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Travelling Wave and Non-Travelling Wave Solutions of Nonlinear Evolution Equation via New Generalized and Improved (G'/G)- Expansion Method

A Thesis that was submitted to

The Department of Mathematics and Natural Sciences, BRAC University,

for partial fulfilment of the requirements of the award of the degree of

Bachelor of Science in Mathematics

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01 March, 2018

Declaration

I do hereby declare that the thesis titled "Travelling Wave and Non-Travelling Wave Solutions of Nonlinear Evolution Equation via New Generalized and Improved (G'/G)-Expansion Method" is submitted to the Department of Mathematics and Natural Sciences of BRAC University in partial fulfilment of the Bachelor of Science in Mathematics. This is my original work and has not been submitted elsewhere for the award of any other degree or any other publication. Every work that has been used as reference for this work has been cited properly.

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Dedication

To my parents for their blessings, love, care, kindness and patience. They are the ones who have supported me, not only during the course of thesis but also my entire life and throughout my whole school career.

Acknowledgement

First and foremost I would like to express my thoughts of gratefulness to Almighty Allah who bestowed blessings, courage and good health, patience and inspiration upon me and without the blessings of Almighty Allah it would not have been possible for me to complete my thesis.

Then I would like to express my deepest gratitude and also would like to acknowledge my honourable and enthusiastic supervisor Dr. Hasibun Naher, Mathematics and Natural Sciences department of BRAC University, whose patience, expertise, understanding, encouragement and invaluable guidance have helped me to complete my thesis, from the beginning to the end of the writing. I extend my thanks to her for making time for me and letting me to do a research work under her guidance even after having such a lot of work pressure.

I sincerely express my gratitude to the entire family of Mathematics and Natural Sciences department and also to the department of BILL of BRAC University for their guidance, motivation, inspiration and for helping me to establish and to become the better version of myself. I would also like to thank my fellow mates for their help, support and encouragement throughout the whole university life.

Last but not the least, I am blessed and grateful to have such great parents to stand beside me, support, inspire, motivate and encourage me all the time. Because of them today I have successfully been able to come this far otherwise I would have given up a long time ago.

Abstract

In this thesis, we have applied the new extension of the generalized and improved $(G'/G)-$ expansion method in order to find the explicit solutions of non-travelling and travelling wave solutions of Fisher equation. Many new general and abundant solutions are obtained and they have been described in five different families in terms of hyperbolic functions, trigonometric functions and rational functions, involving many new and real parameters. It also shows that the method which has been applied is direct, concise and an effective tool for solving any kinds of nonlinear evolution equations, which arises frequently in applied mathematics, mathematical physics, engineering and in many scientific real time application. Furthermore we have presented the graphical solutions of the obtained solutions by using the computational software Maple.

Keywords: Differential equation (DE), ordinary differential equation (ODE), linear ordinary differential equation (LODE), nonlinear ordinary differential equation (NLODE), partial differential equation (PDE), linear partial differential equation (LPDE), nonlinear partial differential equation (NLPDE), nonlinear evolution equation (NLEE), Fisher equation, travelling wave solution, non-travelling wave solution, hyperbolic function, trigonometric function, rational function

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CHAPTER 1

INTRODUCTION

1.1 History of Differential Equation (DE): In pure and applied mathematics world, differential equations have been and still now are one of the foremost topics, ever since it was proposed in the mid-17th century. The history of DE raised from calculus by the English physicist Isaac Newton, German mathematician Gottfried Leibniz, Bernoulli's brother, Euler, Cauchy and many others, right after Newton published " The Method of Fluxions and Infinite series" in 1671. In 1676 when Newton solved his first d.e at the same year Leibniz introduced the term differential equation "*acquotio differential*" in latin by using two variables x and y only and denoting it as dx and dy. Unlike many other mathematical functions, the solutions which are obtained from d.es are not numbers but functions involving one or more variable derivative terms with respect to other. Using this DE many applications were made to biology, chemistry, economics, mechanics, and physics and as well as other areas of natural sciences in real life. In Fluxions and Infinite series Newton classified DE as ordinary differential equation and partial differential equation.

1.1.1 Definition of Differential Equation (DE): A differential equation is an equation which consists of independent variable, dependent variable and differential coefficient of one or more than one dependent variable with respect to one or more than one independent variable is referred to as differential equation. For example:

(1) $\frac{dy}{dx}+(1+y^4)\sin x=0$, (2) $\frac{\delta^2 z}{\delta x^2}=m\left(\frac{\delta^3 z}{\delta x^3}\right)$, (3) $y^3\frac{\delta z}{\delta t}-by\frac{\delta z}{\delta y}=atz$, (4) $\frac{d^2x}{dt^2}=8t^5$

1.1.2 Definition of Ordinary Differential Equation (ODE): A differential equation which consists of one or more than one dependent variable with respect to a single independent variable is known as ordinary differential equation. For example:

$$(1)\ \frac{d^2z}{dx^2}+\frac{dz}{dx}-8y=0,\ (2)\ \frac{dx}{dt}+\frac{dy}{dt}=e^x+y,\ (3)\ \frac{dx}{dt}+10x=\sin t$$

1.1.3 Definition of Linear Ordinary Differential Equation (LODE) and Nonlinear Ordinary Differential Equation (NLODE): An equation

$$a_n(x)\frac{d^n y}{dx^n}+a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}}+a_{n-2}(x)\frac{d^{n-2}y}{dx^{n-2}}+\ldots a_1(x)\frac{dy}{dx}+a_0(x)y=g(x)$$

involving nth order ODE is said to be linear if the power of each term of the dependent variable y and all its derivatives $y^{(n)},y^{(n-1)}\ldots y'',y'$ is 1 or it is of first degree and the function of x in front of every derivative depends on the independent variable x that is the coefficients $a_0, a_1,\ldots a_n$ of $y,y',\ldots y^{(n)}$ depends on independent variable, then it is LODE. For example:

$$(1)\ y''-5y'+5ty=0,\ (2)\ x^3\frac{d^3y}{dx^3}+x^2\frac{d^2y}{dx^2}-2y=e^x$$

An ODE is said to be nonlinear if the coefficients of the dependent variable depends on the dependent variable, or if the equation involves a nonlinear function of y or if the no derivative term is greater than 1 then it is known as NLODE. For example:

$$(1)\ (2-y)y''+3y=e^x,\ (2)\ \frac{d^3y}{dx^3}+\tan y=0$$

1.1.4 Definition of Partial Differential Equation (PDE): A differential equation which consists of one or more than one dependent variables with respect to two or more than two independent variable is called PDE. For example:

$$(1)\ x\frac{dz}{dx}+y\frac{dz}{dy}=e^{z},\ (2)\ \frac{\delta u}{\delta y}=2+\frac{\delta u}{\delta x}$$

1.1.5 Definition of Linear Partial Differential Equation (LPDE) and Nonlinear Partial Differential Equation (NLPDE): An equation is said to be LPDE if the dependent variable and all of its derivatives is of first degree, otherwise it is NLPDE. For example:

Linear PDE:

$$(1)\ \frac{\delta^3 u}{\delta x^3}=\frac{\delta u}{\delta y},\ (2)\ \frac{\delta^2 u}{\delta x\delta y}=\frac{\delta u}{\delta x}\tan t$$

Nonlinear PDE:

$$(1)\ u\frac{\delta^2 u}{\delta x^2}=\frac{\delta u}{\delta t}\tan t,\ (2)\ \frac{\delta^2 u}{\delta x^2}+\left(\frac{\delta u}{\delta x}\right)^2=0$$

Nonlinear equations have led to a most important position, during the twentieth century the development of NLPDEs has increased remarkably because for understanding the complexity of nonlinear models and for solving them that are directed by NLPDEs, great importance has been given. But one of the strong desires of progressing NLPDEs has been the study of nonlinear travelling wave problems and over the past two-three decades nonlinear waves have experienced a great revolution many new remarkable and unexpected phenomena have been observed. One of the major achievements in the investigation of NLPDEs has been the nonlinear evolution equations (NLEEs).

1.1.6 Definition of Nonlinear Evolution Equation (NLEE): A NLPDE with time t, as a single independent variable expressed in terms of space variable $u(x,t)$ is referred to as nonlinear evolution equation (NLEE). For example: the Fisher equation $u_t - u_{xx} - u(1-u) = 0$.

It is one of the most powerful and important modelled equations among all equations in nonlinear sciences, because they reveal a lot of physical information which helps to understand the operation of the physical model better, which is why the explicit solutions of NLEEs plays an important role in the study of physical phenomena and remains a vital field for researchers in the ongoing investigation. Though NLEEs are used in many fields but now a day it is mostly used to research travelling wave solutions.

1.2 Traveling Waves: Wave patterns are formed when a vibrating source disturbs the atoms or particles of medium (water, air, spring, string etc.) continuously that travels along the direction of wave motion with speed and without any change in shape is called traveling waves. When the medium is disturbed the particles get displaced from their rest position temporarily but there is a force that acts on the particle, causing them to return to its original position. This disturbance travels down through the system and through this disturbance energy and momentum is transferred from one end to another without any net motion in the medium. To understand the phenomena of travelling wave solutions, mathematical operations have been used to describe the travelling wave function in the form of $u(x,t) = f(x-ct)$ where $u(x,t)$ represents the wave distress moving along the direction of x when $c > 0$ or $c < 0$ respectively. They are obtained when a NLEE is reduced to ODE, taking $u(x,t) = u(\xi)$, where $\xi = x - ct$ and c is the speed of the waves and they can be solved by using appropriate method. During the past few decades there has been a lot of development for obtaining travelling wave solutions and there happen to appear many types of it and some of them are:

1.2.1 Solitary Waves and Solitons: Solitary waves are a kind of enclosed travelling wave that transforms form state to another state $-\infty < \xi < \infty$ asymptotically, where $\xi = x - ct$ without any change to its shape. It is usually referred to a single soliton but when there is more than one soliton solution then they are called solitons. These solitons are a special kind of solitary wave and hold a particle like characteristics. They always maintain their identities, their shape and speed before and after interaction with each other and they appear either in bell shape or in kink form.

1.2.2 Peakons: The travelling wave solutions of the peakons are known as the peaked solitary waves or peaked solitons because the corner of its crest forms a peak, like the shape of the $e^{-|x|}$ function. Like solitons peakons also maintain their shape and speed after interaction with each other, and also they contain some points where discontinuities occur at the first derivatives as a result they have finite jump in the solution of $u(x,t)$.

1.2.3 Periodic Solution: Periodic solution is another type of travelling wave that moves with a constant speed in one dimensional function such as $\cos(x-t)$. $u_{tt} = u_{xx}$ is the standard wave equation form that gives solution of the periodic waves. These solutions can appear in many mathematical system like cellular automata and reaction-diffusion advection systems. Apart from that they are also used in many mathematical models like, biology, physics and chemistry as well.

CHAPTER 2

LITERATURE

2.1 Analytic Methods for Solving Nonlinear Evolution Equations (NLEEs): In physical sciences all essential equations are equations are nonlinear and are often complicated to interpret, therefore unravelling the exact solution of NLEEs have turned out to be a chief concern for both mathematician and physicists. At the same time the curiosity of finding exact solutions of NLEEs, in order to understand the NLEE modelled phenomena is becoming more intense day by day. Thus to figure out the exact solution of NLEEs substantial work are being made by mathematicians and scientists and have developed effective and convincing methods such as Hirota's bilinear transformation method[16,17], the tanh-function method [18,19], the extended tanh-method[20,21], the Exp-function method [22,23], the F-expansion method [24], the Jacobi elliptic function method [26], Modified Exp-function method [25], Weierstrass elliptic function method[34], the homogeneous balance method [32,33], the modified simple equation method [27,28], the asymptotic methods [29], the variational iteration method [30,31], the sub-ODE method [5,6], the sine–cosine method [37,38], the homotopy perturbation method [35,36], the rank analysis method [39], the ansatz method [2-4], the exp-function expansion method[40], the tanh-coth method [41,42], the direct algebraic method[43], Riccati equation method [44], the Backlund transformation method [45], the theta function method [46,47], the Cole-Hopf transformation method [48,49], the hyperbolic tangent expansion method [50,51], the trial function method [52], the non-linear transform method[53], and so on but no such coalesce method have yet been proven to work out with sorts of NLEEs.

2.2 Literature review of the (G'/G) – Expansion Method: For the past few decades and so on, a vast research has been going on to find explicit solutions of NLEEs which are used as models in order to describe many important and problematic physical phenomena in various fields of science. Thus a various group of scientists has created some exact solution of NLEEs by using vast range of improved and effective methods, for instance, inverse scattering transformation [1], the sub-ODE method [5,6], the exp-function method [7] and so on in order to get to understand the nonlinear phenomena much better and also for developing more new applications of it. But as said earlier that no such coalesce method have yet been proven to work out with sorts of NLEEs.

Later in 2008, Wang *et al* [8] introduced a new method called the $\left(G'/G\right)$- Expansion Method for finding the solutions of traveling waves of NLEEs. Here the travelling wave solutions of NLEEs are expressed in the $\left(G'/G\right)$ polynomial where $G=G\left(\xi\right)$ $\xi = x - Vt$ that leads to a second order linear ordinary differential equation (LODE), $G'' + \lambda G' + \mu G = 0$ where $G' = \dfrac{dG\left(\xi\right)}{d\xi}$ and λ and μ are arbitrary constants and to solve any NLEEs through this process neither initial or boundary conditions nor any trail functions are required at the beginning. To demonstrate this method various NLEES have been used to find traveling waves like the KdV equation, mKdV equation, the reaction-diffusion equation and so on [8,9]. This $\left(G'/G\right)$- expansion method shows that it is one of the most powerful and effective method to solve NLEEs since it gives a clear and short to the point results in terms of hyperbolic functions, trigonometry functions and rational functions which is why scientists have carried out a lot of researches to construct traveling wave solutions via this method.

Further researches of (G'/G)- expansion method has been carried out by many researchers to show the possible productivity of the application. For example Zhang et all [12] developed the original $\left(G'/G\right)$- expansion method from $u(\xi) = \sum_{i=0}^{m} a_i \left(\dfrac{G'}{G}\right)^i$ where $a_m \neq 0$, to

$u(\xi)=\sum_{i=-m}^{m} a_i \left(\frac{G'}{G}\right)^i$, where a_{-m} or a_m may be zero but cannot be zero at the same time, as solutions and named it as the improved (G'/G)- expansion method for finding traveling waves solutions of NLEEs [13,14,68]. Then Akber *et all* [15] introduced the generalized and improved (G'/G)- expansion method in the form $u(\xi)=\sum_{n=-m}^{m} \frac{e^{-n}}{(d+(G'/G))^n}$ where either e_{-m} or e_m can be zero but cannot be zero at the same time and the same second order LODE were used as auxiliary equation to construct travelling wave solution Like this many researchers have carried out and still carrying out experiments using the (G'/G)- expansion method to generate more new traveling wave solutions of NLEEs [69-79].

CHAPTER 3

METHODOLOGY

3.1 Methodology of new extension of the generalized and improved (G'/G)- expansion method:

As it is known that (G'/G)- expansion method is one of the simplest and most powerful method for obtaining travelling wave solutions of NLEEs and so far its application have been used in various ways to solve nonlinear evolution problems. Recently a new application have been introduced called the new extension of the generalized and improved (G'/G)- expansion method for NLEEs. So to demonstrate this method, first a NLPDE is taken with two real independent variables x and t i.e.

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx} ...) = 0 \tag{3.1.1}$$

where P is the polynomial and here $u = u(x,t)$ is an unknown function. In the polynomial P contains, different partial derivatives of the function u itself wherein involves the highest order derivatives and the highest nonlinear terms. Now the prime process of this method is being discussed in steps below.

Step1: Suppose that,

$$u(x,t) = u(\xi), \ \xi = x \pm Wt \tag{3.1.2}$$

where the constant term W is known as the speed of wave, is substituted in eq (3.1.1), which allows a PDE to convert ta an ODE with respect to ξ .

$$Q(u, u', u'', u''' ...) = 0 \tag{3.1.3}$$

Step 2: Eq (3.1.3) is being integrated term by term and if needed it can be integrated more than once and the integral constants maybe set to zero to make it easy to solve. Now the integrated travelling wave solution of eq (3.1.3) can be represented as.

$$u(\xi)=\sum_{j=-N}^{N} a_j\left(d+H\right)^j+\sum_{j=1}^{N}\frac{b_j}{\left(d+H\right)^j} \tag{3.1.4}$$

where a_N, a_{-N} or b_N can be zero but all cannot be zero at the same time, $a_j(j=0,\pm1,\pm2,...\pm N)$, $b_j(j=1,2,3,...N)$, d is the arbitrary constant to be determined later and $H(\xi)$ is

$$H(\xi)=\left(\frac{G'}{G}\right) \tag{3.1.5}$$

where $G=G(\xi)$ satisfies the nonlinear ordinary differential equation (ODE) i.e.

$$\lambda GG''-\mu GG'-\delta(G')^2-\beta(G)^2=0 \tag{3.1.6}$$

where $G''=\frac{d^2G}{d\xi}, G'=\frac{dG}{d\xi}$ and $\lambda,\mu,\delta\ \&\ \beta$ are the real parameters

Step 3: The positive integer N appearing in the integrated solution of eq (3.1.3) is then determined by considering the homogeneous balance between the highest order derivative and the highest nonlinear term. The value of N is substituted in eq (3.1.4) which gives a complete ODE. Then the completed ODE of eq (3.1.4), eq (3.1.5)along with eq (3.1.6) is substituted in the integrated solution of eq (3.1.3) and collecting all the powers of the term $\left(d+H\right)$ in descending order to the left hand side, thus transforms into another polynomial of $\left(d+H\right)$, here $(d+H)^N\ (N=0,\pm1,\pm2,...)$ and $(d+H)^{-N}\ (N=1,2,3...)$.

Step 4: The coefficient of the $(d+H)$ polynomial is then equated to zero, hence generates a set of algebraic equation. By solving the algebraic equation gives the value for $a_j(j=0,\pm1,\pm2,...\pm N)$, $b_j(j=1,2,3,...N)$,d and W obtained from eq (3.1.5). Now by solving eq (3.1.6) we obtain a general solution, which is then substituted with the values of constants into eq (3.1.4) we can achieve more general type and more new travelling wave solutions of NLPDE of eq (3.1.1).

Step 5: Using the general solution of eq (3.1.6), the following solutions for eq (3.1.5) are obtained:

Family 1: When $\mu \neq 0,\ \Psi=\lambda-\delta$ and $\Omega=\mu^2+4\beta(\lambda-\delta)>0,$

$$H(\xi)=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)} \tag{3.1.7}$$

Family 2: When $\mu \neq 0,\ \Psi=\lambda-\delta$ and $\Omega=\mu^2+4\beta(\lambda-\delta)<0,$

$$H(\xi)=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)} \tag{3.1.8}$$

Family 3: When $\mu \neq 0,\ \Psi=\lambda-\delta$ and $\Omega=\mu^2+4\beta(\lambda-\delta)=0,$

$$H(\xi)=\left(\frac{G'}{G}\right)=\frac{B}{2\Psi}+\frac{C_2}{C_1+C_2\xi} \tag{3.1.9}$$

Family 4: When $\mu=0,\ \Psi=\lambda-\delta$ and $\Delta=\Psi\beta>0,$

$$H(\xi)=\left(\frac{G'}{G}\right)=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)} \tag{3.1.10}$$

Family 5: When $\mu=0,\ \Psi=\lambda-\delta$ and $\Delta=\Psi\beta<0,$

$$H(\xi)=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)} \tag{3.1.11}$$

CHAPTER 4

APPLICATION

4.1 The Fisher Equation: The partial differential equation,

$$u_t - u_{xx} - u(1-u) = 0 \tag{4.1.1}$$

is a combination of diffusion with logistic nonlinearity, known as the Fisher equation. It explains the classic interaction between the linear diffusion and nonlinear reaction that takes place in the application of chemistry, biology, population dynamics and medicine, involving nonlinear evolution problems like neutron population in nuclear reaction and population dynamics in one dimensional locus. That is when nonlinearity releases energy and in order to balance, energy is being consumed by diffusion, thus happen to appear traveling waves [66]. Fisher Equation was named after sir Ronald Aylmer Fisher, who was a statistician and a biologist as well. In 1936, Fisher published a paper called 'The wave of advance of advantageous genes' where he first proposed the Fisher equation,

$$u_t = lu_{xx} + ju(1-u) \qquad x \in (-\infty, \infty), \quad t \geq 0$$

(4.1.2) where l is the coefficient of diffusion and j is the growth factor of the species and $u(x,t)$ is the population density, in order to describe the allele gene mutation and diffusion of the advantageous gene in the surroundings of one dimensional habitat and while explaining he discovered infinite travelling wave solutions for Fisher equation [62]. Right after then in 1937 Kolmogorov *et al* [63] used the Fisher equation by taking it as two dimensional equation and showed that unique travelling wave solutions can be obtained by using certain initial and boundary conditions for when the wave speed C, $c \geq 2$. For this Fisher equation is also known as the Fisher-KPP equation. Carey and Shen [64] rescaled eq (4.1.2) to remove the coefficients l and j by introducing scaled variables $\alpha = jt$ and

$\beta = \left(j/l \right)^{1/2} x$, and substituting these scaled variables in eq (4.1.2) eventually converts to eq (4.1.1), so to obtain travelling wave solutions easily.

Fisher equation or the Fisher-KPP equation have been used for solving many mathematical model phenomena and also in many biological systems as well, for example: tissue engineering [54], in a simple wound healing process there occurs a constant travelling speed, so to verify that Fisher equation has been to obtain that constant travelling wave speed, then in nuclear reactor theory as propagation of neutron [55], then again in the context of flame propagation [56] and also in Brownian motion process [57], combustion [58], logistic population growth model [59], neurophysiology [60] etc.

Apart from that many authors have used the Fisher equation or the Fisher-KPP equation and transformed it into more improved version of the equation according to researcher's purpose that can be seen in the articles [61,62].

4.2 Application

Let us consider the Fisher equation to investigate and to construct new travelling wave solutions by using the new extension of the generalized and improved $(G'/G)-$expansion method.

The Fisher equation:

$$u_t - u_{xx} - u(1-u) = 0 \qquad (4.2.1)$$

By using wave transformation of eq (3.1.2), $u(x,t) = u(\xi)$, $\xi = x - Wt$ into eq (4.2.1) we have,

$$u_t = \frac{\delta u}{\delta t} = \frac{\delta u}{\delta \xi}.\frac{\delta \xi}{\delta t} = u'(-W) = -Wu'$$

$$u_{xx} = \frac{\delta}{\delta x}\left(\frac{\delta u}{\delta x}\right) = \frac{\delta}{\delta x}\left(\frac{\delta u}{\delta \xi}.\frac{\delta \xi}{\delta x}\right) = \frac{\delta}{\delta x}(u') = u''$$

The above wave transformations, transform eq (4.2.1) into the following NLODE:

$$u'' + Wu' + u - u^2 = 0 \qquad (4.2.2)$$

Now by making the ansatz (3.1.4) for the solution of eq (4.2.2) and by taking the homogeneous balance between the nonlinear term u^2 and the highest order derivative term u'' in eq (4.2.2) we obtain the value for N i.e. $N = 2$. Therefore the solution of eq (4.2.2) can be written in the form:

$$u(\xi) = a_0 + a_1(d+H) + a_2(d+H)^2 + (a_{-1}+b_1)(d+H)^{-1} + (a_{-2}+b_2)(d+H)^{-2} \qquad (4.2.3)$$

where $a_{-2}, a_{-1,} a_0, a_1, a_2, b_1, b_2$ and d are constants to be determined.

Now substituting eq (4.2.3) along with eq (3.1.5) and (3.1.6) into eq (4.2.2) and by simplifying it, the left hand side is transformed into polynomials in $(d+H)^N$ $(N = 0, \pm 1, \pm 2, ...)$ and $(d+H)^{-N}$ $(N = 1, 2, 3...)$. By collecting the resulted polynomials to zero, yields a set of simultaneous algebraic equations for $a_{-2}, a_{-1,} a_0, a_1, a_2, b_1, b_2, d, c$ and W.

4.3 Systems of Algebraic Equations:

$$\left(d + \frac{G'}{G}\right)^4 : -6a_2\delta^2 + 12a_2\delta\lambda + \lambda^2 a_2^2 - 6a_2\lambda^2 = 0$$

$$\left(d + \frac{G'}{G}\right)^3 : 20\delta^2 da_2 + 10\mu a_2\lambda + 20\lambda^2 da_2 - 2ca_2\lambda\delta + 4\lambda a_1\delta - 2\delta^2 a_1 - 40da_2\lambda\delta - 10\mu a_2\delta$$
$$+ 2c\lambda^2 a_2 + 2\lambda^2 a_1 a_2 - 2a_1\lambda^2 = 0$$

$$\left(d+\frac{G'}{G}\right)^2: -4c\lambda^2 a_2 d - 24a_2\delta^2 d^2 - c\lambda a_1\delta - 12\delta\lambda a_1 d + 2\lambda^2 a_0 a_2 + 24\delta a_2\mu d + 48\delta a_2\lambda d^2$$
$$+6a_1\delta^2 d + 3\lambda a_1\mu + 6a_1\lambda^2 d - 2c\lambda a_2\mu + 4c\lambda a_2\delta d + \lambda^2 a_1^2 - 3\delta a_1\mu + c\lambda^2 a_1 - \lambda^2 a_2$$
$$-4a_2\mu^2 - 24\lambda a_2\mu d - 8\delta a_2\beta - 24a_2\lambda^2 d^2 + 8\lambda a_2\beta = 0$$
$$\left(d+\frac{G'}{G}\right)^1: -24\lambda d^3 a_2\delta + 18\lambda d^2 a_2\mu - 12\lambda d a_2\beta + 12\lambda d^2 a_1\delta - 6\lambda d a_1\mu - 18\delta d^2 a_2\mu + 12\delta d a_2\beta$$
$$+6\delta d a_1\mu + 12\lambda^2 d^3 a_2 - 6\lambda^2 d^2 a_1 + 12\delta^2 d^3 a_2 - 6\delta^2 d^2 a_1 + 6a_2\mu^2 d - 6\mu a_2\beta + 2\beta a_1\lambda$$
$$-2\beta a_1\delta - a_1\lambda^2 - a_1\mu^2 - 2c\lambda a_2\beta - c\lambda a_1\mu + 2c\lambda^2 d^2 a_2 - 2c\lambda^2 d a_1 + 2\lambda^2 a_0 a_1 + 2\lambda^2 a_2 a_{-1}$$
$$+2\lambda^2 a_2 b_1 - 2c\lambda a_2\delta d^2 + 2c\lambda a_2\mu d + 2c\lambda a_1\delta d = 0$$
$$\left(d+\frac{G'}{G}\right)^0: 4\delta d^3 a_2\mu + 4\delta d^4 a_2\lambda - 4\delta d^2 a_2\beta - 4\delta d^3 a_1\lambda - 3\delta d^2 a_1\mu - 4\mu d^3 a_2\lambda + 4\mu d a_2\beta$$
$$+3\mu d^2 a_1\lambda + 4\lambda d^2 a_2\beta - 2\beta a_1\lambda d + 2\beta a_1\delta d - 4\delta\lambda d b_1 - 4\delta\lambda d a_{-1} - c\lambda^2 b_1 - c\lambda^2 a_{-1}$$
$$+2\delta^2 d^3 a_1 - 2\mu^2 d^2 a_2 + \mu^2 d a_1 - 2\lambda^2 d^4 a_2 + 2\lambda^2 d^3 a_1 - \beta a_1\mu + 2\delta^2 d a_{-1} - \delta\mu a_{-1}$$
$$+4\delta\lambda a_{-2} + 4\delta\lambda b_2 + 2\delta^2 d b_1 - \delta\mu b_1 + \lambda\mu a_{-1} + 2\lambda^2 d b_1 + 2\lambda^2 d a_{-1} + \lambda\mu b_1 - 2a_2\beta^2$$
$$-2\delta^2 b_2 - 2\delta^2 a_{-2} - 2\lambda^2 a_{-2} - 2\lambda^2 b_2 - 2\delta^2 d^4 a_2 + c\lambda^2 a_1 d^2 + c\lambda\delta b_1 - c\lambda a_1\beta + c\lambda\delta a_{-1}$$
$$+2\lambda^2 a_1 a_{-1} + 2\lambda^2 a_1 b_1 + 2\lambda^2 a_2 a_{-2} + 2\lambda^2 a_2 b_2 - \lambda^2 a_0 + \lambda^2 a_0^2 - c\lambda a_1\delta d^2 + c\lambda a_1\mu d = 0$$
$$\left(d+\frac{G'}{G}\right)^{-1}: -24\delta\lambda d a_{-2} + 6\delta\mu d b_1 + 12\delta\lambda d^2 a_{-1} - 24\delta\lambda d b_2 + 6\delta\mu d a_{-1} + 12\delta\lambda d^2 b_1 - 6\lambda\mu d b_1$$
$$-6\lambda\mu d a_{-1} + 12\delta^2 d b_2 + 12\delta^2 d a_{-2} - 6\delta\mu a_{-2} - 6\delta\mu b_2 - 2\delta\beta a_{-1} - 2\delta\beta b_1 - 6\delta^2 d^2 a_{-1}$$
$$-6\delta^2 d^2 b_1 + 12\lambda^2 d a_{-2} - 6\lambda^2 d^2 a_{-1} + 12\lambda^2 d b_2 + 6\lambda\mu a_{-2} + 6\lambda\mu b_2 + 2\lambda\beta a_{-1} + 2\lambda\beta b_1$$
$$-6\lambda^2 d^2 b_1 - \mu^2 a_{-1} - \mu^2 b_1 + c\lambda\mu a_{-1} + c\lambda\mu b_1 + 2c\lambda\delta a_{-2} + 2c\lambda\delta b_2 + 2c\lambda^2 d a_{-1} + 2c\lambda^2 d b_1$$
$$-2c\lambda^2 b_2 - 2c\lambda^2 a_{-2} + 2\lambda^2 a_0 a_{-1} + 2\lambda^2 a_0 b_1 + 2\lambda^2 a_1 a_{-2} + 2\lambda^2 a_1 b_2 - \lambda^2 a_{-1} - \lambda^2 b_1 - 2c\delta\lambda d a_{-1}$$
$$-2c\delta\lambda d b_1 = 0$$
$$\left(d+\frac{G'}{G}\right)^{-2}: 48\delta\lambda d^2 b_2 + 48\delta\lambda d^2 a_{-2} + 9\lambda d^2\mu b_1 + 9\lambda d^2\mu a_{-1} - 24\lambda d\mu a_{-2} - 24\lambda d\mu b_2 - 6\lambda d\beta a_{-1}$$
$$-6\lambda d\beta b_1 - 12\lambda d^3\delta a_{-1} - 12\lambda d^3\delta b_1 - 9\delta d^2\mu b_1 - 9\delta d^2\mu a_{-1} + 24\delta d\mu a_{-2} + 24\delta d\mu b_2$$
$$+6\delta d\beta a_{-1} + 6\delta d\beta b_1 - 24\lambda^2 d^2 a_{-2} + 6\lambda^2 d^3 a_{-1} - 24\lambda^2 d^2 b_2 + 6\lambda^2 d^3 b_1 - 24\delta^2 d^2 b_2$$
$$-24\delta^2 d^2 a_{-2} + 6\delta^2 d^3 a_{-1} + 6\delta^2 d^3 b_1 + 3\mu^2 d b_1 + 3\mu^2 d a_{-1} - 3\mu\beta a_{-1} - 3\mu\beta b_1 - 8\delta\beta a_{-2}$$
$$-8\delta\beta b_2 + 8\lambda\beta a_{-2} + 8\lambda\beta b_2 - \lambda^2 a_{-2} - \lambda^2 b_2 - 4\mu^2 a_{-2} - 4\mu^2 b_2 + c\lambda\beta a_{-1} + c\lambda\beta b_1$$
$$+2c\lambda\mu a_{-2} + 2c\lambda\mu b_2 - c\lambda^2 d^2 a_{-1} - c\lambda^2 d^2 b_1 + 4c\lambda^2 d a_{-2} + 4c\lambda^2 d b_2 + 2\lambda^2 a_0 a_{-2}$$
$$+2\lambda^2 a_0 b_2 + 2\lambda^2 a_{-1} b_1 + \lambda^2 a_{-1}^2 + \lambda^2 b_1^2 + c\lambda\delta d^2 a_{-1} + c\lambda\delta d^2 b_1 - c\lambda\mu d a_{-1} - c\lambda\mu d b_1$$
$$-4c\lambda\delta d a_{-2} - 4c\lambda\delta d b_2 = 0$$

$$\left(d+\frac{G'}{G}\right)^{-3}: -40\delta d^3\lambda a_{-2}+4\delta d^3\mu b_1+4\delta d^4\lambda a_{-1}-40\delta d^3\lambda b_2+4\delta d^3\mu a_{-1}-30\delta d^2\mu a_{-2}$$
$$-30\delta d^2\mu b_2-4\delta d^2\beta a_{-1}-4\delta d^2\beta b_1+4\delta d^4\lambda b_1+30\mu d^2\lambda a_{-2}-4\mu d^3\lambda a_{-1}+30\mu d^2\lambda b_2$$
$$+4\mu d\beta a_{-1}+4\mu d\beta b_1-4\mu d^3\lambda b_1+4\lambda d^2\beta a_{-1}+4\lambda d^2\beta b_1+20\beta\delta db_2-20\beta\lambda da_{-2}$$
$$+20\beta\delta da_{-2}-20\beta\lambda db_2+20\delta^2d^3b_2+20\delta^2d^3a_{-2}-2\delta^2d^4a_{-1}-2\delta^2d^4b_1-2\mu^2d^2b_1-2\mu^2d^2a_{-1}$$
$$+10\mu^2da_{-2}+10\mu^2db_2+20\lambda^2d^3a_{-2}-2\lambda^2d^4a_{-1}+20\lambda^2d^3b_2-2\lambda^2d^4b_1-10\beta\mu a_{-2}$$
$$-10\beta\mu b_2-2\beta^2a_{-1}-2\beta^2b_1-2c\lambda^2d^2a_{-2}-2c\lambda^2d^2b_2+2c\lambda\beta a_{-2}+2c\lambda\beta b_2+2\lambda^2a_{-1}a_{-2}$$
$$+2\lambda^2a_{-1}b_2+2\lambda^2b_1a_{-2}+2\lambda^2b_1b_2+2c\lambda\delta d^2a_{-2}+2c\lambda\delta d^2b_2-2c\lambda\mu da_{-2}-2c\lambda\mu db_2=0$$

$$\left(d+\frac{G'}{G}\right)^{-4}: \lambda^2a_{-2}^2+\lambda^2b_2^2-12\mu d^3\lambda a_{-2}-12\mu d^3\lambda b_2+12\mu d\beta a_{-2}+12\mu d\beta b_2+12\lambda d^2\beta a_{-2}$$
$$+12\lambda d^2\beta b_2+12\delta d^4\lambda b_2+12\delta d^3\mu a_{-2}+12\delta d^3\mu b_2+12\delta d^4\lambda a_{-2}-6\lambda^2d^4a_{-2}$$
$$-12\delta d^2\beta a_{-2}-12\delta d^2\beta b_2-6\mu^2d^2b_2-6\lambda^2d^4b_2-6\beta^2b_2+2\lambda^2a_{-2}b_2-6\beta^2a_{-2}$$
$$-6\delta^2d^4a_{-2}-6\mu^2d^2a_{-2}-6\delta^2d^4b_2=0$$

4.4 Results of the Fisher Equation: Solving the above systems of algebraic equations with the aid of Maple, we have obtained the following sets results:

(i) Results of Non Travelling Waves:

Set 1

$$\lambda=\lambda,\ \mu=-2d\Psi,\ \delta=\delta,\ \beta=-\frac{1}{4\Psi}\left(\lambda^2+4d^2\Psi^2\right),\ W=0,\ d=d,\ a_{-2}=-\frac{1}{8\Psi^2}\left(-3\lambda^2+4b_2\Psi^2\right),$$
$$a_{-1}=-b_1,\ a_0=\frac{3}{2},\ a_1=0,\ a_2=0,\ b_1=b_1,\ b_2=b_2; \qquad (4.4.1)$$

where, $\Psi=\lambda-\delta$

Set 2

$$\lambda=\lambda,\ \mu=-2d\Psi,\ \delta=\delta,\ \beta=-\frac{1}{4\Psi}\left(-\lambda^2+4d^2\Psi^2\right),\ W=0,\ d=d,\ a_{-2}=-\frac{1}{8\Psi^2}\left(-3\lambda^2+4b_2\Psi^2\right),$$

$$a_{-1}=-b_1,\ a_0=-\frac{1}{2},\ a_1=0,\ a_2=0,\ b_1=b_1,\ b_2=b_2; \qquad (4.4.2)$$

where, $\Psi=\lambda-\delta$

Set 3

$$\lambda=\lambda,\ \mu=\mu,\ \delta=-\frac{1}{4\beta}\left(\lambda^2-\mu^2-4\lambda\beta\right),\ \beta=\beta,\ W=0,\ d=d,\ a_{-2}=\frac{1}{8\lambda^2\beta^2}\left(-24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2\right.$$
$$\left.-96\mu d\beta^3-8\lambda^2b_2\beta^2+48\beta^4+3\lambda^4d^4-6\lambda^2\mu^2d^4+3d^4\mu^4+24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right),a_{-1}=-\frac{1}{4\lambda^2\beta^2}$$
$$\left(36\mu^2\beta^2d-12\lambda^2\beta^2d+18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d-6\mu^2d^3\lambda^2+3\mu^4d^3+4\lambda^2b_1\beta^2\right),$$

$$a_0=\frac{1}{8\lambda^2\beta^2}\left(3\lambda^4d^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2\right),\ a_2=0,\ b_1=b_1,$$
$$b_2=b_2; \tag{4.4.3}$$

Set 4

$$\lambda=\lambda,\ \mu=\mu,\ \delta=\frac{1}{4\beta}\left(\lambda^2+\mu^2+4\lambda\beta\right),\ \beta=\beta,\ W=0,\ d=d,\ a_{-2}=\frac{1}{8\lambda^2\beta^2}\left(24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2\right.$$
$$\left.-96\mu d\beta^3-8\lambda^2b_2\beta^2+48\beta^4+3\lambda^4d^4+6\lambda^2\mu^2d^4+3d^4\mu^4-24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right),\ a_{-1}=-\frac{1}{4\lambda^2\beta^2}$$
$$\left(36\mu^2\beta^2d+12\lambda^2\beta^2d-18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3+6\mu^2d^3\lambda^2+3\mu^4d^3+4\lambda^2b_1\beta^2\right),$$
$$a_0=\frac{3}{8\lambda^2\beta^2}\left(\lambda^4d^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right),\ a_1=0,\ a_2=0,\ b_1=b_1,$$
$$b_2=b_2; \tag{4.4.4}$$

Set 5

$$\lambda=\mu,\ \mu=\mu,\ \delta=\delta,\ \beta=0,\ W=0,\ d=d,\ a_{-2}=\frac{1}{\mu^2}\left(6d^4\Psi^2-\mu^2b_2+6\mu^2d^2+12\mu d^3\Psi\right),\ a_{-1}=\frac{1}{\mu^2}$$
$$\left(-12d^3\Psi^2-\mu^2b_1-\mu^26d-18\mu d^2\Psi\right),\ a_0=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^26d-6\mu\delta d\right),\ a_1=0,\ a_2=0,\ b_1=b_1,$$
$$b_2=b_2; \tag{4.4.5}$$

where, $\Psi=\lambda-\delta$

Set 6

$$\lambda=-\mu,\ \mu=\mu,\ \delta=\delta,\ \beta=0,\ W=0,\ d=d,\ a_{-2}=\frac{1}{\mu^2}\left(6d^4\Psi^2-\mu^2b_2+6\mu^2d^2+12\mu^2d^3-12\delta\mu d^3\right),$$
$$a_{-1}=-\frac{1}{\mu^2}\left(12d^3\Psi^2+\mu^2b_1+6\mu^2d-18\mu d^2\Psi\right),\ a_0=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2d-6\mu\delta d\right),\ a_1=0,\ a_2=0,$$

$$b_1 = b_1,\, b_2 = b_2; \tag{4.4.6}$$

where, $\Psi = \lambda - \delta$

Set 7

$$\lambda = \lambda,\, \mu = \mu,\, \delta = \frac{1}{4\beta}\left(\lambda^2 + \mu^2 + 4\lambda\beta\right),\, \beta = \beta,\, W = 0,\, d = d,\, a_{-2} = -b_2,\, a_{-1} = -b_1,\, a_0 = \frac{3}{8\lambda^2\beta^2}\left(d^2\lambda^2 + 2\lambda^2\mu^2 d^2 - 4\lambda^2\mu d\beta + 4\lambda^2\beta^2 + \mu^4 d^2 - 4\mu^3 d\beta + 4\beta^2\mu^2\right),\, a_1 = -\frac{3}{4\lambda^2\beta^2}\left(-2\mu\beta\lambda^2 - 2\beta\mu^3 + d\lambda^4 + 2d\lambda^2\mu^2 + \mu^4 d\right),\, a_2 = \frac{3}{8\lambda^2\beta^2}\left(\lambda^2 + \mu^2\right)^2,\, b_1 = b_1,\, b_2 = b_2; \tag{4.4.7}$$

Set 8

$$\lambda = \lambda,\, \mu = \mu,\, \delta = -\frac{1}{4\beta}\left(\lambda^2 - \mu^2 - 4\lambda\beta\right),\, \beta = \beta,\, W = 0,\, d = d,\, a_{-2} = -b_2,\, a_{-1} = -b_1,\, a_0 = \frac{1}{8\lambda^2\beta^2}\left(3d^2\lambda^2 - 6\lambda^2\mu^2 d^2 + 12\lambda^2\mu d\beta - 4\lambda^2\beta^2 + 3\mu^4 d^2 - 12\mu^3 d\beta + 12\beta^2\mu^2\right),\, a_1 = -\frac{3}{4\lambda^2\beta^2}\left(2\mu\beta\lambda^2 - 2\beta\mu^3 + d\lambda^4 - -2d\lambda^2\mu^2 + \mu^4 d\right),\, a_2 = \frac{3}{8\lambda^2\beta^2}\left(\lambda^2 - \mu^2\right)^2,\, b_1 = b_1,\, b_2 = b_2; \tag{4.4.8}$$

Set 9

$$\lambda = \mu,\, \mu = \mu,\, \delta = \delta,\, \beta = 0,\, W = 0,\, d = d,\, a_{-2} = -b_2,\, a_{-1} = -b_1,\, a_0 = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 + 6\mu^2 d - 6\mu\delta d\right),\, a_1 = \frac{1}{\mu^2}\left(-12d\Psi^2 - 6\mu\Psi\right),\, a_2 = \frac{6\Psi^2}{\mu^2},\, b_1 = b_1,\, b_2 = b_2; \tag{4.4.9}$$

where, $\Psi = \lambda - \delta$

Set 10

$$\lambda = -\mu,\, \mu = \mu,\, \delta = \delta,\, \beta = 0,\, W = 0,\, d = d,\, a_{-2} = -b_2,\, a_{-1} = -b_1,\, a_0 = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d\right),\, a_1 = \frac{1}{\mu^2}\left(-12d\Psi^2 + 6\mu^2 - 6\mu\delta\right),\, a_2 = \frac{6\Psi^2}{\mu^2},\, b_1 = b_1,\, b_2 = b_2; \tag{4.4.10}$$

where, $\Psi = \lambda - \delta$

Set 11

$$\lambda=\lambda,\ \mu=-2d\Psi,\ \delta=\delta,\ \beta=-\frac{1}{16\Psi}\left(\lambda^2+16d^2\Psi^2\right),\ W=0,\ d=d,\ a_{-2}=-\frac{1}{128\Psi^2}\left(3\lambda^2+128b_2\Psi^2\right),$$
$$a_{-1}=-b_1,\ a_0=\frac{3}{4},\ a_1=0,\ a_2=\frac{6\Psi^2}{\lambda^2},\ b_1=b_1,\ b_2=b_2; \qquad (4.4.11)$$

where, $\Psi=\lambda-\delta$

Set 12

$$\lambda=\lambda,\ \mu=-2d\Psi,\ \delta=\delta,\ \beta=-\frac{1}{16\Psi}\left(-\lambda^2+16d^2\Psi^2\right),\ W=0,\ d=d,\ a_{-2}=-\frac{1}{128\Psi^2}\left(-3\lambda^2+128b_2\Psi^2\right),$$
$$a_{-1}=-b_1,\ a_0=\frac{1}{4},\ a_1=0,\ a_2=\frac{6\Psi^2}{\lambda^2},\ b_1=b_1,\ b_2=b_2; \qquad (4.4.12)$$

where, $\Psi=\lambda-\delta$

(ii) Results of Travelling Waves:

Set1

$$\lambda=\lambda,\ \mu=-2d\Psi,\ \delta=\delta,\ \beta=-\frac{1}{96\Psi}\left(96d^2\Psi^2-\lambda^2\right),\ W=\pm\frac{5}{\sqrt{6}},\ d=d,\ a_{-2}=-\frac{1}{1536\Psi^2}\left(1536b_2\Psi^2-\lambda^2\right),$$
$$a_{-1}=-\frac{1}{16\Psi}\left(16b_1\Psi\pm\frac{\lambda}{\sqrt{6}}\right),\ a_0=\frac{3}{8},\ a_1=\mp\frac{6\Psi}{\lambda\sqrt{6}},\ a_2=\frac{6\Psi^2}{\lambda^2},\ b_1=b_1,\ b_2=b_2; \qquad (4.4.13)$$

where, $\Psi=\lambda-\delta$

Set 2

$$\lambda=\lambda,\ \mu=-2d\Psi,\ \delta=\delta,\ \beta=-\frac{1}{96\Psi}\left(96d^2\Psi^2+\lambda^2\right),\ W=\pm\frac{5i}{\sqrt{6}},\ d=d,\ a_{-2}=-\frac{1}{1536\Psi^2}\left(1536b_2\Psi^2-\lambda^2\right),$$
$$a_{-1}=\frac{1}{16\Psi}\left(-16b_1\Psi\pm\frac{\lambda}{\sqrt{6}}\right),\ a_0=\frac{5}{8},\ a_1=\mp\frac{6\Psi}{\lambda\sqrt{6}},\ a_2=\frac{6\Psi^2}{\lambda^2},\ b_1=b_1,\ b_2=b_2; \qquad (4.4.14)$$

where, $\Psi=\lambda-\delta$

Set 3

$$\lambda=\lambda,\ \mu=\pm\frac{\lambda}{\sqrt{6}},\ \delta=\frac{\lambda}{d}\left(d\pm\frac{1}{\sqrt{6}}\right),\ \beta=0,\ W=\pm\frac{5}{\sqrt{6}},\ d=d,\ a_{-2}=-b_2,\ a_{-1}=-b_1,\ a_0=0,\ a_1=0,$$
$$a_2=\frac{1}{d^2},\ b_1=b_1,\ b_2=b_2; \quad (4.4.15)$$

Set 4

$$\lambda=\lambda,\ \mu=\pm\frac{\lambda i}{\sqrt{6}},\ \delta=\frac{\lambda}{d}\left(d\pm\frac{i}{\sqrt{6}}\right),\ \beta=0,\ W=\pm\frac{5i}{\sqrt{6}},\ d=d,\ a_{-2}=-b_2,\ a_{-1}=-b_1,\ a_0=1,\ a_1=0,$$
$$a_2=-\frac{1}{d^2},\ b_1=b_1,\ b_2=b_2; \quad (4.4.16)$$

Set5

$$\lambda=\lambda,\ \mu=\mp\frac{\lambda}{\sqrt{6}},\ \delta=\frac{\lambda}{d}\left(d\mp\frac{1}{\sqrt{6}}\right),\ \beta=0,\ W=\pm\frac{5}{\sqrt{6}},\ d=d,\ a_{-2}=-b_2,\ a_{-1}=-b_1,\ a_0=1,\ a_1=-\frac{2}{d},$$
$$a_2=\frac{1}{d^2},\ b_1=b_1,\ b_2=b_2; \quad (4.4.17)$$

Set 6

$$\lambda=\lambda,\ \mu=\mp\frac{\lambda i}{\sqrt{6}},\ \delta=\frac{\lambda}{d}\left(d\mp\frac{i}{\sqrt{6}}\right),\ \beta=0,\ W=\pm\frac{5i}{\sqrt{6}},\ d=d,\ a_{-2}=-b_2,\ a_{-1}=-b_1,\ a_0=0,\ a_1=\frac{2}{d},$$
$$a_2=-\frac{1}{d^2},\ b_1=b_1,\ b_2=b_2; \quad (4.4.18)$$

Set 7

$$\lambda=\lambda,\ \mu=\mp\frac{\lambda}{\sqrt{6}},\ \delta=\delta,\ \beta=0,\ W=\pm\frac{5}{\sqrt{6}},\ d=d,\ a_{-2}=-b_2,\ a_{-1}=-b_1,\ a_0=\frac{6d^2\Psi^2}{\lambda^2},\ a_1=-\frac{12d\Psi^2}{\lambda^2},$$
$$a_2=\frac{6\Psi^2}{\lambda^2},\ b_1=b_1,\ b_2=b_2; \quad (4.4.19)$$

where, $\Psi=\lambda-\delta$

Set 8

$$\lambda=\lambda,\ \mu=\mp\frac{\lambda i}{\sqrt{6}},\ \delta=\delta,\ \beta=0,\ W=\pm\frac{5i}{\sqrt{6}},\ d=d,\ a_{-2}=-b_2,\ a_{-1}=-b_1,\ a_0=\frac{6d^2\Psi^2+\lambda^2}{\lambda^2},$$

$$a_1 = -\frac{12d\Psi^2}{\lambda^2},\ a_2 = \frac{6\Psi^2}{\lambda^2},\ b_1 = b_1,\ b_2 = b_2; \tag{4.4.20}$$

where, $\Psi = \lambda - \delta$

Set 9

$$\lambda = \lambda,\ \mu = \pm\frac{\lambda}{\sqrt{6}},\ \delta = \delta,\ \beta = 0,\ W = \pm\frac{5}{\sqrt{6}},\ d = d,\ a_{-2} = -b_2,\ a_{-1} = -b_1,$$
$$a_0 = \frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2}{\sqrt{6}} \mp \frac{12d\lambda\delta}{\sqrt{6}} + 6d^2\Psi^2\right),\ a_1 = \frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta}{\sqrt{6}} \mp \frac{12\lambda^2}{\sqrt{6}} - d\Psi^2\right),\ a_2 = \frac{6\Psi^2}{\lambda^2},\ b_1 = b_1,\ b_2 = b_2; \tag{4.4.21}$$

where, $\Psi = \lambda - \delta$

Set 10

$$\lambda = \lambda,\ \mu = \pm\frac{\lambda i}{\sqrt{6}},\ \delta = \delta,\ \beta = 0,\ W = \pm\frac{5i}{\sqrt{6}},\ d = d,\ a_{-2} = -b_2,\ a_{-1} = -b_1,$$
$$a_0 = \frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2 i}{\sqrt{6}} \mp \frac{2\lambda\delta i}{\sqrt{6}} + d\Psi^2\right),\ a_1 = \frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta i}{\sqrt{6}} \mp \frac{12\lambda^2 i}{\sqrt{6}} - d\Psi^2\right),\ a_2 = \frac{6\Psi^2}{\lambda^2},\ b_1 = b_1,\ b_2 = b_2; \tag{4.4.22}$$

where, $\Psi = \lambda - \delta$

Set 11

$$\lambda = \pm\mu\sqrt{6},\ \mu = \mu,\ \delta = \pm\mu\sqrt{6},\ \beta = \beta,\ W = \pm\frac{5\sqrt{6}}{6},\ d = d,\ a_{-2} = \frac{1}{\mu^2}\left(\mu^2 d^2 + \beta^2 - 2\mu\beta d - \mu^2 b_2\right),\ a_{-1} = -b_1,$$
$$a_0 = 0,\ a_1 = 0,\ a_2 = 0,\ b_1 = b_1,\ b_2 = b_2; \tag{4.4.23}$$

Set 12

$$\lambda = \pm\mu i\sqrt{6},\ \mu = \mu,\ \delta = \pm\mu i\sqrt{6},\ \beta = \beta,\ W = \mp\frac{5i\sqrt{6}}{6},\ d = d,\ a_{-2} = \frac{1}{\mu^2}\left(2\mu\beta d - \mu^2 d^2 - \beta^2 - \mu^2 b_2\right),\ a_{-1} = -b_1,$$
$$a_0 = 1,\ a_1 = 0,\ a_2 = 0,\ b_1 = b_1,\ b_2 = b_2; \tag{4.4.24}$$

Set 13

$$\lambda = \pm\mu\sqrt{6},\ \mu=\mu,\ \delta=\delta,\ \beta=0,\ W=\frac{\mp 5\mu d\sqrt{6}-\mu+\delta d}{\left(\pm\mu\sqrt{6}\mp d\delta\sqrt{6}+6\mu d\right)},\ d=d,\ a_{-2}=\frac{1}{\mu^2}\Big(\pm 2\mu^2 d^3\sqrt{6}\mp 2\delta\mu d^4\sqrt{6}$$

$$+\mu^2 d^2+\delta^2 d^4-2\delta d^3\mu+6\mu^2 d^4-\mu^2 b_2\Big),\ a_{-1}=\frac{1}{\mu^2}\Big(\pm 4\delta d^3\mu\sqrt{6}\mp 4\mu^2 d^2\sqrt{6}-2d\mu^2+4\delta\mu d^2-2\delta^2 d^3$$

$$-12\mu^2 d^3-\mu^2 b_1\Big),\ a_0=\left\{\frac{18d\mu^3-36\mu^2 d^2\delta+18\mu d^3\delta^2+36\mu^3 d^3\pm 18d^2\mu^3\sqrt{6}\pm\mu^3\sqrt{6}\mp 18\delta d^3\mu^2\sqrt{6}}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d}\right.$$

$$\left.\frac{\mp 3d\delta\mu^2\sqrt{6}\pm 3\mu\delta^2 d^2\sqrt{6}\mp\delta^3 d^3\sqrt{6}}{}\right\},\ a_1=0,\ a_2=0,\ b_1=b_1,\ b_2=b_2; \tag{4.4.25}$$

Set 14

$$\lambda=\pm\mu i\sqrt{6},\mu=\mu,\delta=\delta,\beta=0,W=\frac{\mp 5\mu d i\sqrt{6}+\mu-\delta d}{\left(\pm\mu i\sqrt{6}\mp d\delta i\sqrt{6}-6\mu d\right)},\ d=d,\ a_{-2}=\frac{1}{\mu^2}\Big(\pm 2\mu d^4\delta i\sqrt{6}\mp 2\mu^2 d^3 i\sqrt{6}$$

$$-\mu^2 d^2-\delta^2 d^4+2\delta d^3\mu+6\mu^2 d^4-\mu^2 b_2\Big),\ a_{-1}=\frac{1}{\mu^2}\Big(2d\mu^2-4\delta\mu d^2\mp 4\delta d^3\mu i\sqrt{6}\pm 4\mu^2 d^2 i\sqrt{6}+2\delta^2 d^3$$

$$-12\mu^2 d^3-\mu^2 b_1\Big),\ a_0=\left\{\frac{d\Big(12\mu^3-36\mu^2 d\delta\pm 18d\mu^3 i\sqrt{6}\mp 18\delta d^2\mu^2 i\sqrt{6}+18\mu d^2\delta^2\pm 2\delta\mu^2 i\sqrt{6}-36\mu^3 d^2}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d}\right.$$

$$\left.\frac{\mp 3\mu\delta^2 d i\sqrt{6}\pm\delta^3 d^2 i\sqrt{6}\Big)}{}\right\},\ a_1=0,\ a_2=0,\ b_1=b_1,\ b_2=b_2; \tag{4.4.26}$$

Set 15

$$\lambda=\lambda,\mu=\mu,\delta=-\frac{1}{24\beta}\left(\lambda^2-6\mu^2-24\lambda\beta\right),\beta=\beta,W=\pm\frac{5}{\sqrt{6}},\ d=d,\ a_{-2}=-b_2,\ a_{-1}=-b_1,$$

$$a_0=\frac{1}{96\lambda^2\beta^2}\left(\lambda^4 d^2\pm\frac{24\beta d\lambda^3}{\sqrt{6}}-12\lambda^2\mu^2 d^2+24\lambda^2\beta^2+24\lambda^2\mu\beta d\mp\frac{144\mu^2\beta d}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2}{\sqrt{6}}+144\beta^2\mu^2\right.$$

$$\left.+36\mu^4 d^2-144\mu^3 d\beta\right),\ a_1=-\frac{1}{48\lambda^2\beta^2}\left(\pm\frac{12\lambda^3\beta}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2}{\sqrt{6}}+12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4-12d\lambda^2\mu^2+36\mu^4 d\right),$$

$$a_2=\frac{1}{96\lambda^2\beta^2}\left(\lambda^2-6\mu^2\right)^2,\ b_1=b_1,\ b_2=b_2; \tag{4.4.27}$$

Set 16

$$\lambda=\lambda,\mu=\mu,\delta=\frac{1}{24\beta}\left(\lambda^2+6\mu^2+24\lambda\beta\right),\beta=\beta,W=\pm\frac{5i}{\sqrt{6}},\ d=d,\ a_{-2}=-b_2,\ a_{-1}=-b_1,$$

$$a_0=\frac{1}{96\lambda^2\beta^2}\left(\lambda^4 d^2\mp\frac{24\beta d\lambda^3 i}{\sqrt{6}}+12\lambda^2\mu^2 d^2+72\lambda^2\beta^2-24\lambda^2\mu\beta d\mp\frac{144\lambda\mu^2\beta d i}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2 i}{\sqrt{6}}+144\beta^2\mu^2\right.$$

$$+36\mu^4 d^2 - 144\mu^3 d\beta),\ a_1 = -\frac{1}{48\lambda^2\beta^2}\left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}} \mp \frac{72\lambda\beta\mu^2 i}{\sqrt{6}} - 12\mu\beta\lambda^2 - 72\beta\mu^3 + d\lambda^4 + 12d\lambda^2\mu^2 + 36\mu^4 d\right),$$

$$a_2 = \frac{1}{96\lambda^2\beta^2}\left(\lambda^2 + 6\mu^2\right)^2,\ b_1 = b_1,\ b_2 = b_2; \tag{4.4.28}$$

4.5 Solutions of Fisher Equation: By using the above results of the Fisher Equation we have found out the following non-travelling and travelling wave solutions of the Fisher Equation

4.5.1 Solutions of non-travelling wave:

Hyperbolic form of the non-travelling wave solutions:

Substituting Eq (4.3.1) into Eq (4.1.3), along with Eq (3.1.7) and simplifying, yields the following non travelling wave solutions,

$$u11(x,t) = \frac{3}{2} + \left(-b_1 + b_1\right)\left(d + H\right)^{-1} + \left\{-\frac{1}{8}\left(\frac{-3\lambda^2 + 4b_2\Psi^2}{\Psi^2}\right) + b_2\right\}\left(d + H\right)^{-2}$$

$$\therefore u11(x,t) = \frac{3}{2} + \left(\frac{3\lambda^2 + 4b_2\Psi^2}{8\Psi^2}\right)\left(d + H\right)^{-2} \tag{i}$$

where, $H = (\frac{G'}{G}) = \frac{\mu}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (i) becomes:

$$u11_1(x,t) = \frac{3}{2} + \left(\frac{3\lambda^2 + 4b_2\Psi^2}{8\Psi^2}\right)\left(d + \frac{\mu}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi}\frac{\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}\right)^{-2}$$

$$=\frac{3}{2}+\left(\frac{3\lambda^2+4b_2\Psi^2}{8\Psi^2}\right)\left(d+\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)^{-2}$$

$$=\frac{3}{2}+\left(\frac{3\lambda^2+4b_2\Psi^2}{8\Psi^2}\right)\left(\frac{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{2\Psi}\right)^{-2}$$

$$=\frac{3}{2}+\left(\frac{3\lambda^2+4b_2\Psi^2}{8\Psi^2}\right)\left(\frac{2d\Psi-2d\Psi+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{2\Psi}\right)^{-2}, \quad \text{since } \mu=-2d\Psi$$

$$=\frac{3}{2}+\left(\frac{3\lambda^2+4b_2\Psi^2}{8\Psi^2}\right)\left(\frac{\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{2\Psi}\right)^{-2}$$

$$=\frac{3}{2}+\left(\frac{3\lambda^2+4b_2\Psi^2}{8\Psi^2}\right)\left(\frac{4\Psi^2}{\Omega\coth^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}\right)$$

$$\therefore u1 1_1(x,t)=\frac{3}{2}+\frac{3\lambda^2+4b_2\Psi^2}{2\Omega\coth^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

If $C_1\neq 0$ but $C_2=0$ then equation (i) becomes,

$$u1 1_2(x,t)=\frac{3}{2}+\left(\frac{3\lambda^2+4b_2\Psi^2}{8\Psi^2}\right)\left(d+\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{\sinh\left(\frac{\sqrt{\Omega}}{2\psi}\xi\right)}{\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}\right)^{-2}$$

$$\therefore u1 1_2(x,t)=\frac{3}{2}+\frac{3\lambda^2+4b_2\Psi^2}{2\Omega\tanh^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

Substituting Eq (4.3.2) into Eq (4.1.3), along with Eq (3.1.7) and simplifying, yields the following non travelling wave solutions,

$$u21(x,t) = -\frac{1}{2} + \left(-\frac{1}{8}\frac{-3\lambda^2 + 4b_2\Psi^2}{\Psi^2} + b_2 \right)(d+H)^{-2}$$

$$\therefore u21(x,t) = -\frac{1}{2} + \left(\frac{3\lambda^2 + 4b_2\Psi^2}{8\Psi^2} \right)(d+H)^{-2} \qquad \textbf{(ii)}$$

where, $H = (\frac{G'}{G}) = \frac{\mu}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi} \frac{C_1 \sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2 \cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1 \cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2 \sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (ii) becomes:

$$u21_1(x,t) = -\frac{1}{2} + \left(\frac{3\lambda^2 + 4b_2\Psi^2}{8\Psi^2} \right) \left(d + \frac{\mu}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi} \frac{\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)} \right)^{-2}$$

$$\therefore u21_1(x,t) = -\frac{1}{2} + \frac{3\lambda^2 + 4b_2\Psi^2}{2\Omega \coth^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

If $C_1 \neq 0$ but $C_2 = 0$ then equation (ii) becomes,

$$u21_2(x,t) = -\frac{1}{2} + \left(\frac{3\lambda^2 + 4b_2\Psi^2}{8\Psi^2} \right) \left(d + \frac{\mu}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi} \frac{\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)} \right)^{-2}$$

$$\therefore u21_2(x,t) = -\frac{1}{2} + \frac{3\lambda^2 + 4b_2\Psi^2}{2\Omega \tanh^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

Substituting Eq (4.3.3) into Eq (4.1.3), along with Eq (3.1.7) and simplifying, yields the following non travelling wave solutions,

$$u31(x,t)=\frac{1}{8\lambda^2\beta^2}\left(3\lambda^4d^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2\right)$$
$$+\left\{-\frac{1}{4\lambda^2\beta^2}\left(36\mu^2\beta^2d-12\lambda^2\beta^2d+18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3\right.\right.$$
$$\left.\left.-6\mu^2d^3\lambda^2+3\mu^4d^3+4\lambda^2b_1\beta^2\right)+b_1\right\}(d+H)^{-1}+\left\{\frac{1}{8\lambda^2\beta^2}\left(-24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2\right.\right.$$
$$\left.\left.-96\mu d\beta^3-8\lambda^2b_2\beta^2+48\beta^4+3\lambda^4d^4-6\lambda^2\mu^2d^4+3d^4\mu^4+24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)+b_2\right\}$$
$$(d+H)^{-2}$$

$$\therefore u31(x,t)=\frac{1}{8\lambda^2\beta^2}\left(3\lambda^4d^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2\right)$$
$$-\frac{1}{4\lambda^2\beta^2}\left(36\mu^2\beta^2d-12\lambda^2\beta^2d+18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3\right.$$
$$\left.-6\mu^2d^3\lambda^2+3\mu^4d^3\right)(d+H)^{-1}+\frac{1}{8\lambda^2\beta^2}\left(-24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2-96\mu d\beta^3\right.$$
$$\left.+48\beta^4+3\lambda^4d^4-6\lambda^2\mu^2d^4+3d^4\mu^4+24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)(d+H)^{-2} \qquad \textbf{(iii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (iii) becomes:

$$u31_1(x,t)=\frac{1}{2\lambda^2\beta^2}\left(\frac{3\lambda^4d^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2}{4}-\right.$$
$$\frac{\Psi\left(36\mu^2\beta^2d-12\lambda^2\beta^2d+18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3-6\mu^2d^3\lambda^2+3\mu^4d^3\right)}{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}-$$

$$\left.\frac{\Psi^2\left(24\lambda^2\beta^2d^2-72\mu^2d^2\beta^2+96\mu d\beta^3-48\beta^4-3\lambda^4d^4+6\lambda^2\mu^2d^4-3d^4\mu^4-24\lambda^2\mu\beta d^3+24\mu^3\beta d^3\right)}{\left\{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

If $C_1 \neq 0$ but $C_2 = 0$ then equation (iii) becomes,

$$u31_2(x,t)=\frac{1}{2\lambda^2\beta^2}\left(\frac{3\lambda^4d^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2}{4}-\right.$$
$$\frac{\Psi\left(36\mu^2\beta^2d-12\lambda^2\beta^2d+18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3-6\mu^2d^3\lambda^2+3\mu^4d^3\right)}{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}-$$

$$\left.\frac{\Psi^2\left(24\lambda^2\beta^2d^2-72\mu^2d^2\beta^2+96\mu d\beta^3-48\beta^4-3\lambda^4d^4+6\lambda^2\mu^2d^4-3d^4\mu^4-24\lambda^2\mu\beta d^3+24\mu^3\beta d^3\right)}{\left\{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

Substituting Eq (4.3.4) into Eq (4.1.3), along with Eq (3.1.7) and simplifying, yields the following non travelling wave solutions,

$$u41(x,t)=\frac{3}{8\lambda^2\beta^2}\left(\lambda^4d^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)$$
$$+\left\{-\frac{1}{4\lambda^2\beta^2}\left(36\mu^2\beta^2d+12\lambda^2\beta^2d-18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3\right.\right.$$
$$\left.\left.+6\mu^2d^3\lambda^2+3\mu^4d^3+4\lambda^2b_1\beta^2\right)+b_1\right\}(d+H)^{-1}+\left\{\frac{1}{8\lambda^2\beta^2}\left(24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2\right.\right.$$

$$\left.\left.-96\mu d\beta^3-8\lambda^2b_2\beta^2+48\beta^4+3\lambda^4d^4+6\lambda^2\mu^2d^4+3d^4\mu^4-24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)+b_2\right\}$$
$$(d+H)^{-2}$$
$$\therefore u41(x,t)=\frac{3}{8\lambda^2\beta^2}\left(\lambda^4d^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)$$
$$-\frac{1}{4\lambda^2\beta^2}\left(36\mu^2\beta^2d+12\lambda^2\beta^2d-18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3\right.$$
$$\left.+6\mu^2d^3\lambda^2+3\mu^4d^3\right)(d+H)^{-1}+\frac{1}{8\lambda^2\beta^2}\left(24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2-96\mu d\beta^3+48\beta^4\right.$$
$$\left.+3\lambda^4d^4+6\lambda^2\mu^2d^4+3d^4\mu^4-24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)(d+H)^{-2} \qquad \textbf{(iv)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (iv) becomes:

$$u41_1(x,t)=\frac{1}{2\lambda^2\beta^2}\left(\frac{3\left(\lambda^4d^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)}{4}-\right.$$
$$\frac{\Psi\left(36\mu^2\beta^2d+12\lambda^2\beta^2d-18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3+6\mu^2d^3\lambda^2+3\mu^4d^3\right)}{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}+$$
$$\left.\frac{\Psi^2\left(24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2-96\mu d\beta^3+48\beta^4+3\lambda^4d^4+6\lambda^2\mu^2d^4+3d^4\mu^4-24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)}{\left\{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

If $C_1\neq 0$ but $C_2=0$ then equation (iv) becomes,

$$u41_2(x,t)=\frac{1}{2\lambda^2\beta^2}\left(\frac{3\left(\lambda^4d^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)}{4}-\right.$$
$$\frac{\Psi\left(6\mu^2\beta^2d+12\lambda^2\beta^2d-18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3+6\mu^2d^3\lambda^2+3\mu^4d^3\right)}{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}+$$
$$\left.\frac{\Psi^2\left(24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2-96\mu d\beta^3+48\beta^4+3\lambda^4d^4+6\lambda^2\mu^2d^4+3d^4\mu^4-24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)}{\left\{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

Substituting Eq (4.3.5) into Eq (4.1.3), along with Eq (3.1.7) and simplifying, yields the following non travelling wave solutions,

$$u51(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d\right)+\left\{\frac{1}{\mu^2}\left(-12d^3\Psi^2-\mu^2 b_1-\mu^2 6d-18\mu d^2\Psi\right)+b_1\right\}$$

$$\left(d+H\right)^{-1}+\left\{\frac{1}{\mu^2}\left(6d^4\Psi^2-\mu^2 b_2+\mu^2 6d^2+12\mu d^3\Psi\right)+b_2\right\}\left(d+H\right)^{-2}$$

$$\therefore u51(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d^3\Psi^2-\mu^2 6d-18\mu d^2\Psi\right)\left(d+H\right)^{-1}$$

$$+\frac{1}{\mu^2}\left(6d^4\Psi^2+\mu^2 6d^2+12\mu d^3\Psi\right)\left(d+H\right)^{-2} \quad \textbf{(v)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (v) becomes:

$$u51_1(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d-\frac{2\Psi\left(12d^3\Psi^2+\mu^2 6d+18\mu d^2\Psi\right)}{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}+\frac{4\Psi^2\left(6d^4\Psi^2+\mu^2 6d^2+12\mu d^3\Psi\right)}{\left\{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}\right.$$

If $C_1\neq 0$ but $C_2=0$ then equation (v) becomes:

$$u51_2(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d-\frac{2\Psi\left(12d^3\Psi^2+\mu^2 6d+18\mu d^2\Psi\right)}{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}+\frac{4\Psi^2\left(6d^4\Psi^2+\mu^2 6d^2+12\mu d^3\Psi\right)}{\left\{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

Substituting Eq (4.3.6) into Eq (4.1.3), along with Eq (3.1.7) and simplifying, yields the following non travelling wave solutions,

$$u61(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d\right)+\left\{-\frac{1}{\mu^2}\left(12d^3\Psi^2+\mu^2 b_1+6\mu^2 d-18\mu d^2\Psi\right)+b_1\right\}$$

$$(d+H)^{-1}+\left\{\frac{1}{\mu^2}\left(6d^4\Psi^2-\mu^2 b_2+6\mu^2 d^2+12\mu^2 d^3-12\delta\mu d^3\right)+b_2\right\}(d+H)^{-2}$$

$$\therefore u61(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d\right)-\frac{1}{\mu^2}\left(12d^3\Psi^2+6\mu^2 d-18\mu d^2\Psi\right)(d+H)^{-1}$$

$$+\frac{1}{\mu^2}\left(6d^4\Psi^2+6\mu^2 d^2+12\mu^2 d^3-12\delta\mu d^3\right)(d+H)^{-2} \qquad \textbf{(vi)}$$

$$\text{where, } H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

If $C_1=0$ but $C_2\neq 0$, then the above equation(vi) becomes:

$$u61_1(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d-\frac{2\Psi\left(12d^3\Psi^2+6\mu^2 d-18\mu d^2\Psi\right)}{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}+\right.$$

$$\left.\frac{4\Psi^2\left(6d^4\Psi^2+6\mu^2 d^2+12\mu^2 d^3-12\delta\mu d^3\right)}{\left\{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

If $C_1\neq 0$ but $C_2=0$ then equation(vi) becomes ,

$$u61_2(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d-\frac{2\Psi\left(12d^3\Psi^2+6\mu^2 d-18\mu d^2\Psi\right)}{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}+\right.$$

$$\left.\frac{4\Psi^2\left(6d^4\Psi^2+6\mu^2 d^2+12\mu^2 d^3-12\delta\mu d^3\right)}{\left\{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

Substituting Eq (4.3.7) into Eq (4.1.3), along with Eq (3.1.7) and simplifying, yields the following non travelling wave solutions,

$$u71(x,t)=\frac{3}{8\lambda^2\beta^2}\left(d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)-\frac{3}{4\lambda^2\beta^2}$$
$$\left(-2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4+2d\lambda^2\mu^2+\mu^4d\right)\left(d+H\right)+\frac{3}{8\lambda^2\beta^2}\left(\lambda^2+\mu^2\right)^2\left(d+H\right)^2$$
$$+\left(-b_1+b_1\right)\left(d+H\right)^{-1}+\left(-b_2+b_2\right)\left(d+H\right)^{-2}$$
$$\therefore u71(x,t)=\frac{3}{8\lambda^2\beta^2}\left(d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)-\frac{3}{4\lambda^2\beta^2}$$
$$\left(-2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4+2d\lambda^2\mu^2+\mu^4d\right)\left(d+H\right)+\frac{3}{8\lambda^2\beta^2}\left(\lambda^2+\mu^2\right)^2\left(d+H\right)^2 \quad \textbf{(vii)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (vii) becomes:

$$u71_1(x,t)=\frac{3}{8\lambda^2\beta^2}\left(d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right.$$
$$+\frac{\left(2\mu\beta\lambda^2+2\beta\mu^3-d\lambda^4-2d\lambda^2\mu^2-\mu^4d\right)\left\{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}{\Psi}$$
$$\left.+\frac{\left(\lambda^2+\mu^2\right)^2\left\{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{4\Psi^2}\right)$$

If $C_1\neq 0$ but $C_2=0$ then equation (vii) becomes,

$$u71_2(x,t)=\frac{3}{8\lambda^2\beta^2}\Bigg(d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2+$$

$$\frac{\left(2\mu\beta\lambda^2+2\beta\mu^3-d\lambda^4-2d\lambda^2\mu^2-\mu^4d\right)\left\{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}{\Psi}$$

$$+\frac{\left(\lambda^2+\mu^2\right)^2\left\{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{4\Psi^2}\Bigg)$$

Substituting Eq (4.3.8) into Eq (4.1.3), along with Eq (3.1.7) and simplifying, yields the following non travelling wave solutions,

$$u81(x,t)=\frac{1}{8\lambda^2\beta^2}\left(3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2\right)$$

$$-\frac{3}{4\lambda^2\beta^2}\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4d\right)\left(d+H\right)+\frac{3}{8\lambda^2\beta^2}\left(\lambda^2-\mu^2\right)^2\left(d+H\right)^2$$

$$+\left(-b_1+b_1\right)\left(d+H\right)^{-1}+\left(-b_2+b_2\right)\left(d+H\right)^{-2}$$

$$\therefore u81(x,t)=\frac{1}{8\lambda^2\beta^2}\left(3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2\right)$$

$$-\frac{3}{4\lambda^2\beta^2}\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4d\right)\left(d+H\right)+\frac{3}{8\lambda^2\beta^2}\left(\lambda^2-\mu^2\right)^2\left(d+H\right)^2 \quad \textbf{(viii)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (viii) becomes:

$$u81_1(x,t)=\frac{1}{8\lambda^2\beta^2}\Big(3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2-$$

$$\frac{3\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4 d\right)\left\{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}{\Psi}$$

$$\left.+\frac{3\left(\lambda^2-\mu^2\right)^2\left\{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{4\Psi^2}\right)$$

If $C_1 \neq 0$ but $C_2 = 0$ then equation (viii) becomes,

$$u81_2(x,t)=\frac{1}{8\lambda^2\beta^2}\left(3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2-\right.$$

$$\frac{3\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4 d\right)\left\{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}{\Psi}$$

$$\left.+\frac{3\left(\lambda^2-\mu^2\right)^2\left\{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{4\Psi^2}\right)$$

Substituting Eq (4.3.9) into Eq (4.1.3), along with Eq (3.1.7) and simplifying, yields the following non travelling wave solutions,

$$u91(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2+6\mu^2d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d\Psi^2-6\mu\Psi\right)\left(d+H\right)+\frac{6\Psi^2}{\mu^2}\left(d+H\right)^2$$
$$+\left(-b_1+b_1\right)\left(d+H\right)^{-1}+\left(-b_2+b_2\right)\left(d+H\right)^{-2}$$

$$\therefore u91(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2+6\mu^2d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d\Psi^2-6\mu\Psi\right)\left(d+H\right)+\frac{6\Psi^2}{\mu^2}\left(d+H\right)^2 \quad \textbf{(ix)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (ix) becomes:

$$u91_1(x,t) = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 + 6\mu^2 d - 6\mu\delta d - \frac{\left(6d\Psi^2 + 3\mu\Psi\right)\left\{2d\Psi + \mu + \sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}{\Psi} + \frac{3\left\{2d\Psi + \mu + \sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{2}\right)$$

If $C_1 \neq 0$ but $C_2 = 0$ then equation (ix) becomes ,

$$u91_2(x,t) = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 + 6\mu^2 d - 6\mu\delta d - \frac{\left(6d\Psi^2 + 3\mu\Psi\right)\left\{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}{\Psi} + \frac{3\left\{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{2}\right)$$

Substituting Eq (4.3.10) into Eq (4.1.3), along with Eq (3.1.7) and simplifying, yields the following non travelling wave solutions,

$$u101(x,t) = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d\right) + \frac{1}{\mu^2}\left(-12d\Psi^2 + 6\mu^2 - 6\mu\delta\right)\left(d + H\right) + \frac{6\Psi^2}{\mu^2}\left(d + H\right)^2 + \left(-b_1 + b_1\right)\left(d + H\right)^{-1} + \left(-b_2 + b_2\right)\left(d + H\right)^{-2}$$

$$\therefore u101(x,t) = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d\right) + \frac{1}{\mu^2}\left(-12d\Psi^2 + 6\mu^2 - 6\mu\delta\right)\left(d + H\right) + \frac{6\Psi^2}{\mu^2}\left(d + H\right)^2 \qquad \textbf{(x)}$$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (x) becomes:

$$u101_1(x,t) = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d - \frac{\left(6d\Psi^2 - 3\mu^2 + 3\mu\delta\right)\left\{2d\Psi + \mu + \sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}{\Psi}\right.$$

$$+\frac{3\left\{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^{2}}{2}\Bigg)$$

If $C_1 \neq 0$ but $C_2 = 0$ then equation (x) becomes,

$$u101_2(x,t)=\frac{1}{\mu^2}\Bigg(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d-\frac{\left(6d\Psi^2-3\mu^2+3\mu\delta\right)\left\{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}{\Psi}$$

$$+\frac{3\left\{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^{2}}{2}\Bigg)$$

Substituting Eq (4.3.11) into Eq (4.1.3), along with Eq (3.1.7) and simplifying, yields the following non travelling wave solutions,

$$u11\,1(x,t)=\frac{3}{4}+\left(\frac{6\Psi^2}{\lambda^2}\right)(d+H)^2+\left(-b_1+b_1\right)(d+H)^{-1}+\left(-\frac{1}{128}\frac{-3\lambda^2+128b_2\Psi^2}{\Psi^2}+b_2\right)(d+H)^{-2}$$

$$\therefore u11\,1(x,t)=\frac{3}{4}+\left(\frac{6\Psi^2}{\lambda^2}\right)(d+H)^2+\left(\frac{3\lambda^2}{128\Psi^2}\right)(d+H)^{-2} \quad \textbf{(xi)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (xi) becomes:

$$u11\,1_1(x,t)=\frac{3}{4}+\frac{3\Omega\coth^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{2\lambda^2}+\frac{3\lambda^2}{32\Omega\coth^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

If $C_1 \neq 0$ but $C_2 = 0$ then equation (xi) becomes,

$$u11\,1_2(x,t)=\frac{3}{4}+\frac{3\Omega\tanh^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{2\lambda^2}+\frac{3\lambda^2}{32\Omega\tanh^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

Substituting Eq (4.3.12) into Eq (4.1.3), along with Eq (3.1.7) and simplifying, yields the following non travelling wave solutions,

$$u12\,1(x,t)=\frac{1}{4}+\left(\frac{6\Psi^2}{\lambda^2}\right)(d+H)^2+(-b_1+b_1)(d+H)^{-1}+\left(-\frac{1}{128}\frac{-3\lambda^2+128b_2\Psi^2}{\Psi^2}+b_2\right)(d+H)^{-2}$$

$$\therefore u12\,1(x,t)=\frac{1}{4}+\left(\frac{6\Psi^2}{\lambda^2}\right)(d+H)^2+\left(\frac{3\lambda^2}{128\Psi^2}\right)(d+H)^{-2} \qquad \textbf{(xii)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (xii) becomes:

$$u12\,1_1(x,t)=\frac{1}{4}+\frac{3\Omega\coth^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{2\lambda^2}+\frac{3\lambda^2}{32\Omega\coth^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

If $C_1\neq 0$ but $C_2=0$ then equation (xii) becomes,

$$u12\,1_2(x,t)=\frac{1}{4}+\frac{3\Omega\tanh^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{2\lambda^2}+\frac{3\lambda^2}{32\Omega\tanh^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

Trigonometric form of non-travelling wave solutions:

Substituting Eq (4.3.1) into Eq (4.1.3), along with Eq (3.1.8) and simplifying, yields the following non travelling wave solutions,

$$\therefore u12(x,t)=\frac{3}{2}+\left(\frac{3\lambda^2+4b_2\Psi^2}{8\Psi^2}\right)\left(d+H\right)^{-2} \qquad \textbf{(i)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (i) becomes

$$u12_1(x,t)=\frac{3}{2}+\frac{3\lambda^2+4b_2\Psi^2}{2\Omega i^2\cot^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

If $C_1\neq 0$ but $C_2=0$ then equation (i) becomes,

$$u12_2(x,t)=\frac{3}{2}+\frac{3\lambda^2+4b_2\Psi^2}{2\Omega i^2\tan^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

Substituting Eq (4.3.2) into Eq (4.1.3), along with Eq (3.1.8) and simplifying, yields the following non travelling wave solutions,

$$\therefore u22(x,t)=-\frac{1}{2}+\left(\frac{3\lambda^2+4b_2\Psi^2}{8\Psi^2}\right)\left(d+H\right)^{-2} \qquad \textbf{(ii)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (ii) becomes:

$$u22_1(x,t)=-\frac{1}{2}+\frac{3\lambda^2+4b_2\Psi^2}{2\Omega i^2\cot^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

If $C_1\neq 0$ but $C_2=0$ then equation (ii) becomes,

$$u22_2(x,t)=-\frac{1}{2}+\frac{3\lambda^2+4b_2\Psi^2}{2\Omega i^2\tan^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

Substituting Eq (4.3.3) into Eq (4.1.3), along with Eq (3.1.8) and simplifying, yields the following non travelling wave solutions,

$$\begin{aligned}\therefore u32(x,t)=&\frac{1}{8\lambda^2\beta^2}\left(3\lambda^4d^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2\right)\\&-\frac{1}{4\lambda^2\beta^2}\left(36\mu^2\beta^2d-12\lambda^2\beta^2d+18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3\right.\\&\left.-6\mu^2d^3\lambda^2+3\mu^4d^3\right)\ (d+H)^{-1}+\frac{1}{8\lambda^2\beta^2}\left(-24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2-96\mu d\beta^3\right.\\&\left.+48\beta^4+3\lambda^4d^4-6\lambda^2\mu^2d^4+3d^4\mu^4+24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)(d+H)^{-2}\end{aligned}\qquad \textbf{(iii)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (iii) becomes:

$$u32_1(x,t) = \frac{1}{2\lambda^2\beta^2}\left(\frac{3\lambda^4 d^2 - 6\lambda^2\mu^2 d^2 + 12\lambda^2\mu d\beta - 4\lambda^2\beta^2 + 3\mu^4 d^2 - 12\mu^3 d\beta + 12\beta^2\mu^2}{4} - \frac{\Psi\left(36\mu^2\beta^2 d - 12\lambda^2\beta^2 d + 18\lambda^2 d^2\beta\mu - 18\mu^3\beta d^2 - 24\mu\beta^3 + 3\lambda^4 d\right)}{2d\Psi + \mu + \sqrt{\Omega} i \cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)} - \frac{\Psi^2\left(24\lambda^2\beta^2 d^2 - 72\mu^2 d^2\beta^2 + 96\mu d\beta^3 - 48\beta^4 - 3\lambda^4 d^4 + 6\lambda^2\mu^2 d^4 - 3d^4\mu^4 - 24\lambda^2\mu\beta d^3 + 24\mu^3\beta d^3\right)}{\left\{2d\Psi + \mu + \sqrt{\Omega} i \cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

If $C_1 \neq 0$ but $C_2 = 0$ then equation (iii) becomes,

$$u32_2(x,t) = \frac{1}{2\lambda^2\beta^2}\left(\frac{\left(3\lambda^4 d^2 - 6\lambda^2\mu^2 d^2 + 12\lambda^2\mu d\beta - 4\lambda^2\beta^2 + 3\mu^4 d^2 - 12\mu^3 d\beta + 12\beta^2\mu^2\right)}{4} + \frac{\Psi\left(36\mu^2\beta^2 d - 12\lambda^2\beta^2 d + 18\lambda^2 d^2\beta\mu - 18\mu^3\beta d^2 - 24\mu\beta^3 + 3\lambda^4 d^3 - 6\mu^2 d^3\lambda^2 + 3\mu^4 d^3\right)}{2d\Psi + \mu + \sqrt{\Omega} i \tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)} + \frac{\Psi^2\left(-24\lambda^2\beta^2 d^2 + 72\mu^2 d^2\beta^2 - 96\mu d\beta^3 + 48\beta^4 + 3\lambda^4 d^4 - 6\lambda^2\mu^2 d^4 + 3d^4\mu^4 + 24\lambda^2\mu\beta d^3 - 24\mu^3\beta d^3\right)}{\left\{2d\Psi + \mu + \sqrt{\Omega} i \tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

Substituting Eq (4.3.4) into Eq (4.1.3), along with Eq (3.1.8) and simplifying, yields the following non travelling wave solutions,

$$\begin{aligned}\therefore u42(x,t) &= \frac{3}{8\lambda^2\beta^2}\left(\lambda^4 d^2 + 2\lambda^2\mu^2 d^2 - 4\lambda^2\mu d\beta + 4\lambda^2\beta^2 + \mu^4 d^2 - 4\mu^3 d\beta + 4\beta^2\mu^2\right) \\ &- \frac{1}{4\lambda^2\beta^2}\left(36\mu^2\beta^2 d + 12\lambda^2\beta^2 d - 18\lambda^2 d^2\beta\mu - 18\mu^3\beta d^2 - 24\mu\beta^3 + 3\lambda^4 d^3\right. \\ &\left.+ 6\mu^2 d^3\lambda^2 + 3\mu^4 d^3\right)(d+H)^{-1} + \frac{1}{8\lambda^2\beta^2}\left(24\lambda^2\beta^2 d^2 + 72\mu^2 d^2\beta^2 - 96\mu d\beta^3 + 48\beta^4\right. \\ &\left.+ 3\lambda^4 d^4 + 6\lambda^2\mu^2 d^4 + 3d^4\mu^4 - 24\lambda^2\mu\beta d^3 - 24\mu^3\beta d^3\right)(d+H)^{-2} \qquad \textbf{(iv)}\end{aligned}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (iv) becomes:

$$u42_1(x,t)=\frac{1}{2\lambda^2\beta^2}\left(\frac{3\left(\lambda^4d^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)}{4}-\right.$$
$$\frac{\Psi\left(36\mu^2\beta^2d+12\lambda^2\beta^2d-18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3+6\mu^2d^3\lambda^2+3\mu^4d^3\right)}{2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}+$$
$$\left.\frac{\Psi^2\left(24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2-96\mu d\beta^3+48\beta^4+3\lambda^4d^4+6\lambda^2\mu^2d^4+3d^4\mu^4-24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)}{\left\{2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

If $C_1\neq 0$ but $C_2=0$ then equation (iv) becomes,

$$u42_2(x,t)=\frac{1}{2\lambda^2\beta^2}\left(\frac{3\left(\lambda^4d^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)}{4}+\right.$$
$$\frac{\Psi\left(36\mu^2\beta^2d+12\lambda^2\beta^2d-18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d+6\mu^2d^3\lambda^2+3\mu^4d^3\right)}{2d\Psi+\mu+\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}+$$
$$\left.\frac{\Psi^2\left(24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2-96\mu d\beta^3+48\beta^4+3\lambda^4d^4+6\lambda^2\mu^2d^4+3d^4\mu^4-24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)}{\left\{2d\Psi+\mu+\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

Substituting Eq (4.3.5) into Eq (4.1.3), along with Eq (3.1.8) and simplifying, yields the following non travelling wave solutions,

$$\therefore u52(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d^3\Psi^2-\mu^2 6d-18\mu d^2\Psi\right)(d+H)^{-1}$$
$$+\frac{1}{\mu^2}\left(6d^4\Psi^2+\mu^2 6d^2+12\mu d^3\Psi\right)(d+H)^{-2} \qquad \text{(v)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (v) becomes:

$$u52_1(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d-\frac{2\Psi\left(12d^3\Psi^2+\mu^2 6d+18\mu d^2\Psi\right)}{2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}\right.$$
$$\left.+\frac{4\Psi^2\left(6d^4\Psi^2+\mu^2 6d^2+12\mu d^3\Psi\right)}{\left\{2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

If $C_1\neq 0$ but $C_2=0$ then equation (v) becomes,

$$u52_2(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d+\frac{2\Psi\left(12d^3\Psi^2+\mu^2 6d+18\mu d^2\Psi\right)}{2d\Psi+\mu+\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}\right.$$
$$\left.+\frac{4\Psi^2\left(6d^4\Psi^2+\mu^2 6d^2+12\mu d^3\Psi\right)}{\left\{2d\Psi+\mu+\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

Substituting Eq (4.3.6) into Eq (4.1.3), along with Eq (3.1.8) and simplifying, yields the following non travelling wave solutions,

$$\therefore u62(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d\right)-\frac{1}{\mu^2}\left(12d^3\Psi^2+6\mu^2 d-18\mu d^2\Psi\right)(d+H)^{-1}$$
$$+\frac{1}{\mu^2}\left(6d^4\Psi^2+6\mu^2 d^2+12\mu^2 d^3-12\delta\mu d^3\right)(d+H)^{-2} \qquad \textbf{(vi)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (vi) becomes:

$$u62_1(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2d-6\mu\delta d-\frac{2\Psi\left(12d^3\Psi^2+6\mu^2d-18\mu d^2\Psi\right)}{2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}+\right.$$
$$\left.\frac{4\Psi^2\left(6d^4\Psi^2+6\mu^2d^2+12\mu^2d^3-12\delta\mu d^3\right)}{\left\{2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

If $C_1\neq 0$ but $C_2=0$ then equation (vi) becomes,

$$u62_2(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2d-6\mu\delta d+\frac{2\Psi\left(12d^3\Psi^2+6\mu^2d-18\mu d^2\Psi\right)}{2d\Psi+\mu+\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}+\right.$$
$$\left.\frac{4\Psi^2\left(6d^4\Psi^2+6\mu^2d^2+12\mu^2d^3-12\delta\mu d^3\right)}{\left\{2d\Psi+\mu+\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}\right)$$

Substituting Eq (4.3.7) into Eq (4.1.3), along with Eq (3.1.8) and simplifying, yields the following non travelling wave solutions,

$$\therefore u72(x,t)=\frac{3}{8\lambda^2\beta^2}\left(d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)-\frac{3}{4\lambda^2\beta^2}$$
$$\left(-2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4+2d\lambda^2\mu^2+\mu^4d\right)(d+H)+\frac{3}{8\lambda^2\beta^2}\left(\lambda^2+\mu^2\right)^2(d+H)^2 \quad \textbf{(vii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (vii) becomes:

$$u72_1(x,t)=\frac{3}{8\lambda^2\beta^2}\Bigg(d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2+\frac{\left(2\mu\beta\lambda^2+2\beta\mu^3-d\lambda^4-2d\lambda^2\mu^2-\mu^4d\right)\left(2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)}{\Psi}+\frac{\left(\lambda^2+\mu^2\right)^2\left\{2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{4\Psi^2}\Bigg)$$

If $C_1\neq 0$ but $C_2=0$ then equation (vii) becomes,

$$u72_2(x,t)=\frac{3}{8\lambda^2\beta^2}\Bigg(d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2-\frac{\left(2\mu\beta\lambda^2+2\beta\mu^3-d\lambda^4-2d\lambda^2\mu^2-\mu^4d\right)\left(2d\Psi+\mu+\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)}{\Psi}+\frac{\left(\lambda^2+\mu^2\right)^2\left\{2d\Psi+\mu+\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{4\Psi^2}\Bigg)$$

Substituting Eq (4.3.8) into Eq (4.1.3), along with Eq (3.1.8) and simplifying, yields the following non travelling wave solutions,

$$\therefore u82(x,t)=\frac{1}{8\lambda^2\beta^2}\left(3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2\right)$$
$$-\frac{3}{4\lambda^2\beta^2}\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4d\right)(d+H)+\frac{3}{8\lambda^2\beta^2}\left(\lambda^2-\mu^2\right)^2(d+H)^2 \quad \textbf{(viii)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (viii) becomes:

$$u82_1(x,t)=\frac{1}{8\lambda^2\beta^2}\Bigg(3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2$$
$$-\frac{3\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4d\right)\left(2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)}{\Psi}$$
$$+\frac{3\left(\lambda^2-\mu^2\right)^2\left\{2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{4\Psi^2}\Bigg)$$

If $C_1\neq 0$ but $C_2=0$ then equation (viii) becomes,

$$u82_2(x,t)=\frac{1}{8\lambda^2\beta^2}\Bigg(3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2$$
$$+\frac{3\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4d\right)\left(2d\Psi+\mu+\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)}{\Psi}$$
$$+\frac{3\left(\lambda^2-\mu^2\right)^2\left\{2d\Psi+\mu+\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{4\Psi^2}\Bigg)$$

Substituting Eq (4.3.9) into Eq (4.1.3), along with Eq (3.1.8) and simplifying, yields the following non travelling wave solutions,

$$\therefore u92(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2+6\mu^2d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d\Psi^2-6\mu\Psi\right)\left(d+H\right)+\frac{6\Psi^2}{\mu^2}\left(d+H\right)^2$$

(ix)

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (ix) becomes:

$$u92_1(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2+6\mu^2d-6\mu\delta d-\frac{\left(6d\Psi^2+3\mu\Psi\right)\left(2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)}{\Psi}+\frac{3\left\{2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{2}\right)$$

If $C_1\neq 0$ but $C_2=0$ then equation (ix) becomes,

$$u92_2(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2+6\mu^2d-6\mu\delta d+\frac{\left(6d\Psi^2+3\mu\Psi\right)\left(2d\Psi+\mu+\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)}{\Psi}+\frac{3\left\{2d\Psi+\mu+\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{2}\right)$$

Substituting Eq (4.3.10) into Eq (4.1.3), along with Eq (3.1.8) and simplifying, yields the following non travelling wave solutions,

$$\therefore u102(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d\Psi^2+6\mu^2-6\mu\delta\right)\left(d+H\right)+\frac{6\Psi^2}{\mu^2}\left(d+H\right)^2 \quad \textbf{(x)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (x) becomes:

$$u102_1(x,t) = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d - \frac{\left(6d\Psi^2 - 3\mu^2 + 3\mu\delta\right)\left(2d\Psi + \mu + \sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)}{\Psi} + \frac{3\left\{2d\Psi + \mu + \sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{2}\right)$$

If $C_1 \neq 0$ but $C_2 = 0$ then equation (x) becomes ,

$$u102_2(x,t) = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d + \frac{\left(6d\Psi^2 - 3\mu^2 + 3\mu\delta\right)\left(2d\Psi + \mu + \sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)}{\Psi} - \frac{3\left\{2d\Psi + \mu + \sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{2}\right)$$

Substituting Eq (4.3.11) into Eq (4.1.3), along with Eq (3.1.8) and simplifying, yields the following non travelling wave solutions,

$$\therefore u112(x,t) = \frac{3}{4} + \left(\frac{6\Psi^2}{\lambda^2}\right)\left(d + H\right)^2 + \left(\frac{3\lambda^2}{128\Psi^2}\right)\left(d + H\right)^{-2} \qquad \textbf{(xi)}$$

where, $$H = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) + C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) + C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (xi) becomes,

$$u112_1(x,t) = \frac{3}{4} - \frac{3\Omega\cot^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{2\lambda^2} + \frac{3\lambda^2}{32\Omega i^2\cot^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

If $C_1 \neq 0$ but $C_2 = 0$ then equation (xi) becomes,

$$u112_2(x,t) = \frac{3}{4} - \frac{3\Omega \tan^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{2\lambda^2} + \frac{3\lambda^2}{32\Omega i^2 \tan^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

Substituting Eq (4.3.12) into Eq (4.1.3), along with Eq (3.1.8) and simplifying, yields the following non travelling wave solutions,

$$\therefore u122(x,t) = \frac{1}{4} + \left(\frac{6\Psi^2}{\lambda^2}\right)(d+H)^2 + \left(\frac{3\lambda^2}{128\Psi^2}\right)(d+H)^{-2} \qquad \textbf{(xii)}$$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (xii) becomes:

$$u122_1(x,t) = \frac{1}{4} - \frac{3\Omega \cot^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{2\lambda^2} + \frac{3\lambda^2}{32\Omega i^2 \cot^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

If $C_1 \neq 0$ but $C_2 = 0$ then equation (xii) becomes,

$$u122_2(x,t) = \frac{1}{4} - \frac{3\Omega \tan^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{2\lambda^2} + \frac{3\lambda^2}{32\Omega i^2 \tan^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

Rational form of the non-travelling wave solutions:

Substituting Eq (4.3.1) into Eq (4.1.3), along with Eq (3.1.9) and simplifying, yields the following non travelling wave solutions,

$$\therefore u13(x,t) = \frac{3}{2} + \left(\frac{3\lambda^2 + 4b_2\Psi^2}{8\Psi^2}\right)(d+H)^{-2} \qquad \textbf{(i)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (i) becomes,

$$u13_1(x,t)=\frac{3}{2}+\frac{\left(3\lambda^2+4b_2\Psi^2\right)\left(C_1+C_2\xi\right)^2}{8\Psi^2C_2^2}$$

Substituting Eq (4.3.2) into Eq (4.1.3), along with Eq (3.1.9) and simplifying, yields the following non travelling wave solutions,

$$u3_2(x,t)=-\frac{1}{2}+\left(\frac{3\lambda^2+4b_2\Psi^2}{8\Psi^2}\right)\left(d+H\right)^{-2} \qquad \textbf{(ii)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (ii) becomes,

$$u23_1(x,t)=-\frac{1}{2}+\frac{\left(3\lambda^2+4b_2\Psi^2\right)\left(C_1+C_2\xi\right)^2}{8\Psi^2C_2^2}$$

Substituting Eq (4.3.3) into Eq (4.1.3), along with Eq (3.1.9) and simplifying, yields the following non travelling wave solutions,

$$\begin{aligned}\therefore u33\left(x,t\right)=&\frac{1}{8\lambda^2\beta^2}\left(3\lambda^4d^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2\right)\\&-\frac{1}{4\lambda^2\beta^2}\left(36\mu^2\beta^2d-12\lambda^2\beta^2d+18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3\right.\\&\left.-6\mu^2d^3\lambda^2+3\mu^4d^3\right)\ \left(d+H\right)^{-1}+\frac{1}{8\lambda^2\beta^2}\left(-24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2-96\mu d\beta^3\right.\\&\left.+48\beta^4+3\lambda^4d^4-6\lambda^2\mu^2d^4+3d^4\mu^4+24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)\left(d+H\right)^{-2}\end{aligned} \qquad \textbf{(iii)}$$

where, $H = (\frac{G'}{G}) = \frac{\mu}{2\Psi} + \frac{C_2}{C_1 + C_2\xi}$,

then the above equation (iii) becomes,

$$u33_1(x,t) = \frac{1}{2\lambda^2\beta^2}\left(\frac{3\lambda^4 d^2 - 6\lambda^2\mu^2 d^2 + 12\lambda^2\mu d\beta - 4\lambda^2\beta^2 + 3\mu^4 d^2 - 12\mu^3 d\beta + 12\beta^2\mu^2}{4} - \right.$$
$$\Psi\left(36\mu^2\beta^2 d - 12\lambda^2\beta^2 d + 18\lambda^2 d^2\beta\mu - 18\mu^3\beta d^2 - 24\mu\beta^3 + 3\lambda^4 d^3 - 6\mu^2 d^3\lambda^2 + 3\mu^4 d^3\right)$$
$$\frac{(C_1 + C_2\xi)}{\{2d\Psi(C_1 + C_2\xi) + \mu(C_1 + C_2\xi) + 2\Psi C_2\}} + \Psi^2\left(-24\lambda^2\beta^2 d^2 + 72\mu^2 d^2\beta^2 - 96\mu d\beta^3 + 48\beta^4 + 3\lambda^4 d^4\right.$$
$$\left.-6\lambda^2\mu^2 d^4 + 3d^4\mu^4 + 24\lambda^2\mu\beta d^3 - 24\mu^3\beta d^3\right)\frac{(C_1 + C_2\xi)^2}{\{2d\Psi(C_1 + C_2\xi) + \mu(C_1 + C_2\xi) + 2\Psi C_2\}^2}$$

Substituting Eq (4.3.4) into Eq (4.1.3), along with Eq (3.1.9) and simplifying, yields the following non travelling wave solutions,

$$\therefore u43(x,t) = \frac{3}{8\lambda^2\beta^2}\left(\lambda^4 d^2 + 2\lambda^2\mu^2 d^2 - 4\lambda^2\mu d\beta + 4\lambda^2\beta^2 + \mu^4 d^2 - 4\mu^3 d\beta + 4\beta^2\mu^2\right)$$
$$-\frac{1}{4\lambda^2\beta^2}\left(36\mu^2\beta^2 d + 12\lambda^2\beta^2 d - 18\lambda^2 d^2\beta\mu - 18\mu^3\beta d^2 - 24\mu\beta^3 + 3\lambda^4 d^3\right.$$
$$\left.+6\mu^2 d^3\lambda^2 + 3\mu^4 d^3\right)(d+H)^{-1} + \frac{1}{8\lambda^2\beta^2}\left(24\lambda^2\beta^2 d^2 + 72\mu^2 d^2\beta^2 - 96\mu d\beta^3 + 48\beta^4\right.$$
$$\left.+3\lambda^4 d^4 + 6\lambda^2\mu^2 d^4 + 3d^4\mu^4 - 24\lambda^2\mu\beta d^3 - 24\mu^3\beta d^3\right)(d+H)^{-2} \qquad \textbf{(iv)}$$

where, $H = (\frac{G'}{G}) = \frac{\mu}{2\Psi} + \frac{C_2}{C_1 + C_2\xi}$,

then the above equation (iv) becomes,

$$u43_1(x,t) = \frac{1}{2\lambda^2\beta^2}\left(\frac{3\left(\lambda^4 d^2 + 2\lambda^2\mu^2 d^2 - 4\lambda^2\mu d\beta + 4\lambda^2\beta^2 + \mu^4 d^2 - 4\mu^3 d\beta + 4\beta^2\mu^2\right.}{4} - \right.$$
$$\Psi\left(36\mu^2\beta^2 d + 12\lambda^2\beta^2 d - 18\lambda^2 d^2\beta\mu - 18\mu^3\beta d^2 - 24\mu\beta^3 + 3\lambda^4 d^3 + 6\mu^2 d^3\lambda^2 + 3\mu^4 d^3\right)$$
$$\frac{C_1 + C_2\xi}{\{2d\Psi(C_1 + C_2\xi) + \mu(C_1 + C_2\xi) + 2\Psi C_2\}} + \frac{\Psi^2\left(24\lambda^2\beta^2 d^2 + 72\mu^2 d^2\beta^2 - 96\mu d\beta^3 + 48\beta^4\right.}{2}$$
$$\frac{\left.+3\lambda^4 d^4 + 6\lambda^2\mu^2 d^4 + 3d^4\mu^4 - 24\lambda^2\mu\beta d^3 - 24\mu^3\beta d^3\right)}{}\frac{(C_1 + C_2\xi)^2}{\{2d\Psi(C_1 + C_2\xi) + \mu(C_1 + C_2\xi) + 2\Psi C_2\}^2}$$

Substituting Eq (4.3.5) into Eq (4.1.3), along with Eq (3.1.9) and simplifying, yields the following non travelling wave solutions,

$$\therefore u35(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d^3\Psi^2-\mu^2 6d-18\mu d^2\Psi\right)\left(d+H\right)^{-1}$$
$$+\frac{1}{\mu^2}\left(6d^4\Psi^2+\mu^2 6d^2+12\mu d^3\Psi\right)\left(d+H\right)^{-2} \qquad \textbf{(v)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (v) becomes,

$$u53_1(x,t)=\frac{1}{\mu^2}\Bigg(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d-\left(12d^3\Psi^2+6\mu^2 d+18\mu d^2\Psi\right)$$
$$\frac{2\Psi\left(C_1+C_2\xi\right)}{\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}}+\left(6d^4\Psi^2+\mu^2 6d^2+12\mu d^3\Psi\right)$$
$$\frac{4\Psi^2\left(C_1+C_2\xi\right)^2}{\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}^2}\Bigg)$$

Substituting Eq (4.3.6) into Eq (4.1.3), along with Eq (3.1.9) and simplifying, yields the following non travelling wave solutions,

$$\therefore u63(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d\right)-\frac{1}{\mu^2}\left(12d^3\Psi^2+6\mu^2 d-18\mu d^2\Psi\right)\left(d+H\right)^{-1}$$
$$+\frac{1}{\mu^2}\left(6d^4\Psi^2+6\mu^2 d^2+12\mu^2 d^3-12\delta\mu d^3\right)\left(d+H\right)^{-2} \qquad \textbf{(vi)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (vi) becomes,

$$u63_1(x,t)=\frac{1}{\mu^2}\Big(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d-\left(12d^3\Psi^2+6\mu^2 d-18\mu d^2\Psi\right)$$

$$\frac{2\Psi\left(C_1+C_2\xi\right)}{\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}}+\left(6d^4\Psi^2+6\mu^2d^2+12\mu^2d^3-12\delta\mu d^3\right)$$

$$\left.\frac{4\Psi^2\left(C_1+C_2\xi\right)^2}{\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}^2}\right)$$

Substituting Eq (4.3.7) into Eq (4.1.3), along with Eq (3.1.9) and simplifying, yields the following non travelling wave solutions,

$$\therefore u73(x,t)=\frac{3}{8\lambda^2\beta^2}\left(d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)-\frac{3}{4\lambda^2\beta^2}$$

$$\left(-2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4+2d\lambda^2\mu^2+\mu^4d\right)\left(d+H\right)+\frac{3}{8\lambda^2\beta^2}\left(\lambda^2+\mu^2\right)^2\left(d+H\right)^2 \quad \text{(vii)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (vii) becomes,

$$u73_1(x,t)=\frac{3}{8\lambda^2\beta^2}\left(d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2-\right.$$

$$\frac{\left(2\mu\beta\lambda^2+2\beta\mu^3-d\lambda^4-2d\lambda^2\mu^2-\mu^4d\right)\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}}{\Psi\left(C_1+C_2\xi\right)}$$

$$\left.+\frac{\left(\lambda^2+\mu^2\right)^2\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}^2}{4\Psi^2\left(C_1+C_2\xi\right)^2}\right)$$

Substituting Eq (4.3.8) into Eq (4.1.3), along with Eq (3.1.9) and simplifying, yields the following non travelling wave solutions,

$$u83(x,t)=\frac{1}{8\lambda^2\beta^2}\left(3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2\right)$$

$$-\frac{3}{4\lambda^2\beta^2}\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4d\right)\left(d+H\right)+\frac{3}{8\lambda^2\beta^2}\left(\lambda^2-\mu^2\right)^2\left(d+H\right)^2 \quad \text{(viii)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$, then the above equation (viii) becomes,

$$u83_1(x,t)=\frac{1}{8\lambda^2\beta^2}\Big(3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2$$
$$-3\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4d\right)\frac{\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}}{\Psi\left(C_1+C_2\xi\right)}$$
$$+\frac{3\left(\lambda^2-\mu^2\right)^2\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}^2}{4\Psi^2\left(C_1+C_2\xi\right)^2}\Big)$$

Substituting Eq (4.3.9) into Eq (4.1.3), along with Eq (3.1.9) and simplifying, yields the following non travelling wave solutions,

$$\therefore u93(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2+6\mu^2d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d\Psi^2-6\mu\Psi\right)\left(d+H\right)+\frac{6\Psi^2}{\mu^2}\left(d+H\right)^2 \quad \textbf{(ix)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (ix) becomes,

$$u93_1(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2+6\mu^2d-6\mu\delta d-\frac{\left(6d\Psi^2+3\mu\Psi\right)\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}}{\Psi\left(C_1+C_2\xi\right)}+\frac{3\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}^2}{2\left(C_1+C_2\xi\right)^2}\right)$$

Substituting Eq (4.3.10) into Eq (4.1.3), along with Eq (3.1.9) and simplifying, yields the following non travelling wave solutions,

$$\therefore u103(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d\Psi^2+6\mu^2-6\mu\delta\right)\left(d+H\right)+\frac{6\Psi^2}{\mu^2}\left(d+H\right)^2 \quad \textbf{(x)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (x) becomes,

$$u103_1(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d-\frac{\left(6d\Psi^2-3\mu^2+3\mu\delta\right)\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}}{\Psi\left(C_1+C_2\xi\right)}+\frac{3\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}^2}{2\left(C_1+C_2\xi\right)^2}\right)$$

Substituting Eq (4.3.11) into Eq (4.1.3), along with Eq (3.1.9) and simplifying, yields the following non travelling wave solutions,

$$\therefore u113(x,t)=\frac{3}{4}+\left(\frac{6\Psi^2}{\lambda^2}\right)\left(d+H\right)^2+\left(\frac{3\lambda^2}{128\Psi^2}\right)\left(d+H\right)^{-2} \qquad \text{(xi)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (xi) becomes,

$$u113_1(x,t)=\frac{3}{4}+\left(\frac{6\Psi^2}{\lambda^2}\right)\frac{C_2^2}{\left(C_1+C_2\xi\right)^2}+\left(\frac{3\lambda^2}{128\Psi^2}\right)\frac{\left(C_1+C_2\xi\right)^2}{C_2^2}$$

Substituting Eq (4.3.12) into Eq (4.1.3), along with Eq (3.1.9) and simplifying, yields the following non travelling wave solutions,

$$\therefore u123(x,t)=\frac{1}{4}+\left(\frac{6\Psi^2}{\lambda^2}\right)\left(d+H\right)^2+\left(\frac{3\lambda^2}{128\Psi^2}\right)\left(d+H\right)^{-2} \qquad \text{(xii)}$$

where, $H=(\frac{G'}{G})=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (xii) becomes,

$$u123_1(x,t)=\frac{1}{4}+\left(\frac{6\Psi^2}{\lambda^2}\right)\frac{C_2^2}{\left(C_1+C_2\xi\right)^2}+\left(\frac{3\lambda^2}{128\Psi^2}\right)\frac{\left(C_1+C_2\xi\right)^2}{C_2^2}$$

Hyperbolic form of the non-travelling wave solutions:

Substituting Eq (4.3.1) into Eq (4.1.3), along with Eq (3.1.10) and simplifying, yields the following non travelling wave solutions,

$$\therefore u14(x,t) = \frac{3}{2} + \left(\frac{3\lambda^2 + 4b_2\Psi^2}{8\Psi^2}\right)(d+H)^{-2} \qquad \textbf{(i)}$$

where, $H = (\frac{G'}{G}) = \frac{\sqrt{\Delta}}{\Psi} \frac{C_1 \sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2 \cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1 \cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2 \sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (i) becomes:

$$u14_1(x,t) = \frac{3}{2} + \frac{3\lambda^2 + 4b_2\Psi^2}{8\left\{d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then equation (i) becomes,

$$u14_2(x,t) = \frac{3}{2} + \frac{3\lambda^2 + 4b_2\Psi^2}{8\left\{d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.3.2) into Eq (4.1.3), along with Eq (3.1.10) and simplifying, yields the following non travelling wave solutions,

$$\therefore u24(x,t) = -\frac{1}{2} + \left(\frac{3\lambda^2 + 4b_2\Psi^2}{8\Psi^2}\right)(d+H)^{-2} \qquad \textbf{(ii)}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (ii) becomes,

$$u24_1(x,t)=-\frac{1}{2}+\frac{3\lambda^2+4b_2\Psi^2}{8\left\{d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1\neq 0$ and $C_2=0$ then equation (ii) becomes,

$$u24_2(x,t)=-\frac{1}{2}+\frac{3\lambda^2+4b_2\Psi^2}{8\left\{d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.3.3) into Eq (4.1.3), along with Eq (3.1.10) and simplifying, yields the following non travelling wave solutions,

$$\begin{aligned}\therefore u34(x,t)=&\frac{1}{8\lambda^2\beta^2}\left(3\lambda^4d^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2\right)\\&-\frac{1}{4\lambda^2\beta^2}\left(36\mu^2\beta^2d-12\lambda^2\beta^2d+18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3\right.\\&\left.-6\mu^2d^3\lambda^2+3\mu^4d^3\right)\left(d+H\right)^{-1}+\frac{1}{8\lambda^2\beta^2}\left(-24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2-96\mu d\beta^3\right.\\&\left.+48\beta^4+3\lambda^4d^4-6\lambda^2\mu^2d^4+3d^4\mu^4+24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)\left(d+H\right)^{-2}\qquad\textbf{(iii)}\end{aligned}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (iii) becomes:

$$u34_1(x,t) = \frac{1}{4\lambda^2\beta^2}\left(\frac{3\lambda^4 d^2 - 6\lambda^2\mu^2 d^2 + 12\lambda^2\mu d\beta - 4\lambda^2\beta^2 + 3\mu^4 d^2 - 12\mu^3 d\beta + 12\beta^2\mu^2}{2} - \frac{\Psi\left(36\mu^2\beta^2 d - 12\lambda^2\beta^2 d + 18\lambda^2 d^2\beta\mu - 18\mu^3\beta d^2 - 24\mu\beta^3 + 3\lambda^4 d - 6\mu^2 d^3\lambda^2 + 3\mu^4 d^3\right)}{d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)} - \frac{\Psi^2\left(24\lambda^2\beta^2 d^2 - 72\mu^2 d^2\beta^2 + 96\mu d\beta^3 - 48\beta^4 - 3\lambda^4 d^4 + 6\lambda^2\mu^2 d^4 - 3d^4\mu^4 - 24\lambda^2\mu\beta d^3 + 24\mu^3\beta d^3\right)}{2\left(d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

If $C_1 \neq 0$ and $C_2 = 0$ then equation (iii) becomes,

$$u34_2(x,t) = \frac{1}{4\lambda^2\beta^2}\left(\frac{3\lambda^4 d^2 - 6\lambda^2\mu^2 d^2 + 12\lambda^2\mu d\beta - 4\lambda^2\beta^2 + 3\mu^4 d^2 - 12\mu^3 d\beta + 12\beta^2\mu^2}{2} - \frac{\Psi\left(36\mu^2\beta^2 d - 12\lambda^2\beta^2 d + 18\lambda^2 d^2\beta\mu - 18\mu^3\beta d^2 - 24\mu\beta^3 + 3\lambda^4 d\right)}{d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)} - \frac{\Psi^2\left(24\lambda^2\beta^2 d^2 - 72\mu^2 d^2\beta^2 + 96\mu d\beta^3 - 48\beta^4 - 3\lambda^4 d^4 + 6\lambda^2\mu^2 d^4 - 3d^4\mu^4 - 24\lambda^2\mu\beta d^3 + 24\mu^3\beta d^3\right)}{2\left(d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

Substituting Eq (4.3.4) into Eq (4.1.3), along with Eq (3.1.10) and simplifying, yields the following non travelling wave solutions,

$$\begin{aligned}\therefore u44(x,t) &= \frac{3}{8\lambda^2\beta^2}\left(\lambda^4 d^2 + 2\lambda^2\mu^2 d^2 - 4\lambda^2\mu d\beta + 4\lambda^2\beta^2 + \mu^4 d^2 - 4\mu^3 d\beta + 4\beta^2\mu^2\right) \\ &- \frac{1}{4\lambda^2\beta^2}\left(36\mu^2\beta^2 d + 12\lambda^2\beta^2 d - 18\lambda^2 d^2\beta\mu - 18\mu^3\beta d^2 - 24\mu\beta^3 + 3\lambda^4 d^3\right. \\ &\left. + 6\mu^2 d^3\lambda^2 + 3\mu^4 d^3\right)(d+H)^{-1} + \frac{1}{8\lambda^2\beta^2}\left(24\lambda^2\beta^2 d^2 + 72\mu^2 d^2\beta^2 - 96\mu d\beta^3 + 48\beta^4\right. \\ &\left. + 3\lambda^4 d^4 + 6\lambda^2\mu^2 d^4 + 3d^4\mu^4 - 24\lambda^2\mu\beta d^3 - 24\mu^3\beta d^3\right)(d+H)^{-2} \qquad \textbf{(iv)}\end{aligned}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (iv) becomes:

$$u44_1(x,t)=\frac{1}{4\lambda^2\beta^2}\left(\frac{3\left(\lambda^4d^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)}{2}-\right.$$

$$\frac{\Psi\left(36\mu^2\beta^2d+12\lambda^2\beta^2d-18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3+6\mu^2d^3\lambda^2+3\mu^4d^3\right)}{d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}+$$

$$\left.\frac{\Psi^2\left(24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2-96\mu d\beta^3+48\beta^4+3\lambda^4d^4+6\lambda^2\mu^2d^4+3d^4\mu^4-24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)}{2\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

If $C_1\neq 0$ and $C_2=0$ then equation (iv) becomes,

$$u44_2(x,t)=\frac{1}{4\lambda^2\beta^2}\left(\frac{3\left(\lambda^4d^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)}{2}-\right.$$

$$\frac{\Psi\left(36\mu^2\beta^2d+12\lambda^2\beta^2d-18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3+6\mu^2d^3\lambda^2+3\mu^4d^3\right)}{d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}+$$

$$\left.\frac{\Psi^2\left(24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2-96\mu d\beta^3+48\beta^4+3\lambda^4d^4+6\lambda^2\mu^2d^4+3d^4\mu^4-24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)}{2\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

Substituting Eq (4.3.5) into Eq (4.1.3), along with Eq (3.1.10) and simplifying, yields the following non travelling wave solutions,

$$\therefore u54(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d^3\Psi^2-\mu^2 6d-18\mu d^2\Psi\right)\left(d+H\right)^{-1}$$
$$+\frac{1}{\mu^2}\left(6d^4\Psi^2+\mu^2 6d^2+12\mu d^3\Psi\right)\left(d+H\right)^{-2} \qquad \textbf{(v)}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (v) becomes:

$$u54_1(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d-\frac{\Psi\left(12d^3\Psi^2+\mu^2 6d+18\mu d^2\Psi\right)}{d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}+\frac{\Psi^2\left(6d^4\Psi^2+\mu^2 6d^2+12\mu d^3\Psi\right)}{\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

If $C_1\neq 0$ and $C_2=0$ then equation (v) becomes,

$$u54_2(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d-\frac{\Psi\left(12d^3\Psi^2+\mu^2 6d+18\mu d^2\Psi\right)}{d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}+\frac{\Psi^2\left(6d^4\Psi^2+\mu^2 6d^2+12\mu d^3\Psi\right)}{\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

Substituting Eq (4.3.6) into Eq (4.1.3), along with Eq (3.1.10) and simplifying, yields the following non travelling wave solutions,

$$\therefore u64(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d\right)-\frac{1}{\mu^2}\left(12d^3\Psi^2+6\mu^2 d-18\mu d^2\Psi\right)\left(d+H\right)^{-1}$$
$$+\frac{1}{\mu^2}\left(6d^4\Psi^2+6\mu^2 d^2+12\mu^2 d^3-12\delta\mu d^3\right)\left(d+H\right)^{-2} \qquad \textbf{(vi)}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (vi) becomes:

$$u64_1(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d-\frac{\Psi\left(12d^3\Psi^2+6\mu^2 d-18\mu d^2\Psi\right)}{d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}+\frac{\Psi^2\left(6d^4\Psi^2+6\mu^2 d^2+12\mu^2 d^3-12\delta\mu d^3\right)}{\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

If $C_1\neq 0$ and $C_2=0$ then equation (vi) becomes,

$$u64_2(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d-\frac{\Psi\left(12d^3\Psi^2+6\mu^2 d-18\mu d^2\Psi\right)}{d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}+\frac{\Psi^2\left(6d^4\Psi^2+6\mu^2 d^2+12\mu^2 d^3-12\delta\mu d^3\right)}{\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

Substituting Eq (4.3.7) into Eq (4.1.3), along with Eq (3.1.10) and simplifying, yields the following non travelling wave solutions,

$$\therefore u74(x,t)=\frac{3}{8\lambda^2\beta^2}\left(d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)-\frac{3}{4\lambda^2\beta^2}$$
$$\left(-2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4+2d\lambda^2\mu^2+\mu^4d\right)\left(d+H\right)+\frac{3}{8\lambda^2\beta^2}\left(\lambda^2+\mu^2\right)^2\left(d+H\right)^2 \qquad \text{(vii)}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (vii) becomes:

$$u74_1(x,t)=\frac{3}{4\lambda^2\beta^2}\left(\frac{d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2}{2}+\right.$$
$$\frac{\left(2\mu\beta\lambda^2+2\beta\mu^3-d\lambda^4-2d\lambda^2\mu^2-\mu^4d\right)}{\Psi}\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)$$
$$\left.+\frac{\left(\lambda^2+\mu^2\right)^2}{2\Psi^2}\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2\right)$$

If $C_1\neq 0$ and $C_2=0$ then equation (vii) becomes,

$$u74_2(x,t)=\frac{3}{4\lambda^2\beta^2}\left(\frac{d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2}{2}+\right.$$
$$\frac{\left(2\mu\beta\lambda^2+2\beta\mu^3-d\lambda^4-2d\lambda^2\mu^2-\mu^4d\right)}{\Psi}\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)$$
$$\left.+\frac{\left(\lambda^2+\mu^2\right)^2}{2\Psi^2}\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2\right)$$

Substituting Eq (4.3.8) into Eq (4.1.3), along with Eq (3.1.10) and simplifying, yields the following non travelling wave solutions,

$$\therefore u84(x,t)=\frac{1}{8\lambda^2\beta^2}\left(3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2\right)$$
$$-\frac{3}{4\lambda^2\beta^2}\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4d\right)\left(d+H\right)+\frac{3}{8\lambda^2\beta^2}\left(\lambda^2-\mu^2\right)^2\left(d+H\right)^2 \qquad \textbf{(viii)}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (viii) becomes:

$$u84_1(x,t)=\frac{1}{4\lambda^2\beta^2}\left(\frac{3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2}{2}-\right.$$

$$\frac{3\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4d\right)}{\Psi}\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)$$
$$\left.+\frac{3\left(\lambda^2-\mu^2\right)^2}{\Psi^2}\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2\right)$$

If $C_1\neq 0$ and $C_2=0$ then equation (viii) becomes,

$$u84_2(x,t)=\frac{1}{4\lambda^2\beta^2}\left(\frac{3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2}{2}-\right.$$

$$\frac{3\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4d\right)}{\Psi}\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)$$
$$\left.+\frac{3\left(\lambda^2-\mu^2\right)^2}{\Psi^2}\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2\right)$$

Substituting Eq (4.3.9) into Eq (4.1.3), along with Eq (3.1.10) and simplifying, yields the following non travelling wave solutions,

$$\therefore u94(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2+6\mu^2d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d\Psi^2-6\mu\Psi\right)\left(d+H\right)+\frac{6\Psi^2}{\mu^2}\left(d+H\right)^2$$

(ix)

where, $H=(\frac{G'}{G})=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (ix) becomes:

$$u94_1(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2+6\mu^2d-6\mu\delta d-\frac{12d\Psi^2+6\mu\Psi}{\Psi}\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)\right.$$
$$\left.+6\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2\right)$$

If $C_1\neq 0$ and $C_2=0$ then equation (ix) becomes,

$$u94_2(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2+6\mu^2d-6\mu\delta d-\frac{12d\Psi^2+6\mu\Psi}{\Psi}\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)\right.$$
$$\left.+6\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2\right)$$

Substituting Eq (4.3.10) into Eq (4.1.3), along with Eq (3.1.10) and simplifying, yields the following non travelling wave solutions,

$$\therefore u104(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d\Psi^2+6\mu^2-6\mu\delta\right)\left(d+H\right)+\frac{6\Psi^2}{\mu^2}\left(d+H\right)^2 \quad \textbf{(x)}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (x) becomes,

$$u104_1(x,t) = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d - \frac{12d\Psi^2 - 6\mu^2 + 6\mu\delta}{\Psi}\left(d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)\right.$$
$$\left. + 6\left(d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2\right)$$

If $C_1 \neq 0$ and $C_2 = 0$ then equation (x) becomes,

$$u104_2(x,t) = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d - \frac{12d\Psi^2 - 6\mu^2 + 6\mu\delta}{\Psi}\left(d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)\right.$$
$$\left. + 6\left(d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2\right)$$

Substituting Eq (4.3.11) into Eq (4.1.3), along with Eq (3.1.10) and simplifying, yields the following non travelling wave solutions,

$$\therefore u114(x,t) = \frac{3}{4} + \left(\frac{6\Psi^2}{\lambda^2}\right)(d+H)^2 + \left(\frac{3\lambda^2}{128\Psi^2}\right)(d+H)^{-2} \qquad \textbf{(xi)}$$

where, $H = \left(\frac{G'}{G}\right) = \frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (xi) becomes:

$$u114_1(x,t) = \frac{3}{4} + \frac{6\left\{d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}{\lambda^2} + \frac{3\lambda^2}{128\left\{d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then equation (xi) becomes,

$$u114_2(x,t) = \frac{3}{4} + \frac{6\left\{d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}{\lambda^2} + \frac{3\lambda^2}{128\left\{d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.3.12) into Eq (4.1.3), along with Eq (3.1.10) and simplifying, yields the following non travelling wave solutions,

$$\therefore u124(x,t) = \frac{1}{4} + \left(\frac{6\Psi^2}{\lambda^2}\right)(d+H)^2 + \left(\frac{3\lambda^2}{128\Psi^2}\right)(d+H)^{-2} \qquad \textbf{(xii)}$$

where, $H = (\frac{G'}{G}) = \frac{\sqrt{\Delta}}{\Psi} \frac{C_1 \sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2 \cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1 \cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2 \sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (xii) becomes:

$$u124_1(x,t) = \frac{1}{4} + \frac{6\left\{d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}{\lambda^2} + \frac{3\lambda^2}{128\left\{d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then equation (xii) becomes,

$$u124_2(x,t) = \frac{1}{4} + \frac{6\left\{d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}{\lambda^2} + \frac{3\lambda^2}{128\left\{d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}$$

Trigonometric form of the non-travelling wave solutions:

Substituting Eq (4.3.1) into Eq (4.1.3), along with Eq (3.1.11) and simplifying, yields the following non travelling wave solutions,

$$\therefore u15(x,t) = \frac{3}{2} + \left(\frac{3\lambda^2 + 4b_2\Psi^2}{8\Psi^2}\right)(d+H)^{-2} \qquad \textbf{(i)}$$

where, $H = (\frac{G'}{G}) = \frac{\sqrt{-\Delta}}{\Psi} \frac{-C_1 \sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right) + C_2 \cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1 \cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right) + C_2 \sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (i) becomes,

$$u15_1(x,t) = \frac{3}{2} + \frac{3\lambda^2 + 4b_2\Psi^2}{8\left\{d\Psi + \sqrt{\Delta} i \cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then equation (i) becomes ,

$$u15_2(x,t) = \frac{3}{2} + \frac{3\lambda^2 + 4b_2\Psi^2}{8\left\{d\Psi - \sqrt{\Delta} i \tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.3.2) into Eq (4.1.3), along with Eq (3.1.11) and simplifying, yields the following non travelling wave solutions,

$$\therefore u25(x,t) = -\frac{1}{2} + \left(\frac{3\lambda^2 + 4b_2\Psi^2}{8\Psi^2}\right)(d+H)^{-2} \qquad \textbf{(ii)}$$

where, $H = (\frac{G'}{G}) = \frac{\sqrt{-\Delta}}{\Psi} \frac{-C_1 \sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right) + C_2 \cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1 \cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right) + C_2 \sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (ii) becomes:

$$u25_1(x,t) = -\frac{1}{2} + \frac{3\lambda^2 + 4b_2\Psi^2}{8\left\{d\Psi + \sqrt{\Delta} i \cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then equation (ii) becomes,

$$u25_2(x,t) = -\frac{1}{2} + \frac{3\lambda^2 + 4b_2\Psi^2}{8\left\{d\Psi - \sqrt{\Delta} i \tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.3.3) into Eq (4.1.3), along with Eq (3.1.11) and simplifying, yields the following non travelling wave solutions,

$$\begin{aligned}\therefore u35(x,t) = &\frac{1}{8\lambda^2\beta^2}\left(3\lambda^4 d^2 - 6\lambda^2\mu^2 d^2 + 12\lambda^2\mu d\beta - 4\lambda^2\beta^2 + 3\mu^4 d^2 - 12\mu^3 d\beta + 12\beta^2\mu^2\right)\\
&-\frac{1}{4\lambda^2\beta^2}\left(36\mu^2\beta^2 d - 12\lambda^2\beta^2 d + 18\lambda^2 d^2\beta\mu - 18\mu^3\beta d^2 - 24\mu\beta^3 + 3\lambda^4 d^3\right.\\
&\left.-6\mu^2 d^3\lambda^2 + 3\mu^4 d^3\right)\ (d+H)^{-1} + \frac{1}{8\lambda^2\beta^2}\left(-24\lambda^2\beta^2 d^2 + 72\mu^2 d^2\beta^2 - 96\mu d\beta^3\right.\\
&\left.+48\beta^4 + 3\lambda^4 d^4 - 6\lambda^2\mu^2 d^4 + 3d^4\mu^4 + 24\lambda^2\mu\beta d^3 - 24\mu^3\beta d^3\right)(d+H)^{-2} \qquad \textbf{(iii)}\end{aligned}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (iii) becomes:

$$u35_1(x,t)=\frac{1}{4\lambda^2\beta^2}\left(\frac{3\lambda^4d^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2}{2}-\right.$$
$$\frac{\Psi\left(36\mu^2\beta^2d-12\lambda^2\beta^2d+18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3-6\mu^2d^3\lambda^2+3\mu^4d^3\right)}{d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}-$$
$$\left.\frac{\Psi^2\left(24\lambda^2\beta^2d^2-72\mu^2d^2\beta^2+96\mu d\beta^3-48\beta^4-3\lambda^4d^4+6\lambda^2\mu^2d^4-3d^4\mu^4-24\lambda^2\mu\beta d^3+24\mu^3\beta d^3\right)}{2\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

If $C_1\neq 0$ and $C_2=0$ then equation (iii) becomes,

$$u35_2(x,t)=\frac{1}{4\lambda^2\beta^2}\left(\frac{3\lambda^4d^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2}{2}-\right.$$
$$\frac{\Psi\left(36\mu^2\beta^2d-12\lambda^2\beta^2d+18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3-6\mu^2d^3\lambda^2+3\mu^4d^3\right)}{d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}-$$
$$\left.\frac{\Psi^2\left(24\lambda^2\beta^2d^2-72\mu^2d^2\beta^2+96\mu d\beta^3-48\beta^4-3\lambda^4d^4+6\lambda^2\mu^2d^4-3d^4\mu^4-24\lambda^2\mu\beta d^3+24\mu^3\beta d^3\right)}{2\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

Substituting Eq (4.3.4) into Eq (4.1.3), along with Eq (3.1.11) and simplifying, yields the following non travelling wave solutions,

$$\therefore u45(x,t)=\frac{3}{8\lambda^2\beta^2}\left(\lambda^4d^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)$$
$$-\frac{1}{4\lambda^2\beta^2}\left(36\mu^2\beta^2d+12\lambda^2\beta^2d-18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3\right.$$
$$\left.+6\mu^2d^3\lambda^2+3\mu^4d^3\right)(d+H)^{-1}+\frac{1}{8\lambda^2\beta^2}\left(24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2-96\mu d\beta^3+48\beta^4\right.$$
$$\left.+3\lambda^4d^4+6\lambda^2\mu^2d^4+3d^4\mu^4-24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)(d+H)^{-2} \qquad \textbf{(iv)}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (iv) becomes,

$$u45_1(x,t)=\frac{1}{4\lambda^2\beta^2}\left(\frac{3\left(\lambda^4d^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)}{2}-\right.$$
$$\frac{\Psi\left(36\mu^2\beta^2d+12\lambda^2\beta^2d-18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3+6\mu^2d^3\lambda^2+3\mu^4d^3\right)}{d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}+$$
$$\left.\frac{\Psi^2\left(24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2+48\beta^4-96\mu d\beta^3+3\lambda^4d^4+6\lambda^2\mu^2d^4+3d^4\mu^4-24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)}{2\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

If $C_1\neq 0$ and $C_2=0$ then equation (iv) becomes,

$$u45_2(x,t)=\frac{1}{4\lambda^2\beta^2}\left(\frac{3\left(\lambda^4d^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)}{2}-\right.$$
$$\frac{\Psi\left(36\mu^2\beta^2d+12\lambda^2\beta^2d-18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3+6\mu^2d^3\lambda^2+3\mu^4d^3\right)}{d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}+$$
$$\left.\frac{\Psi^2\left(24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2+48\beta^4-96\mu d\beta^3+3\lambda^4d^4+6\lambda^2\mu^2d^4+3d^4\mu^4-24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right)}{2\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

Substituting Eq (4.3.5) into Eq (4.1.3), along with Eq (3.1.11) and simplifying, yields the following non travelling wave solutions,

$$\therefore u55(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d^3\Psi^2-\mu^2 6d-18\mu d^2\Psi\right)\left(d+H\right)^{-1}$$
$$+\frac{1}{\mu^2}\left(6d^4\Psi^2+\mu^2 6d^2+12\mu d^3\Psi\right)\left(d+H\right)^{-2} \qquad \textbf{(v)}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (v) becomes,

$$u55_1(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d-\frac{\Psi\left(12d^3\Psi^2+6\mu^2 d+18\mu d^2\Psi\right)}{d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}+\frac{\Psi^2\left(6d^4\Psi^2+\mu^2 6d^2+12\mu d^3\Psi\right)}{\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

If $C_1\neq 0$ and $C_2=0$ then equation (v) becomes,

$$\therefore u55_2(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^2 6d-6\mu\delta d-\frac{\Psi\left(12d^3\Psi^2+6\mu^2 d+18\mu d^2\Psi\right)}{d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}+\frac{\Psi^2\left(6d^4\Psi^2+\mu^2 6d^2+12\mu d^3\Psi\right)}{\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

Substituting Eq (4.3.6) into Eq (4.1.3), along with Eq (3.1.11) and simplifying, yields the following non travelling wave solutions,

$$\therefore u65(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d\right)-\frac{1}{\mu^2}\left(12d^3\Psi^2+6\mu^2 d-18\mu d^2\Psi\right)\left(d+H\right)^{-1}$$
$$+\frac{1}{\mu^2}\left(6d^4\Psi^2+6\mu^2 d^2+12\mu^2 d^3-12\delta\mu d^3\right)\left(d+H\right)^{-2} \qquad \text{(vi)}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (vi) becomes,

$$u65_1(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d-\frac{\Psi\left(12d^3\Psi^2+6\mu^2 d-18\mu d^2\Psi\right)}{d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}+\frac{\Psi^2\left(6d^4\Psi^2+6\mu^2 d^2+12\mu^2 d^3-12\delta\mu d^3\right)}{\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

If $C_1\neq 0$ and $C_2=0$ then equation (vi) becomes,

$$u65_2(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d-\frac{\Psi\left(12d^3\Psi^2+6\mu^2 d-18\mu d^2\Psi\right)}{d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}+\frac{\Psi^2\left(6d^4\Psi^2+6\mu^2 d^2+12\mu^2 d^3-12\delta\mu d^3\right)}{\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}\right)$$

Substituting Eq (4.3.7) into Eq (4.1.3), along with Eq (3.1.11) and simplifying, yields the following non travelling wave solutions,

$$\therefore u75(x,t)=\frac{3}{8\lambda^2\beta^2}\left(d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right)-\frac{3}{4\lambda^2\beta^2}$$
$$\left(-2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4+2d\lambda^2\mu^2+\mu^4d\right)\left(d+H\right)+\frac{3}{8\lambda^2\beta^2}\left(\lambda^2+\mu^2\right)^2\left(d+H\right)^2 \qquad \textbf{(vii)}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (vii) becomes:

$$u75_1(x,t)=\frac{3}{4\lambda^2\beta^2}\left(\frac{d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2}{2}+\right.$$
$$\frac{\left(2\mu\beta\lambda^2+2\beta\mu^3-d\lambda^4-2d\lambda^2\mu^2-\mu^4d\right)}{\Psi}\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)$$
$$\left.+\frac{\left(\lambda^2+\mu^2\right)^2}{2\Psi^2}\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right)$$

If $C_1\neq 0$ and $C_2=0$ then equation (vii) becomes,

$$u75_2(x,t)=\frac{3}{4\lambda^2\beta^2}\left(\frac{d^2\lambda^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2}{2}+\right.$$
$$\frac{\left(2\mu\beta\lambda^2+2\beta\mu^3-d\lambda^4-2d\lambda^2\mu^2-\mu^4d\right)}{\Psi}\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)$$
$$\left.+\frac{\left(\lambda^2+\mu^2\right)^2}{2\Psi^2}\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right)$$

Substituting Eq (4.3.8) into Eq (4.1.3), along with Eq (3.1.11) and simplifying, yields the following non travelling wave solutions,

$$\therefore u85(x,t)=\frac{1}{8\lambda^2\beta^2}\left(3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2\right)$$

$$-\frac{3}{4\lambda^2\beta^2}\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4d\right)\left(d+H\right)+\frac{3}{8\lambda^2\beta^2}\left(\lambda^2-\mu^2\right)^2\left(d+H\right)^2 \quad \textbf{(viii)}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (viii) becomes:

$$u85_1(x,t)=\frac{1}{4\lambda^2\beta^2}\left(\frac{3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2}{2}-\frac{3\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4d\right)}{\Psi}\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)+\frac{3\left(\lambda^2-\mu^2\right)^2}{\Psi^2}\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right)$$

If $C_1\neq 0$ and $C_2=0$ then equation (viii) becomes,

$$u85_2(x,t)=\frac{1}{4\lambda^2\beta^2}\left(\frac{3d^2\lambda^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2}{2}-\frac{3\left(2\mu\beta\lambda^2-2\beta\mu^3+d\lambda^4-2d\lambda^2\mu^2+\mu^4d\right)}{\Psi}\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)+\frac{3\left(\lambda^2-\mu^2\right)^2}{\Psi^2}\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right)$$

Substituting Eq (4.3.9) into Eq (4.1.3), along with Eq (3.1.11) and simplifying, yields the following non travelling wave solutions,

$$\therefore u95(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2+6\mu^2d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d\Psi^2-6\mu\Psi\right)\left(d+H\right)+\frac{6\Psi^2}{\mu^2}\left(d+H\right)^2 \quad \textbf{(ix)}$$

where, $H=(\frac{G'}{G})=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (ix) becomes:

$$u95_1(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2+6\mu^2 d-6\mu\delta d-\frac{12d\Psi^2+6\mu\Psi}{\Psi}\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)\right.$$
$$\left.+6\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right)$$

If $C_1\neq 0$ and $C_2=0$ then equation (ix) becomes,

$$u95_2(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2+6\mu^2 d-6\mu\delta d-\frac{12d\Psi^2+6\mu\Psi}{\Psi}\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)\right.$$
$$\left.+6\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right)$$

Substituting Eq (4.3.10) into Eq (4.1.3), along with Eq (3.1.11) and simplifying, yields the following non travelling wave solutions,

$$\therefore u105(x,t)=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-6\mu^2 d-6\mu\delta d\right)+\frac{1}{\mu^2}\left(-12d\Psi^2+6\mu^2-6\mu\delta\right)\left(d+H\right)+\frac{6\Psi^2}{\mu^2}\left(d+H\right)^2$$

(x)

where, $H=(\frac{G'}{G})=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (x) becomes

$$u105_1(x,t) = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d - \frac{12d\Psi^2 - 6\mu^2 + 6\mu\delta}{\Psi}\left(d\Psi + \sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)\right.$$
$$\left. + 6\left(d\Psi + \sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right)$$

If $C_1 \neq 0$ and $C_2 = 0$ then equation (x) becomes ,

$$u105_2(x,t) = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d - \frac{12d\Psi^2 - 6\mu^2 + 6\mu\delta}{\Psi}\left(d\Psi - \sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)\right.$$
$$\left. + 6\left(d\Psi - \sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right)$$

Substituting Eq (4.3.11) into Eq (4.1.3), along with Eq (3.1.11) and simplifying, yields the following non travelling wave solutions,

$$\therefore u115(x,t) = \frac{3}{4} + \left(\frac{6\Psi^2}{\lambda^2}\right)(d+H)^2 + \left(\frac{3\lambda^2}{128\Psi^2}\right)(d+H)^{-2} \qquad \textbf{(xi)}$$

where, $H = (\frac{G'}{G}) = \frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right) + C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right) + C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (xi) becomes

$$u115_1(x,t) = \frac{3}{4} + \frac{6\left\{d\Psi + \sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}{\lambda^2} + \frac{3\lambda^2}{128\left\{d\Psi + \sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then equation (xi) becomes,

$$u115_2(x,t) = \frac{3}{4} + \frac{6\left\{d\Psi - \sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}{\lambda^2} + \frac{3\lambda^2}{128\left\{d\Psi - \sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.3.12) into Eq (4.1.3), along with Eq (3.1.11) and simplifying, yields the following non travelling wave solutions,

$$\therefore u125(x,t) = \frac{1}{4} + \left(\frac{6\Psi^2}{\lambda^2}\right)(d+H)^2 + \left(\frac{3\lambda^2}{128\Psi^2}\right)(d+H)^{-2} \qquad \text{(xii)}$$

where, $H = (\frac{G'}{G}) = \frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right) + C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right) + C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (xii) becomes:

$$u125_1(x,t) = \frac{1}{4} + \frac{6\left\{d\Psi + \sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}{\lambda^2} + \frac{3\lambda^2}{128\left\{d\Psi + \sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then equation (xii) becomes

$$u125_2(x,t) = \frac{1}{4} + \frac{6\left\{d\Psi - \sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}{\lambda^2} + \frac{3\lambda^2}{128\left\{d\Psi - \sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

4.5.2 Solutions of travelling wave:

Hyperbolic form of travelling wave solutions

Substituting Eq (4.4.13) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v11(x,t)=\frac{3}{8}\mp\frac{6\Psi}{\lambda\sqrt{6}}\left(d+H\right)+\frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2\mp\frac{\lambda}{16\Psi\sqrt{6}}\left(d+H\right)^{-1}+\frac{\lambda^2}{1536\Psi^2}\left(d+H\right)^{-2}\qquad \textbf{(i)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (i) becomes:

$$v1 1_1(x,t)=\frac{3}{8}\mp\frac{3\sqrt{\Omega}}{\lambda\sqrt{6}}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+\frac{3\Omega}{2\lambda^2}\coth^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\mp\frac{\lambda}{8\sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}+\frac{\lambda^2}{384\Omega\coth^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

If $C_1\neq 0$ but $C_2=0$ then,

$$v1 1_2(x,t)=\frac{3}{8}\mp\frac{3\sqrt{\Omega}}{\lambda\sqrt{6}}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+\frac{3\Omega}{2\lambda^2}\tanh^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\mp\frac{\lambda}{8\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}+\frac{\lambda^2}{384\Omega\tanh^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

Substituting Eq (4.4.14) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore u(x,t)=\frac{5}{8}\mp\frac{6\Psi}{\lambda\sqrt{6}}\left(d+H\right)+\frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2\pm\frac{\lambda}{16\Psi\sqrt{6}}\left(d+H\right)^{-1}+\frac{\lambda^2}{1536\Psi^2}\left(d+H\right)^{-2}\qquad \textbf{(ii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (ii) becomes: since $\mu = -2d\Psi$

$$v21_1(x,t) = \frac{5}{8} \mp \frac{3\sqrt{\Omega}}{\lambda\sqrt{6}} \coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + \frac{3\Omega}{2\lambda^2}\coth^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) \pm \frac{\lambda}{8\sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)} + \frac{\lambda^2}{384\Omega\coth^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$u21_2(x,t) = \frac{5}{8} \mp \frac{3\sqrt{\Omega}}{\lambda\sqrt{6}} \tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + \frac{3\Omega}{2\lambda^2}\tanh^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) \pm \frac{\lambda}{8\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)} + \frac{\lambda^2}{384\Omega\tanh^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

Substituting Eq (4.4.15) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v31(x,t) = \frac{1}{d^2}\left(d + H\right)^2 \qquad \textbf{(iii)}$$

where, $$H = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (iii) becomes:

$$v31_1(x,t) = \frac{\left\{2d\Psi\sqrt{6} \pm \lambda + \sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$u31_2(x,t) = \frac{\left\{2d\Psi\sqrt{6} \pm \lambda + \sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

Substituting Eq (4.4.16) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v41(x,t)=1-\frac{1}{d^2}\left(d+H\right)^2 \qquad \textbf{(iv)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (iv) becomes:

$$v41_1(x,t)=1-\frac{\left\{2d\Psi\sqrt{6}\pm\lambda i+\sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

If $C_1\neq 0$ but $C_2=0$ then,

$$v41_2(x,t)=1-\frac{\left\{2d\Psi\sqrt{6}\pm\lambda i+\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

Substituting Eq (4.4.17) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v51(x,t)=1-\frac{2}{d}\left(d+H\right)+\frac{1}{d^2}\left(d+H\right)^2 \qquad \textbf{(v)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (v) becomes:

$$v51_1(x,t) = 1 - \frac{2d\Psi\sqrt{6} \mp \lambda + \sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{d\Psi\sqrt{6}} + \frac{\left\{2d\Psi\sqrt{6} \mp \lambda + \sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v51_2(x,t) = 1 - \frac{2d\Psi\sqrt{6} \mp \lambda + \sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{d\Psi\sqrt{6}} + \frac{\left\{2d\Psi\sqrt{6} \mp \lambda + \sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

Substituting Eq (4.4.18) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v61(x,t) = \frac{2}{d}(d+H) - \frac{1}{d^2}(d+H)^2 \qquad \textbf{(vi)}$$

where, $H = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (vi) becomes:

$$v61_1(x,t) = \frac{2d\Psi\sqrt{6} \mp \lambda i + \sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{d\Psi\sqrt{6}} - \frac{\left\{2d\Psi\sqrt{6} \mp \lambda i + \sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v61_2(x,t) = \frac{2d\Psi\sqrt{6} \mp \lambda i + \sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{d\Psi\sqrt{6}} - \frac{\left\{2d\Psi\sqrt{6} \mp \lambda i + \sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

Substituting Eq (4.4.19) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v71(x,t)=\frac{6d^2\Psi^2}{\lambda^2}-\frac{12d\Psi^2}{\lambda^2}\left(d+H\right)+\frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2 \qquad \textbf{(vii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (vii) becomes,

$$v71_1(x,t)=\frac{1}{\lambda^2}\left\{6d^2\Psi^2-\frac{6d\Psi\left\{2d\Psi\sqrt{6}\mp\lambda+\sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}{\sqrt{6}}+\frac{\left\{2d\Psi\sqrt{6}\mp\lambda+\sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{4}\right\}$$

If $C_1\neq 0$ but $C_2=0$ then,

$$v71_{12}(x,t)=\frac{1}{\lambda^2}\left\{6d^2\Psi^2-\frac{6d\Psi\left\{2d\Psi\sqrt{6}\mp\lambda+\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}{\sqrt{6}}+\frac{\left\{2d\Psi\sqrt{6}\mp\lambda+\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{4}\right\}$$

Substituting Eq (4.4.20) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v81(x,t)=1+\frac{6d^2\Psi^2}{\lambda^2}-\frac{12d\Psi^2}{\lambda^2}\left(d+H\right)+\frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2 \qquad \textbf{(viii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (viii) becomes:

$$v81_1(x,t) = 1 + \frac{1}{\lambda^2}\left\{6d^2\Psi^2 - \frac{6d\Psi\left\{2d\Psi\sqrt{6} \mp \lambda i + \sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}{\sqrt{6}} + \frac{\left\{2d\Psi\sqrt{6} \mp \lambda i + \sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{4}\right\}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$u81_2(x,t) = 1 + \frac{1}{\lambda^2}\left\{6d^2\Psi^2 - \frac{6d\Psi\left\{2d\Psi\sqrt{6} \mp \lambda i + \sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}{\sqrt{6}} + \frac{\left\{2d\Psi\sqrt{6} \mp \lambda i + \sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{4}\right\}$$

Substituting Eq (4.4.21) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v91(x,t) = \frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2}{\sqrt{6}} \mp \frac{12d\lambda\delta}{\sqrt{6}} + 6d^2\Psi^2\right) + \frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta}{\sqrt{6}} \mp \frac{12\lambda^2}{\sqrt{6}} - d\Psi^2\right)(d+H) + \frac{6\Psi^2}{\lambda^2}(d+H)^2 \quad \textbf{(ix)}$$

where, $H = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (ix) becomes:

$$v91_1(x,t) = \frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2}{\sqrt{6}} \mp \frac{12d\lambda\delta}{\sqrt{6}} + 6d^2\Psi^2 + \frac{\left(\pm\frac{\lambda\delta}{\sqrt{6}} \mp \frac{12\lambda^2}{\sqrt{6}} - d\Psi^2\right)\left(2d\Psi\sqrt{6} \pm \lambda + \sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)}{2\Psi\sqrt{6}} + \frac{\left(2d\Psi\sqrt{6} \pm \lambda + \sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)^2}{4}\right\}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v91_2(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2}{\sqrt{6}}\mp\frac{12d\lambda\delta}{\sqrt{6}}+6d^2\Psi^2+\frac{\left(\pm\frac{\lambda\delta}{\sqrt{6}}\mp\frac{12\lambda^2}{\sqrt{6}}-d\Psi^2\right)\left(2d\Psi\sqrt{6}\pm\lambda+\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)}{2\Psi\sqrt{6}}\right.$$

$$\left.+\frac{\left(2d\Psi\sqrt{6}\pm\lambda+\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)^2}{4}\right\}$$

Substituting Eq (4.4.22) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v101(x,t)=\frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2\right)+\frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right)(d+H)+\frac{6\Psi^2}{\lambda^2}(d+H)^2$$

(x)

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (x) becomes:

$$v101_1(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2+\frac{\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right)\left(2d\Psi\sqrt{6}\pm\lambda i+\sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)}{2\Psi\sqrt{6}}\right.$$

$$\left.+\frac{\left(2d\Psi\sqrt{6}\pm\lambda i+\sqrt{6\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)^2}{4}\right\}$$

If $C_1\neq 0$ but $C_2=0$ then,

$$v101_2(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2+\frac{\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right)\left(2d\Psi\sqrt{6}\pm\lambda i+\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)}{2\Psi\sqrt{6}}+\frac{\left(2d\Psi\sqrt{6}\pm\lambda i+\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)^2}{4}\right\}$$

Substituting Eq (4.4.23) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v11\,1(x,t)=\left(\frac{\mu^2 d^2+\beta^2-2\mu\beta d}{\mu^2}\right)(d+H)^{-2} \qquad \textbf{(xi)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (xi) becomes:

$$u11\,1_1(x,t)=\frac{4\Psi^2\left(\mu^2 d^2+\beta^2-2\mu\beta d\right)}{\mu^2\left\{2d\Psi+\mu+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}$$

If $C_1\neq 0$ but $C_2=0$ then,

$$u11\,1_2(x,t)=\frac{4\Psi^2\left(\mu^2 d^2+\beta^2-2\mu\beta d\right)}{\mu^2\left\{2d\Psi+\mu+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.4.24) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v121(x,t) = 1 + \left(\frac{2\mu\beta d - \mu^2 d^2 - \beta^2}{\mu^2} \right) (d + H)^{-2} \qquad \textbf{(xii)}$$

where, $H = \left(\frac{G'}{G} \right) = \frac{\mu}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi} \frac{C_1 \sinh\left(\frac{\sqrt{\Omega}}{2\Psi} \xi \right) + C_2 \cosh\left(\frac{\sqrt{\Omega}}{2\Psi} \xi \right)}{C_1 \cosh\left(\frac{\sqrt{\Omega}}{2\Psi} \xi \right) + C_2 \sinh\left(\frac{\sqrt{\Omega}}{2\Psi} \xi \right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (xii) becomes:

$$v121_1(x,t) = 1 + \frac{4\Psi^2 \left(2\mu\beta d - \mu^2 d^2 - \beta^2 \right)}{\mu^2 \left\{ 2d\Psi + \mu + \sqrt{\Omega} \coth\left(\frac{\sqrt{\Omega}}{2\Psi} \xi \right) \right\}^2}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v121_2(x,t) = 1 + \frac{4\Psi^2 \left(2\mu\beta d - \mu^2 d^2 - \beta^2 \right)}{\mu^2 \left\{ 2d\Psi + \mu + \sqrt{\Omega} \tanh\left(\frac{\sqrt{\Omega}}{2\Psi} \xi \right) \right\}^2}$$

Substituting Eq (4.4.25) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v131(x,t) = \left\{ \frac{18d\mu^3 - 36\mu^2 d^2 \delta + 18\mu d^3 \delta^2 + 36\mu^3 d^3 \pm 18 d^2 \mu^3 \sqrt{6} \pm \mu^3 \sqrt{6} \mp 18\delta d^3 \mu^2 \sqrt{6} \mp 3d\delta\mu^2 \sqrt{6} \pm 3\mu\delta^2 d^2 \sqrt{6} \mp \delta^3 d^3 \sqrt{6}}{\pm \mu^3 \sqrt{6} \mp d\delta\sqrt{6} + 6\mu d} \right\} + \frac{1}{\mu^2} \left(\pm 4\delta d^3 \mu \sqrt{6} \mp 4\mu^2 d^2 \sqrt{6} - 2d\mu^2 + 4\delta\mu d^2 - 2\delta^2 d^3 - 12\mu^2 d^3 \right)$$

$$(d + H)^{-1} + \frac{1}{\mu^2} \left(\pm 2\mu^2 d^3 \sqrt{6} \mp 2\delta\mu d^4 \sqrt{6} + \mu^2 d^2 + \delta^2 d^4 - 2\delta d^3 \mu + 6\mu^2 d^4 \right) (d + H)^{-2} \qquad \textbf{(xiii)}$$

where, $H = \left(\frac{G'}{G} \right) = \frac{\mu}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi} \frac{C_1 \sinh\left(\frac{\sqrt{\Omega}}{2\Psi} \xi \right) + C_2 \cosh\left(\frac{\sqrt{\Omega}}{2\Psi} \xi \right)}{C_1 \cosh\left(\frac{\sqrt{\Omega}}{2\Psi} \xi \right) + C_2 \sinh\left(\frac{\sqrt{\Omega}}{2\Psi} \xi \right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (xiii) becomes:

$$v13\,1_1(x,t) = \frac{18d\mu^3 - 36\mu^2 d^2\delta + 18\mu d^3\delta^2 + 36\mu^3 d^3 \pm 18d^2\mu^3\sqrt{6} \pm \mu^3\sqrt{6} \mp 18\delta d^3\mu^2\sqrt{6} \mp 3d\delta\mu^2\sqrt{6}}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d}$$
$$\frac{\pm 3\mu\delta^2 d^2\sqrt{6} \mp \delta^3 d^3\sqrt{6}}{} + \frac{2\Psi\left(\pm 4\delta d^3\mu\sqrt{6} \mp 4\mu^2 d^2\sqrt{6} - 2d\mu^2 + 4\delta\mu d^2 - 2\delta^2 d^3 - 12\mu^2 d^3\right)}{\mu^2\left\{2d\Psi + \mu + \sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}$$
$$+\frac{4\Psi^2\left(\pm 2\mu^2 d^3\sqrt{6} \mp 2\delta\mu d^4\sqrt{6} + \mu^2 d^2 + \delta^2 d^4 - 2\delta d^3\mu + 6\mu^2 d^4\right)}{\mu^2\left\{2d\Psi + \mu + \sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v131_2(x,t) = \frac{18d\mu^3 - 36\mu^2 d^2\delta + 18\mu d^3\delta^2 + 36\mu^3 d^3 \pm 18d^2\mu^3\sqrt{6} \pm \mu^3\sqrt{6} \mp 18\delta d^3\mu^2\sqrt{6} \mp 3d\delta\mu^2\sqrt{6}}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d}$$
$$\frac{\pm 3\mu\delta^2 d^2\sqrt{6} \mp \delta^3 d^3\sqrt{6}}{} + \frac{2\Psi\left(\pm 4\delta d^3\mu\sqrt{6} \mp 4\mu^2 d^2\sqrt{6} - 2d\mu^2 + 4\delta\mu d^2 - 2\delta^2 d^3 - 12\mu^2 d^3\right)}{\mu^2\left\{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}$$
$$+\frac{4\Psi^2\left(\pm 2\mu^2 d^3\sqrt{6} \mp 2\delta\mu d^4\sqrt{6} + \mu^2 d^2 + \delta^2 d^4 - 2\delta d^3\mu + 6\mu^2 d^4\right)}{\mu^2\left\{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.4.26) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v14(x,t) = \left\{\frac{d\left(12\mu^3 - 36\mu^2 d\delta \pm 18d\mu^3 i\sqrt{6} \mp 18\delta d^2\mu^2 i\sqrt{6} + 18\mu d^2\delta^2 \pm 2\delta\mu^2 i\sqrt{6} - 36\mu^3 d^2 \mp 3\mu\delta^2 di\sqrt{6}\right.}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d}\right.$$
$$\left.\frac{\left.\pm\delta^3 d^2 i\sqrt{6}\right)}{}\right\} + \left\{\frac{1}{\mu^2}\left(2d\mu^2 - 4\delta\mu d^2 \mp 4\delta d^3\mu i\sqrt{6} \pm 4\mu^2 d^2 i\sqrt{6} + 2\delta^2 d^3 - 12\mu^2 d^3\right)\right\}(d+H)^{-1}$$
$$+\left\{\frac{1}{\mu^2}\left(\pm 2\mu d^4\delta i\sqrt{6} \mp 2\mu^2 d^3 i\sqrt{6} - \mu^2 d^2 - \delta^2 d^4 + 2\delta d^3\mu + 6\mu^2 d^4\right)\right\}(d+H)^{-2} \qquad \textbf{(xiv)}$$

where, $$H = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (xiv) becomes:

$$v141_1(x,t) = \frac{d\left(12\mu^3 - 36\mu^2 d\delta \pm 18d\mu^3 i\sqrt{6} \mp 18\delta d^2\mu^2 i\sqrt{6} + 18\mu d^2\delta^2 \pm 2\delta\mu^2 i\sqrt{6} - 36\mu^3 d^2 \mp 3\mu\delta^2 di\sqrt{6}}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d}$$

$$\frac{\pm\delta^3 d^2 i\sqrt{6}}{} + \frac{2\Psi\left(2d\mu^2 - 4\delta\mu d^2 \mp 4\delta d^3\mu i\sqrt{6} \pm 4\mu^2 d^2 i\sqrt{6} + 2\delta^2 d^3 - 12\mu^2 d^3\right)}{\mu^2\left\{2d\Psi + \mu + \sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}$$

$$+\frac{4\Psi^2\left(\pm 2\mu d^4\delta i\sqrt{6} \mp 2\mu^2 d^3 i\sqrt{6} - \mu^2 d^2 - \delta^2 d^4 + 2\delta d^3\mu + 6\mu^2 d^4\right)}{\mu^2\left\{2d\Psi + \mu + \sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v141_2(x,t) = \frac{d\left(12\mu^3 - 36\mu^2 d\delta \pm 18d\mu^3 i\sqrt{6} \mp 18\delta d^2\mu^2 i\sqrt{6} + 18\mu d^2\delta^2 \pm 2\delta\mu^2 i\sqrt{6} - 36\mu^3 d^2 \mp 3\mu\delta^2 di\sqrt{6}}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d}$$

$$\frac{\pm\delta^3 d^2 i\sqrt{6}}{} + \frac{2\Psi\left(2d\mu^2 - 4\delta\mu d^2 \mp 4\delta d^3\mu i\sqrt{6} \pm 4\mu^2 d^2 i\sqrt{6} + 2\delta^2 d^3 - 12\mu^2 d^3\right)}{\mu^2\left\{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}$$

$$+\frac{4\Psi^2\left(\pm 2\mu d^4\delta i\sqrt{6} \mp 2\mu^2 d^3 i\sqrt{6} - \mu^2 d^2 - \delta^2 d^4 + 2\delta d^3\mu + 6\mu^2 d^4\right)}{\mu^2\left\{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.4.27) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v151(x,t) = \frac{1}{96\lambda^2\beta^2}\left(\lambda^4 d^2 \pm \frac{24\beta d\lambda^3}{\sqrt{6}} - 12\lambda^2\mu^2 d^2 + 24\lambda^2\beta^2 + 24\lambda^2\mu\beta d \mp \frac{144\mu^2\beta d}{\sqrt{6}} \pm \frac{288\lambda\mu\beta^2}{\sqrt{6}} + 144\beta^2\mu^2\right.$$

$$\left. + 36\mu^4 d^2 - 144\mu^3 d\beta\right) - \frac{1}{48\lambda^2\beta^2}\left(\pm\frac{12\lambda^3\beta}{\sqrt{6}} \mp \frac{72\lambda\beta\mu^2}{\sqrt{6}} + 12\mu\beta\lambda^2 - 72\beta\mu^3 + d\lambda^4 - 12d\lambda^2\mu^2 + 36\mu^4 d\right)$$

$$(d+H) + \frac{1}{96\lambda^2\beta^2}\left(\lambda^2 - 6\mu^2\right)^2 (d+H)^2 \qquad \textbf{(xv)}$$

where, $$H = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi}\,\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (xv) becomes,

$$v151_1(x,t) = \frac{1}{96\lambda^2\beta^2}\left\{\lambda^4 d^2 \pm \frac{24\beta d\lambda^3}{\sqrt{6}} - 12\lambda^2\mu^2 d^2 + 24\lambda^2\beta^2 + \lambda^2\mu\beta d \mp \frac{144\mu^2\beta d}{\sqrt{6}} \pm \frac{288\lambda\mu\beta^2}{\sqrt{6}} + 144\beta^2\mu^2 \right.$$
$$+36\mu^4 d^2 - 144\mu^3 d\beta - \left(\pm\frac{12\lambda^3\beta}{\sqrt{6}} \mp \frac{72\lambda\beta\mu^2}{\sqrt{6}} + 12\mu\beta\lambda^2 - 72\beta\mu^3 + d\lambda^4 - 12d\lambda^2\mu^2 + 36\mu^4 d\right)$$
$$\left.\left(\frac{2d\Psi + \mu + \sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{\Psi}\right) + \frac{\left(\lambda^2 - 6\mu^2\right)^2\left(2d\Psi + \mu + \sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)^2}{4\Psi^2}\right\}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v151_2(x,t) = \frac{1}{96\lambda^2\beta^2}\left\{\lambda^4 d^2 \pm \frac{24\beta d\lambda^3}{\sqrt{6}} - 12\lambda^2\mu^2 d^2 + 24\lambda^2\beta^2 + \lambda^2\mu\beta d \mp \frac{144\mu^2\beta d}{\sqrt{6}} \pm \frac{288\lambda\mu\beta^2}{\sqrt{6}} + 144\beta^2\mu^2 \right.$$
$$+36\mu^4 d^2 - 144\mu^3 d\beta - \left(\pm\frac{12\lambda^3\beta}{\sqrt{6}} \mp \frac{72\lambda\beta\mu^2}{\sqrt{6}} + 12\mu\beta\lambda^2 - 72\beta\mu^3 + d\lambda^4 - 12d\lambda^2\mu^2 + 36\mu^4 d\right)$$
$$\left.\left(\frac{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{\Psi}\right) + \frac{\left(\lambda^2 - 6\mu^2\right)^2\left(2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)^2}{4\Psi^2}\right\}$$

Substituting Eq (4.4.28) into Eq (4.2.3), along with Eq (3.1.7) and simplifying, yields the following travelling wave solutions,

$$\therefore v16\,1(x,t) = \frac{1}{96\lambda^2\beta^2}\left(\lambda^4 d^2 \mp \frac{24\beta d\lambda^3 i}{\sqrt{6}} + 12\lambda^2\mu^2 d^2 + 72\lambda^2\beta^2 - 24\lambda^2\mu\beta d \mp \frac{144\lambda\mu^2\beta di}{\sqrt{6}} \pm \frac{288\lambda\mu\beta^2 i}{\sqrt{6}} + 144\beta^2\mu^2\right.$$
$$\left. + 36\mu^4 d^2 - 144\mu^3 d\beta\right) - \frac{1}{48\lambda^2\beta^2}\left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}} \mp \frac{72\lambda\beta\mu^2 i}{\sqrt{6}} - 12\mu\beta\lambda^2 - 72\beta\mu^3 + d\lambda^4 + 12d\lambda^2\mu^2 + 36\mu^4 d\right)$$
$$(d+H) + \frac{1}{96\lambda^2\beta^2}\left(\lambda^2 + 6\mu^2\right)^2 (d+H)^2 \qquad \textbf{(xvi)}$$

where, $H = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (xvi) becomes:

$$v16\,1_1(x,t) = \frac{1}{96\lambda^2\beta^2}\left\{\lambda^4 d^2 \mp \frac{24\beta d\lambda^3 i}{\sqrt{6}} + 12\lambda^2\mu^2 d^2 + 72\lambda^2\beta^2 - 24\lambda^2\mu\beta d \mp \frac{144\lambda\mu^2\beta di}{\sqrt{6}} \pm \frac{288\lambda\mu\beta^2 i}{\sqrt{6}} + 144\beta^2\mu^2 \right.$$
$$+36\mu^4 d^2 - 144\mu^3 d\beta - \left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}} \mp \frac{72\lambda\beta\mu^2 i}{\sqrt{6}} - 12\mu\beta\lambda^2 - 72\beta\mu^3 + d\lambda^4 + 12d\lambda^2\mu^2 + 36\mu^4 d\right)$$
$$\left.\left(\frac{2d\Psi + \mu + \sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{\Psi}\right) + \frac{\left(\lambda^2+6\mu^2\right)^2\left(2d\Psi + \mu + \sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)^2}{4\Psi^2}\right\}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v161_2(x,t) = \frac{1}{96\lambda^2\beta^2}\left\{\lambda^4 d^2 \mp \frac{24\beta d\lambda^3 i}{\sqrt{6}} + 12\lambda^2\mu^2 d^2 + 72\lambda^2\beta^2 - 24\lambda^2\mu\beta d \mp \frac{144\lambda\mu^2\beta di}{\sqrt{6}} \pm \frac{288\lambda\mu\beta^2 i}{\sqrt{6}} + 144\beta^2\mu^2 \right.$$
$$+36\mu^4 d^2 - 144\mu^3 d\beta - \left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}} \mp \frac{72\lambda\beta\mu^2 i}{\sqrt{6}} - 12\mu\beta\lambda^2 - 72\beta\mu^3 + d\lambda^4 + 12d\lambda^2\mu^2 + 36\mu^4 d\right)$$
$$\left.\left(\frac{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{\Psi}\right) + \frac{\left(\lambda^2+6\mu^2\right)^2\left(2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)^2}{4\Psi^2}\right\}$$

Trigonometric form of travelling wave solutions

Substituting Eq (4.4.13) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore v12(x,t) = \frac{3}{8} \mp \frac{6\Psi}{\lambda\sqrt{6}}(d+H) + \frac{6\Psi^2}{\lambda^2}(d+H)^2 \mp \frac{\lambda}{16\Psi\sqrt{6}}(d+H)^{-1} + \frac{\lambda^2}{1536\Psi^2}(d+H)^{-2} \quad \textbf{(i)}$$

where, $H = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{\sqrt{-\Omega}}{2\Psi}\,\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) + C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) + C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (i) becomes:

$$v12_1(x,t) = \frac{3}{8} \mp \frac{3\sqrt{\Omega}i}{\lambda\sqrt{6}} \cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) - \frac{3\Omega}{2\lambda^2}\cot^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) \mp \frac{\lambda}{8\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

$$+ \frac{\lambda^2}{384\Omega i^2 \cot^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v12_2(x,t) = \frac{3}{8} \pm \frac{3\sqrt{\Omega}i}{\lambda\sqrt{6}} \tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) - \frac{3\Omega}{2\lambda^2}\tan^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) \pm \frac{\lambda}{8\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

$$+ \frac{\lambda^2}{384\Omega i^2 \tan^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

Substituting Eq (4.4.14) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions:

$$\therefore v22(x,t) = \frac{5}{8} \mp \frac{6\Psi}{\lambda\sqrt{6}}(d+H) + \frac{6\Psi^2}{\lambda^2}(d+H)^2 \pm \frac{\lambda}{16\Psi\sqrt{6}}(d+H)^{-1} + \frac{\lambda^2}{1536\Psi^2}(d+H)^{-2} \quad \textbf{(ii)}$$

where, $H = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{\sqrt{-\Omega}}{2\Psi} \frac{-C_1 \sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) + C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) + C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (ii) becomes:

$$v22_1(x,t) = \frac{5}{8} \mp \frac{3\sqrt{\Omega}i}{\lambda\sqrt{6}} \cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) - \frac{3\Omega}{2\lambda^2}\cot^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) \pm \frac{\lambda}{8\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)} + \frac{\lambda^2}{384\Omega i^2 \cot^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v22_2(x,t)=\frac{5}{8}\mp\frac{3\sqrt{\Omega}i}{\lambda\sqrt{6}}\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+\frac{3\Omega}{2\lambda^2}^{2}\tan^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\mp\frac{\lambda}{8\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}+\frac{\lambda^2}{384\Omega i^2\cot^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

Substituting Eq (4.4.15) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore v32(x,t)=\frac{1}{d^2}(d+H)^2 \qquad \textbf{(iii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (iii) becomes:

$$v32_1(x,t)=\frac{\left\{2d\Psi\sqrt{6}\pm\lambda+\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

If $C_1\neq 0$ but $C_2=0$ then,

$$v32_2(x,t)=\frac{\left\{2d\Psi\sqrt{6}\pm\lambda-\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

Substituting Eq (4.4.16) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore v42(x,t)=1-\frac{1}{d^2}(d+H)^2 \qquad \textbf{(iv)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (iv) becomes:

$$v42_1(x,t)=1-\frac{\left\{2d\Psi\sqrt{6}\pm\lambda i+\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

If $C_1\neq 0$ but $C_2=0$ then,

$$v42_2(x,t)=1-\frac{\left\{2d\Psi\sqrt{6}\pm\lambda i-\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

Substituting Eq (4.4.17) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore v52(x,t)=1-\frac{2}{d}(d+H)+\frac{1}{d^2}(d+H)^2 \qquad \textbf{(v)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (v) becomes:

$$v52_1(x,t)=1-\frac{2d\Psi\sqrt{6}\mp\lambda+\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{d\Psi\sqrt{6}}+\frac{\left\{2d\Psi\sqrt{6}\mp\lambda+\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v52_2(x,t) = 1 - \frac{2d\Psi\sqrt{6} \mp \lambda - \sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{d\Psi\sqrt{6}} + \frac{\left\{2d\Psi\sqrt{6} \mp \lambda - \sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

Substituting Eq (4.4.18) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore v62(x,t) = \frac{2}{d}(d+H) - \frac{1}{d^2}(d+H)^2 \qquad \textbf{(vi)}$$

where, $H = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{\sqrt{-\Omega}}{2\Psi} \frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) + C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) + C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (vi) becomes:

$$v62_1(x,t) = \frac{2d\Psi\sqrt{6} \mp \lambda i + \sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{d\Psi\sqrt{6}} - \frac{\left\{2d\Psi\sqrt{6} \mp \lambda i + \sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v62_2(x,t) = \frac{2d\Psi\sqrt{6} \mp \lambda i - \sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{d\Psi\sqrt{6}} - \frac{\left\{2d\Psi\sqrt{6} \mp \lambda i - \sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2}$$

Substituting Eq (4.4.19) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore v72(x,t) = \frac{6d^2\Psi^2}{\lambda^2} - \frac{12d\Psi^2}{\lambda^2}(d+H) + \frac{6\Psi^2}{\lambda^2}(d+H)^2 \qquad \textbf{(vii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (vii) becomes,

$$v72_1(x,t)=\frac{1}{\lambda^2}\left\{6d^2\Psi^2-\frac{6d\Psi\left\{2d\Psi\sqrt{6}\mp\lambda+\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}}{\sqrt{6}}+\frac{\left\{2d\Psi\sqrt{6}\mp\lambda+\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{4}\right\}$$

If $C_1\neq 0$ but $C_2=0$ then,

$$v72_2(x,t)=\frac{1}{\lambda^2}\left\{6d^2\Psi^2-\frac{6d\Psi\left\{2d\Psi\sqrt{6}\mp\lambda-\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}}{\sqrt{6}}+\frac{\left\{2d\Psi\sqrt{6}\mp\lambda-\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{4}\right\}$$

Substituting Eq (4.4.20) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore v82(x,t)=1+\frac{6d^2\Psi^2}{\lambda^2}-\frac{12d\Psi^2}{\lambda^2}\left(d+H\right)+\frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2 \qquad \textbf{(viii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (viii) becomes,

$$v82_1(x,t)=1+\frac{1}{\lambda^2}\left\{6d^2\Psi^2-\frac{6d\Psi\left\{2d\Psi\sqrt{6}\mp\lambda i+\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}}{\sqrt{6}}+\frac{\left\{2d\Psi\sqrt{6}\mp\lambda i+\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{4}\right\}$$

If $C_1\neq 0$ but $C_2=0$ then,

$$v82_2(x,t)=1+\frac{1}{\lambda^2}\left\{6d^2\Psi^2-\frac{6d\Psi\left\{2d\Psi\sqrt{6}\mp\lambda i-\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}}{\sqrt{6}}+\frac{\left\{2d\Psi\sqrt{6}\mp\lambda i-\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{4}\right\}$$

Substituting Eq (4.4.21) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore v92(x,t)=\frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2}{\sqrt{6}}\mp\frac{12d\lambda\delta}{\sqrt{6}}+6d^2\Psi^2\right)+\frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta}{\sqrt{6}}\mp\frac{12\lambda^2}{\sqrt{6}}-d\Psi^2\right)(d+H)+\frac{6\Psi^2}{\lambda^2}(d+H)^2$$

(ix)

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (ix) becomes:

$$v92_1(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2}{\sqrt{6}}\mp\frac{12d\lambda\delta}{\sqrt{6}}+6d^2\Psi^2+\frac{\left(\pm\frac{\lambda\delta}{\sqrt{6}}\mp\frac{12\lambda^2}{\sqrt{6}}-d\Psi^2\right)\left(2d\Psi\sqrt{6}\pm\lambda+\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)}{2\Psi\sqrt{6}}\right.$$
$$\left.+\frac{\left(2d\Psi\sqrt{6}\pm\lambda+\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)^2}{4}\right\}$$

If $C_1\neq 0$ but $C_2=0$ then,

$$v92_2(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2}{\sqrt{6}}\mp\frac{12d\lambda\delta}{\sqrt{6}}+6d^2\Psi^2+\frac{\left(\pm\frac{\lambda\delta}{\sqrt{6}}\mp\frac{12\lambda^2}{\sqrt{6}}-d\Psi^2\right)\left(2d\Psi\sqrt{6}\pm\lambda-\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)}{2\Psi\sqrt{6}}\right.$$
$$\left.+\frac{\left(2d\Psi\sqrt{6}\pm\lambda-\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)^2}{4}\right\}$$

Substituting Eq (4.4.22) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore u102(x,t)=\frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2\right)+\frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right)(d+H)+\frac{6\Psi^2}{\lambda^2}(d+H)^2 \qquad \textbf{(x)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (x) becomes,

$$v102_1(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2+\frac{\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right)\left(2d\Psi\sqrt{6}\pm\lambda i+\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)}{2\Psi\sqrt{6}}+\frac{\left(2d\Psi\sqrt{6}\pm\lambda i+\sqrt{6\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)^2}{4}\right\}$$

If $C_1\neq 0$ but $C_2=0$ then,

$$v102_2(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2+\frac{\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right)\left(2d\Psi\sqrt{6}\pm\lambda i-\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)}{2\Psi\sqrt{6}}+\frac{\left(2d\Psi\sqrt{6}\pm\lambda i-\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)^2}{4}\right\}$$

Substituting Eq (4.4.23) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore v112(x,t)=\left(\frac{\mu^2d^2+\beta^2-2\mu\beta d}{\mu^2}\right)(d+H)^{-2} \qquad \textbf{(xi)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq 0$, then the above equation (xi) becomes:

$$v112_1(x,t)=\frac{4\Psi^2\left(\mu^2d^2+\beta^2-2\mu\beta d\right)}{\mu^2\left\{2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}$$

If $C_1\neq 0$ but $C_2=0$ then,

$$v112_2(x,t)=\frac{4\Psi^2\left(\mu^2d^2+\beta^2-2\mu\beta d\right)}{\mu^2\left\{2d\Psi+\mu-\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.4.24) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore v122(x,t)=1+\left(\frac{2\mu\beta d-\mu^2d^2-\beta^2}{\mu^2}\right)(d+H)^{-2} \qquad \textbf{(xii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (xii) becomes:

$$v122_1(x,t) = 1 + \frac{4\Psi^2\left(2\mu\beta d - \mu^2 d^2 - \beta^2\right)}{\mu^2\left\{2d\Psi + \mu + \sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v122_2(x,t) = 1 + \frac{4\Psi^2\left(2\mu\beta d - \mu^2 d^2 - \beta^2\right)}{\mu^2\left\{2d\Psi + \mu - \sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.4.25) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore v132(x,t) = \left\{\frac{18d\mu^3 - 36\mu^2 d^2\delta + 18\mu d^3\delta^2 + 36\mu^3 d^3 \pm 18d^2\mu^3\sqrt{6} \pm \mu^3\sqrt{6} \mp 18\delta d^3\mu^2\sqrt{6} \mp 3d\delta\mu^2\sqrt{6} \pm 3\mu\delta^2 d^2\sqrt{6} \mp \delta^3 d^3\sqrt{6}}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d}\right\} + \frac{1}{\mu^2}\left(\pm 4\delta d^3\mu\sqrt{6} \mp 4\mu^2 d^2\sqrt{6} - 2d\mu^2 + 4\delta\mu d^2 - 2\delta^2 d^3 - 12\mu^2 d^3\right)$$
$$(d+H)^{-1} + \frac{1}{\mu^2}\left(\pm 2\mu^2 d^3\sqrt{6} \mp 2\delta\mu d^4\sqrt{6} + \mu^2 d^2 + \delta^2 d^4 - 2\delta d^3\mu + 6\mu^2 d^4\right)(d+H)^{-2} \qquad \textbf{(xiii)}$$

where, $$H = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{\sqrt{-\Omega}}{2\Psi}\,\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) + C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) + C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (xiii) becomes:

$$v132_1(x,t) = \frac{18d\mu^3 - 36\mu^2 d^2\delta + 18\mu d^3\delta^2 + 36\mu^3 d^3 \pm 18d^2\mu^3\sqrt{6} \pm \mu^3\sqrt{6} \mp 18\delta d^3\mu^2\sqrt{6} \mp 3d\delta\mu^2\sqrt{6}}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d}$$

$$\frac{\pm 3\mu\delta^2 d^2\sqrt{6} \mp \delta^3 d^3\sqrt{6}}{} + \frac{2\Psi\left(\pm 4\delta d^3\mu\sqrt{6} \mp 4\mu^2 d^2\sqrt{6} - 2d\mu^2 + 4\delta\mu d^2 - 2\delta^2 d^3 - 12\mu^2 d^3\right)}{\mu^2\left\{2d\Psi + \mu + \sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}}$$

$$\frac{4\Psi^2\left(\pm 2\mu^2 d^3\sqrt{6} \mp 2\delta\mu d^4\sqrt{6} + \mu^2 d^2 + \delta^2 d^4 - 2\delta d^3\mu + 6\mu^2 d^4\right)}{\mu^2\left\{2d\Psi + \mu + \sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v132_2(x,t) = \frac{18d\mu^3 - 36\mu^2 d^2\delta + 18\mu d^3\delta^2 + 36\mu^3 d^3 \pm 18d^2\mu^3\sqrt{6} \pm \mu^3\sqrt{6} \mp 18\delta d^3\mu^2\sqrt{6} \mp 3d\delta\mu^2\sqrt{6}}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d}$$

$$\frac{\pm 3\mu\delta^2 d^2\sqrt{6} \mp \delta^3 d^3\sqrt{6}}{} + \frac{2\Psi\left(\pm 4\delta d^3\mu\sqrt{6} \mp 4\mu^2 d^2\sqrt{6} - 2d\mu^2 + 4\delta\mu d^2 - 2\delta^2 d^3 - 12\mu^2 d^3\right)}{\mu^2\left\{2d\Psi + \mu - \sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}}$$

$$\frac{4\Psi^2\left(\pm 2\mu^2 d^3\sqrt{6} \mp 2\delta\mu d^4\sqrt{6} + \mu^2 d^2 + \delta^2 d^4 - 2\delta d^3\mu + 6\mu^2 d^4\right)}{\mu^2\left\{2d\Psi + \mu - \sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.4.26) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore v142(x,t) = \left\{\frac{d\left(12\mu^3 - 36\mu^2 d\delta \pm 18d\mu^3 i\sqrt{6} \mp 18\delta d^2\mu^2 i\sqrt{6} + 18\mu d^2\delta^2 \pm 2\delta\mu^2 i\sqrt{6} - 36\mu^3 d^2 \mp 3\mu\delta^2 di\sqrt{6}\right.}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d}\right.$$

$$\left.\frac{\left.\pm\delta^3 d^2 i\sqrt{6}\right)}{}\right\} + \left\{\frac{1}{\mu^2}\left(2d\mu^2 - 4\delta\mu d^2 \mp 4\delta d^3\mu i\sqrt{6} \pm 4\mu^2 d^2 i\sqrt{6} + 2\delta^2 d^3 - 12\mu^2 d^3\right)\right\}(d+H)^{-1}$$

$$+\left\{\frac{1}{\mu^2}\left(\pm 2\mu d^4\delta i\sqrt{6} \mp 2\mu^2 d^3 i\sqrt{6} - \mu^2 d^2 - \delta^2 d^4 + 2\delta d^3\mu + 6\mu^2 d^4\right)\right\}(d+H)^{-2} \qquad \textbf{(xiv)}$$

where, $H = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{\sqrt{-\Omega}}{2\Psi}\,\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) + C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) + C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1 = 0$ but $C_2 \neq 0$, then the above equation (xiv) becomes:

$$v142_1(x,t)=\frac{d\left(12\mu^3-36\mu^2d\delta\pm18d\mu^3i\sqrt{6}\mp18\delta d^2\mu^2i\sqrt{6}+18\mu d^2\delta^2\pm2\delta\mu^2i\sqrt{6}-36\mu^3d^2\mp3\mu\delta^2di\sqrt{6}\right.}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d}$$

$$\frac{\left.\pm\delta^3d^2i\sqrt{6}\right)}{}+\frac{2\Psi\left(2d\mu^2-4\delta\mu d^2\mp4\delta d^3\mu i\sqrt{6}\pm4\mu^2d^2i\sqrt{6}+2\delta^2d^3-12\mu^2d^3\right)}{\mu^2\left\{2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}}$$

$$+\frac{4\Psi^2\left(\pm2\mu d^4\delta i\sqrt{6}\mp2\mu^2d^3i\sqrt{6}-\mu^2d^2-\delta^2d^4+2\delta d^3\mu+6\mu^2d^4\right)}{\mu^2\left\{2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}$$

If $C_1\neq0$ but $C_2=0$ then,

$$v142_2(x,t)=\frac{d\left(12\mu^3-36\mu^2d\delta\pm18d\mu^3i\sqrt{6}\mp18\delta d^2\mu^2i\sqrt{6}+18\mu d^2\delta^2\pm2\delta\mu^2i\sqrt{6}-36\mu^3d^2\mp3\mu\delta^2di\sqrt{6}\right.}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d}$$

$$\frac{\left.\pm\delta^3d^2i\sqrt{6}\right)}{}+\frac{2\Psi\left(2d\mu^2-4\delta\mu d^2\mp4\delta d^3\mu i\sqrt{6}\pm4\mu^2d^2i\sqrt{6}+2\delta^2d^3-12\mu^2d^3\right)}{\mu^2\left\{2d\Psi+\mu-\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}}$$

$$+\frac{4\Psi^2\left(\pm2\mu d^4\delta i\sqrt{6}\mp2\mu^2d^3i\sqrt{6}-\mu^2d^2-\delta^2d^4+2\delta d^3\mu+6\mu^2d^4\right)}{\mu^2\left\{2d\Psi+\mu-\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.4.27) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore v152(x,t)=\frac{1}{96\lambda^2\beta^2}\left(\lambda^4d^2\pm\frac{24\beta d\lambda^3}{\sqrt{6}}-12\lambda^2\mu^2d^2+24\lambda^2\beta^2+24\lambda^2\mu\beta d\mp\frac{144\mu^2\beta d}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2}{\sqrt{6}}+144\beta^2\mu^2\right.$$
$$\left.+36\mu^4d^2-144\mu^3d\beta\right)-\frac{1}{48\lambda^2\beta^2}\left(\pm\frac{12\lambda^3\beta}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2}{\sqrt{6}}+12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4-12d\lambda^2\mu^2+36\mu^4d\right)$$
$$(d+H)+\frac{1}{96\lambda^2\beta^2}\left(\lambda^2-6\mu^2\right)^2(d+H)^2 \qquad \textbf{(xv)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$

If $C_1=0$ but $C_2\neq0$, then the above equation (xv) becomes:

$$v152_1(x,t)=\frac{1}{96\lambda^2\beta^2}\left\{\lambda^4d^2\pm\frac{24\beta d\lambda^3}{\sqrt{6}}-12\lambda^2\mu^2d^2+24\lambda^2\beta^2+\lambda^2\mu\beta d\mp\frac{144\mu^2\beta d}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2}{\sqrt{6}}+144\beta^2\mu^2\right.$$
$$+36\mu^4d^2-144\mu^3d\beta-\left(\pm\frac{12\lambda^3\beta}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2}{\sqrt{6}}+12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4-12d\lambda^2\mu^2+36\mu^4d\right)$$
$$\left.\left(\frac{2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{\Psi}\right)+\frac{\left(\lambda^2-6\mu^2\right)^2\left(2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)^2}{4\Psi^2}\right\}$$

If $C_1\neq 0$ but $C_2=0$ then,

$$v152_2(x,t)=\frac{1}{96\lambda^2\beta^2}\left\{\lambda^4d^2\pm\frac{24\beta d\lambda^3}{\sqrt{6}}-12\lambda^2\mu^2d^2+24\lambda^2\beta^2+\lambda^2\mu\beta d\mp\frac{144\mu^2\beta d}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2}{\sqrt{6}}+144\beta^2\mu^2\right.$$
$$+36\mu^4d^2-144\mu^3d\beta-\left(\pm\frac{12\lambda^3\beta}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2}{\sqrt{6}}+12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4-12d\lambda^2\mu^2+36\mu^4d\right)$$
$$\left.\left(\frac{2d\Psi+\mu+\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{\Psi}\right)+\frac{\left(\lambda^2-6\mu^2\right)^2\left(2d\Psi+\mu+\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)^2}{4\Psi^2}\right\}$$

Substituting Eq (4.4.28) into Eq (4.2.3), along with Eq (3.1.8) and simplifying, yields the following travelling wave solutions,

$$\therefore v162(x,t)=\frac{1}{96\lambda^2\beta^2}\left(\lambda^4d^2\mp\frac{24\beta d\lambda^3 i}{\sqrt{6}}+12\lambda^2\mu^2d^2+72\lambda^2\beta^2-24\lambda^2\mu\beta d\mp\frac{144\lambda\mu^2\beta di}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2 i}{\sqrt{6}}+144\beta^2\mu^2\right.$$
$$\left.+36\mu^4d^2-144\mu^3d\beta\right)-\frac{1}{48\lambda^2\beta^2}\left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2 i}{\sqrt{6}}-12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4+12d\lambda^2\mu^2+36\mu^4d\right)$$
$$(d+H)+\frac{1}{96\lambda^2\beta^2}\left(\lambda^2+6\mu^2\right)^2(d+H)^2 \qquad \textbf{(xvi)}$$

$$\text{where, } H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\,\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}$$

If $C_1=0$ but $C_2\neq 0$, then the above equation (xvi) becomes:

$$v162_1(x,t)=\frac{1}{96\lambda^2\beta^2}\Bigg\{\lambda^4d^2\mp\frac{24\beta d\lambda^3 i}{\sqrt{6}}+12\lambda^2\mu^2d^2+72\lambda^2\beta^2-24\lambda^2\mu\beta d\mp\frac{144\lambda\mu^2\beta di}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2 i}{\sqrt{6}}+144\beta^2\mu^2$$
$$+36\mu^4d^2-144\mu^3d\beta-\left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2 i}{\sqrt{6}}-12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4+12d\lambda^2\mu^2+36\mu^4 d\right)$$
$$\left(\frac{2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{\Psi}\right)+\frac{\left(\lambda^2+6\mu^2\right)^2\left(2d\Psi+\mu+\sqrt{\Omega}i\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)^2}{4\Psi^2}\Bigg\}$$

If $C_1 \neq 0$ but $C_2 = 0$ then,

$$v162_2(x,t)=\frac{1}{96\lambda^2\beta^2}\Bigg\{\lambda^4d^2\mp\frac{24\beta d\lambda^3 i}{\sqrt{6}}+12\lambda^2\mu^2d^2+72\lambda^2\beta^2-24\lambda^2\mu\beta d\mp\frac{144\lambda\mu^2\beta di}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2 i}{\sqrt{6}}+144\beta^2\mu^2$$
$$+36\mu^4d^2-144\mu^3d\beta-\left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2 i}{\sqrt{6}}-12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4+12d\lambda^2\mu^2+36\mu^4 d\right)$$
$$\left(\frac{2d\Psi+\mu-\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{\Psi}\right)+\frac{\left(\lambda^2+6\mu^2\right)^2\left(2d\Psi+\mu-\sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)^2}{4\Psi^2}\Bigg\}$$

Rational form travelling waves

Substituting Eq (4.4.13) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions,

$$\therefore v13(x,t)=\frac{3}{8}\mp\frac{6\Psi}{\lambda\sqrt{6}}\left(d+H\right)+\frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2\mp\frac{\lambda}{16\Psi\sqrt{6}}\left(d+H\right)^{-1}+\frac{\lambda^2}{1536\Psi^2}\left(d+H\right)^{-2} \qquad \textbf{(i)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (i) becomes,

$$v13_1(x,t)=\frac{3}{8}\mp\frac{6\Psi C_2}{\lambda\sqrt{6}\left(C_1+C_2\xi\right)}+\frac{6\Psi^2C_2^2}{\lambda^2\left(C_1+C_2\xi\right)^2}\mp\frac{\lambda\left(C_1+C_2\xi\right)}{16\Psi\sqrt{6}C_2}+\frac{\lambda^2\left(C_1+C_2\xi\right)^{2\;-2}}{1536\Psi^2C_2^2}$$

Substituting Eq (4.4.14) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions,

$$\therefore v23(x,t)=\frac{5}{8}\mp\frac{6\Psi}{\lambda\sqrt{6}}\left(d+H\right)+\frac{6\Psi^{2}}{\lambda^{2}}\left(d+H\right)^{2}\pm\frac{\lambda}{16\Psi\sqrt{6}}\left(d+H\right)^{-1}+\frac{\lambda^{2}}{1536\Psi^{2}}\left(d+H\right)^{-2} \quad \textbf{(ii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (ii) becomes,

$$v23_1(x,t)=\frac{5}{8}\mp\frac{6\Psi C_2}{\lambda\sqrt{6}\left(C_1+C_2\xi\right)}+\frac{6\Psi^2C_2^2}{\lambda^2\left(C_1+C_2\xi\right)^2}\pm\frac{\lambda\left(C_1+C_2\xi\right)}{16\Psi\sqrt{6}C_2}+\frac{\lambda^2\left(C_1+C_2\xi\right)^2}{1536\Psi^2C_2^2}$$

Substituting Eq (4.4.15) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions,

$$\therefore v33(x,t)=\frac{1}{d^2}\left(d+H\right)^2 \quad \textbf{(iii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (iii) becomes,

$$v33_1(x,t)=\frac{\left\{2d\Psi\sqrt{6}\left(C_1+C_2\xi\right)\pm\lambda\left(C_1+C_2\xi\right)+2\Psi\sqrt{6}C_2\right\}^2}{24d^2\Psi^2\left(C_1+C_2\xi\right)^2}$$

Substituting Eq (4.4.16) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions,

$$\therefore v43(x,t)=1-\frac{1}{d^2}\left(d+H\right)^2 \quad \textbf{(iv)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (iv) becomes,

$$v43_1(x,t)=1-\frac{\left\{2d\Psi\sqrt{6}\left(C_1+C_2\xi\right)\pm\lambda i\left(C_1+C_2\xi\right)+2\Psi\sqrt{6}C_2\right\}^2}{24d^2\Psi^2\left(C_1+C_2\xi\right)^2}$$

Substituting Eq (4.4.17) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions,

$$\therefore v53(x,t)=1-\frac{2}{d}\left(d+H\right)+\frac{1}{d^2}\left(d+H\right)^2 \qquad \textbf{(v)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (v) becomes,

$$v53_1(x,t)=1-\frac{2d\Psi\sqrt{6}\left(C_1+C_2\xi\right)\mp\lambda\left(C_1+C_2\xi\right)+2\Psi\sqrt{6}C_2}{d\Psi\sqrt{6}\left(C_1+C_2\xi\right)}+\frac{\left\{2d\Psi\sqrt{6}\left(C_1+C_2\xi\right)\mp\lambda\left(C_1+C_2\xi\right)+2\Psi\sqrt{6}C_2\right\}^2}{24d^2\Psi^2\left(C_1+C_2\xi\right)^2}$$

Substituting Eq (4.4.18) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions,

$$\therefore v63(x,t)=\frac{2}{d}\left(d+H\right)-\frac{1}{d^2}\left(d+H\right)^2 \qquad \textbf{(vi)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (vi) becomes,

$$v63_1(x,t)=\frac{2d\Psi\sqrt{6}\left(C_1+C_2\xi\right)\mp\lambda i\left(C_1+C_2\xi\right)+2\Psi\sqrt{6}C_2}{d\Psi\sqrt{6}\left(C_1+C_2\xi\right)}-\frac{\left\{2d\Psi\sqrt{6}\left(C_1+C_2\xi\right)\mp\lambda i\left(C_1+C_2\xi\right)+2\Psi\sqrt{6}C_2\right\}^2}{24d^2\Psi^2\left(C_1+C_2\xi\right)^2}$$

Substituting Eq (4.4.19) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions,

$$\therefore v73(x,t) = \frac{6d^2\Psi^2}{\lambda^2} - \frac{12d\Psi^2}{\lambda^2}\left(d+H\right) + \frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2 \qquad \textbf{(vii)}$$

where, $H = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{C_2}{C_1 + C_2\xi}$,

then the above equation (vii) becomes,

$$v73_1(x,t) = \frac{1}{\lambda^2}\left(6d^2\Psi^2 - \frac{6d\Psi\left\{2d\Psi\sqrt{6}\left(C_1+C_2\xi\right) \mp \lambda\left(C_1+C_2\xi\right) + 2\Psi\sqrt{6}C_2\right\}}{\sqrt{6}\left(C_1+C_2\xi\right)} + \frac{\left\{2d\Psi\sqrt{6}\left(C_1+C_2\xi\right) \mp \lambda\left(C_1+C_2\xi\right) + 2\Psi\sqrt{6}C_2\right\}^2}{4\left(C_1+C_2\xi\right)^2} \right)$$

Substituting Eq (4.4.20) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions,

$$\therefore v83(x,t) = 1 + \frac{6d^2\Psi^2}{\lambda^2} - \frac{12d\Psi^2}{\lambda^2}\left(d+H\right) + \frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2 \qquad \textbf{(viii)}$$

where, $H = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{C_2}{C_1 + C_2\xi}$,

then the above equation (viii) becomes,

$$v83_1(x,t) = 1 + \frac{1}{\lambda^2}\left(6d^2\Psi^2 - \frac{6d\Psi\left\{2d\Psi\sqrt{6}\left(C_1+C_2\xi\right) \mp \lambda i\left(C_1+C_2\xi\right) + 2\Psi\sqrt{6}C_2\right\}}{\sqrt{6}\left(C_1+C_2\xi\right)} + \frac{\left\{2d\Psi\sqrt{6}\left(C_1+C_2\xi\right) \mp \lambda i\left(C_1+C_2\xi\right) + 2\Psi\sqrt{6}C_2\right\}^2}{4\left(C_1+C_2\xi\right)^2} \right)$$

Substituting Eq (4.4.21) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions

$$\therefore v93(x,t)=\frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2}{\sqrt{6}}\mp\frac{12d\lambda\delta}{\sqrt{6}}+6d^2\Psi^2\right)+\frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta}{\sqrt{6}}\mp\frac{12\lambda^2}{\sqrt{6}}-d\Psi^2\right)(d+H)+\frac{6\Psi^2}{\lambda^2}(d+H)^2 \quad \textbf{(ix)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (ix) becomes,

$$v93_1(x,t)=\frac{1}{\lambda^2}\Bigg\{\pm\frac{12d\lambda^2}{\sqrt{6}}\mp\frac{12d\lambda\delta}{\sqrt{6}}+6d^2\Psi^2+\left(\pm\frac{\lambda\delta}{\sqrt{6}}\mp\frac{12\lambda^2}{\sqrt{6}}-d\Psi^2\right)$$
$$\left(\frac{2d\Psi\sqrt{6}(C_1+C_2\xi)\pm\lambda(C_1+C_2\xi)+2\Psi\sqrt{6}C_2}{2\Psi\sqrt{6}(C_1+C_2\xi)}\right)+\frac{\left(2d\Psi\sqrt{6}(C_1+C_2\xi)\pm\lambda(C_1+C_2\xi)+2\Psi\sqrt{6}C_2\right)^2}{4(C_1+C_2\xi)^2}\Bigg\}$$

Substituting Eq (4.4.22) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions

$$\therefore v103(x,t)=\frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2\right)+\frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right)(d+H)+\frac{6\Psi^2}{\lambda^2}(d+H)^2 \quad \textbf{(x)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (x) becomes,

$$v103_1(x,t)=\frac{1}{\lambda^2}\Bigg\{\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2+\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right)$$
$$\left(\frac{2d\Psi\sqrt{6}(C_1+C_2\xi)\pm\lambda i(C_1+C_2\xi)+2\Psi\sqrt{6}C_2}{2\Psi\sqrt{6}(C_1+C_2\xi)}\right)+\frac{\left(2d\Psi\sqrt{6}(C_1+C_2\xi)\pm\lambda i(C_1+C_2\xi)+2\Psi\sqrt{6}C_2\right)^2}{4(C_1+C_2\xi)^2}\Bigg\}$$

Substituting Eq (4.4.23) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions,

$$\therefore v113(x,t)=\left(\frac{\mu^2 d^2+\beta^2-2\mu\beta d}{\mu^2}\right)(d+H)^{-2} \quad \textbf{(xi)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (xi) becomes,

$$v113_1(x,t)=\frac{4\Psi^2\left(\mu^2d^2+\beta^2-2\mu\beta d\right)\left(C_1+C_2\xi\right)^2}{\mu^2\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}^2}$$

Substituting Eq (4.4.24) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions,

$$\therefore v123(x,t)=1+\left(\frac{2\mu\beta d-\mu^2d^2-\beta^2}{\mu^2}\right)(d+H)^{-2} \qquad \textbf{(xii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (xii) becomes,

$$u123_1(x,t)=1+\frac{4\Psi^2\left(2\mu\beta d-\mu^2d^2-\beta^2\right)\left(C_1+C_2\xi\right)^2}{\mu^2\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}^2}$$

Substituting Eq (4.4.25) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions,

$$\therefore v133(x,t)=\left\{\frac{18d\mu^3-36\mu^2d^2\delta+18\mu d^3\delta^2+36\mu^3d^3\pm18d^2\mu^3\sqrt{6}\pm\mu^3\sqrt{6}\mp18\delta d^3\mu^2\sqrt{6}\mp3d\delta\mu^2\sqrt{6}\pm3\mu\delta^2d^2\sqrt{6}\mp\delta^3d^3\sqrt{6}}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d}\right\}+\frac{1}{\mu^2}\left(\pm4\delta d^3\mu\sqrt{6}\mp4\mu^2d^2\sqrt{6}-2d\mu^2+4\delta\mu d^2-2\delta^2d^3-12\mu^2d^3\right)$$

$$(d+H)^{-1}+\frac{1}{\mu^2}\left(\pm2\mu^2d^3\sqrt{6}\mp2\delta\mu d^4\sqrt{6}+\mu^2d^2+\delta^2d^4-2\delta d^3\mu+6\mu^2d^4\right)(d+H)^{-2} \qquad \textbf{(xiii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (xiii) becomes,

$$u133_1(x,t)=\frac{18d\mu^3-36\mu^2d^2\delta+18\mu d^3\delta^2+36\mu^3d^3\pm18d^2\mu^3\sqrt{6}\pm\mu^3\sqrt{6}\mp18\delta d^3\mu^2\sqrt{6}\mp3d\delta\mu^2\sqrt{6}}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d}$$
$$\frac{\pm3\mu\delta^2d^2\sqrt{6}\mp\delta^3d^3\sqrt{6}}{}+\frac{2\Psi\left(\pm4\delta d^3\mu\sqrt{6}\mp4\mu^2d^2\sqrt{6}-2d\mu^2+4\delta\mu d^2-2\delta^2d^3-12\mu^2d^3\right)}{\mu^2\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}}$$
$$+\frac{4\Psi^2\left(\pm2\mu^2d^3\sqrt{6}\mp2\delta\mu d^4\sqrt{6}+\mu^2d^2+\delta^2d^4-2\delta d^3\mu+6\mu^2d^4\right)}{\mu^2\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}^2}$$

Substituting Eq (4.4.26) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions,

$$\therefore v143(x,t)=\left\{\frac{d\left(12\mu^3-36\mu^2d\delta\pm18d\mu^3i\sqrt{6}\mp18\delta d^2\mu^2i\sqrt{6}+18\mu d^2\delta^2\pm2\delta\mu^2i\sqrt{6}-36\mu^3d^2\mp3\mu\delta^2di\sqrt{6}\right.}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d}\right.$$
$$\left.\frac{\left.\pm\delta^3d^2i\sqrt{6}\right)}{}\right\}+\left\{\frac{1}{\mu^2}\left(2d\mu^2-4\delta\mu d^2\mp4\delta d^3\mu i\sqrt{6}\pm4\mu^2d^2i\sqrt{6}+2\delta^2d^3-12\mu^2d^3\right)\right\}\left(d+H\right)^{-1}$$
$$+\left\{\frac{1}{\mu^2}\left(\pm2\mu d^4\delta i\sqrt{6}\mp2\mu^2d^3i\sqrt{6}-\mu^2d^2-\delta^2d^4+2\delta d^3\mu+6\mu^2d^4\right)\right\}\left(d+H\right)^{-2} \qquad \textbf{(xiv)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (xiv) becomes,

$$v143_1(x,t)=\frac{d\left(12\mu^3-36\mu^2d\delta\pm18d\mu^3i\sqrt{6}\mp18\delta d^2\mu^2i\sqrt{6}+18\mu d^2\delta^2\pm2\delta\mu^2i\sqrt{6}-36\mu^3d^2\mp3\mu\delta^2di\sqrt{6}\right.}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d}$$
$$\frac{\left.\pm\delta^3d^2i\sqrt{6}\right)}{}+\frac{2\Psi\left(2d\mu^2-4\delta\mu d^2\mp4\delta d^3\mu i\sqrt{6}\pm4\mu^2d^2i\sqrt{6}+2\delta^2d^3-12\mu^2d^3\right)}{\mu^2\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}}+$$
$$\frac{4\Psi^2\left(\pm2\mu d^4\delta i\sqrt{6}\mp2\mu^2d^3i\sqrt{6}-\mu^2d^2-\delta^2d^4+2\delta d^3\mu+6\mu^2d^4\right)}{\mu^2\left\{2d\Psi\left(C_1+C_2\xi\right)+\mu\left(C_1+C_2\xi\right)+2\Psi C_2\right\}^2}$$

Substituting Eq (4.4.27) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions,

$$\therefore v153(x,t)=\frac{1}{96\lambda^2\beta^2}\left(\lambda^4d^2\pm\frac{24\beta d\lambda^3}{\sqrt{6}}-12\lambda^2\mu^2d^2+24\lambda^2\beta^2+24\lambda^2\mu\beta d\mp\frac{144\mu^2\beta d}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2}{\sqrt{6}}+144\beta^2\mu^2\right.$$
$$\left.+36\mu^4d^2-144\mu^3d\beta\right)-\frac{1}{48\lambda^2\beta^2}\left(\pm\frac{12\lambda^3\beta}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2}{\sqrt{6}}+12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4-12d\lambda^2\mu^2+36\mu^4d\right)$$
$$\left(d+H\right)+\frac{1}{96\lambda^2\beta^2}\left(\lambda^2-6\mu^2\right)^2\left(d+H\right)^2 \qquad \textbf{(xv)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (xv) becomes,

$$v153_1(x,t)=\frac{1}{96\lambda^2\beta^2}\left\{\lambda^4d^2\pm\frac{24\beta d\lambda^3}{\sqrt{6}}-12\lambda^2\mu^2d^2+24\lambda^2\beta^2+24\lambda^2\mu\beta d\mp\frac{144\mu^2\beta d}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2}{\sqrt{6}}+144\beta^2\mu^2\right.$$
$$+36\mu^4d^2-144\mu^3d\beta-\left(\pm\frac{12\lambda^3\beta}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2}{\sqrt{6}}+12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4-12d\lambda^2\mu^2+36\mu^4d\right)$$
$$\left.\left(\frac{2d\Psi(C_1+C_2\xi)+\mu(C_1+C_2\xi)+2\Psi C_2}{\Psi(C_1+C_2\xi)}\right)+\frac{(\lambda^2-6\mu^2)^2\{2d\Psi(C_1+C_2\xi)+\mu(C_1+C_2\xi)+2\Psi C_2\}^2}{4\Psi^2(C_1+C_2\xi)^2}\right\}$$

Substituting Eq (4.4.28) into Eq (4.2.3), along with Eq (3.1.9) and simplifying, yields the following travelling wave solutions,

$$\therefore v163(x,t)=\frac{1}{96\lambda^2\beta^2}\left(\lambda^4d^2\mp\frac{24\beta d\lambda^3 i}{\sqrt{6}}+12\lambda^2\mu^2d^2+72\lambda^2\beta^2-24\lambda^2\mu\beta d\mp\frac{144\lambda\mu^2\beta di}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2 i}{\sqrt{6}}+144\beta^2\mu^2\right.$$
$$\left.+36\mu^4d^2-144\mu^3d\beta\right)-\frac{1}{48\lambda^2\beta^2}\left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2 i}{\sqrt{6}}-12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4+12d\lambda^2\mu^2+36\mu^4d\right)$$
$$(d+H)+\frac{1}{96\lambda^2\beta^2}(\lambda^2+6\mu^2)^2(d+H)^2 \qquad \textbf{(xvi)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi}$,

then the above equation (xvi) becomes,

$$v163_1(x,t)=\frac{1}{96\lambda^2\beta^2}\left\{\lambda^4d^2\mp\frac{24\beta d\lambda^3 i}{\sqrt{6}}+12\lambda^2\mu^2d^2+72\lambda^2\beta^2-24\lambda^2\mu\beta d\mp\frac{144\lambda\mu^2\beta di}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2 i}{\sqrt{6}}+144\beta^2\mu^2\right.$$
$$+36\mu^4d^2-144\mu^3d\beta-\left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2 i}{\sqrt{6}}-12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4+12d\lambda^2\mu^2+36\mu^4d\right)$$
$$\left.\left(\frac{2d\Psi(C_1+C_2\xi)+\mu(C_1+C_2\xi)+2\Psi C_2}{\Psi(C_1+C_2\xi)}\right)+\frac{(\lambda^2+6\mu^2)^2\{2d\Psi(C_1+C_2\xi)+\mu(C_1+C_2\xi)+2\Psi C_2\}^2}{4\Psi^2(C_1+C_2\xi)^2}\right\}$$

Hyperbolic form of travelling waves:

Substituting Eq (4.4.13) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v14(x,t)=\frac{3}{8}\mp\frac{6\Psi}{\lambda\sqrt{6}}\left(d+H\right)+\frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2\mp\frac{\lambda}{16\Psi\sqrt{6}}\left(d+H\right)^{-1}+\frac{\lambda^2}{1536\Psi^2}\left(d+H\right)^{-2} \quad \textbf{(i)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (i) becomes:

$$v14_1(x,t)=\frac{3}{8}\mp\frac{6\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{\lambda\sqrt{6}}+\frac{6\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{\lambda^2}\mp\frac{\lambda}{16\Psi\sqrt{6}\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}$$
$$+\frac{\lambda^2}{1536\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v14_2(x,t)=\frac{3}{8}\mp\frac{6\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{\lambda\sqrt{6}}+\frac{6\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{\lambda^2}\mp\frac{\lambda}{16\Psi\sqrt{6}\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}$$
$$+\frac{\lambda^2}{1536\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}$$

Substituting Eq (4.4.14) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solution:

$$\therefore v24(x,t)=\frac{5}{8}\mp\frac{6\Psi}{\lambda\sqrt{6}}\left(d+H\right)+\frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2\pm\frac{\lambda}{16\Psi\sqrt{6}}\left(d+H\right)^{-1}+\frac{\lambda^2}{1536\Psi^2}\left(d+H\right)^{-2} \quad \textbf{(ii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (ii) becomes:

$$v24_1(x,t) = \frac{5}{8} \mp \frac{6\left(d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{\lambda\sqrt{6}} + \frac{6\left(d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{\lambda^2} \pm \frac{\lambda}{16\Psi\sqrt{6}\left(d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)} + \frac{\lambda^2}{1536\left(d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v24_2(x,t) = \frac{5}{8} \mp \frac{6\left(d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{\lambda\sqrt{6}} + \frac{6\left(d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{\lambda^2} \pm \frac{\lambda}{16\Psi\sqrt{6}\left(d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)} + \frac{\lambda^2}{1536\left(d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}$$

Substituting Eq (4.4.15) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v34(x,t) = \frac{1}{d^2}(d+H)^2 \qquad \textbf{(iii)}$$

where, $H = \left(\frac{G'}{G}\right) = \frac{\sqrt{\Delta}}{\Psi} \frac{C_1 \sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2 \cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1 \cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2 \sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (iii) becomes,

$$v34_1(x,t) = \frac{\left(d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v34_2(x,t)=\frac{\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

Substituting Eq (4.4.16) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v44(x,t)=1-\frac{1}{d^2}(d+H)^2 \qquad \textbf{(iv)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (iv) becomes:

$$v44_1(x,t)=1-\frac{\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v44_2(x,t)=1-\frac{\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

Substituting Eq (4.4.17) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v54(x,t)=1-\frac{2}{d}(d+H)+\frac{1}{d^2}(d+H)^2 \qquad \textbf{(v)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (v) becomes:

$$v54_1(x,t)=1-\frac{2\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{d\Psi}+\frac{\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v54_2(x,t)=1-\frac{2\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{d\Psi}+\frac{\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

Substituting Eq (4.4.18) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v64(x,t)=\frac{2}{d}(d+H)-\frac{1}{d^2}(d+H)^2 \qquad \textbf{(vi)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (vi) becomes,

$$v64_1(x,t)=\frac{2\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{d\Psi}-\frac{\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v64_2(x,t) = \frac{2\left(d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{d\Psi} - \frac{\left(d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

Substituting Eq (4.4.19) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v74(x,t) = \frac{6d^2\Psi^2}{\lambda^2} - \frac{12d\Psi^2}{\lambda^2}(d+H) + \frac{6\Psi^2}{\lambda^2}(d+H)^2 \qquad \textbf{(vii)}$$

where, $H = \left(\frac{G'}{G}\right) = \frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (vii) becomes:

$$v74_1(x,t) = \frac{1}{\lambda^2}\left\{6d^2\Psi^2 - 12d\Psi\left(d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right) + 6\left(d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2\right\}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v74_2(x,t) = \frac{1}{\lambda^2}\left\{6d^2\Psi^2 - 12d\Psi\left(d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right) + 6\left(d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2\right\}$$

Substituting Eq (4.4.20) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v84(x,t) = 1 + \frac{6d^2\Psi^2}{\lambda^2} - \frac{12d\Psi^2}{\lambda^2}(d+H) + \frac{6\Psi^2}{\lambda^2}(d+H)^2 \qquad \textbf{(viii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (viii) becomes,

$$v84_1(x,t)=1+\frac{1}{\lambda^2}\left\{6d^2\Psi^2-12d\Psi\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)+6\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2\right\}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v84_2(x,t)=1+\frac{1}{\lambda^2}\left\{6d^2\Psi^2-12d\Psi\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)+6\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2\right\}$$

Substituting Eq (4.4.21) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v94(x,t)=\frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2}{\sqrt{6}}\mp\frac{12d\lambda\delta}{\sqrt{6}}+6d^2\Psi^2\right)+\frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta}{\sqrt{6}}\mp\frac{12\lambda^2}{\sqrt{6}}-d\Psi^2\right)(d+H)+\frac{6\Psi^2}{\lambda^2}(d+H)^2$$

(ix)

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (ix) becomes

$$v94_1(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2}{\sqrt{6}}\mp\frac{12d\lambda\delta}{\sqrt{6}}+6d^2\Psi^2+\frac{\left(\pm\frac{\lambda\delta}{\sqrt{6}}\mp\frac{12\lambda^2}{\sqrt{6}}-d\Psi^2\right)\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{\Psi}\right.$$

$$+6\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^{2}\Bigg\}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v94_2(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2}{\sqrt{6}}\mp\frac{12d\lambda\delta}{\sqrt{6}}+6d^2\Psi^2+\frac{\left(\pm\frac{\lambda\delta}{\sqrt{6}}\mp\frac{12\lambda^2}{\sqrt{6}}-d\Psi^2\right)\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{\Psi}\right.$$

$$+6\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^{2}\Bigg\}$$

Substituting Eq (4.4.22) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v104(x,t)=\frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2\right)+\frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right)(d+H)+\frac{6\Psi^2}{\lambda^2}(d+H)^2$$

(x)

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (x) becomes:

$$v104_1(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2+\frac{\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right)\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{\Psi}\right.$$

$$+6\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^{2}\Bigg\}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v104_2(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2+\frac{\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right)\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{\Psi}\right.$$

$$\left.+6\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2\right\}$$

Substituting Eq (4.4.23) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v114(x,t)=\left(\frac{\mu^2 d^2+\beta^2-2\mu\beta d}{\mu^2}\right)(d+H)^{-2} \qquad \textbf{(xi)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (xi) becomes:

$$v114_1(x,t)=\frac{\Psi^2\left(\mu^2 d^2+\beta^2-2\mu\beta d\right)}{\mu^2\left\{d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v114_2(x,t)=\frac{\Psi^2\left(\mu^2 d^2+\beta^2-2\mu\beta d\right)}{\mu^2\left\{d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.4.24) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v124(x,t) = 1 + \left(\frac{2\mu\beta d - \mu^2 d^2 - \beta^2}{\mu^2} \right) (d + H)^{-2} \qquad \textbf{(xii)}$$

where, $H = \left(\frac{G'}{G} \right) = \frac{\sqrt{\Delta}}{\Psi} \frac{C_1 \sinh\left(\frac{\sqrt{\Delta}}{\Psi} \xi \right) + C_2 \cosh\left(\frac{\sqrt{\Delta}}{\Psi} \xi \right)}{C_1 \cosh\left(\frac{\sqrt{\Delta}}{\Psi} \xi \right) + C_2 \sinh\left(\frac{\sqrt{\Delta}}{\Psi} \xi \right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (xii) becomes:

$$v124_1(x,t) = 1 + \frac{\Psi^2 \left(2\mu\beta d - \mu^2 d^2 - \beta^2 \right)}{\mu^2 \left\{ d\Psi + \sqrt{\Delta} \coth\left(\frac{\sqrt{\Delta}}{\Psi} \xi \right) \right\}^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v124_2(x,t) = 1 + \frac{\Psi^2 \left(2\mu\beta d - \mu^2 d^2 - \beta^2 \right)}{\mu^2 \left\{ d\Psi + \sqrt{\Delta} \tanh\left(\frac{\sqrt{\Delta}}{\Psi} \xi \right) \right\}^2}$$

Substituting Eq (4.4.25) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v134(x,t) = \left\{ \frac{18d\mu^3 - 36\mu^2 d^2 \delta + 18\mu d^3 \delta^2 + 36\mu^3 d^3 \pm 18d^2 \mu^3 \sqrt{6} \pm \mu^3 \sqrt{6} \mp 18\delta d^3 \mu^2 \sqrt{6} \mp 3d\delta\mu^2 \sqrt{6} \pm 3\mu\delta^2 d^2 \sqrt{6} \mp \delta^3 d^3 \sqrt{6}}{\pm\mu^3 \sqrt{6} \mp d\delta\sqrt{6} + 6\mu d} \right\} + \frac{1}{\mu^2} \left(\pm 4\delta d^3 \mu \sqrt{6} \mp 4\mu^2 d^2 \sqrt{6} - 2d\mu^2 + 4\delta\mu d^2 - 2\delta^2 d^3 - 12\mu^2 d^3 \right)$$

$$(d + H)^{-1} + \frac{1}{\mu^2} \left(\pm 2\mu^2 d^3 \sqrt{6} \mp 2\delta\mu d^4 \sqrt{6} + \mu^2 d^2 + \delta^2 d^4 - 2\delta d^3 \mu + 6\mu^2 d^4 \right) (d + H)^{-2} \qquad \textbf{(xiii)}$$

where, $H = \left(\frac{G'}{G} \right) = \frac{\sqrt{\Delta}}{\Psi} \frac{C_1 \sinh\left(\frac{\sqrt{\Delta}}{\Psi} \xi \right) + C_2 \cosh\left(\frac{\sqrt{\Delta}}{\Psi} \xi \right)}{C_1 \cosh\left(\frac{\sqrt{\Delta}}{\Psi} \xi \right) + C_2 \sinh\left(\frac{\sqrt{\Delta}}{\Psi} \xi \right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (xiii) becomes:

$$v134_1(x,t) = \frac{18d\mu^3 - 36\mu^2 d^2\delta + 18\mu d^3\delta^2 + 36\mu^3 d^3 \pm 18d^2\mu^3\sqrt{6} \pm \mu^3\sqrt{6} \mp 18\delta d^3\mu^2\sqrt{6} \mp 3d\delta\mu^2\sqrt{6}}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d}$$

$$\frac{\pm 3\mu\delta^2 d^2\sqrt{6} \mp \delta^3 d^3\sqrt{6}}{} + \frac{\Psi\left(\pm 4\delta d^3\mu\sqrt{6} \mp 4\mu^2 d^2\sqrt{6} - 2d\mu^2 + 4\delta\mu d^2 - 2\delta^2 d^3 - 12\mu^2 d^3\right)}{\mu^2\left\{d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}}$$

$$+\frac{\Psi^2\left(\pm 2\mu^2 d^3\sqrt{6} \mp 2\delta\mu d^4\sqrt{6} + \mu^2 d^2 + \delta^2 d^4 - 2\delta d^3\mu + 6\mu^2 d^4\right)}{\mu^2\left\{d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v134_2(x,t) = \frac{18d\mu^3 - 36\mu^2 d^2\delta + 18\mu d^3\delta^2 + 36\mu^3 d^3 \pm 18d^2\mu^3\sqrt{6} \pm \mu^3\sqrt{6} \mp 18\delta d^3\mu^2\sqrt{6} \mp 3d\delta\mu^2\sqrt{6}}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d}$$

$$\frac{\pm 3\mu\delta^2 d^2\sqrt{6} \mp \delta^3 d^3\sqrt{6}}{} + \frac{\Psi\left(\pm 4\delta d^3\mu\sqrt{6} \mp 4\mu^2 d^2\sqrt{6} - 2d\mu^2 + 4\delta\mu d^2 - 2\delta^2 d^3 - 12\mu^2 d^3\right)}{\mu^2\left\{d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}}$$

$$+\frac{\Psi^2\left(\pm 2\mu^2 d^3\sqrt{6} \mp 2\delta\mu d^4\sqrt{6} + \mu^2 d^2 + \delta^2 d^4 - 2\delta d^3\mu + 6\mu^2 d^4\right)}{\mu^2\left\{d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.4.26) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions

$$\therefore v144(x,t) = \left\{\frac{d\left(12\mu^3 - 36\mu^2 d\delta \pm 18d\mu^3 i\sqrt{6} \mp 18\delta d^2\mu^2 i\sqrt{6} + 18\mu d^2\delta^2 \pm 2\delta\mu^2 i\sqrt{6} - 36\mu^3 d^2 \mp 3\mu\delta^2 di\sqrt{6}}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d}\right.$$

$$\left.\frac{\pm\delta^3 d^2 i\sqrt{6}}{}\right\} + \left\{\frac{1}{\mu^2}\left(2d\mu^2 - 4\delta\mu d^2 \mp 4\delta d^3\mu i\sqrt{6} \pm 4\mu^2 d^2 i\sqrt{6} + 2\delta^2 d^3 - 12\mu^2 d^3\right)\right\}(d+H)^{-1}$$

$$+\left\{\frac{1}{\mu^2}\left(\pm 2\mu d^4\delta i\sqrt{6} \mp 2\mu^2 d^3 i\sqrt{6} - \mu^2 d^2 - \delta^2 d^4 + 2\delta d^3\mu + 6\mu^2 d^4\right)\right\}(d+H)^{-2} \qquad \textbf{(xiv)}$$

where, $$H = \left(\frac{G'}{G}\right) = \frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (xiv) becomes,

$$v144_1(x,t) = \frac{d\left(12\mu^3 - 36\mu^2 d\delta \pm 18d\mu^3 i\sqrt{6} \mp 18\delta d^2\mu^2 i\sqrt{6} + 18\mu d^2\delta^2 \pm 2\delta\mu^2 i\sqrt{6} - 36\mu^3 d^2 \mp 3\mu\delta^2 di\sqrt{6}\right.}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d}$$

$$\frac{\left.\pm\delta^3 d^2 i\sqrt{6}\right)}{} + \frac{\Psi\left(2d\mu^2 - 4\delta\mu d^2 \mp 4\delta d^3\mu i\sqrt{6} \pm 4\mu^2 d^2 i\sqrt{6} + 2\delta^2 d^3 - 12\mu^2 d^3\right)}{\mu^2\left\{d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}} +$$

$$\frac{\Psi^2\left(\pm 2\mu d^4\delta i\sqrt{6} \mp 2\mu^2 d^3 i\sqrt{6} - \mu^2 d^2 - \delta^2 d^4 + 2\delta d^3\mu + 6\mu^2 d^4\right)}{\mu^2\left\{d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v144_2(x,t) = \frac{d\left(12\mu^3 - 36\mu^2 d\delta \pm 18d\mu^3 i\sqrt{6} \mp 18\delta d^2\mu^2 i\sqrt{6} + 18\mu d^2\delta^2 \pm 2\delta\mu^2 i\sqrt{6} - 36\mu^3 d^2 \mp 3\mu\delta^2 di\sqrt{6}\right.}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d}$$

$$\frac{\left.\pm\delta^3 d^2 i\sqrt{6}\right)}{} + \frac{\Psi\left(2d\mu^2 - 4\delta\mu d^2 \mp 4\delta d^3\mu i\sqrt{6} \pm 4\mu^2 d^2 i\sqrt{6} + 2\delta^2 d^3 - 12\mu^2 d^3\right)}{\mu^2\left\{d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}} +$$

$$\frac{\Psi^2\left(\pm 2\mu d^4\delta i\sqrt{6} \mp 2\mu^2 d^3 i\sqrt{6} - \mu^2 d^2 - \delta^2 d^4 + 2\delta d^3\mu + 6\mu^2 d^4\right)}{\mu^2\left\{d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.4.27) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solution,

$$\therefore v154(x,t) = \frac{1}{96\lambda^2\beta^2}\left(\lambda^4 d^2 \pm \frac{24\beta d\lambda^3}{\sqrt{6}} - 12\lambda^2\mu^2 d^2 + 24\lambda^2\beta^2 + 24\lambda^2\mu\beta d \mp \frac{144\mu^2\beta d}{\sqrt{6}} \pm \frac{288\lambda\mu\beta^2}{\sqrt{6}} + 144\beta^2\mu^2\right.$$

$$\left. + 36\mu^4 d^2 - 144\mu^3 d\beta\right) - \frac{1}{48\lambda^2\beta^2}\left(\pm\frac{12\lambda^3\beta}{\sqrt{6}} \mp \frac{72\lambda\beta\mu^2}{\sqrt{6}} + 12\mu\beta\lambda^2 - 72\beta\mu^3 + d\lambda^4 - 12d\lambda^2\mu^2 + 36\mu^4 d\right)$$

$$(d+H) + \frac{1}{96\lambda^2\beta^2}\left(\lambda^2 - 6\mu^2\right)^2(d+H)^2 \qquad \textbf{(xv)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (xv) becomes

$$v154_1(x,t)=\frac{\lambda^4d^2\pm\frac{24\beta d\lambda^3}{\sqrt{6}}-12\lambda^2\mu^2d^2+24\lambda^2\beta^2+24\lambda^2\mu\beta d\mp\frac{144\mu^2\beta d}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2}{\sqrt{6}}+144\beta^2\mu^2}{96\lambda^2\beta^2}$$

$$\frac{+36\mu^4d^2-144\mu^3d\beta}{}-\frac{\left(\pm\frac{12\lambda^3\beta}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2}{\sqrt{6}}+12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4-12d\lambda^2\mu^2+36\mu^4d\right)}{48\Psi\lambda^2\beta^2}$$

$$\frac{\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{}+\frac{\left(\lambda^2-6\mu^2\right)^2\left(d\Psi+\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{96\Psi^2\lambda^2\beta^2}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v154_2(x,t)=\frac{\lambda^4d^2\pm\frac{24\beta d\lambda^3}{\sqrt{6}}-12\lambda^2\mu^2d^2+24\lambda^2\beta^2+24\lambda^2\mu\beta d\mp\frac{144\mu^2\beta d}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2}{\sqrt{6}}+144\beta^2\mu^2}{96\lambda^2\beta^2}$$

$$\frac{+36\mu^4d^2-144\mu^3d\beta}{}-\frac{\left(\pm\frac{12\lambda^3\beta}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2}{\sqrt{6}}+12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4-12d\lambda^2\mu^2+36\mu^4d\right)}{48\Psi\lambda^2\beta^2}$$

$$\frac{\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{}+\frac{\left(\lambda^2-6\mu^2\right)^2\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{96\Psi^2\lambda^2\beta^2}$$

Substituting Eq (4.4.28) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v164(x,t)=\frac{1}{96\lambda^2\beta^2}\left(\lambda^4d^2\mp\frac{24\beta d\lambda^3 i}{\sqrt{6}}+12\lambda^2\mu^2d^2+72\lambda^2\beta^2-24\lambda^2\mu\beta d\mp\frac{144\lambda\mu^2\beta di}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2 i}{\sqrt{6}}+144\beta^2\mu^2\right.$$

$$+36\mu^4 d^2 - 144\mu^3 d\beta\Big) - \frac{1}{48\lambda^2\beta^2}\left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}} \mp \frac{72\lambda\beta\mu^2 i}{\sqrt{6}} - 12\mu\beta\lambda^2 - 72\beta\mu^3 + d\lambda^4 + 12d\lambda^2\mu^2 + 36\mu^4 d\right)$$

$$(d+H) + \frac{1}{96\lambda^2\beta^2}\left(\lambda^2+6\mu^2\right)^2 (d+H)^2 \qquad \textbf{(xvi)}$$

where, $H = \left(\frac{G'}{G}\right) = \frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (xvi) becomes,

$$v164_1(x,t) = \frac{\lambda^4 d^2 \mp \frac{24\beta d\lambda^3 i}{\sqrt{6}} + 12\lambda^2\mu^2 d^2 + 72\lambda^2\beta^2 - 24\lambda^2\mu\beta d \mp \frac{144\lambda\mu^2\beta d i}{\sqrt{6}} \pm \frac{288\lambda\mu\beta^2 i}{\sqrt{6}} + 144\beta^2\mu^2}{96\lambda^2\beta^2}$$

$$\frac{+36\mu^4 d^2 - 144\mu^3 d\beta}{} - \frac{\left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}} \mp \frac{72\lambda\beta\mu^2 i}{\sqrt{6}} - 12\mu\beta\lambda^2 - 72\beta\mu^3 + d\lambda^4 + 12d\lambda^2\mu^2 + 36\mu^4 d\right)}{48\Psi\lambda^2\beta^2}$$

$$\frac{\left(d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{} + \frac{\left(\lambda^2+6\mu^2\right)^2\left(d\Psi + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{96\Psi^2\lambda^2\beta^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v164_2(x,t) = \frac{\lambda^4 d^2 \mp \frac{24\beta d\lambda^3 i}{\sqrt{6}} + 12\lambda^2\mu^2 d^2 + 72\lambda^2\beta^2 - 24\lambda^2\mu\beta d \mp \frac{144\lambda\mu^2\beta d i}{\sqrt{6}} \pm \frac{288\lambda\mu\beta^2 i}{\sqrt{6}} + 144\beta^2\mu^2}{96\lambda^2\beta^2}$$

$$\frac{+36\mu^4 d^2 - 144\mu^3 d\beta}{} - \frac{\left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}} \mp \frac{72\lambda\beta\mu^2 i}{\sqrt{6}} - 12\mu\beta\lambda^2 - 72\beta\mu^3 + d\lambda^4 + 12d\lambda^2\mu^2 + 36\mu^4 d\right)}{48\Psi\lambda^2\beta^2}$$

$$\frac{\left(d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{} + \frac{\left(\lambda^2+6\mu^2\right)^2\left(d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{96\Psi^2\lambda^2\beta^2}$$

Trigonometric form of travelling waves

Substituting Eq (4.4.13) into Eq (4.2.3), along with Eq (3.1.11) and simplifying, yields the following travelling wave solutions,

$$\therefore v15(x,t)=\frac{3}{8}\mp\frac{6\Psi}{\lambda\sqrt{6}}\left(d+H\right)+\frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2\mp\frac{\lambda}{16\Psi\sqrt{6}}\left(d+H\right)^{-1}+\frac{\lambda^2}{1536\Psi^2}\left(d+H\right)^{-2} \quad \textbf{(i)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (i) becomes,

$$v15_1(x,t)=\frac{3}{8}\mp\frac{6\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{\lambda\sqrt{6}}+\frac{6\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{\lambda^2}\mp\frac{\lambda}{16\Psi\sqrt{6}\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}$$
$$+\frac{\lambda^2}{1536\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v15_2(x,t)=\frac{3}{8}\mp\frac{6\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{\lambda\sqrt{6}}+\frac{6\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{\lambda^2}\mp\frac{\lambda}{16\Psi\sqrt{6}\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}$$
$$+\frac{\lambda^2}{1536\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}$$

Substituting Eq (4.4.14) into Eq (4.2.3), along with Eq (3.1.11) and simplifying, yields the following travelling wave solutions,

$$\therefore v25(x,t)=\frac{5}{8}\mp\frac{6\Psi}{\lambda\sqrt{6}}\left(d+H\right)+\frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2\pm\frac{\lambda}{16\Psi\sqrt{6}}\left(d+H\right)^{-1}+\frac{\lambda^2}{1536\Psi^2}\left(d+H\right)^{-2} \quad \textbf{(ii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (ii) becomes:

$$v25_1(x,t)=\frac{5}{8}\mp\frac{6\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{\lambda\sqrt{6}}+\frac{6\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{\lambda^2}\pm\frac{\lambda}{16\Psi\sqrt{6}\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}$$
$$+\frac{\lambda^2}{1536\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v25_2(x,t)=\frac{5}{8}\mp\frac{6\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{\lambda\sqrt{6}}+\frac{6\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{\lambda^2}\pm\frac{\lambda}{16\Psi\sqrt{6}\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}$$
$$+\frac{\lambda^2}{1536\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}$$

Substituting Eq (4.4.15) into Eq (4.2.3), along with Eq (3.1.11) and simplifying, yields the following travelling wave solutions,

$$\therefore v35(x,t)=\frac{1}{d^2}\left(d+H\right)^2 \qquad \textbf{(iii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (iii) becomes:

$$v35_1(x,t)=\frac{\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v35_2(x,t)=\frac{\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

Substituting Eq (4.4.16) into Eq (4.2.3), along with Eq (3.1.11) and simplifying, yields the following travelling wave solutions,

$$\therefore v45(x,t)=1-\frac{1}{d^2}\left(d+H\right)^2 \qquad \textbf{(iv)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (iv) becomes:

$$v45_1(x,t)=1-\frac{\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v45_2(x,t)=1-\frac{\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

Substituting Eq (4.4.17) into Eq (4.2.3), along with Eq (3.1.11) and simplifying, yields the following travelling wave solutions,

$$\therefore v55(x,t)=1-\frac{2}{d}\left(d+H\right)+\frac{1}{d^2}\left(d+H\right)^2 \qquad \textbf{(v)}$$

where, $$H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$$

If $C_1=0$ and $C_2\neq 0$, then the above equation (v) becomes,

$$v55_1(x,t)=1-\frac{2\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{d\Psi}+\frac{\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v55_2(x,t)=1-\frac{2\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{d\Psi}+\frac{\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

Substituting Eq (4.4.18) into Eq (4.2.3), along with Eq (3.1.11) and simplifying, yields the following travelling wave solutions,

$$\therefore v65(x,t)=\frac{2}{d}\left(d+H\right)-\frac{1}{d^2}\left(d+H\right)^2 \qquad \textbf{(vi)}$$

where, $$H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (vi) becomes:

$$v65_1(x,t) = \frac{2\left(d\Psi + \sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{d\Psi} - \frac{\left(d\Psi + \sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v65_2(x,t) = \frac{2\left(d\Psi - \sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{d\Psi} - \frac{\left(d\Psi - \sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2}$$

Substituting Eq (4.4.19) into Eq (4.2.3), along with Eq (3.1.11) and simplifying, yields the following travelling wave solutions,

$$\therefore v75(x,t) = \frac{6d^2\Psi^2}{\lambda^2} - \frac{12d\Psi^2}{\lambda^2}\left(d+H\right) + \frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2 \qquad \textbf{(vii)}$$

where, $H = \left(\frac{G'}{G}\right) = \frac{\sqrt{-\Delta}}{\Psi}\,\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right) + C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right) + C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (vii) becomes:

$$v75_1(x,t) = \frac{1}{\lambda^2}\left\{6d^2\Psi^2 - 12d\Psi\left(d\Psi + \sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right) + 6\left(d\Psi + \sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right\}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v75_2(x,t) = \frac{1}{\lambda^2}\left\{6d^2\Psi^2 - 12d\Psi\left(d\Psi - \sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right) + 6\left(d\Psi - \sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right\}$$

Substituting Eq (4.4.20) into Eq (4.2.3), along with Eq (3.1.11) and simplifying, yields the following travelling wave solutions,

$$\therefore v85(x,t)=1+\frac{6d^2\Psi^2}{\lambda^2}-\frac{12d\Psi^2}{\lambda^2}\left(d+H\right)+\frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2 \qquad \textbf{(viii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (viii) becomes:

$$v85_1(x,t)=1+\frac{1}{\lambda^2}\left\{6d^2\Psi^2-12d\Psi\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)+6\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right\}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v85_2(x,t)=1+\frac{1}{\lambda^2}\left\{6d^2\Psi^2-12d\Psi\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)+6\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right\}$$

Substituting Eq (4.4.21) into Eq (4.2.3), along with Eq (3.1.11) and simplifying, yields the following travelling wave solutions,

$$\therefore v95\left(x,t\right)=\frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2}{\sqrt{6}}\mp\frac{12d\lambda\delta}{\sqrt{6}}+6d^2\Psi^2\right)+\frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta}{\sqrt{6}}\mp\frac{12\lambda^2}{\sqrt{6}}-d\Psi^2\right)\left(d+H\right)+\frac{6\Psi^2}{\lambda^2}\left(d+H\right)^2$$

(ix)

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (ix) becomes:

$$v95_1(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2}{\sqrt{6}}\mp\frac{12d\lambda\delta}{\sqrt{6}}+6d^2\Psi^2+\frac{\left(\pm\frac{\lambda\delta}{\sqrt{6}}\mp\frac{12\lambda^2}{\sqrt{6}}-d\Psi^2\right)\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{\Psi}\right.$$

$$\left.+6\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right\}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v95_2(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2}{\sqrt{6}}\mp\frac{12d\lambda\delta}{\sqrt{6}}+6d^2\Psi^2+\frac{\left(\pm\frac{\lambda\delta}{\sqrt{6}}\mp\frac{12\lambda^2}{\sqrt{6}}-d\Psi^2\right)\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{\Psi}\right.$$

$$\left.+6\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right\}$$

Substituting Eq (4.4.22) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v105(x,t)=\frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2\right)+\frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right)(d+H)+\frac{6\Psi^2}{\lambda^2}(d+H)^2 \quad \textbf{(x)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (x) becomes:

$$v105_1(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2+\frac{\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right)\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{\Psi}\right.$$

$$+6\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\Bigg\}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v105_2(x,t)=\frac{1}{\lambda^2}\left\{\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2+\frac{\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right)\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{\Psi}\right.$$

$$\left.+6\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right\}$$

Substituting Eq (4.4.23) into Eq (4.2.3), along with Eq (3.1.11) and simplifying, yields the following travelling wave solutions,

$$\therefore v115(x,t)=\left(\frac{\mu^2 d^2+\beta^2-2\mu\beta d}{\mu^2}\right)(d+H)^{-2} \qquad \textbf{(xi)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1 = 0$ and $C_2 \neq 0$, then the above equation (xi) becomes:

$$v115_1(x,t)=\frac{\Psi^2\left(\mu^2 d^2+\beta^2-2\mu\beta d\right)}{\mu^2\left\{d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1 \neq 0$ and $C_2 = 0$ then,

$$v115_2(x,t)=\frac{\Psi^2\left(\mu^2d^2+\beta^2-2\mu\beta d\right)}{\mu^2\left\{d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.4.24) into Eq (4.2.3), along with Eq (3.1.11) and simplifying, yields the following travelling wave solutions,

$$\therefore v125(x,t)=1+\left(\frac{2\mu\beta d-\mu^2d^2-\beta^2}{\mu^2}\right)(d+H)^{-2} \qquad \textbf{(xii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (xii) becomes:

$$v125_1(x,t)=1+\frac{\Psi^2\left(2\mu\beta d-\mu^2d^2-\beta^2\right)}{\mu^2\left\{d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v125_2(x,t)=1+\frac{\Psi^2\left(2\mu\beta d-\mu^2d^2-\beta^2\right)}{\mu^2\left\{d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.4.25) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v135(x,t)=\left\{\frac{18d\mu^3-36\mu^2d^2\delta+18\mu d^3\delta^2+36\mu^3d^3\pm 18d^2\mu^3\sqrt{6}\pm\mu^3\sqrt{6}\mp 18\delta d^3\mu^2\sqrt{6}\mp 3d\delta\mu^2\sqrt{6}}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d}\right.$$

$$\left.\frac{\pm 3\mu\delta^2 d^2\sqrt{6}\mp\delta^3 d^3\sqrt{6}}{}\right\}+\frac{1}{\mu^2}\left(\pm 4\delta d^3\mu\sqrt{6}\mp 4\mu^2 d^2\sqrt{6}-2d\mu^2+4\delta\mu d^2-2\delta^2 d^3-12\mu^2 d^3\right)$$

$$\left(d+H\right)^{-1}+\frac{1}{\mu^2}\left(\pm 2\mu^2 d^3\sqrt{6}\mp 2\delta\mu d^4\sqrt{6}+\mu^2 d^2+\delta^2 d^4-2\delta d^3\mu+6\mu^2 d^4\right)\left(d+H\right)^{-2} \quad \textbf{(xiii)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (xiii) becomes:

$$v135_1(x,t)=\frac{18d\mu^3-36\mu^2 d^2\delta+18\mu d^3\delta^2+36\mu^3 d^3\pm 18d^2\mu^3\sqrt{6}\pm\mu^3\sqrt{6}\mp 18\delta d^3\mu^2\sqrt{6}\mp 3d\delta\mu^2\sqrt{6}}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d}$$

$$\frac{\pm 3\mu\delta^2 d^2\sqrt{6}\mp\delta^3 d^3\sqrt{6}}{}+\frac{\Psi\left(\pm 4\delta d^3\mu\sqrt{6}\mp 4\mu^2 d^2\sqrt{6}-2d\mu^2+4\delta\mu d^2-2\delta^2 d^3-12\mu^2 d^3\right)}{\mu^2\left\{d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}}$$

$$+\frac{\Psi^2\left(\pm 2\mu^2 d^3\sqrt{6}\mp 2\delta\mu d^4\sqrt{6}+\mu^2 d^2+\delta^2 d^4-2\delta d^3\mu+6\mu^2 d^4\right)}{\mu^2\left\{d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v135_2(x,t)=\frac{18d\mu^3-36\mu^2 d^2\delta+18\mu d^3\delta^2+36\mu^3 d^3\pm 18d^2\mu^3\sqrt{6}\pm\mu^3\sqrt{6}\mp 18\delta d^3\mu^2\sqrt{6}\mp 3d\delta\mu^2\sqrt{6}}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d}$$

$$\frac{\pm 3\mu\delta^2 d^2\sqrt{6}\mp\delta^3 d^3\sqrt{6}}{}+\frac{\Psi\left(\pm 4\delta d^3\mu\sqrt{6}\mp 4\mu^2 d^2\sqrt{6}-2d\mu^2+4\delta\mu d^2-2\delta^2 d^3-12\mu^2 d^3\right)}{\mu^2\left\{d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}}$$

$$+\frac{\Psi^2\left(\pm 2\mu^2 d^3\sqrt{6}\mp 2\delta\mu d^4\sqrt{6}+\mu^2 d^2+\delta^2 d^4-2\delta d^3\mu+6\mu^2 d^4\right)}{\mu^2\left\{d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.4.26) into Eq (4.2.3), along with Eq (3.1.11) and simplifying, yields the following travelling wave solutions,

$$\therefore v145(x,t)=\left\{\frac{d\left(12\mu^3-36\mu^2 d\delta\pm 18d\mu^3 i\sqrt{6}\mp 18\delta d^2\mu^2 i\sqrt{6}+18\mu d^2\delta^2\pm 2\delta\mu^2 i\sqrt{6}-36\mu^3 d^2\mp 3\mu\delta^2 di\sqrt{6}\right.}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d}\right.$$

$$\left.\frac{\pm\delta^3 d^2 i\sqrt{6}}{}\right\}+\left\{\frac{1}{\mu^2}\left(2d\mu^2-4\delta\mu d^2\mp 4\delta d^3\mu i\sqrt{6}\pm 4\mu^2 d^2 i\sqrt{6}+2\delta^2 d^3-12\mu^2 d^3\right)\right\}(d+H)^{-1}$$

$$+\left\{\frac{1}{\mu^2}\left(\pm 2\mu d^4\delta i\sqrt{6}\mp 2\mu^2 d^3 i\sqrt{6}-\mu^2 d^2-\delta^2 d^4+2\delta d^3\mu+6\mu^2 d^4\right)\right\}(d+H)^{-2} \qquad \textbf{(xiv)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (xiv) becomes:

$$v145_1(x,t)=\frac{d\left(12\mu^3-36\mu^2 d\delta\pm 18d\mu^3 i\sqrt{6}\mp 18\delta d^2\mu^2 i\sqrt{6}+18\mu d^2\delta^2\pm 2\delta\mu^2 i\sqrt{6}-36\mu^3 d^2\mp 3\mu\delta^2 di\sqrt{6}\right.}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d}$$

$$\frac{\left.\pm\delta^3 d^2 i\sqrt{6}\right)}{}+\frac{\Psi\left(2d\mu^2-4\delta\mu d^2\mp 4\delta d^3\mu i\sqrt{6}\pm 4\mu^2 d^2 i\sqrt{6}+2\delta^2 d^3-12\mu^2 d^3\right)}{\mu^2\left\{d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}}+$$

$$\frac{\Psi^2\left(\pm 2\mu d^4\delta i\sqrt{6}\mp 2\mu^2 d^3 i\sqrt{6}-\mu^2 d^2-\delta^2 d^4+2\delta d^3\mu+6\mu^2 d^4\right)}{\mu^2\left\{d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$\therefore v145_2(x,t)=\frac{d\left(12\mu^3-36\mu^2 d\delta\pm 18d\mu^3 i\sqrt{6}\mp 18\delta d^2\mu^2 i\sqrt{6}+18\mu d^2\delta^2\pm 2\delta\mu^2 i\sqrt{6}-36\mu^3 d^2\mp 3\mu\delta^2 di\sqrt{6}\right.}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d}$$

$$\frac{\left.\pm\delta^3 d^2 i\sqrt{6}\right)}{}+\frac{\Psi\left(2d\mu^2-4\delta\mu d^2\mp 4\delta d^3\mu i\sqrt{6}\pm 4\mu^2 d^2 i\sqrt{6}+2\delta^2 d^3-12\mu^2 d^3\right)}{\mu^2\left\{d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}}+$$

$$\frac{\Psi^2\left(\pm 2\mu d^4\delta i\sqrt{6}\mp 2\mu^2 d^3 i\sqrt{6}-\mu^2 d^2-\delta^2 d^4+2\delta d^3\mu+6\mu^2 d^4\right)}{\mu^2\left\{d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}$$

Substituting Eq (4.4.27) into Eq (4.2.3), along with Eq (3.1.11) and simplifying, yields the following travelling wave solutions,

$$\therefore v155(x,t)=\frac{1}{96\lambda^2\beta^2}\left(\lambda^4d^2\pm\frac{24\beta d\lambda^3}{\sqrt{6}}-12\lambda^2\mu^2d^2+24\lambda^2\beta^2+24\lambda^2\mu\beta d\mp\frac{144\mu^2\beta d}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2}{\sqrt{6}}+144\beta^2\mu^2\right.$$
$$\left.+36\mu^4d^2-144\mu^3d\beta\right)-\frac{1}{48\lambda^2\beta^2}\left(\pm\frac{12\lambda^3\beta}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2}{\sqrt{6}}+12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4-12d\lambda^2\mu^2+36\mu^4d\right)$$
$$(d+H)+\frac{1}{96\lambda^2\beta^2}\left(\lambda^2-6\mu^2\right)^2(d+H)^2 \qquad \textbf{(xv)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (xv) becomes:

$$v155_1(x,t)=\frac{\lambda^4d^2\pm\frac{24\beta d\lambda^3}{\sqrt{6}}-12\lambda^2\mu^2d^2+24\lambda^2\beta^2+24\lambda^2\mu\beta d\mp\frac{144\mu^2\beta d}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2}{\sqrt{6}}+144\beta^2\mu^2}{96\lambda^2\beta^2}$$
$$\frac{+36\mu^4d^2-144\mu^3d\beta}{}-\frac{\left(\pm\frac{12\lambda^3\beta}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2}{\sqrt{6}}+12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4-12d\lambda^2\mu^2+36\mu^4d\right)}{48\Psi\lambda^2\beta^2}$$
$$\frac{\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{}+\frac{\left(\lambda^2-6\mu^2\right)^2\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{96\Psi^2\lambda^2\beta^2}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v155_2(x,t)=\frac{\lambda^4d^2\pm\frac{24\beta d\lambda^3}{\sqrt{6}}-12\lambda^2\mu^2d^2+24\lambda^2\beta^2+24\lambda^2\mu\beta d\mp\frac{144\mu^2\beta d}{\sqrt{6}}\pm\frac{288\lambda\mu\beta^2}{\sqrt{6}}+144\beta^2\mu^2}{96\lambda^2\beta^2}$$
$$\frac{+36\mu^4d^2-144\mu^3d\beta}{}-\frac{\left(\pm\frac{12\lambda^3\beta}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2}{\sqrt{6}}+12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4-12d\lambda^2\mu^2+36\mu^4d\right)}{48\Psi\lambda^2\beta^2}$$
$$\frac{\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{}+\frac{\left(\lambda^2-6\mu^2\right)^2\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{96\Psi^2\lambda^2\beta^2}$$

Substituting Eq (4.4.28) into Eq (4.2.3), along with Eq (3.1.10) and simplifying, yields the following travelling wave solutions,

$$\therefore v165(x,t)=\frac{1}{96\lambda^2\beta^2}\left(\lambda^4 d^2 \mp \frac{24\beta d\lambda^3 i}{\sqrt{6}}+12\lambda^2\mu^2 d^2+72\lambda^2\beta^2-24\lambda^2\mu\beta d \mp \frac{144\lambda\mu^2\beta di}{\sqrt{6}} \pm \frac{288\lambda\mu\beta^2 i}{\sqrt{6}}+144\beta^2\mu^2\right.$$

$$\left.+36\mu^4 d^2-144\mu^3 d\beta\right)-\frac{1}{48\lambda^2\beta^2}\left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2 i}{\sqrt{6}}-12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4+12d\lambda^2\mu^2+36\mu^4 d\right)$$

$$(d+H)+\frac{1}{96\lambda^2\beta^2}\left(\lambda^2+6\mu^2\right)^2(d+H)^2 \qquad \textbf{(xvi)}$$

where, $H=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}$

If $C_1=0$ and $C_2\neq 0$, then the above equation (xvi) becomes:

$$v165_1(x,t)=\frac{\lambda^4 d^2 \mp \frac{24\beta d\lambda^3 i}{\sqrt{6}}+12\lambda^2\mu^2 d^2+72\lambda^2\beta^2-24\lambda^2\mu\beta d \mp \frac{144\lambda\mu^2\beta di}{\sqrt{6}} \pm \frac{288\lambda\mu\beta^2 i}{\sqrt{6}}+144\beta^2\mu^2}{96\lambda^2\beta^2}$$

$$\frac{+36\mu^4 d^2-144\mu^3 d\beta}{}-\frac{\left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2 i}{\sqrt{6}}-12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4+12d\lambda^2\mu^2+36\mu^4 d\right)}{48\Psi\lambda^2\beta^2}$$

$$\frac{\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{}+\frac{\left(\lambda^2+6\mu^2\right)^2\left(d\Psi+\sqrt{\Delta}i\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{96\Psi^2\lambda^2\beta^2}$$

If $C_1\neq 0$ and $C_2=0$ then,

$$v165_2(x,t)=\frac{\lambda^4 d^2 \mp \frac{24\beta d\lambda^3 i}{\sqrt{6}}+12\lambda^2\mu^2 d^2+72\lambda^2\beta^2-24\lambda^2\mu\beta d \mp \frac{144\lambda\mu^2\beta di}{\sqrt{6}} \pm \frac{288\lambda\mu\beta^2 i}{\sqrt{6}}+144\beta^2\mu^2}{96\lambda^2\beta^2}$$

$$\frac{+36\mu^4 d^2-144\mu^3 d\beta}{}-\frac{\left(\mp\frac{12\lambda^3\beta i}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2 i}{\sqrt{6}}-12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4+12d\lambda^2\mu^2+36\mu^4 d\right)}{48\Psi\lambda^2\beta^2}$$

$$\frac{\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{}+\frac{\left(\lambda^2+6\mu^2\right)^2\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{96\Psi^2\lambda^2\beta^2}$$

4.6 Graphical representation of Fisher Equation: With the help of computational software, Maple, we have plotted some of the obtained solutions for both non-travelling and travelling wave solutions in below.

4.6.1 Graphs of non-travelling wave solutions:

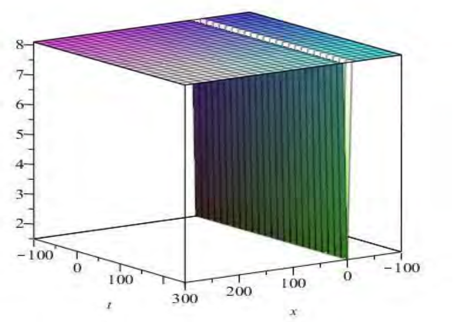

Fig-I: Periodic Solution

Eq - $u11_1$, for $\lambda = 20,\ \mu = 30,\ \delta = 6, \beta = 2,\ b_2 = 17$ and $x = -100..300,\ t = -100..250$

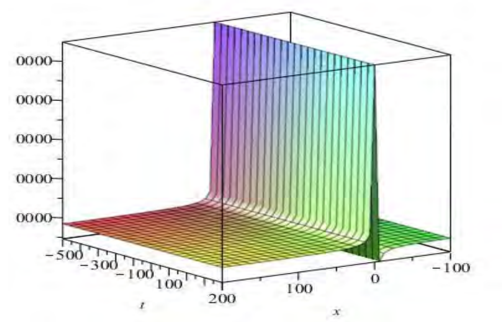

Fig-II: Periodic Solution

Eq- $u73_1$, for $\lambda = 0.597,\ \mu = 12.56, \beta = 2, d = 1, C_1 = 0.85, C_2 = 1.25$ and $x = -100..200,\ t = -500..400$

4.6.2 Graphs of travelling wave solutions:

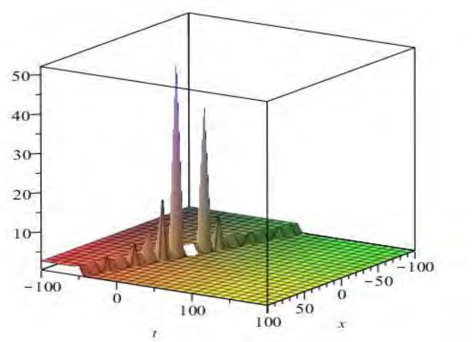

Fig-III: Soliton

Eq - $v71_1$, for $\lambda = 8,\ \mu = 3,\ \delta = 2.4,\ \beta = 2, d = 1.7$, $W = -\frac{5}{\sqrt{6}}$ and $x = -100..100,\ \ t = -100..200$

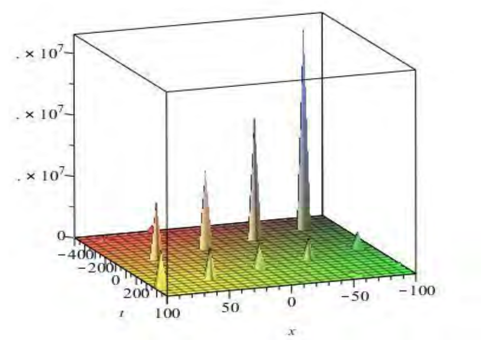

Fig-IV: Soliton

Eq- $v52_1$, for $\lambda = 0.1,\ \mu = 0.7, \delta = 0.04, \beta = -8$, $W = -\frac{5}{\sqrt{6}}$ and $x = -100..100,\ \ t = -100..200$

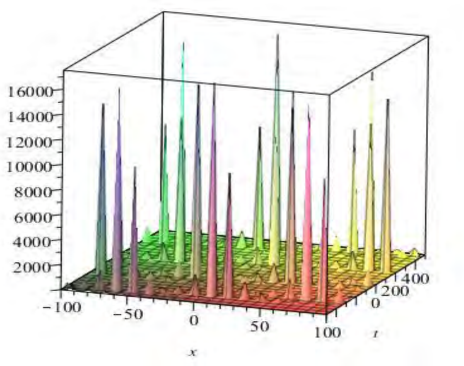

Fig-V: Solitons

Eq- $v72_2$,for $\lambda = 9,\ \mu = 3,\ \delta = 2.4, \beta = -2,$

$W = -\frac{5}{\sqrt{6}}, d = 1.7$ and $x = -100..100,\ t = -500..500$

CHAPTER 5

CONCLUSION

5.1 Conclusion:

In this thesis, we have found both travelling and non travelling wave solutions for the Fisher equation by using the further extension of the generalized and the improved (G'/G)-expansion method. The auxiliary equation used in the method, which involves many arbitrary parameters can take any real values and then the NLODE produces many new solutions. By using this method we have successfully found five types of solutions in terms of hyperbolic, trigonometric and rational functions. Through this method the solutions can be investigated with the help of symbolic computational software like the Maple or Matlab to facilitate the computation of the complex systems of algebraic equations.

In conclusion we can say that the new extension of the generalized and the improved (G'/G)- expansion method has been applied successfully in the Fisher's equation and based on the solutions and graphs we can conclude that this method is a powerful method for obtaining various types of wave solutions of the NLEEs that arises in every application of mathematical field.

5.2 Future work:

The method that has been used in this thesis has great advantages, as it provides many new and more abundant general and explicit travelling wave solutions along with non travelling wave solutions, with many real parameters, which are straightforward and precise. At the same time it also discloses more of the vital insight mechanism of the complex physical phenomena of NLEEs. Therefore this method can be used to solve different and all sorts of

NLPDEs that arises continually in engineering sciences, mathematical physics and in many scientific real time application fields. Moreover the solutions that we have found from the given results can be used for solving purpose in the application of chemistry, biology, population dynamics and medicine where nonlinear evolution problems involve.

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