s per shipment multiplied by weight w_u per unit, where $d_t \cdot W$ counted as the total short tons-miles and EF_t is the amount of CO₂ emitted in kg for transporting one unit of such weight for one mile (in kg CO₂/short ton-mile).

- Annual carbon emission cost for holding inventory at a manufacturer: $\left(\frac{D \cdot Q}{2P}\right) \cdot e_{hm} \cdot \lambda$

The carbon emission from inventory stored at the manufacturer's facility occur because of electricity consumption together with storage maintenance activities performed. The annual carbon emission costs are determined by the average inventory level $\frac{D \cdot Q}{2P}$, multiplying with

carbon emission coefficient e_{hm} (in kg), multiplied by per unit carbon emission cost coefficient λ .

Manufacturer's carbon emission coefficient per unit inventory holding e_{hm} (in kg):

$$e_{hm} = (E_{sm} \cdot EF_e)$$

Here, E_{sm} represent the electricity consumption (in kWh) per unit of inventory holding, which is multiplied with the carbon emission coefficient of EF_e (in kg CO₂/kWh) to find the amount of CO₂ emitted per unit inventory holding. The measurement e_{hm} (in kg) determines the carbon emissions resulting from electricity consumption required for per unit holding inventory at the manufacturer site.

- Annual carbon emission cost due to retailer's inventory holding: $\left(\frac{Q}{2}\right) \cdot e_{hr} \cdot \lambda$

The carbon emission from inventory stored at retailer follow the same calculation methodology used by a manufacturer. The retailer determines the annual carbon emission cost by multiplying its annual average inventory $\frac{Q}{2}$ with carbon emission coefficient e_{hr} (in kg) and then multiplied by the per unit carbon emission cost coefficient λ .

Retailer's per unit inventory holding emission coefficient e_{hr} (in kg):

$$e_{hr} = (E_{sr} \cdot EF_e)$$

The term, E_{sr} denotes the amount of electricity (in kWh) needed to maintain per unit inventory holding in retailer storage, which is multiplied with the carbon emission coefficient of EF_e (in kg CO₂/kWh) to find the amount of CO₂ emitted per unit inventory holding. The measurement e_{hr} (in kg) determines the carbon emissions resulting from electricity consumption required for per unit holding inventory at the retailer site.

- Annual carbon emissions cost for lots production: $\left(\frac{D}{Q}\right) \cdot e_p \cdot \lambda$

The carbon emission output from production procedures depends on lot sizes with operational performance efficiency factors. Carbon emission per lot is denoted by e_p (in kg), combined with the number of annual production cycles D/Q . The carbon emission cost coefficient per unit (in kg) is denoted by λ , which results in an annual carbon emission expense of $\frac{D}{Q} \cdot e_p \cdot \lambda$

Production carbon emission coefficient per setup, e_p (in kg):

$$e_p = (E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})$$

The coefficients examine carbon emissions per setup via several critical elements which are integrated in this calculation. The total electricity consumption E_c (in kWh) for setting up a process is multiplied by the CO₂ emission coefficient EF_e (in kg CO₂/kWh) for electricity. The calculation provides the total carbon emission that results from electrical power usage.

The calculation combines N_{gc} , average natural gas usage (in m³) per setup with EF_{ng} , the carbon emission coefficient (in kg CO₂/m³) representing CO₂ emissions per cubic meter of natural gas used, to determine gas-related emissions during production setup. Similarly, the carbon emission from diesel usage in the setup stage can be determined by multiplying the total diesel usage D_{ic} , (in liters) with EF_{di} , carbon emission coefficient (in kg CO₂/liter) per unit of diesel use.

The calculation of total carbon emissions for waste treatment is obtained by the product of W_{st} , representing setup waste amount (in short tons/setup) and EF_{st} , representing the CO₂ emission coefficient (in kg CO₂/short ton) per unit of waste treatment. Waste disposal specially landfilling as well as treatment operations lead to the release of carbon emission that is accounted by EF_{st} multiplied with W_{st} data.

The calculation of e_p represents a complete approach to assessing production carbon emission since it includes all energy consumption through electricity, natural gas and diesel together with the calculation for waste emission.

Thus, initially the total cost function TC can be expressed as:

$$TC = \left(\frac{D}{Q}\right) \cdot S_m + \left(\frac{D \cdot Q}{2P}\right) \cdot h_m + \left(\frac{D}{Q}\right) \cdot T + \left(\frac{D}{Q}\right) \cdot A_r + \left(\frac{Q}{2}\right) \cdot h_r + \left(\frac{D}{Q}\right) \cdot e_t \cdot \lambda + \left(\frac{D \cdot Q}{2P}\right) \cdot e_{hm} \cdot \lambda + \left(\frac{Q}{2}\right) \cdot e_{hr} \cdot \lambda + \left(\frac{D}{Q}\right) \cdot e_p \cdot \lambda$$

After substituting for the carbon emission coefficients into the total cost function obtain:

$$TC = \left(\frac{D}{Q}\right) \cdot S_m + \left(\frac{D \cdot Q}{2P}\right) \cdot h_m + \left(\frac{D}{Q}\right) \cdot T + \left(\frac{D}{Q}\right) \cdot A_r + \left(\frac{Q}{2}\right) \cdot h_r + \left(\frac{D}{Q}\right) \cdot \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda + \left(\frac{D \cdot Q}{2P}\right) \cdot (E_{sm} \cdot EF_e) \cdot \lambda + \left(\frac{Q}{2}\right) \cdot (E_{sr} \cdot EF_e) \cdot \lambda + \left(\frac{D}{Q}\right) \cdot \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda$$

However, following the MOQ contract (Tuncel et al., 2022), the order quantity Q is constrained by α times of the annual demand as $Q = \alpha D$, where $\alpha > 0$.

The total cost function TC includes the costs of setup, holding inventory, transportation, ordering, and the carbon emissions linked to transportation and operational activities concerning with production. Substituting for this Q value into the total cost function enables complete examination of operational and carbon emission cost to support operations that are efficient and sustainable. Inventory decision-making becomes more informed through this approach because it demonstrates the importance of optimal order quantity optimization for lowering expenses and environmental impact.

3.4 Optimization and Validation Approach

Note that the order quantity Q , that is, the lot size Q is restricted to the demand D by the α value with the equation, $Q = \alpha D$, where $\alpha > 0$. Substituting for $Q = \alpha D$ into the total cost function and simplifying it transforms to

$$\begin{aligned}
TC = & \left(\frac{D}{\alpha D}\right) \cdot S_m + \left(\frac{D \cdot \alpha D}{2P}\right) \cdot h_m + \left(\frac{D}{\alpha D}\right) \cdot T + \left(\frac{D}{\alpha D}\right) \cdot A_r + \left(\frac{\alpha D}{2}\right) \cdot h_r \\
& + \left(\frac{D}{\alpha D}\right) \cdot \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda + \left(\frac{D \cdot \alpha D}{2P}\right) \cdot (E_{sm} \cdot EF_e) \cdot \lambda \\
& + \left(\frac{\alpha D}{2}\right) \cdot (E_{sr} \cdot EF_e) \cdot \lambda + \left(\frac{D}{\alpha D}\right) \cdot \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda
\end{aligned}$$

Now substituting for $Q = \alpha D$ derive different cost components in the variable α as shown below:

- Setup cost manufacturer: $\left(\frac{D}{\alpha D}\right) \cdot S_m = \frac{S_m}{\alpha}$
- Holding cost manufacturer: $\left(\frac{D \cdot \alpha D}{2P}\right) \cdot h_m = \left(\frac{h_m \cdot D^2}{2P}\right) \cdot \alpha$
- Transportation cost: $\left(\frac{D}{\alpha D}\right) \cdot T = \frac{T}{\alpha}$
- Retailer ordering cost: $\left(\frac{D}{\alpha D}\right) \cdot A_r = \frac{A_r}{\alpha}$
- Retailer holding cost: $\left(\frac{\alpha D}{2}\right) \cdot h_r = \left(\frac{h_r D}{2}\right) \cdot \alpha$
- Transportation carbon emissions cost: $\left(\frac{D}{\alpha D}\right) \cdot \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda$
 $= \frac{1}{\alpha} \cdot \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda$
- Manufacturer inventory holding carbon emissions cost: $\left(\frac{D \cdot \alpha D}{2P}\right) \cdot (E_{sm} \cdot EF_e) \cdot \lambda$
 $= \left\{ \frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P} \right\} \alpha$
- Retailer inventory holding carbon emissions cost: $\frac{\alpha D}{2} \cdot (E_{sr} \cdot EF_e) \cdot \lambda$
 $= \left\{ \frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e) \right\} \alpha$
- Batches production carbon emissions cost: $\left(\frac{D}{\alpha D}\right) \cdot \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda$
 $= \frac{1}{\alpha} \cdot \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda$

Now, combining these simplified terms, arrived at the final form of the total cost function is:

$$\begin{aligned}
TC(\alpha) &= \frac{S_m}{\alpha} + \left(\frac{h_m \cdot D^2}{2P}\right) \cdot \alpha + \frac{T}{\alpha} + \frac{A_r}{\alpha} + \left(\frac{h_r D}{2}\right) \cdot \alpha \\
&+ \frac{1}{\alpha} \cdot \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda + \left\{\frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P}\right\} \alpha \\
&+ \left\{\frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e)\right\} \alpha + \frac{1}{\alpha} \cdot \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda \\
\\
TC(\alpha) &= \left[\frac{h_m \cdot D^2}{2P} + \frac{h_r D}{2} + \frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P} + \frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e) \right] \cdot \alpha + \\
&\left[\frac{S_m + T + A_r + \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda + \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda}{\alpha} \right]
\end{aligned}$$

Step 1: Differentiating $TC(\alpha)$ with respect to α as follows showing each term separately.

- Derivative of $\frac{S_m}{\alpha}$, that is, $\frac{d}{d\alpha} \left(\frac{S_m}{\alpha} \right) = -\frac{S_m}{\alpha^2}$
- Derivative of $\left(\frac{h_m \cdot D^2}{2P}\right) \cdot \alpha$, that is, $\frac{d}{d\alpha} \left(\frac{h_m \cdot D^2}{2P} \right) \cdot \alpha = \frac{h_m D^2}{2P}$
- Derivative of $\frac{T}{\alpha}$, that is, $\frac{d}{d\alpha} \left(\frac{T}{\alpha} \right) = -\frac{T}{\alpha^2}$
- Derivative of $\frac{A_r}{\alpha}$, that is, $\frac{d}{d\alpha} \left(\frac{A_r}{\alpha} \right) = -\frac{A_r}{\alpha^2}$
- Derivative of $\left(\frac{h_r D}{2}\right) \cdot \alpha$, that is, $\frac{d}{d\alpha} \left(\frac{h_r D}{2} \right) \cdot \alpha = \frac{h_r D}{2}$
- Derivative of $\frac{1}{\alpha} \cdot \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda$, that is, $\frac{d}{d\alpha} \left[\frac{1}{\alpha} \cdot \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda \right]$
 $= -\frac{1}{\alpha^2} \cdot \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda$
- Derivative of $\left\{\frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P}\right\} \alpha$, that is, $\frac{d}{d\alpha} \left\{ \frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P} \right\} \alpha = \frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P}$
- Derivative of $\left\{\frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e)\right\} \alpha$, that is, $\frac{d}{d\alpha} \left\{ \frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e) \right\} \alpha = \frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e)$
- Derivative of $\frac{1}{\alpha} \cdot \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda$,
that is, $\frac{d}{d\alpha} \left[\frac{1}{\alpha} \cdot \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda \right]$
 $= -\frac{1}{\alpha^2} \cdot \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda$

Step 2: Combining the derivatives

Now let's combine all these derivatives into one equation:

$$\begin{aligned} \frac{d(TC)}{d\alpha} &= -\frac{S_m}{\alpha^2} + \frac{h_m D^2}{2P} - \frac{T}{\alpha^2} - \frac{A_r}{\alpha^2} + \frac{h_r D}{2} \\ &\quad - \frac{1}{\alpha^2} \cdot \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda + \frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P} \\ &\quad + \frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e) - \frac{1}{\alpha^2} \cdot \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda \\ \frac{d(TC)}{d\alpha} &= \left[\frac{h_m D^2}{2P} + \frac{h_r D}{2} + \frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P} + \frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e) \right] \\ &\quad + \left[\frac{-S_m - T - A_r - \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda - \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda}{\alpha^2} \right] \end{aligned}$$

Step 3: Differentiating each of the term of $\frac{d(TC)}{d\alpha}$ again with respect to α , that is, $\frac{d}{d\alpha} \left(\frac{d(TC)}{d\alpha} \right) =$

$$\frac{d^2(TC)}{d\alpha^2}$$

- Differentiating the first term of $TC(\alpha)$, that is $\frac{d^2(TC)}{d\alpha^2} \left[\frac{h_m D^2}{2P} + \frac{h_r D}{2} + \frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P} + \frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e) \right] = 0$

- Differentiating the second term of $TC(\alpha)$, that is

$$\begin{aligned} &\frac{d^2(TC)}{d\alpha^2} \left[\frac{-S_m - T - A_r - \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda - \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda}{\alpha^2} \right] \\ &= \frac{-2}{\alpha^3} \left[-S_m - T - A_r - \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda - \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda \right] \\ &= \frac{2[S_m + T + A_r + \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda + \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda]}{\alpha^3} \\ \frac{d^2(TC)}{d\alpha^2} &= \frac{2[S_m + T + A_r + \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda + \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda]}{\alpha^3} > 0, \end{aligned}$$

Since the second derivative of the total cost function is positive with respect to α , this total cost function clearly shows the convexity in α . So the optimal α can be found by equating $\frac{d(TC)}{d\alpha}$ equal to zero.

Step 4: Setting the $\frac{d(TC)}{d\alpha} = 0$

Put this derivative equal to zero in order to find the optimal value of α :

$$\left[\frac{h_m D^2}{2P} + \frac{h_r D}{2} + \frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P} + \frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e) \right] + \left[\frac{-S_m - T - A_r - \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda - \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda}{\alpha^2} \right] = 0$$

$$\left[\frac{h_m D^2}{2P} + \frac{h_r D}{2} + \frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P} + \frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e) \right] - \left[\frac{S_m + T + A_r + \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda + \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda}{\alpha^2} \right] = 0$$

Step 5: Solving for α

Isolating the $\frac{1}{\alpha^2}$ terms on one side and the constant term on the other side, we have

$$\frac{S_m + T + A_r + \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda + \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda}{\alpha^2} = \frac{h_m D^2}{2P} + \frac{h_r D}{2} + \frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P} + \frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e)$$

Multiplying throughout by α^2 :

$$S_m + T + A_r + \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda + \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda = \alpha^2 \left\{ \frac{h_m D^2}{2P} + \frac{h_r D}{2} + \frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P} + \frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e) \right\}$$

Finally, we have

$$\alpha^2 = \frac{S_m + T + A_r + \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda + \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda}{\frac{h_m D^2}{2P} + \frac{h_r D}{2} + \frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P} + \frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e)}$$

Taking the square root, the optimal α , denoted by α^* is given by

$$\alpha^* = \sqrt{\frac{S_m + T + A_r + \{d_t \cdot (u_s \cdot w_u) \cdot EF_t\} \cdot \lambda + \{(E_c \cdot EF_e) + (N_{gc} \cdot EF_{ng}) + (D_{ic} \cdot EF_{di}) + (W_{st} \cdot EF_{st})\} \cdot \lambda}{\frac{h_m D^2}{2P} + \frac{h_r D}{2} + \frac{D^2 \cdot (E_{sm} \cdot EF_e) \cdot \lambda}{2P} + \frac{\lambda D}{2} \cdot (E_{sr} \cdot EF_e)}}$$

This optimal α value enables SCM professionals to identify cost-effective order or lot size Q via the equation $Q = \alpha D$. The optimal α combines many supply chain costs such as setup, inventory holding for both the manufacturer and the retailer, transportation, ordering and several carbon emissions. The calculated optimal value of α can immediately be used in the formula $Q = \alpha D$ to determine the optimal order quantity Q .

3.5 Model Implementation and Sensitivity Analysis

The Python implementation of the mathematical model systematically determines the optimal order quantity (Q) by evaluating operational expenses along with carbon emission costs. The initial step requires defining multiple input variables including annual demand (D), annual production rate (P), setup costs manufacturer (S_m), transportation costs (T), retailer ordering costs (A_r), holding costs both manufacturer and retailer (h_m) and (h_r), etc. and the value of carbon emission-related coefficients are taken from published article and from EPA such as (EF_t), (EF_e), (EF_{ng}), (EF_{di}) and (EF_{st}).

By using these parameters, the optimization coefficient (α) is calculated by harmonizing costs associated to operational, and carbon emissions. The optimal order quantity (Q) is then resulting as $Q = \alpha * D$, allowing efficient inventory management as well as supply chain.

Computed annual operational costs are summing setup costs, transportation costs, retailer ordering costs, and as well as holding costs at both the manufacturer and retailer ends, all adjusted to the optimized order quantity (Q). The evaluation of carbon emissions costs involves calculating emissions from transportation, manufacturer inventory holding, retailer inventory holding and production based on energy use and waste handling. The total cost calculation combines operational costs with carbon emissions costs to deliver a complete view of supply chain expenses.

Sensitivity analysis represents the effects of changing key parameters such as (D) , (P) , (S_m) , (T) , (A_r) , (h_m) and (h_r) from -50% to +50% of their baseline values. Each variation results in again calculations of optimal (α) , optimal order quantity (Q) , operational costs, carbon emissions costs, and overall total costs. The study demonstrates how different parameter variation affect both system operational efficiency and cost structures. The analysis presents findings through tables and graphs that showcase trends and sensitivities in a clear manner.

The analysis demonstrates that the model provides a reliable method to optimize supply chain operations and environmental sustainability. The findings enable stakeholders to make decisions that strike a balance between economic effectiveness and environmental sustainability targets. These details python code both main model and sensitivity are provided in the Appendix A section.

Chapter 4 Results, Discussion and Sensitivity Analysis

4.1 Optimal Order Quantity Decisions Balancing Operations and Carbon Emission Costs

Effective management of supply chain complexity requires organizations to focus on both the operational efficiency and the environmental sustainability. By applying the developed mathematical model for obtaining the optimal order quantity (Q) determination, organizations can control inventory by reducing both the operational costs and the environmental impact (since carbon emission cost is proportional to the carbon emission). We illustrate it here by a numerical example problem. Various carbon emission coefficients are taken such as transportation (EF_t), natural gas (EF_{ng}), and diesel (EF_{di}) from (EPA, 2025), electricity (EF_e) from (Lee & Lee, 2021), landfilling waste treatment (EF_{st}) from (Mohsen et al., 2019), and per unit carbon emission cost coefficient (λ) from (Nordhaus, 2017) and others parameters value are taken as deterministic conditions.

The data of the numerical example problem are as follows:

- Annual demand rate of a product (D) = 100000 units
- Annual production rate at a manufacturer (P) = 110000 units
- Setup cost per setup of a production process (S_m) = 30000 BDT
- Transportation cost per shipment (T) = 20,000 BDT
- Retailer ordering cost per order (A_r) = 8000 BDT
- Holding cost per unit per year at a manufacturer (h_m) = 25 BDT
- Holding cost per unit per year at a retailer (h_r) = 45 BDT
- Total distance traversed per shipment (d_t) = 200 miles
- Total number of units carried per shipment (u_s) = 2500 units
- Weight per unit item (w_u) = 0.005 short ton/unit
- CO₂ emission coefficient for diesel driven medium-and heavy-duty truck freight transportation (EF_t) = 0.186 kg CO₂/short ton-mile

- Electricity consumed for storage a unit of inventory at a manufacturer (E_{sm}) = 1.5 kWh/unit
- CO₂ emission coefficient for usage of electricity (EF_e) = 0.437 kg CO₂/kWh
- Electricity consumed for storing one unit of inventory at a retailer (E_{sr}) = 2 kWh/unit
- Average electricity consumption per setup (E_c) = 600 kWh/setup
- Average natural gas consumption per setup (N_{gc}) = 25 m³/setup
- CO₂ emission coefficient for natural gas (EF_{ng}) = 1.923 kg CO₂/m³
- Average diesel consumption per setup (D_{ic}) = 20 liters/setup
- CO₂ emission coefficient for diesel (EF_{di}) = 2.697 kg CO₂/liter
- Average waste generated per setup (W_{st}) = 0.20 short ton/setup
- CO₂ emission coefficient for landfilling waste treatment (EF_{st}) = 353.80 kg CO₂ /short ton
- Cost emission coefficient per kg of CO₂ released (λ)= 5.08 BDT

Using the above parameter values with Python Implementation, we find the optimal subsequent results as shown in Table 1.

Table 1: Optimal α value and the other subsequent results

Descriptions	Results
Optimal α value	0.1290
Optimal order/Batch quantity (Q)	12,900 units
Annual operational cost total	886,453.31 BDT
Annual total carbon emissions cost	83,603.92 BDT
Annual total cost (Operational + Carbon emissions)	970,057.23 BDT

The optimal α value is found as 0.1290 and hence the optimal order quantity Q stands at 12,900 units through the equation $Q = \alpha D$. The calculated value suggests a direct correlation between the quantity of ordered goods Q and the market demand D which reinforces the need for precise inventory forecasting before placing orders.

The calculation of optimal order quantity includes several operational costs such as setup, holding, transportation, and ordering together with the costs connected to carbon emissions. Such integrated analysis enables businesses to bring together environmental and financial considerations for making better business decisions.

The assessment of costs linked to the optimal order quantity reveals valuable insights about sustainable procurement practices as follows:

- The annual cost of operations amounts to 886,453.31 BDT which incorporates all costs related to inventory management and logistics. This cost highlights the need for optimal stock movement and storage systems.
- The assessment of environmental impact relies on monetary quantification of carbon emission during carrying out operations of transportation, inventory holding and production by businesses. The annual carbon emission cost of 83,603.92 BDT verify that sustainability measures can become part of holistic operational approaches instead of remaining isolated initiatives.
- The optimal annual operational and carbon emission expenses 970,057.23 BDT, provides a complete overview of production-delivery supply chain management expenses. The scenario of combined financial and environmental perspectives enables businesses to perform a comprehensive assessment of supply chain performance while maintaining economic effectiveness. The breakdown of the expenses demonstrates the domination of the budget by the operational expenses, while the total emission cost remains substantial due to the importance of sustainability aspects in supply chain frameworks.

Determination of the optimal order quantity helps businesses minimize carbon waste while controlling stock quantities better to support market adjustments and achieve sustainability targets. The study demonstrates that operational efficiency should be united with

environmental responsibility to create an effective strategic system. The total cost minimization indicates that sustainability should become essential for such a supply chain decision-making process.

4.2 Costs and Emissions Breakdown of the Optimal Cost Solution

Table 2: Operational and carbon emission costs breakdown

Descriptions	Results
Operational costs breakdown:	
Annual setup cost at the manufacturer	232,558.14 BDT
Annual inventory holding cost at the manufacturer	146,590.91 BDT
Annual transportation cost	155,038.76 BDT
Annual retailer's ordering cost	62,015.50 BDT
Annual inventory holding cost at the retailer	290,250.00 BDT
Total annual operational cost:	886,453.31 BDT
Carbon emissions costs breakdown:	
Annual carbon emission cost during transportation	18,311.63 BDT
Annual carbon emission cost during inventory holding at the manufacturer	19,525.56 BDT
Annual carbon emission cost during inventory holding at the retailer	28,637.48 BDT
Annual carbon emission cost during production	17,129.25 BDT
Annual total carbon emissions cost:	83,603.92 BDT
Annual total cost (Operational + Carbon emissions):	970,057.23 BDT

The cost components of the optimal cost are depicted by bar charts in Figure 1.

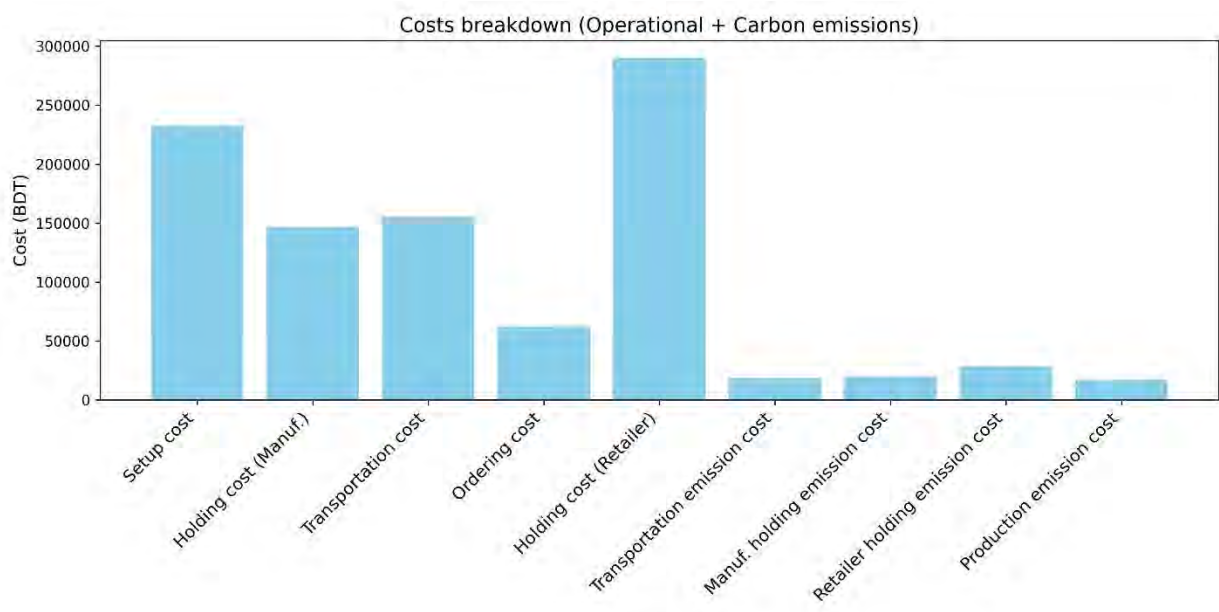


Figure 1: Breakdown of operations and carbon emissions costs

A detailed breakdown of operational and carbon emissions costs is crucial to understand the interaction between financial and environmental effects in the supply chain management. Analyzing individual cost components enables businesses to spot optimization possibilities and integrate operations with environmental priorities while making informed decisions.

Operational costs breakdown:

Annual setup cost at the manufacturer:

Annual total number of productions lots to be produced by the manufacturer must spend 232,558.14 BDT that cover the expenses of setting up the processes per year. The large value of this money demonstrates that manufacturers must properly manage batch frequency to avoid unnecessary setups despite meeting customer requirements.

Annual inventory holding cost at the manufacturer:

The annual expense of 146,590.91 BDT incurred when inventory remain in storage facilities of the manufacturer. The cost of using space along with energy expenses and maintaining inventory at the storage time establishes this expenditure. The efficient management of inventory turnover stands as the primary solution to minimize these related expenses.

Annual transportation cost:

The movement of goods between the manufacturer facility and the retailer location results in transportation cost of 155,038.76 BDT per year. This expense depends on how much is shipped and the distance traversed as well as the shipping methods used. The increase in lot size results in decreased transportation requirements but produces higher storage expenses.

Annual retailer ordering cost:

The expense of 62,015.50 BDT needed to the issue of purchasing requests from the retailer represent the ordering costs per year. Effective order management practices are necessary because these costs consist of operational and administrative expenses.

Annual inventory holding cost at retailer:

The annual expense of 290,250.00 BDT for storing inventory at the retail location constitutes the largest operational cost component. The cost demonstrates the effects of holding larger stock quantities by considering both the facility requirement together with the obsolete inventory risks.

Annual total operational cost:

The consolidated view of annual total operational cost of 886,453.31 BDT reveals the total financial pressure upon supply chain management operations. This data demonstrates why organizations need to focus on optimizing individual unit costs to minimize their total operational expenses.

Carbon emissions costs breakdown:

Annual carbon emission cost for transporting products:

Transportation emission mainly results from the burning of fuel by vehicles that transport goods. The annual transportation emission cost of 18,311.63 BDT is the direct increase with the number of shipments increases. This particular cost increase means the more carbon emitting by the logistics activities. Organizing deliveries into optimal batch quantities while changing to environmentally friendly transportation solutions helps minimize this expense.

Annual manufacturer inventory holding carbon emission cost:

The electricity requirements to maintain storage facilities at the manufacturing site determine its associated carbon emissions cost of 19,525.56 BDT per year. Effective energy-saving technology implementation along with strategic storage space optimization can minimize expense for this carbon emission cost.

Annual carbon emission cost for retailer's holding inventory:

The cost 28,637.48 BDT represents the carbon emission cost for stocking products at the retailer site. The retail level needs sustainable storage practices since this category has the most expensive carbon emission cost.

Annual carbon emission cost due to processing products:

Manufacturing activities result in annual carbon emissions cost of 17,129.25 BDT which represents the environmental consequences of the production phase. Techniques in advanced manufacturing along with sustainable energy sources provide potential solutions to reduce this cost.

Annual total carbon emissions cost:

Supply chain operations lead to the environmental impact through an evaluation of the annual total combined carbon emissions cost of 83,603.92 BDT. The analysis exposes business opportunities where they can implement environmental solution throughout their operational supply chain.

Combined annual total cost:

Total yearly costs reached to 970,057.23 BDT which includes operational expenses alongside carbon emission expenses. The combination of financial and environmental perspectives enables businesses to perform a comprehensive assessment of supply chain performance while

maintaining economic effectiveness. The breakdown of expenses demonstrates the domination of the budget of operational expenses while the total emission cost remains substantial because sustainability aspects are gaining importance in supply chain frameworks.

An organization can discover essential performance enhancement opportunities through detailed analysis of their cost units. The implementation of energy-efficient methods together with optimized order quantities and green transportation services may lead to reduction in various operational costs as well as various environmental emission costs. The complete analysis enables organizations to find practical solutions for creating sustainable economic-environmental equilibrium in supply chain operations. Sensitivity analysis, discussed in the next section, will showcase the model's adaptability to changes in operational parameters.

4.3 Sensitivity Analysis Results

Sensitivity analyses are performed to see the effects of changing values of key parameters such as (D) , (P) , (S_m) , (T) , (A_r) , (h_m) and (h_r) upon the overall results, that is, the effect on the optimal (α) , optimal order quantity (Q) , operational costs, carbon emissions costs, and overall total costs. The study demonstrates how variations in different parameter values affect both system operational efficiency and cost structures. The analyses present findings through tables and graphs that showcase trends and sensitivities in a clear manner. The findings enable stakeholders to make decisions that strike a balance between economic effectiveness and environmental sustainability targets. This section shows outcomes from the sensitivity analyses performed based on different supply chain scenarios and their effects first on the operational costs, then carbon emission cost and the total costs (TC) . First, we analyze the effect of changes in the demand (D) upon the result. We increase the demand as shown in Table 3 but keeping the other parameter values the same and find the corresponding optimal solutions. The results are shown in Table 3.

Table 3: Sensitivity analysis performed by increasing the annual demand (D)

Parameter value (D)	Optimal α	Optimal Q	Operational cost (BDT)	Carbon emissions cost (BDT)	Total cost (TC) (BDT)
50000	0.2004	10020	571,802.98	52,640.92	624,443.90
60000	0.1793	10758	638,885.20	59,150.83	698,036.03
70000	0.1628	11396	703,325.36	65,455.78	768,781.14
80000	0.1494	11952	765,794.09	71,607.03	837,401.12
90000	0.1384	12456	826,726.05	77,653.75	904,379.80
100000	0.1290	12900	886,453.31	83,603.92	970,057.23
110000	0.1209	13299	945,200.32	89,481.02	1,034,681.34
120000	0.1139	13668	1,003,130.43	95,307.37	1,098,437.80
130000	0.1077	14001	1,060,388.42	101,081.39	1,161,469.81
140000	0.1023	14322	1,117,054.92	106,834.18	1,223,889.10
150000	0.0973	14595	1,173,260.46	112,524.40	1,285,784.86

An increase in the annual demand (D) decreases the optimal α value and increases the optimal order quantity (Q) successively. Increases in the demand result in rising total operational and environmental emissions costs and hence overall total cost. All trends are shown in Figure 2.

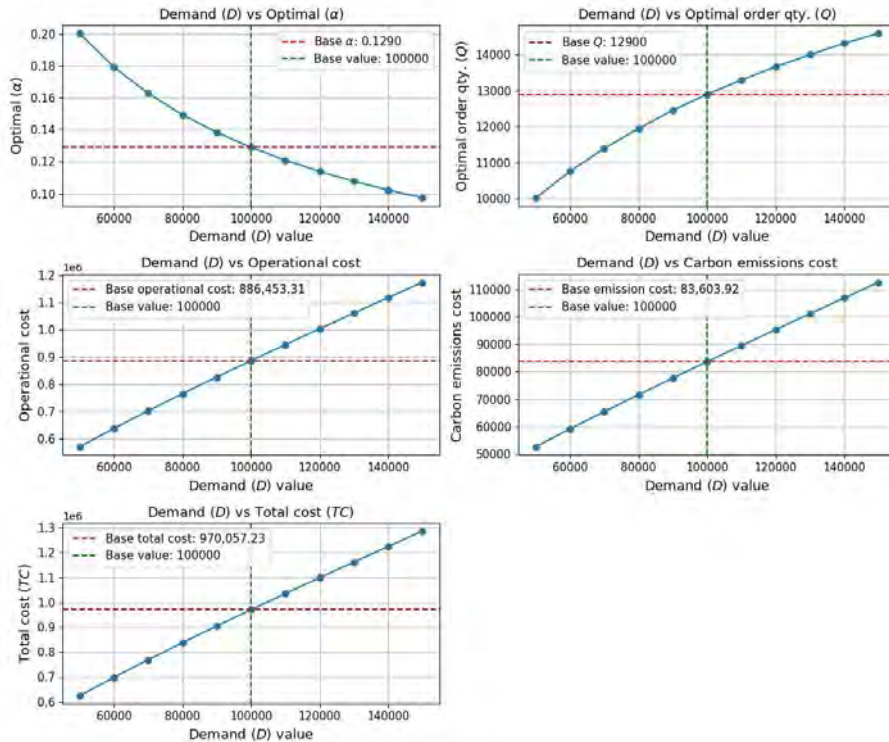


Figure 2: Sensitivity analysis by increasing annual demand (D)

From Figure 2 it can be observed that as the demand increases the optimal α value decreases with decreasing rate successively but the optimal order quantity increases with decreasing rate, while the operational and the carbon emission costs and the total cost increases linearly.

Comment: Effective strategies including better demand forecasting and efficient inventory management should be implemented to respond to changes in customer demand. An efficient demand management approach helps business organizations control costs while minimizing their environmental footprint.

Next, we perform the sensitivity analysis by increasing the production rate (P) but keeping the other parameter values the same. The results corresponding to the successively increasing production rates are given in Table 4.

Table 4: Sensitivity analysis by increasing the annual production rate (P)

Parameter value (P)	Optimal α	Optimal Q	Operational cost (BDT)	Carbon emissions cost (BDT)	Total cost (TC) (BDT)
55000	0.1113	11130	1,024,493.65	99,478.12	1,123,971.77
66000	0.1164	11640	980,636.33	94,481.61	1,075,117.94
77000	0.1205	12050	948,069.68	90,747.08	1,038,816.76
88000	0.1238	12380	922,899.85	87,835.71	1,010,735.56
99000	0.1266	12660	902,834.35	85,508.86	988,343.21
110000	0.1290	12900	886,453.31	83,603.92	970,057.23
121000	0.1311	13110	872,819.26	82,016.33	954,835.59
132000	0.1329	13290	861,295.63	80,667.34	941,962.97
143000	0.1344	13440	851,430.14	79,501.56	930,931.70
154000	0.1358	13580	842,875.95	78,495.29	921,371.24
165000	0.1371	13710	835,387.51	77,617.04	913,004.55

The increase in the annual production rate (P) causes a small rise in α leading to the higher optimal order quantities (Q), which in turn leads to the reduction in the operational and carbon emissions costs and hence the total cost, which are depicted in Figure 3.

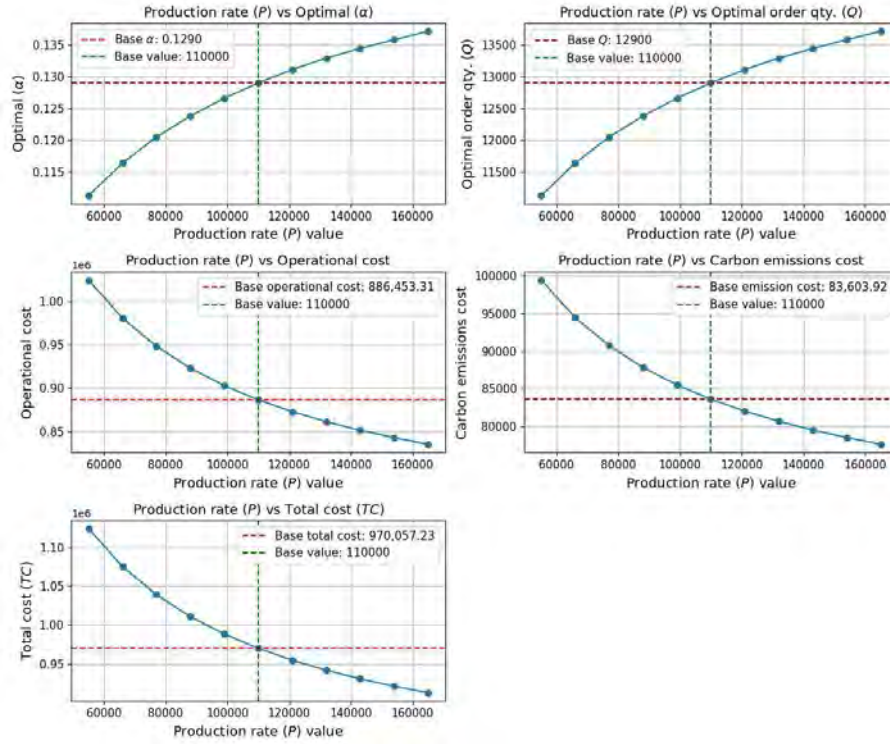


Figure 3: Sensitivity analysis for the increased annual production rate (P)

It can be observed from Figure 3 that as the production rate increases both the optimal α and the optimal order quantity increases with decreasing rate, while the operational and carbon emission costs decrease with decreasing rate and hence the total cost.

Comment: The business organizations should enhance its efficient production capacity since it will decrease both the operational and carbon emission costs and hence the total cost. Updated production process allows organizations to maintain a better balance between production rates and the optimal production quantities, leading to overall cost reduction.

Next, we perform the sensitivity analysis by increasing the setup cost (S_m) at the manufacturer by keeping the other parameter values the same. The results corresponding to these changes are given in Table 5.

Table 5: Sensitivity analysis by increasing manufacturer's setup cost (S_m)

Parameter value (S_m)	Optimal α	Optimal Q	Operational cost (BDT)	Carbon emissions cost (BDT)	Total cost (TC) (BDT)
15000	0.1125	11250	763,188.13	82,641.52	845,829.65
18000	0.1160	11600	789,369.91	82,722.11	872,092.02
21000	0.1194	11940	814,717.08	82,869.20	897,586.28
24000	0.1227	12270	839,304.70	83,071.46	922,376.16
27000	0.1259	12590	863,197.83	83,319.17	946,517.00
30000	0.1290	12900	886,453.31	83,603.92	970,057.23
33000	0.1321	13210	909,110.02	83,929.63	993,039.65
36000	0.1351	13510	931,220.90	84,281.18	1,015,502.08
39000	0.1380	13800	952,825.43	84,652.77	1,037,478.20
42000	0.1408	14080	973,959.09	85,039.35	1,058,998.44
45000	0.1436	14360	994,638.36	85,451.61	1,080,089.97

It can be observed from Table 5 that as the manufacturer's setup cost (S_m) increases the α value rises with an increased optimal order quantity (Q), leading to increased operational and carbon emissions costs and hence the total costs, which are shown in Figure 4.

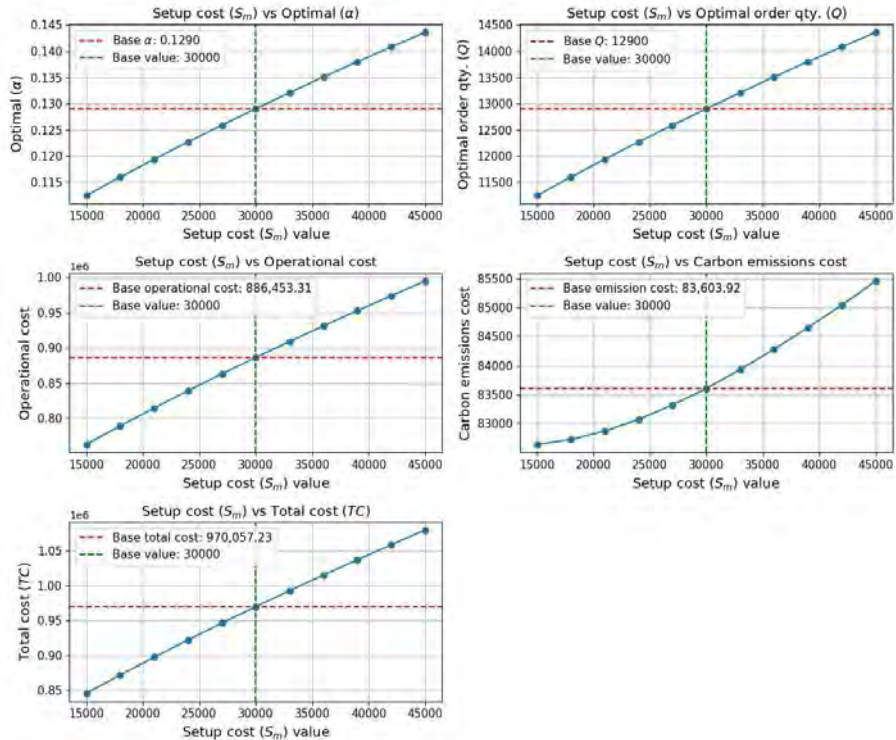


Figure 4: Sensitivity analysis by increasing manufacturer's setup cost (S_m)

From Figure 4 it can be observed easily that as the manufacturer's setup cost increases, the optimal α , optimal order quantity, the operational cost and the total cost increases linearly. However, the carbon emission cost increases non-linearly with successively increasing rates.

Comment: Manufacturers should evaluate and streamline their setup operations to decrease both the cost and the environmental impact. An efficient setup procedure combined with reduced setup times can deliver substantial cost savings alongside reduced environmental impact.

Then, we increased the transportation costs per shipment (T) by keeping the other parameter values the same and found the results. The results are given in Table 6.

Table 6: Sensitivity analysis by increasing transportation costs per shipment (T)

Parameter value (T)	Optimal α	Optimal Q	Operational cost (BDT)	Carbon emissions cost (BDT)	Total cost (TC) (BDT)
10000	0.1182	11820	806,359.55	82,809.92	889,169.47
12000	0.1205	12050	822,994.58	82,930.37	905,924.95
14000	0.1227	12270	839,304.70	83,071.46	922,376.16
16000	0.1248	12480	855,310.49	83,228.53	938,539.02
18000	0.1269	12690	871,021.90	83,406.36	954,428.26
20000	0.1290	12900	886,453.31	83,603.92	970,057.23
22000	0.1311	13110	901,618.18	83,820.28	985,438.46
24000	0.1331	13310	916,540.18	84,042.97	1,000,583.15
26000	0.1351	13510	931,220.90	84,281.18	1,015,502.08
28000	0.1370	13700	945,683.64	84,521.23	1,030,204.87
30000	0.1389	13890	959,926.74	84,774.12	1,044,700.86

Increasing transportation cost per shipment (T) results in higher optimal α value generating an increased optimal order quantity (Q). As (T) increases, both the operations cost, the carbon emission cost increases and hence the total cost, which are shown in Figure 5.

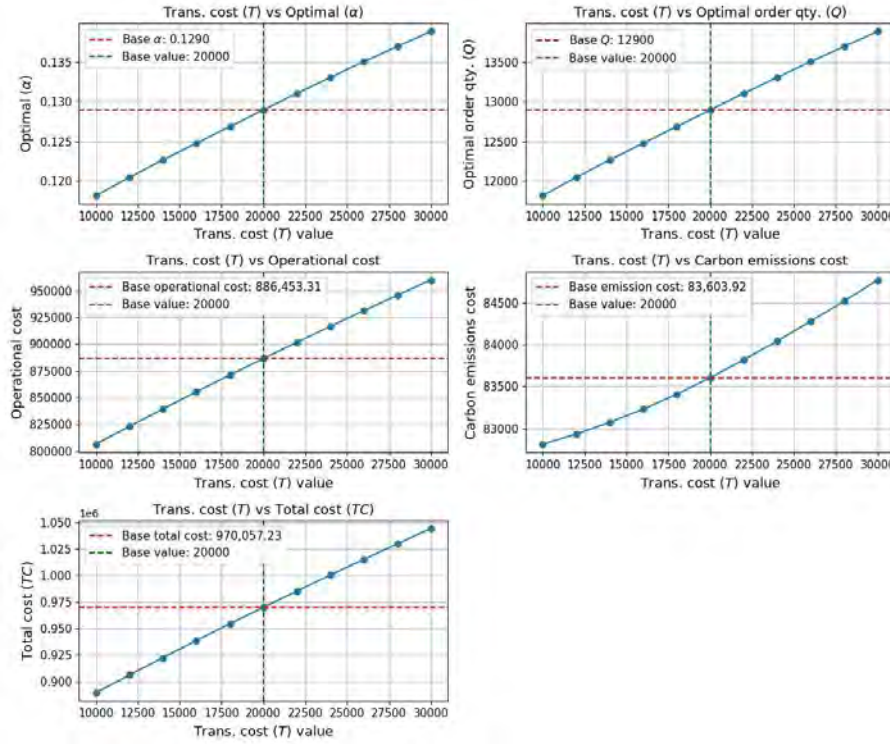


Figure 5: Sensitivity analysis by increasing transportation costs per shipment (T)

It can be observed from Figure 5 that as the transportation cost increases the optimal α , Q and total operations cost increases linearly but the carbon emission cost increases non-linearly with increasing rate. However, it has a negligible effect on the total cost since the total cost increases at the same rate.

Comment: Organizations should try for sustainable and economical transportation solutions to reduce their operational costs along with carbon emission. Implementing advanced logistics systems together with advanced transportation systems should result in reduced total costs and environmental impact.

Thereafter, we carry out the sensitivity analysis by increasing retailer's ordering cost (A_r) but keeping the other parameter values the same. The results are given in Table 7.

Table 7: Sensitivity analysis by increasing retailer's ordering cost (A_r)

Parameter value (A_r)	Optimal α	Optimal Q	Operational cost (BDT)	Carbon emissions cost (BDT)	Total cost (TC) (BDT)
4000	0.1248	12480	855,310.49	83,228.53	938,539.02
4800	0.1257	12570	861,624.54	83,302.27	944,926.81
5600	0.1265	12650	867,900.69	83,370.94	951,271.63
6400	0.1273	12730	874,132.01	83,442.50	957,574.51
7200	0.1282	12820	880,309.67	83,526.40	963,836.07
8000	0.1290	12900	886,453.31	83,603.92	970,057.23
8800	0.1298	12980	892,554.62	83,684.17	976,238.79
9600	0.1306	13060	898,614.37	83,767.10	982,381.47
10400	0.1315	13150	904,622.41	83,863.53	988,485.94
11200	0.1323	13230	910,600.94	83,951.99	994,552.93
12000	0.1331	13310	916,540.18	84,042.97	1,000,583.15

Increases in the retailer's ordering cost (A_r) increases the optimal α value which results in a higher optimal order quantity (Q). The resulting higher operational cost combined with the increased carbon emissions cost, increases the total costs, all of which are shown in Figure 6.

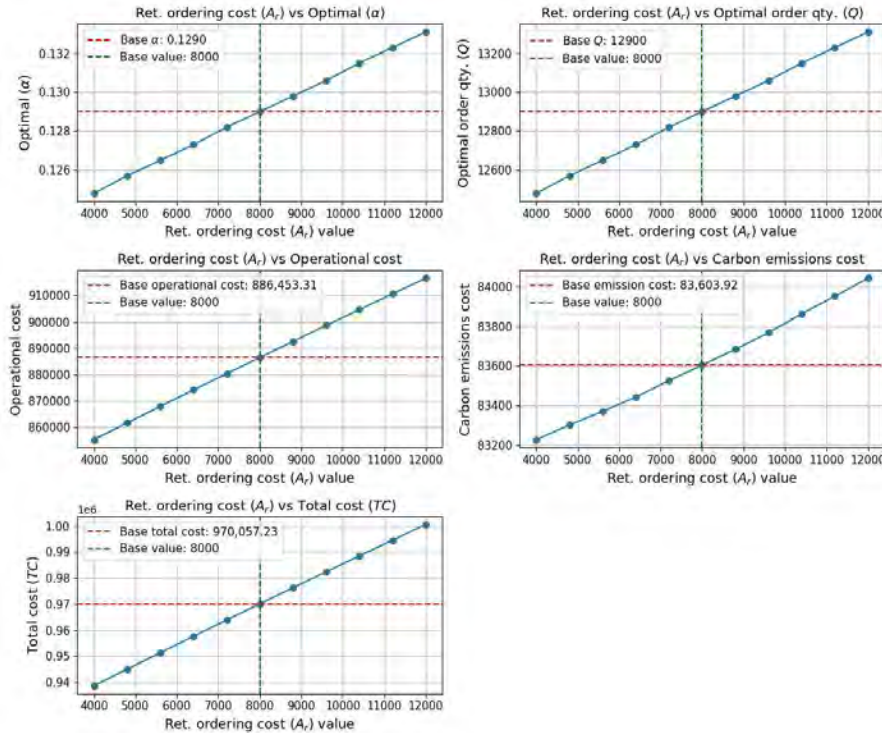


Figure 6: Sensitivity analysis for retailer's increased ordering cost (A_r)

From Figure 6 it can be noted that as the ordering cost increases the optimal α value, the optimal order quantity, the operations cost and the total cost increases almost linearly while the carbon emission cost increases slightly non-linearly with increasing rate. Hence, the carbon emission cost has negligible effect on the total cost.

Comment: Organizations should enhance their ordering systems because it helps to reduce both the cost and the environmental impact. Strategic improvements to ordering systems may improve the situation.

Next, we perform the sensitivity analysis by increasing the manufacturer's holding cost (h_m) but keeping the other parameter values the same, and found optimal solutions as provided in Table 8.

Table 8: Sensitivity analysis by increasing manufacturer's holding cost (h_m)

Parameter value (h_m)	Optimal α	Optimal Q	Operational cost (BDT)	Carbon emissions cost (BDT)	Total cost (TC) (BDT)
12.5	0.1400	14000	808,831.17	84,926.21	893,757.38
15.0	0.1376	13760	824,929.81	84,599.73	909,529.54
17.5	0.1353	13530	840,727.01	84,305.83	925,032.84
20.0	0.1331	13310	856,237.58	84,042.97	940,280.55
22.5	0.1310	13100	871,475.36	83,809.55	955,284.91
25.0	0.1290	12900	886,453.31	83,603.92	970,057.23
27.5	0.1271	12710	901,183.60	83,424.34	984,607.94
30.0	0.1253	12530	915,677.70	83,269.03	998,946.73
32.5	0.1235	12350	929,953.81	83,128.80	1,013,082.61
35.0	0.1219	12190	944,006.65	83,017.31	1,027,023.96
37.5	0.1202	12020	957,865.48	82,913.05	1,040,778.53

Increase in manufacturer's annual unit inventory holding cost (h_m) results in reduced optimal α values and (Q) values leading to higher operational cost but lower carbon emission costs successively, however the optimal total cost increases, all of which are highlighted in Figure 7.

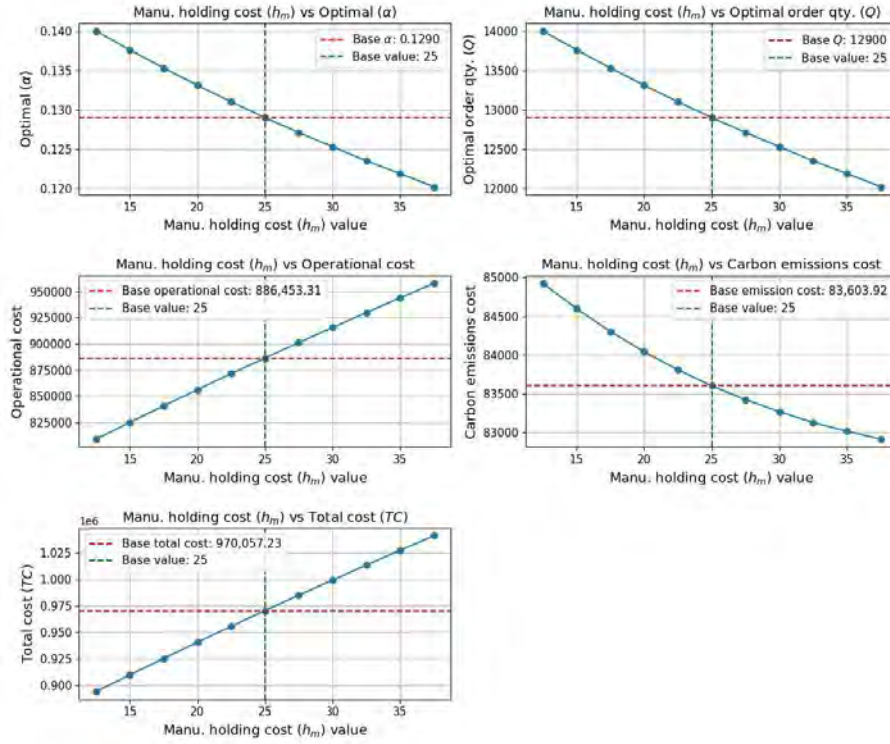


Figure 7: Sensitivity analysis for the manufacturer's increased holding cost (h_m)

It can be noted from Figure 7 that as the manufacturer's holding cost increases the optimal α value, the optimal order quantity (Q), the carbon emission cost decreases with decreasing rates, while the total operations cost and the total cost increases linearly. Hence, the decrease in the carbon emission cost has negligible effect on the total cost. So, the increases in the holding cost increases the operations cost and hence the total cost.

Comment: Organizations should try to reduce the manufacturer's holding cost by employing an efficient inventory management system in order to reduce total operational cost.

Then we perform the sensitivity analysis by increasing the retailer's holding cost (h_r) by keeping the other parameter values the same. The found optimal solutions are given in Table 9.

Table 9: Sensitivity analysis by increasing retailer's holding cost (h_r)

Parameter value (h_r)	Optimal α	Optimal Q	Operational cost (BDT)	Carbon emissions cost (BDT)	Total cost (TC) (BDT)
22.5	0.1541	15410	724,855.11	87,202.52	812,057.63
27.0	0.1479	14790	759,890.04	86,131.41	846,021.45
31.49	0.1424	14240	793,401.55	85,271.87	878,673.42
36.0	0.1375	13750	825,568.18	84,586.56	910,154.74
40.5	0.1330	13300	856,551.59	84,031.45	940,583.04
45.0	0.1290	12900	886,453.31	83,603.92	970,057.23
49.5	0.1253	12530	915,392.93	83,269.03	998,661.96
54.0	0.1219	12190	943,452.56	83,017.31	1,026,469.87
58.5	0.1188	11880	970,705.49	82,838.58	1,053,544.07
62.99	0.1159	11590	997,220.95	82,718.77	1,079,939.72
67.5	0.1132	11320	1,023,053.85	82,651.57	1,105,705.42

When the retailer's annual unit holding cost (h_r) increases the optimal α value decreases by reducing the optimal order quantity (Q), leading to increased operational cost but reduced carbon emission cost and successively increased total costs. All trends are shown in Figure 8.

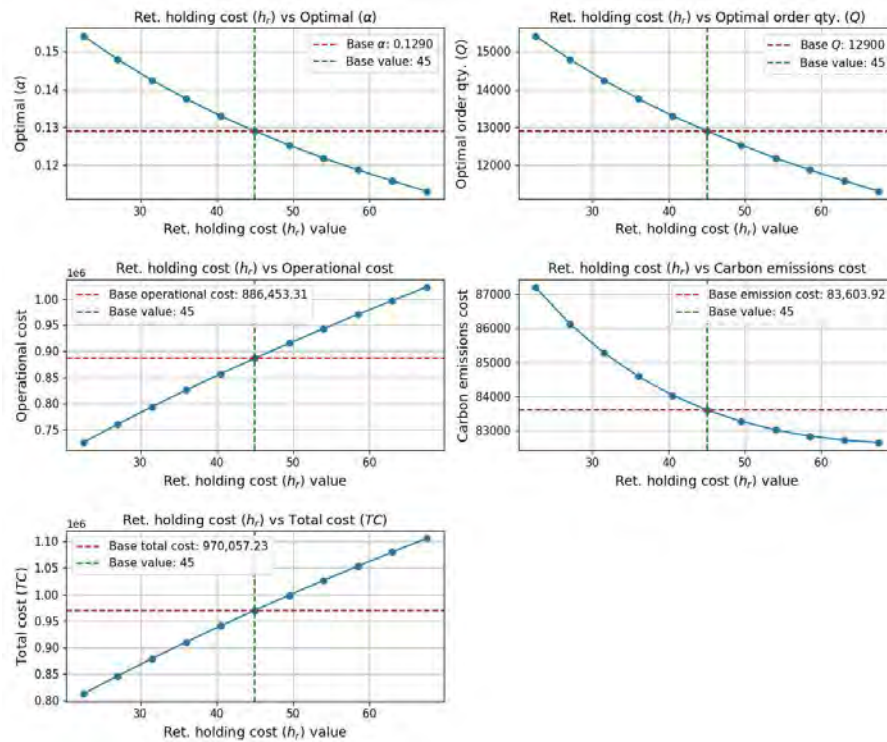


Figure 8: Sensitivity analysis for the retailer's increased holding cost (h_r)

It can be observed from Figure 8 that as the retailer's holding cost increases the optimal α value, the optimal order quantity (Q) and the carbon emission cost decrease non-linearly with a small decreasing rate, while the operations cost and the total cost increase slightly with decreasing rate. Hence the decreasing carbon emission cost has a small effect on the total cost.

Comment: The retailers should improve their inventory management systems to maintain proper inventory balance while reducing holding costs and hence the operations cost. The establishment of effective inventory control methods allows lowering operations cost (due to decreased inventory costs) by decreasing the environmental impact.

The sensitivity analyses carried out demonstrates that different parameter values affect the optimization coefficient (α) and order quantity (Q) together with operational cost and carbon emissions cost and hence the total cost. Strategic management of these parameters leads to both financial savings and a reduction to the environmental impact of carbon emission. The implementation of efficient processes across all stages from demand management through production setup and transportation to ordering and inventory control optimizes supply chain operations and supports sustainable business practices.

4.4 Discussion on Practical and Theoretical Implications

Practical implications:

Integrating operational costs along with the carbon emission costs in the model for obtaining optimal order quantity provides multiple significant advantages to SCM practices. Organizations can minimize their expenses through optimal order quantity management that decreases setup costs and reduces both manufacturer and retailer holding costs and transportation costs, ordering expenses and emission cost. Companies that integrate carbon emissions into their decision-making process achieve two benefits: they adopt green practices while reducing their total carbon footprint in order to fulfill their sustainability objectives. The solution approach to the model strengthens decision-making through its ability to show how

different supply chain parameters influence both operational cost and emissions thus enabling organizations to make better strategic supply chain choices.

Theoretical implications:

The developed model enhances SCM theory by merging economic practices with environmental considerations. By incorporating carbon emission costs the model expands the boundaries of traditional cost optimization model to deliver an integrated perspective of supply chain optimization. This model establishes connections between operations management professionals and environmental sustainability experts by implementing interdisciplinary standards. The model offers application versatility across multiple industries which positions it as a useful tool for ongoing research. This model can act as the basis for continuing research that incorporates economic responsibilities alongside environmental concerns within SCM to implement sustainable business practices.

In summary, this model provides organizations with a valuable operational tool to achieve cost reduction and environmental improvement through its applications in practice and also aids the theoretical research of supply chain optimization. The application of this model enables businesses to make wiser choices resulting in sustainable practices which produce efficient and environmentally sensitive supply chain networks.

Chapter 5 Conclusion and Recommendations

5.1 Summary of Findings

This research developed a mathematical model that incorporates both the operational and the carbon emission costs to find the optimal order quantity (Q) in a production delivery supply chain. For the studied numerical example problem, the results demonstrate an optimal α value of 0.1290 which produces an optimal order quantity (Q) 12,900 units. The optimum order quantity achieves the minimum balanced combination of the operational and the carbon emission costs. For this example, problem, the study showed an annual operational cost totaling to 886,453.31 BDT and an annual carbon emission cost to 83,603.92 BDT. The total annual cost reached 970,057.23 BDT when operational costs were combined with carbon emission costs.

A sensitivity analysis revealed how different parameters including demand, production rate, setup costs, transportation costs, ordering costs and inventory holding cost both at the manufacturer and the retailer change the optimal order quantity and overall total expense levels. The analysis delivered useful guidelines for handling these changes effectively. The integration of carbon emission cost within the model promotes eco-friendly business decisions and enables the implementation of sustainable practices by business organizations.

5.2 Contributions to Sustainable Supply Chain Optimization

The research introduced multiple essential additions to sustainable supply chain optimization methods. The model extended traditional cost optimization frameworks by incorporating wider aspect of carbon emission costs to offer increased SCM abilities. The developed approach here presented a real-world operational framework that helps organizations save costs while achieving environmental reduction targets. This study provided the tools to help companies make better decisions regarding their order quantities and transportation schedules and inventory management and production rate decisions. The sensitivity analyses help businesses

in gaining practical insights about different factors which influence costs and emissions so they can make strategic adaptive decisions.

This model supports organizations meeting their sustainable supply chain goals by ensuring both economic efficiency and ecological responsibility as core priorities. Implementing this model, organizations may build their green practices which in turn can attract customers and investors who value sustainability. Besides, this research serves as a solid platform for researchers to carry out further research work regarding the integration of economic and environmental factors in SCM. The provided model serves as an initial platform for additional research studies involving different industrial elements alongside its validation across multiple contexts.

Supply chain optimization through operational and carbon emission cost integration leads to savings while building sustainable business operations. Thus, implementing this model one can get better business success alongside environmentally friendly supply chain operations.

5.3 Practical Recommendations for Order Quantity and Carbon Emission Control

Organizations need to adopt several strategic approaches to improve order quantity optimization and carbon emission control effectiveness. In this regard the foundation of optimization includes both the demand forecasting systems and inventory management. Organizations can lower operational costs and carbon emissions by using precise demand forecasting to shift order quantity in action with market demand reality. Monitoring inventory efficiently reduces holding costs which results in cost savings and produces positive environmental outcomes.

The recommendation to enhance production efficiency represents a critical element in minimizing environmental impact and operational costs. Higher production rates combined

with improved manufacturing procedures enable businesses to reach reasonable order quantities that result in lowered operational and carbon emission expenses. The assessment followed by enhancement of setup procedures proving crucial for improvement. Efficient setup procedures alongside minimum setup times leads to major savings while cutting down environmental impact during operations.

The future call is for defining a better environmentally safe and less expensive transportation approaches. The implementation of efficient logistics and transportation systems produces reduced transportation expenses while cutting down carbon emissions. The optimization of ordering procedures leads to reasonably reduced order quantity coupling with diminished expenses. Businesses that use more effective ordering processes together with fewer order demands generate financial savings while provoking lower carbon emissions.

The most essential part in sustainably management in supply chains involves optimizing inventory management operations throughout it. A well-managed inventory system uses cost-effective solutions between storage expenses and greenhouse gas emissions to provide companies with financial and environmental benefits. Adopting the mentioned practical recommendations by businesses can maximize order quantity control as well as carbon emissions management to achieve sustainable supply chain operations.

5.4 Limitations and Future Research Scope

The research delivered important insight about sustainable supply chain improvement while facing issues that may reduce its wider practical scope. The deterministic parameter values used in the mathematical model analyses restrict its effectiveness for modeling advanced supply chain interactions. The elimination of disruptions along with fixed lead times and specific economic factors reduce the scope of finding applicability across different settings. Only the operational and the carbon emissions expenses were calculated in the analyses while social and ethical factors remained unexamined.

Research exploring the supply chain requires additional exploration of environmental factors such as disruptions, changes in material prices and regulatory policies together with green logistics principles. Long-term studies of sustainability measures against adaptability and resilience would boost the comprehension in this field. A comparative study of industry practices and geographical locations will lead to enhanced best practices development. Performance evaluation metrics that include comprehensive sustainability indicators assist organizations to maintain a balance between financial gains and environmental targets. Future research will enhance theoretical frameworks as well as real-world applications resulting in a sustainable global economy. Therefore, future research can be carried out in the indicated directions.

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Appendix A

1. Main code of the mathematical model

```
import math
```

```
import matplotlib.pyplot as plt
```

```
# Step 1: Defining all input parameters
```

```
D = 100000
```

```
P = 110000
```

```
S_m = 30000
```

```
T = 20000
```

```
A_r = 8000
```

```
h_m = 25
```

```
h_r = 45
```

```
d_t = 200
```

```
u_s = 2500
```

```
w_u = 0.005
```

```
EF_t = 0.186
```

```
E_sm = 1.5
```

```
EF_e = 0.437
```

```
E_sr = 2
```

```
E_c = 600
```

```
N_gc = 25
```

```
EF_ng = 1.923
```

```
D_ic = 20
```

```
EF_di = 2.697
```

```
W_st = 0.20
```

```
EF_st = 353.80
```

$$\lambda = 5.08$$

Step 2: Calculating optimal α value

$$\begin{aligned} \text{numerator} &= (S_m + T + A_r + (d_t * (u_s * w_u) * EF_t) * \lambda + \\ &\quad (E_c * EF_e + N_gc * EF_ng + D_ic * EF_di + W_st * EF_st) * \lambda) \\ \text{denominator} &= (h_m * D^{**2} / (2 * P) + (D / 2) * h_r + \\ &\quad (D^{**2} * E_sm * EF_e) / (2 * P) * \lambda + (D / 2) * (E_sr * EF_e) * \lambda) \\ \alpha &= \text{round}(\text{math.sqrt}(\text{numerator} / \text{denominator}), 4) \end{aligned}$$

Step 3: Calculating optimal order/batch quantity Q by using the rounded α

$$Q = \text{round}(\alpha * D)$$

Step 4: Calculating Operational Costs using the rounded α

$$\begin{aligned} \text{operational_cost} &= (D / Q) * S_m + h_m * (D * Q / (2 * P)) + (D / Q) * T + \backslash \\ &\quad (D / Q) * A_r + (Q / 2) * h_r \end{aligned}$$

Step 5: Calculating Each Carbon Emissions Costs by using rounded α

$$\begin{aligned} e_t &= (D / Q) * (d_t * (u_s * w_u) * EF_t) * \lambda \\ e_hm &= (D * Q / (2 * P)) * (E_sm * EF_e) * \lambda \\ e_hr &= (Q / 2) * (E_sr * EF_e) * \lambda \\ e_p &= (D / Q) * (E_c * EF_e + N_gc * EF_ng + D_ic * EF_di + W_st * EF_st) * \lambda \end{aligned}$$

Total Carbon Emissions Cost of the model

$$\text{total_carbon_emissions_cost} = e_t + e_hm + e_hr + e_p$$

Step 6: Total Cost (Operational Cost + Carbon Emissions Cost)

$$\text{total_cost} = \text{operational_cost} + \text{total_carbon_emissions_cost}$$

Step 7: Displaying results in a table

```
print("\nResults Summary:")
print(f'Optimal  $\alpha$  value of the model: { $\alpha$ :.4f}')
print(f'Optimal order/Batch quantity Q of the model: {Q} units\n')
print(f'Supply chain operational costs breakdown:')

```

```

print(f' - Annual setup cost of the manufacturer: BDT {(D / Q) * S_m:.2f}')
print(f' - Annual holding cost of the manufacturer: BDT {h_m * (D * Q / (2 * P)):.2f}')
print(f' - Annual transportation cost: BDT {(D / Q) * T:.2f}')
print(f' - Annual ordering cost of the retailer: BDT {(D / Q) * A_r:.2f}')
print(f' - Annual holding cost of the retailer: BDT {(Q / 2) * h_r:.2f}')
print(f' - Total annual operational cost: BDT {operational_cost:.2f}')
print("\nSupply chain carbon emissions costs breakdown:")
print(f' - Annual transportation carbon emission cost: BDT {e_t:.2f}')
print(f' - Annual manufacturer holding carbon emission cost: BDT {e_hm:.2f}')
print(f' - Annual retailer holding carbon emission cost: BDT {e_hr:.2f}')
print(f' - Annual production carbon emission cost: BDT {e_p:.2f}')
print(f' - Total annual carbon emissions cost: BDT {total_carbon_emissions_cost:.2f}')
print(f"\nAnnual total cost (Operational + Carbon emissions): BDT {total_cost:.2f}\n")

# Step 8: Visualizing costs using a bar graph
categories = ['Setup cost', 'Holding cost (Manuf.)', 'Transportation cost',
              'Ordering cost', 'Holding cost (Retailer)',
              'Transportation emission cost', 'Manuf. holding emission cost',
              'Retailer holding emission cost', 'Production emission cost']

values = [
    (D / Q) * S_m,
    h_m * (D * Q / (2 * P)),
    (D / Q) * T,
    (D / Q) * A_r,
    (Q / 2) * h_r,
    e_t,
    e_hm,

```

```

    e_hr,
    e_p
]
plt.figure(figsize=(12, 6))
plt.bar(categories, values, color='skyblue')
plt.xticks(rotation=45, ha='right')
plt.title('Costs breakdown (Operational + Carbon emissions)')
plt.ylabel('Cost (BDT)')
plt.tight_layout()
plt.show()

```

2. Sensitivity analysis code of the mathematical model

```

import math

import matplotlib.pyplot as plt

import pandas as pd

import os

base_parameters = {

    "D": 100000, # Annual demand rate of a product

    "P": 110000, # Annual production rate of a product

    "S_m": 30000, # Setup cost of the manufacturer per lot

    "T": 20000, # Transportation Cost

    "A_r": 8000, # Retailer ordering cost per order per year

    "h_m": 25, # Manufacturer holding cost per unit per year

    "h_r": 45, # Retailer holding cost per unit per year

}

```

```

parameter_titles = {
    "D": "Demand ($D$)",
    "P": "Production rate ($P$)",
    "S_m": "Setup cost ($S_m$)",
    "T": "Trans. cost ($T$)",
    "A_r": "Ret. ordering cost ($A_r$)",
    "h_m": "Manu. holding cost ($h_m$)",
    "h_r": "Ret. holding cost ($h_r$)",
}

```

```

def calculate_values(params):

```

```

    D = params["D"]
    P = params["P"]
    S_m = params["S_m"]
    T = params["T"]
    A_r = params["A_r"]
    h_m = params["h_m"]
    h_r = params["h_r"]
    u_s = 2500
    d_t = 200
    w_u = 0.005
    EF_t = 0.186
    E_sm = 1.5
    EF_e = 0.437

```

$$E_{sr} = 2$$

$$E_c = 600$$

$$N_{gc} = 25$$

$$EF_{ng} = 1.923$$

$$D_{ic} = 20$$

$$EF_{di} = 2.697$$

$$W_{st} = 0.20$$

$$EF_{st} = 353.80$$

$$\lambda = 5.08$$

$$\text{numerator} = S_m + T + A_r + (d_t * u_s * w_u * EF_t * \lambda) + \backslash$$

$$(E_c * EF_e + N_{gc} * EF_{ng} + D_{ic} * EF_{di} + W_{st} * EF_{st}) * \lambda$$

$$\text{denominator} = h_m * D^2 / (2 * P) + D * h_r / 2 + \backslash$$

$$D^2 * E_{sm} * EF_e * \lambda / (2 * P) + D * E_{sr} * EF_e * \lambda / 2$$

$$\alpha = \text{math.sqrt}(\text{numerator} / \text{denominator})$$

$$\alpha_{\text{rounded}} = \text{round}(\alpha, 4)$$

$$Q = \text{round}(\alpha_{\text{rounded}} * D)$$

$$\text{operational_cost} = \text{round}((D / Q) * S_m + h_m * (D * Q / (2 * P)) + (D / Q) * T + \backslash$$

$$(D / Q) * A_r + (Q / 2) * h_r, 2)$$

$$e_t = \text{round}((D / Q) * (d_t * u_s * w_u * EF_t * \lambda), 2)$$

$$e_{hm} = \text{round}((D * Q / (2 * P)) * (E_{sm} * EF_e * \lambda), 2)$$

$$e_{hr} = \text{round}((Q / 2) * (E_{sr} * EF_e * \lambda), 2)$$

$$e_p = \text{round}((D / Q) * (E_c * EF_e + N_{gc} * EF_{ng} + D_{ic} * EF_{di} + W_{st} * EF_{st}) * \lambda, 2)$$

```
total_carbon_emissions_cost = round(e_t + e_hm + e_hr + e_p, 2)
```

```
total_cost = round(operational_cost + total_carbon_emissions_cost, 2)
```

```
return alpha_rounded, Q, operational_cost, total_carbon_emissions_cost, total_cost
```

```
def sensitivity_analysis(parameter_name, base_value):
```

```
    variations = range(-50, 51, 10)
```

```
    results = []
```

```
    alphas = []
```

```
    qs = []
```

```
    operational_costs = []
```

```
    emission_costs = []
```

```
    total_costs = []
```

```
    base_alpha, base_Q, base_operational, base_emission, base_total =  
    calculate_values(base_parameters)
```

```
    for variation in variations:
```

```
        percentage_change = variation / 100
```

```
        new_value = base_value * (1 + percentage_change)
```

```
        temp_params = base_parameters.copy()
```

```
        temp_params[parameter_name] = new_value
```

```
        alpha, Q, operational_cost, total_carbon_emissions_cost, total_cost =  
        calculate_values(temp_params)
```

```

results.append([variation, new_value, f'{alpha:.4f}', Q, f'{operational_cost:,.2f}',
                f'{total_carbon_emissions_cost:,.2f}', f'{total_cost:,.2f}'])

alphas.append(alpha)

qs.append(Q)

operational_costs.append(operational_cost)

emission_costs.append(total_carbon_emissions_cost)

total_costs.append(total_cost)


df = pd.DataFrame(results,

                  columns=["Varying %", "Parameter Value", r"$\alpha$", r"$Q$", "Operational
Cost",

                          "Carbon Emissions Cost", r"$Total Cost ($TC$)"])

print(f"\nSensitivity Analysis for {parameter_titles[parameter_name]}:\n")

print(df.to_markdown(index=False, numalign="left", stralign="left",
tablefmt="fancy_grid"))


plt.figure(figsize=(12, 10))

title_font = {'size': 13}

label_font = {'size': 13}

tick_font = {'size': 10.5}

legend_font = {'size': 11}


param_values = [base_value * (1 + v / 100) for v in variations]


plt.subplot(3, 2, 1)

```

```

plt.plot(param_values, alphas, marker='o')

plt.axhline(y=base_alpha, color='r', linestyle='--', label=r'Base  $\alpha$ : ' +
f'{base_alpha:.4f}')

plt.axvline(x=base_value, color='g', linestyle='--', label=f'Base value: {base_value}')

plt.title(f'{parameter_titles[parameter_name]} vs Optimal " + r"( $\alpha$ )",
fontdict=title_font)

plt.xlabel(f'{parameter_titles[parameter_name]} value', fontdict=label_font)

plt.ylabel(r"Optimal ( $\alpha$ )", fontdict=label_font)

plt.xticks(fontsize=tick_font['size'])

plt.yticks(fontsize=tick_font['size'])

plt.legend(prop=legend_font)

plt.grid(True)

```

```

plt.subplot(3, 2, 2)

plt.plot(param_values, qs, marker='o')

plt.axhline(y=base_Q, color='r', linestyle='--', label=r'Base  $Q$ : ' + f'{base_Q}')

plt.axvline(x=base_value, color='g', linestyle='--', label=f'Base value: {base_value}')

plt.title(f'{parameter_titles[parameter_name]} vs Optimal order qty. " + r"( $Q$ )",
fontdict=title_font)

plt.xlabel(f'{parameter_titles[parameter_name]} value', fontdict=label_font)

plt.ylabel(r"Optimal order qty. ( $Q$ )", fontdict=label_font)

plt.xticks(fontsize=tick_font['size'])

plt.yticks(fontsize=tick_font['size'])

plt.legend(prop=legend_font)

plt.grid(True)

```

```

plt.subplot(3, 2, 3)

```

```

plt.plot(param_values, operational_costs, marker='o')

plt.axhline(y=base_operational, color='r', linestyle='--',

            label=f'Base operational cost: {base_operational:,.2f}')

plt.axvline(x=base_value, color='g', linestyle='--', label=f'Base value: {base_value}')

plt.title(f'{parameter_titles[parameter_name]} vs Operational cost', fontdict=title_font)

plt.xlabel(f'{parameter_titles[parameter_name]} value', fontdict=label_font)

plt.ylabel("Operational cost", fontdict=label_font)

plt.xticks(fontsize=tick_font['size'])

plt.yticks(fontsize=tick_font['size'])

plt.legend(prop=legend_font)

plt.grid(True)


plt.subplot(3, 2, 4)

plt.plot(param_values, emission_costs, marker='o')

plt.axhline(y=base_emission, color='r', linestyle='--',

            label=f'Base emission cost: {base_emission:,.2f}')

plt.axvline(x=base_value, color='g', linestyle='--', label=f'Base value: {base_value}')

plt.title(f'{parameter_titles[parameter_name]} vs Carbon emissions cost',
fontdict=title_font)

plt.xlabel(f'{parameter_titles[parameter_name]} value', fontdict=label_font)

plt.ylabel("Carbon emissions cost", fontdict=label_font)

plt.xticks(fontsize=tick_font['size'])

plt.yticks(fontsize=tick_font['size'])

plt.legend(prop=legend_font)

plt.grid(True)

```

```

plt.subplot(3, 2, 5)

plt.plot(param_values, total_costs, marker='o')

plt.axhline(y=base_total, color='r', linestyle='--', label=f'Base total cost: {base_total:,.2f}')

plt.axvline(x=base_value, color='g', linestyle='--', label=f'Base value: {base_value}')

plt.title(f'{parameter_titles[parameter_name]} vs Total cost " + r"($TC$)",
fontdict=title_font)

plt.xlabel(f'{parameter_titles[parameter_name]} value", fontdict=label_font)

plt.ylabel(r"Total cost ($TC$)", fontdict=label_font)

plt.xticks(fontsize=tick_font['size'])

plt.yticks(fontsize=tick_font['size'])

plt.legend(prop=legend_font)

plt.grid(True)


plt.tight_layout()


plt.show() # added plt.show() to show the plot


for param_name in base_parameters.keys():

    sensitivity_analysis(param_name, base_parameters[param_name])

```