A New Approach to Select Adaptive Intrinsic Mode Functions (IMFs) of Empirical Mode Decomposition (EMD)

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DECLARATION

We, hereby declare that this thesis is based on the results found by ourselves. Materials of work found by other researcher are mentioned by reference and through citation. This thesis, neither in whole nor in part, has been previously submitted for any degree.

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ABSTRACT

In the field of signal processing an adaptive algorithm for the selection of Intrinsic Mode Functions (IMF) of Empirical Mode Decomposition (EMD) is a time demand. In this paper, we propose an effective model for adaptive selection of IMFs after decomposition. This proposed algorithm decomposes an input signal using EMD, then the resultant IMF's are passed through a trained Support Vector Machine (SVM) for the separation of relevant and irrelevant IMF's. The irrelevant IMF's are then de-noised. And all IMFs are then reconstructed. The proposed model selects IMF adaptively without any human supervision and helps achieving higher Signal to Noise Ratio (SNR) while keeping Percentage RMS Difference (PRD) and Max Error low. Experiment results show up to 36.16% SNR value, PRD and Max Error are reduced to 1.557% and 0.085%, respectively.

Chapter 1

INTRODUCTION

1.1 Introduction

In this age of technological advancement signal processing has left its foot print in almost every technological sectors. From home automation to deep space exploration, advance robotics to medical research signal processing is working as backbone to those respective sectors. Signal processing is also used in artificial intelligence. Signal analysis helps in acquiring desired information's from a raw input data set [14]. In regards of performance limitation of Fourier based method in analysis of nonstationary signals Wavelet transform are commonly used [1]. Although widely used wavelet has locality and adaptability issues. EMD has proven itself as powerful algorithm of non-stationary signal analysis [2,3,6]. It decomposes a signal into finite set of frequency modulated components known as IMF. Without leaving the time domain EMD is adaptive and efficient in decomposing a signal. The resultant IMFs from the EMD are then separated in two sets, noise free and noise dominant IMFs. Various researchers have used different approaches on separation. Phuong et al. proposes Naïve Bayes classifier [3] a probabilistic and decision based classifier for the separation of noise-free and noise dominant IMFs. On the other hand, an energy based thresholding method has been proposed by [5] and [6]. Based on the results found by Douglas [4] comparison of the two we have used the proposed method by [6] for our SVM training. SVM has immense popularity in the field of machine learning due to many attractive features and excellent potential empirical performance. Moreover, SVM does not suffer from the limitations of data dimensionality and limited samples [7,8]. Traditional statistical classification techniques provide ideal results when sample size tends to infinity, when in most real cases samples are small and limited [9]. Thus we have proposed SVM as the IMF classifier and Savitzky-Golay filter (SGF), also known as least-squares smoothing filter is used to smoothen noisy IMFs. In [11], a comparison of de-noising methods is presented, where SGF works better in low to medium range SNR conditions.

1.2 Contribution Summary

In this paper we have proposed an adaptive IMF selection algorithm which exploits the features of energy based thresholding and empirical potential of SVM for the classification of noise-free IMFs. Then the adaptively selected noise dominant IMFs are de-noised through SGF. Noise free and de-noised IMFs are then reconstructed. Later conclusion is drawn from the results found by the comparison of SNR, PRD and Max Error of the input and reconstructed signal.

1.3 Thesis Orientation

The rest of this thesis is organized as follows:

- Chapter II discusses on the premises of our thesis – Algorithm to adaptively select Intrinsic Mode Functions(IMFs) after Empirical Mode Decomposition (EMD) using Support Vector Machine (SVM), Pearson's Correlation Coefficient and Savitzky-Golay filter to de-noise the rest of the IMFs. All the IMFs are then reconstructed for evaluation.
- Chapter III discusses our proposed model.
- Chapter IV describes experimental result analysis to determine the proposed model's efficiency.
- Chapter VI will conclude this paper with some words on the future scope regarding this research as well as its limitations.

Chapter 2

BACKGROUND STUDY

2.1 Hilbert Huang Transformation (HHT)

The Hilbert–Huang transform (HHT) is an empirically based data-analysis method. Its source of expansion is adaptive, for which it can produce physically meaningful representations of data from non-linear and non-stationary processes. It has a limitation as it can't be laid to a firm theoretical foundation.

2.2 Empirical Mode Decomposition (EMD)

The Empirical Mode Decomposition (EMD) is an important step to reduce any given data into a collection of intrinsic mode functions (IMF) to which the Hilbert spectral analysis can be applied. EMD has proven its strength over many comparative studies [15]. IMF represents a simple oscillatory mode as an equivalent to the simple harmonic function, but it is much more general: instead of constant amplitude and frequency in a simple harmonic component, an IMF can have variable amplitude and frequency along the time axis.

The procedure of extracting an IMF is called sifting. The sifting process is as follows:

- Identify all the local extrema in the test data X(t).
- Connect all the local maxima by a cubic spline line as the upper envelope.
- Repeat the procedure for the local minima to produce the lower envelope.

The upper and lower envelopes should cover all the data between them. Their mean is m_1. The difference between the data and m_1 is the first component h_1:

$$h_1 = X(t) - m_1 \quad \textbf{[1]}$$

Ideally, h_1 should satisfy the definition of an IMF, since the construction of h_1 described above should have made it symmetric and having all maxima positive and all minima negative. After the first round of sifting, a crest may become a local maximum. New extrema generated in this way actually reveal the proper modes lost in the initial examination. In the subsequent sifting process, h_1 can only be treated as a proto-IMF. In the next step, h_1 is treated as data:

$$h_2 = h_1 - m_2 \qquad \textbf{[2]}$$

After repeated sifting up to k times, h_1 becomes an IMF, that is:

$$h_1 = h_{k-1} - m_k \qquad \textbf{[3]}$$

Then, h_k is designated as the first IMF component of the data:

$$c_1 = h_k \qquad \textbf{[4]}$$

2.3 Intrinsic Mode Function (IMF)

Intrinsic Mode Functions (IMF) is defined as a function that fulfils the following requirements:

i. In the whole data set, the number of extrema and the number of zero-crossings must either be equal or differ at most by one.

ii. At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

It represents a generally simple oscillatory mode as an equivalent to the simple harmonic function. By definition, an IMF is any function with the identical number of extrema and zero crossings, whose envelopes are symmetric with respect to zero. This definition assurance's a well-behaved Hilbert transform of the IMF.

2.4 Pearson's Correlation Coefficient

A statistic measuring the linear interdependence between two variables or two sets of data. This is implemented in order to identify relevant and irrelevant IMFs. The relevant IMFs are selected on the basis of their coefficient value compared to the threshold value. If $P_i \geq T$ the IMF is relevant, irrelevant if not. This is taken as an aid to train the SVM.

2.5 Support Vector Machine (SVM)

In machine learning, support vector machines (SVMs), also known as support vector networks are supervised learning models with associated learning algorithms that analyse data used for classification and regression analysis. Given a set of training examples, each marked for belonging to one of two categories, an SVM training algorithm builds a model that assigns new examples into one category or the other, making it a non-probabilistic binary linear classifier. An SVM model is a representation of the examples as points in space, mapped so that the examples of the separate categories are divided by a clear gap that is as wide as possible. New examples are then mapped into that same space and predicted to belong to a category based on which side of the gap they fall on.

In addition to performing linear classification, SVMs can efficiently perform a non-linear classification using what is called the kernel trick, implicitly mapping their inputs into high-dimensional feature spaces. For that reason, SVM is applied in many critical medical diagnostic approaches like EEG [16], ECG [17] signal classification, cancer identification, face recognition, and speech disorder.

When data are not labelled, supervised learning is not possible, and an unsupervised learning approach is required, which attempts to find natural clustering of the data to groups, and then map new data to these formed groups. The clustering algorithm which provides an improvement to the support vector machines is called support vector clustering and is often used in industrial applications either when data is not labelled or when only some data is labelled as a pre-processing for a classification pass.

2.6 Savitzky-Golay filter

Savitzky-Golay smoothing filters (also called digital smoothing polynomial filters or least-squares smoothing filters) are typically used to "smooth out" a noisy signal whose frequency span (without noise) is large [18,19]. In this type of application, Savitzky-Golay smoothing filters perform much better than standard averaging FIR filters, which tend to filter out a significant portion of the signal's high frequency content along with the noise. Although Savitzky-Golay filters are more effective at preserving the pertinent high frequency components of the signal, they are less successful than standard averaging FIR filters at rejecting noise. Savitzky-Golay filters are optimal in the sense that they minimize the least-squares error in fitting a polynomial to frames of noisy data [20].

2.7 Degree of Accuracy

2.7.1 Signal to Noise Ratio (SNR)

SNR is a measurement unit that compares the level of a desired signal to the level of background noise. It is defined as the ratio of signal power to the noise power, often expressed in decibels when normalized. A ratio higher than 1:1 (greater than 0 dB) indicates more signal than noise. Signal-to-noise ratio is sometimes used informally to refer to the ratio of useful information to false or irrelevant data in a conversation or exchange. For any de-noising techniques therefore higher SNR values are desired and low values represent poor performance. However, pure SNR values are not always used. The industry standards measure SNR in decibels(dB) of power and therefore apply a log rule. Here we have used the 20 log rule along with the pure SNR ratio to yield better sensitivity in the values. This allows us to normalize the values for better illustration purposes. According to industry standards an SNR value of 32 dB and above means excellent image quality and SNR values around 20 dB is defined as acceptable image quality.

2.7.2 Percentage RMS Difference (PRD)

The RMS value of any varying signal provides a measure of the amount of energy deliverable by the source if it were instead constant in nature. The difference in RMS value of the signal and the that of the processed signal allows us to evaluate how similar they are and therefore can be used as a measure of the quality of the de-noising technique(SGF) used. The smaller the difference is the better the resemblance to the original signal and hence a sign of better performance. Instead of directly using the difference value, a percentage difference makes more sense as no two signals have the same energy content. The percentage RMS difference normalizes the variety of signals, and provides consistent values that would otherwise be erratic in nature. Therefore, lower PRD value represents better performance and higher values represent bad performance.

2.7.3 Max Error

Max error represents the maximum value of the absolute difference between two signals. Now, if we consider the original signal and the de-noised signal the max error between the two signals represents how far off one is from in other in terms of their similarity. Larger values would indicate that the smoothing was too much or too little. Hence, lower max error values are desired. Lower Max Error values would represent better performance and higher values would represent bad performance.

Chapter 3

PROSPOSED MODEL

3.1 Proposed model

A complete block diagram of the proposed model is shown in Figure 1. We have used EMD to decompose the input signal into a finite number of IMFs. There is no change in the traditional EMD algorithm. A statistical threshold value is derived from the IMFs using the Pearson's correlation coefficient and the IMFs are classified into noise free and noise dominant IMFs. A minor subset of the classification results is used to train the SVM. Then a larger set of IMFs are fed into the SVM classifier. The Noise-free IMFs and the filtered Noise-dominant IMFs classified by the SVM are used to reconstruct the signal.

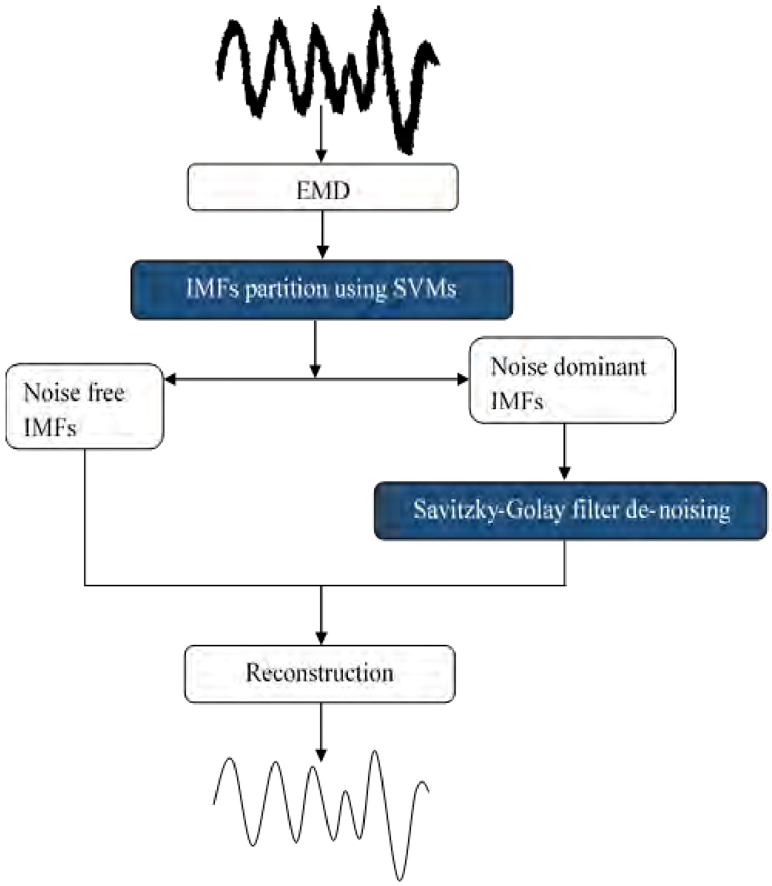

Figure 1: A flow diagram of the proposed model.

3.1.1 Empirical Decomposition and Threshold Derivation

The input signal is first decomposed into finite set of intrinsic mode functions. The Pearson correlation coefficient of the resultant IMFs are then calculated. Following the proposal of [6] a threshold value is derived (eqn.5). If Pearson's correlation coefficient of the IMFs is μ_i and the threshold is expressed as τ, expression of the threshold is:

$$\tau = \frac{\max(\mu_i)}{10 * \max(\mu_i) - 3} \quad \textbf{[5]}$$

The relevant IMFs are selected on the basis of their coefficient values calculated earlier in compare to the threshold value. If $\mu_i \geq \tau$ the IMF is relevant, irrelevant if not. This is taken as an aid to train the SVM.

3.1.2 SVM training and IMF Classification

Using a subset of the decomposed IMFs, we train an SVM object. To train the SVM we used standard deviation [13], and root mean square (RMS) [12] values as features of the IMFs for training the SVM object. Then a larger set of IMFs are classified by using the trained SVM object. The SVM algorithm classifies the IMFs into noise-free and noise dominant IMFs.

3.1.3 Denoising the Noise Dominant IMFs

Using the Savitzky-Golay filter, the set of noise dominant IMFs are then smoothened using an 3^{rd} order polynomial and a frame size of 41. A low order polynomial is used so that a good amount of smoothing is achieved, higher orders may lead to better metrics but at the cost of reduced noise reduction therefore, a high polynomial value is avoided. Neither a low frame size nor a high frame size is used as this would over smoothen the data or destroy signal properties respectively. The polynomial and frame size were chosen after a few close combinations were used to find the optimal one that suits our input data. The third order polynomial and a frame size of forty-one yielded better results.

3.1.4 Reconstruction of the Signal

The noise-free IMFs and de-noised IMFs are used to reconstruct the signal. Summation of the IMFs gives us the new filtered signal.

$$X = \sum noise\ free\ IMFs + \sum de - noised\ IMFs \qquad \textbf{[6]}$$

Where *'R'* is the reconstructed signal, which we later use for the comparison between the original and newly constructed signal also for comparison between different methods.

Chapter 4

EXPERIMENTAL SETUP AND RESULTS

4.1 Experimental setup

To evaluate the performance of proposed model we used MATLAB® version 14 simulation tools. The data set used in this simulation is the same as of the paper [10]. A randomized white noise has been added to the original signal in order to evaluate the de-noising techniques based on the added noise estimate. In the experiment we used the three parameters PRD, SNR and Max Error value to evaluate the results. Figure 2, 3, 4 and 5 demonstrate 4 sample signals, their SVM only de-noised version, their Pearson's coefficient based thresholding de-noised version, and finally using our proposed model consisting of SVM classification and de-noising using the Savitzky-Golay smoothing filter.

4.2 Results

Table 1: Calculated PRD values of the different methods

Signal No's	Construct signal using first 3 IMFs	Construct signal using first 7 IMFs	Construct signal using SVM classified IMFs	Construct signal using PCC classified IMFs	Construct signal using proposed Model
1	33.23221	7.838449	16.09981	16.09981	1.557232
2	87.25636	10.92784	26.86902	26.86902	20.83046
3	34.41056	7.861017	16.45177	16.45177	1.731638
4	85.81789	9.857918	15.99813	15.99813	1.881234
5	442.0039	17.15239	17.16053	17.16053	1.889945
6	367.1038	15.93682	15.95132	15.95132	1.803857
7	393.5864	15.70988	15.72584	15.72584	1.594879
8	339.2442	15.68831	15.69245	15.69245	1.777524
9	317.163	16.16164	16.15663	16.15663	1.734342
10	361.1604	16.90043	16.9242	16.9242	1.995055

The mathematical formula of PRD is as follows:

$$PRD = \sqrt{\frac{\sum_{n=0}^{N}(V(n) - V_R)^2}{\sum_{n=0}^{N}(V(n))^2}} \times 100\% \quad \textbf{[7]}$$

where V_n: original signal and V_R: reconstructed signal.

Table 1 shows PRD values for signals constructed using five different methods. Lower PRD value represents closer resemblance to original signal. Each of the columns represent values of respective methods headed by their name. Rows represents the first10 signals out of the used input signal set. The values show a clear trend of efficiency as the columns progress. Our proposed model, SVM classification and de-noising through SGF, establishes a clean dominance over other methods used.

Table 2: Calculated SNR values of the different methods

Signal No's	Construct signal using first 3 IMFs	Construct signal using first 7 IMFs	Construct signal using SVM classified IMFs	Construct signal using PCC classified IMFs	Construct signal using proposed Model
1	9.861508	22.1167	15.89195	15.89195	36.15505
2	3.602976	19.2322	11.4458	11.4458	13.63161
3	9.593254	22.09255	15.71066	15.71066	35.23365
4	3.699018	20.12644	15.94906	15.94906	34.51364
5	0.233363	15.34763	15.34373	15.34373	34.47421
6	0.347342	15.98028	15.97261	15.97261	34.87866
7	0.292333	16.10222	16.09367	16.09367	35.94732
8	0.360645	16.11375	16.11163	16.11163	35.0058
9	0.401466	15.85567	15.8585	15.8585	35.21942
10	0.323476	15.47803	15.466	15.466	34.00385

The mathematical equation of SNR is as follows:

$$SNR_{db} = 20 \times \log_{10} \frac{signal}{noise} \quad \textbf{[8]}$$

The proposed model performs exceptionally well over all the other methods, which is evident from the three tables (Table 1,2,3). Table 2 and Figure 3 demonstrate the performance of the proposed model with other conventional models considering SNR. The proposed method stays consistent even for large set data inputs, but for better readability only 10 have been shown.

The Table 3 presents the experimental results of proposed model and other state-of-art models considering the Max Error values.

Table 3: Calculated Max Errors values of the different methods

Signal No's	**Construct signal using first 3 IMFs**	**Construct signal using first 7 IMFs**	**Construct signal using SVM classified IMFs**	**Construct signal using PCC classified IMFs**	**Construct signal using proposed Model**
1	3.352951	0.915177	1.958808	1.958808	0.103132
2	6.285758	1.269061	4.366918	4.366918	4.355913
3	3.787007	0.92158	2.519456	2.519456	0.344366
4	6.158683	1.053266	1.804797	1.804797	0.103655
5	8.311906	1.865461	1.863248	1.863248	0.135298
6	8.500025	1.952262	1.956835	1.956835	0.093569
7	8.445439	1.274218	1.279523	1.279523	0.125799
8	8.784585	1.602733	1.604387	1.604387	0.099009
9	8.623053	1.693574	1.694402	1.694402	0.085452
10	9.908853	1.824299	1.828911	1.828911	0.11097

Max Error also a good parameter for performance evaluation as suggested in [11]. The formula used here is quite simple. R_i being the result signal and Si representing the original signal, the normalized equation is:

$$\Delta_{max} = |R_i - S_i| \quad \textbf{[9]}$$

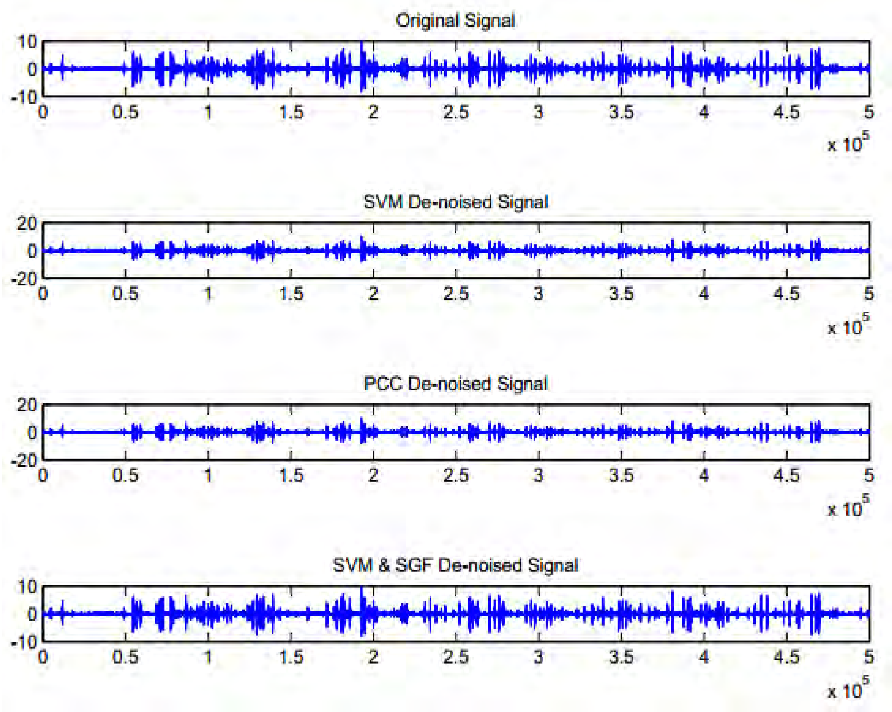

Figure 2: Original and Reconstructed Signal 1.

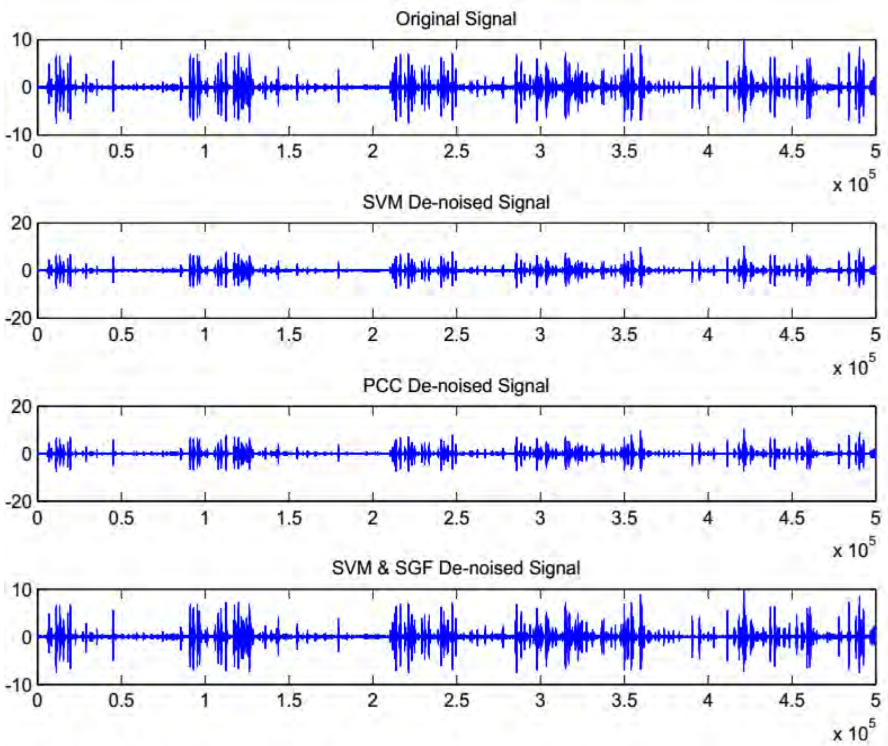

Figure 3: Original and Reconstructed Signal 2.

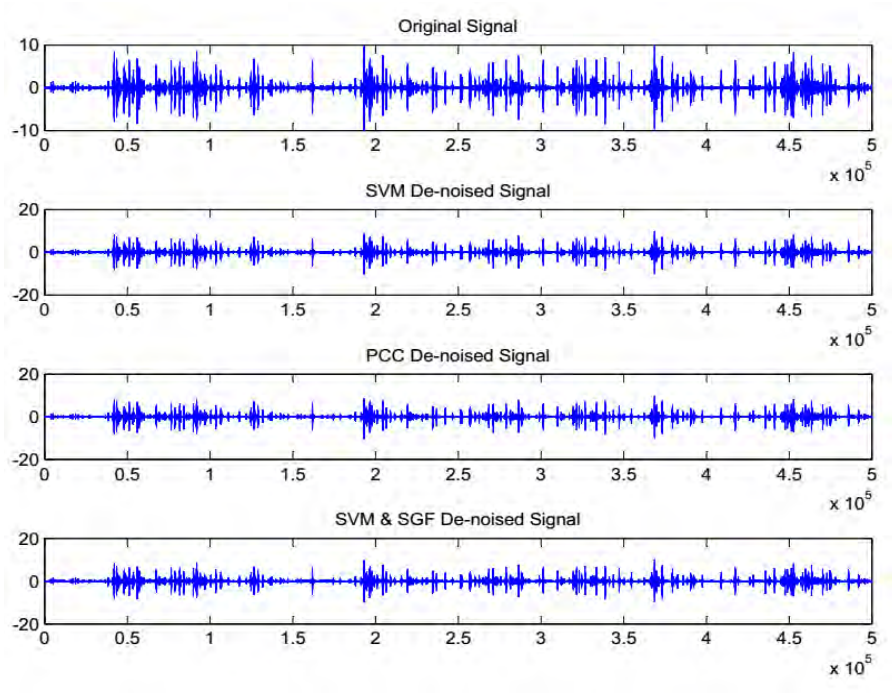

Figure 4: Original and Reconstructed Signal 3.

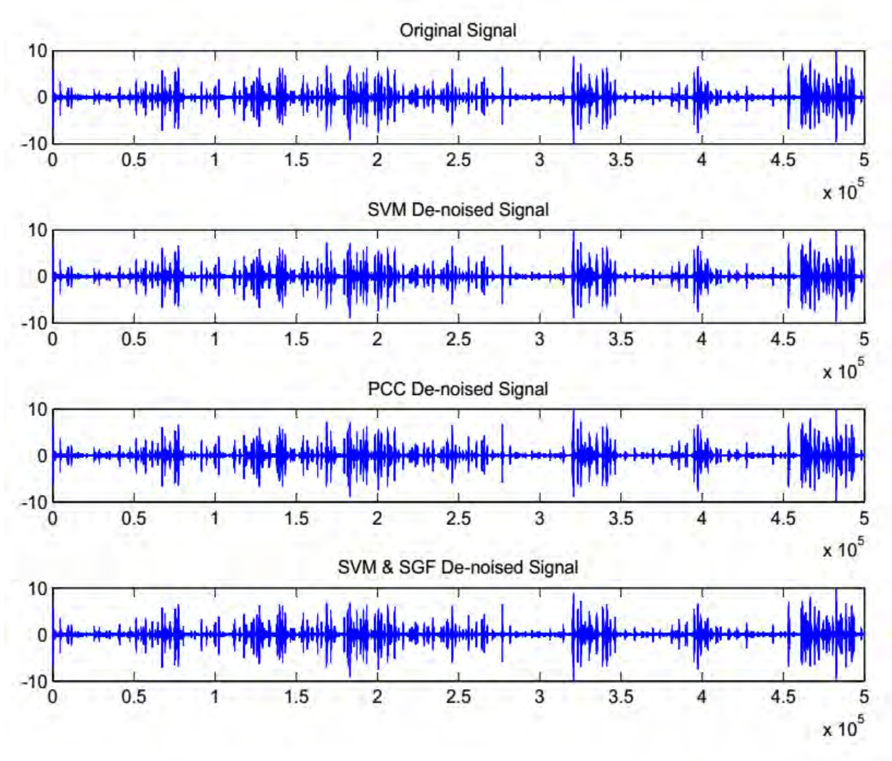

Figure 5: Original and Reconstructed Signal 4.

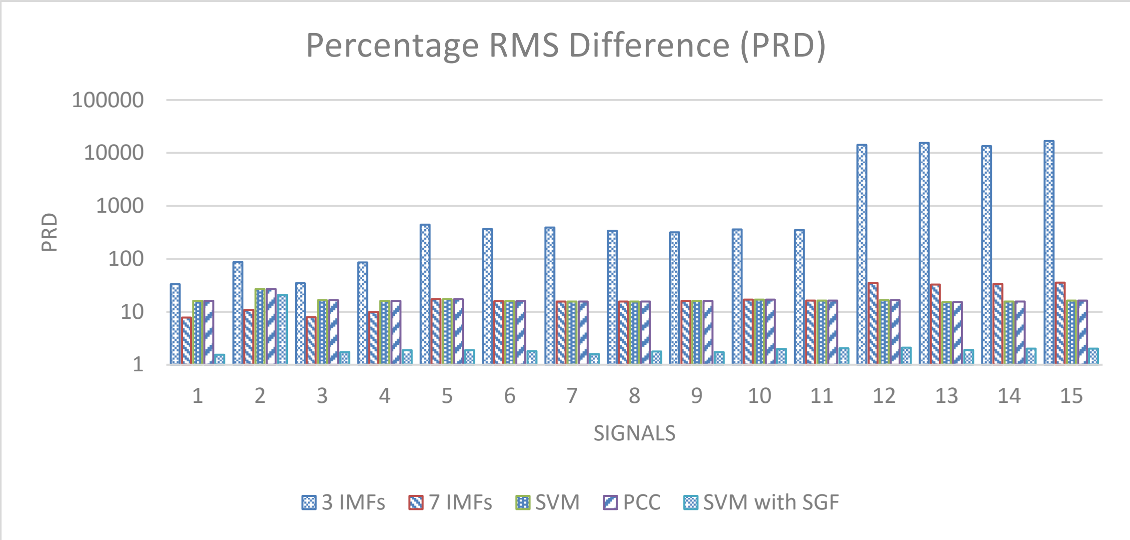

Figure 6: PRD values for different methods.

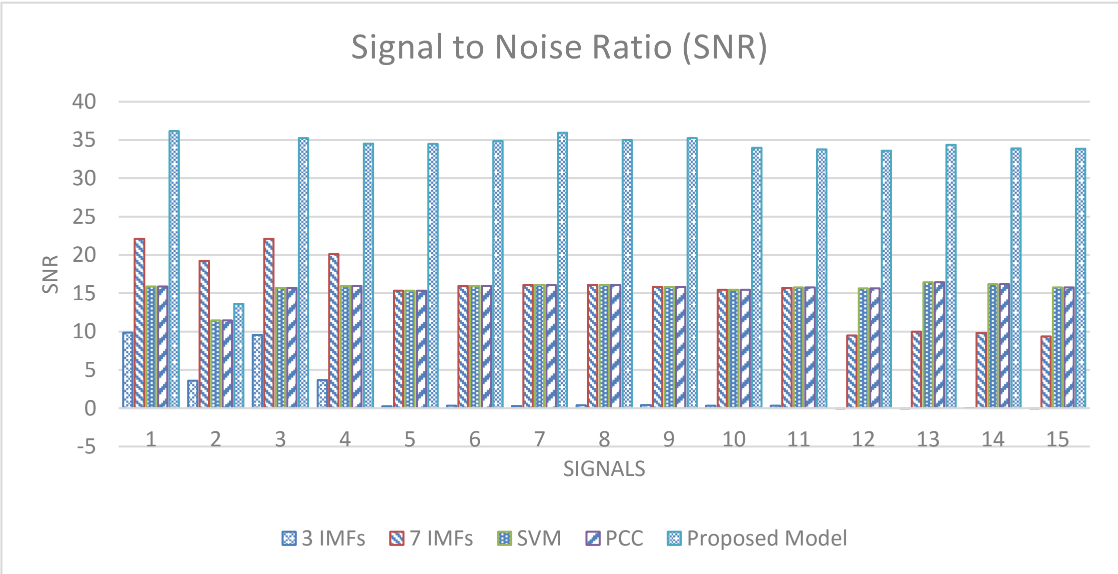

Figure 7: SNR values for different methods.

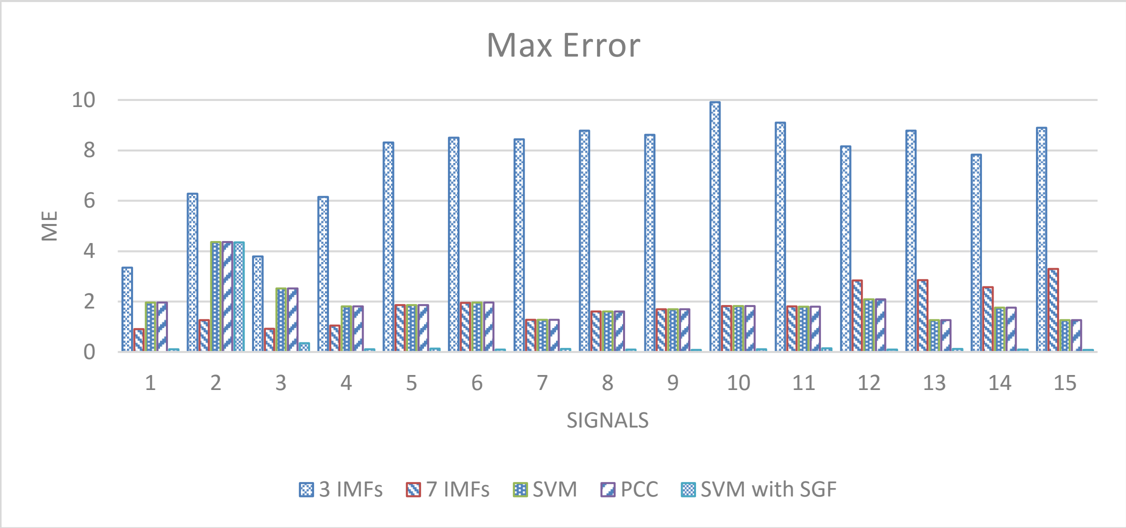

Figure 8: ME values for different methods.

Chapter 5

CONCLUSION

5.1 Conclusion

This paper presents a noble method of adaptive IMF selection for EMD. In this model we implement SVM for the classification of decomposed IMFs. The classified noise dominant IMFs are de-noised using the Savitzky-Golay filter. Later noise free and smoothened IMFs are reconstructed. The experiment shows that the proposed model achieved a maximum 36.16% SNR, PRD and Max Error are reduced to 1.557% and 0.085% respectively in a medium noisy environment, a considerable improvement over all other models.

5.2 Future works

The classification of IMFs, de-noising them and re-constructing back to a clean version of the original signal is a field where lots of effort have been put and some remarkable results obtained. But there are far too many methods and their combinations to obtain the final results. Not all have been tried out yet. This is an attempt to implement, obtain results and compare a combination of our choice, an attempt to contribute in the field of signal de-noising. Our future target is to do the same with various other methods and group all results, and make a comparison study between them. As different classifications would lead to IMFs better suited for a specific use case. After successful completion, a general idea about SVM's capabilities in noise reduction can be gained. But it also raises the question are there any better classifiers available? Our future work is to discover the answers to the above question to find which classifiers and de-noising methods work best on which types of data sets. The proposed model in of itself is adaptive in nature but as noise levels change for better results other different methods may have different results to offer.

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