COMPARISON OF DIFFERENT MODELS FOR THE ANALYSIS OF RAYLEIGH FADING CHANNELS

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December 2007

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DECLARATION

We hereby declare that this thesis is based on the results found by us. Materials of work found by other researchers are mentioned by reference. This thesis, neither in whole nor in part, has been previously submitted for any degree.

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ACKNOWLEDGEMENTS

Special thanks to Dr. M A Matin who taught us how to think of fading processes clearly when we were fortunate enough to be doing our thesis work under his supervision, to Dr. Tarik A Chowdhury, our co-supervisor who has helped us with our work throughout the research period and all other respected faculty members of the Department of Computer Science and Engineering who have provided their all out support during this time.

ABSTRACT

These days, computer simulation is increasingly being used for design and performance evaluation of communication systems. When simulating a mobile wireless channel for communication systems, it is usually assumed that the fading process is a random variate with Rayleigh distribution. In this paper, we have tried to understand Rayleigh Fading (which best describes the scenario in an urban or macrocell environment) better by comparing different models and observing the changes that are caused by varying different parameters.

At present, there are a number of methods to generate the Rayleigh fading process, some of them quite recently proposed. Due to the use of different Rayleigh fading generators, different simulations of the same communication system yield different results. Two methods, viz., the Jakes method and the Dent (Modified Jakes) method have been studied, simulated and compared based on the Rayleigh fading process properties. Various communication systems have been simulated using the Rayleigh fading generators and the differences in the results, if any, have been analyzed. It was seen that the Dent method is usually seen to better describe a Rayleigh fading process compared to the Jakes method.

Furthermore, BER analysis of a Rayleigh Fading Process was also done to see the effects on the change of different parameters on the variance of BER with respect to the SNR. Also, the Rayleigh Fading Process was compared to the phenomenon of AWGN using this process.

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CHAPTER I

INTRODUCTION

Simulation of wireless channels accurately is very important for the design and performance evaluation of wireless communication systems and components. Fading or loss of signals is a very important phenomenon and must be well understood by all engineers related to the Wireless Communications Field. That leads us to the fading models which try to describe the fading patterns in different environments and conditions. Although no model can ‘perfectly’ describe an environment, they strive to obtain as much precision as possible. The better a model can describe a fading environment, the better can it be compensated with other signals, so that, on the receiving end, the signal is error free or at least close to being error free. This would mean higher clarity of voice and higher accuracy of data transmitted over wireless medium.

When simulating the wireless channel for mobile and macro cellular communications, it is usually assumed that the fading process is a Rayleigh fading process. The discrete samples of the Rayleigh fading process have a Rayleigh distribution and are correlated. For generating the Rayleigh fading coefficients, two different methods of random variate generations are used. The model proposed by Jakes is a commonly accepted model of a multipath fading environment. The initial simulation method that was used is the Sum of sinusoids method proposed by Jakes. The fading process can also be realized by a different model, the Dent Model, which is basically and improvement over the Jakes Model. This report attempts to find a well suited Rayleigh fading process generation technique for a given communication system.

Chapter II gives a background on Wireless Communications and Fading Channels.

Chapter III does a BER Analysis of the Rayleigh fading process and compares the differences in BER between Rayleigh Fading and AWGN.

Chapter IV compares the two generation methods based on the Rayleigh fading in terms of their behavior when the different input parameters (including the Speed *v*, the Central Carrier Frequency *fc*, the Symbol Frequency *fs* and the Number of Channel Coefficients to Generate *M*) were changed.

Chapter V compares the Jakes and Dent processes in terms of the value of the MSE (Mean Square Error) obtained from the Autocorrelation and Cross-correlation of the signals.

Chapter VI concludes the work.

CHAPTER II

BACKGROUND ON WIRELESS COMMUNICATIONS AND FADING CHANNELS

2.1 Fading and Multipath

Fading refers to the distortion that a carrier-modulated telecommunication signal experiences over certain propagation media [7]. In wireless systems, fading is due to multipath propagation and is sometimes referred to as multipath induced fading. To understand fading, it is essential to understand multipath. In wireless telecommunications, multipath is the propagation phenomenon that results in radio signals' reaching the receiving antenna by two or more paths. Causes of multipath include atmospheric ducting, ionospheric reflection and refraction, and reflection from terrestrial objects, such as mountains and buildings. The effects of multipath include constructive and destructive interference, and phase shifting of the signal. This distortion of signals caused by multipath is known as fading.

In other words it can be said that in the real world, multipath occurs when there is more than one path available for radio signal propagation. The phenomenon of reflection, diffraction and scattering all give rise to additional radio propagation paths beyond the direct optical LOS (Line of Sight) path between the radio transmitter and receiver.

2.2. Fading Channels

A Fading Channel is a communications channel which has to face different fading phenomenons while the signal is carried from the transmitter to the receiver. Fading Channels face a phenomenon called multipath (as described above) which occurs when all the radio propagation effects combine in a real world environment. In other words, when multiple signal propagation paths exist, caused by whatever phenomenon, the actual received signal level is the vector sum of all the signals incident from any direction or angle of arrival. Some signals will aid the direct path, while other signals will subtract (or tend to vector cancel) from the direct signal path. The total composite phenomenon is thus called Multipath.

2.3. Causes of Fading

Fading is caused by different physical phenomemon, some of which have been discussed below.

2.3.1 Doppler Shift

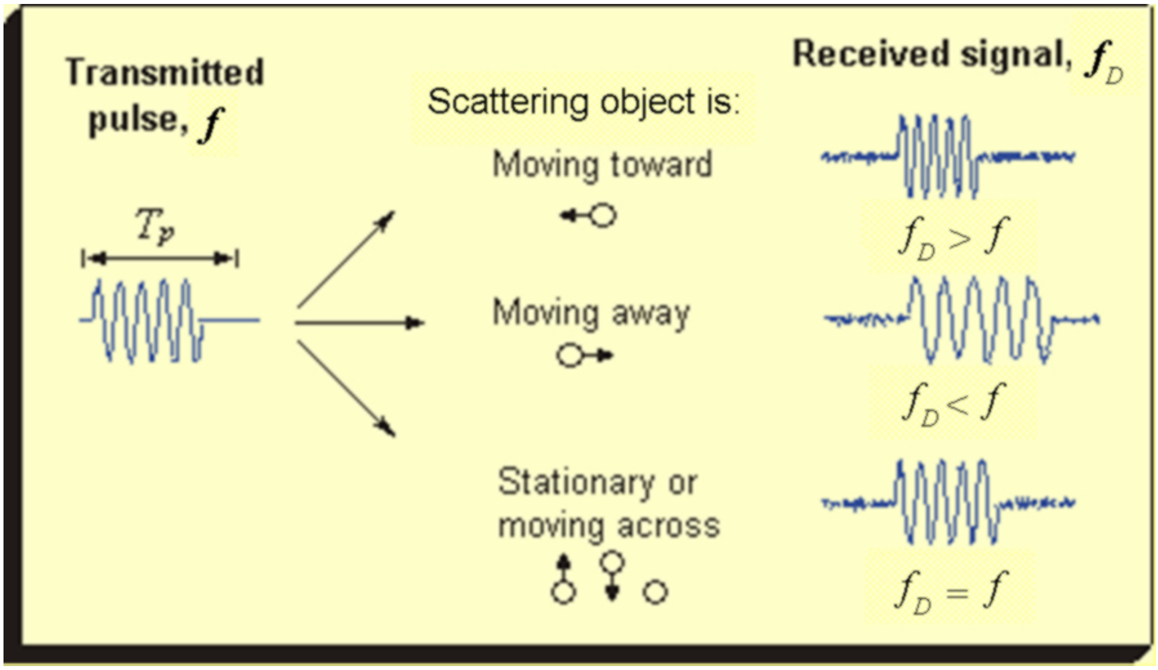

Fig:2.1. Diagrammatic Representation of Doppler Shift

(Picture Courtesy: Research at University of Southern Maine)

Let us consider a situation where a mobile is moving at a constant velocity *v* along a path and the source is moving at a velocity of v_s [8]. The observed frequency *f'* and the emitted frequency *f* will be related by the following equation:

$$f' = \left(\frac{v}{v \pm v_s}\right) f \tag{2.1}$$

It is evident from the formula that the detected frequency increases for objects moving towards the observer and decreases when the source moves away. This is known as the Doppler Effect.

2.3.2. Reflection

Reflection occurs when a propagating electromagnetic wave impinges upon an object which has very large dimensions when compared to the wavelength of the propagating wave. In other words it can be said that if the plane wave is incident on a perfect dielectric, part of the energy is transmitted and part of the energy is reflected back into the first medium. If the second medium is a perfect conductor, all the energy is reflected back. Reflections occur from the surface of the earth and from buildings and walls. In practice, not only metallic materials cause reflections, but dielectrics also cause this phenomenon.

2.3.3 Diffraction

Diffraction occurs when the radio path between the transmitter and receiver is obstructed by a surface that has sharp irregularities (edges). The secondary waves resulting from the obstructing surface are present throughout the space and even behind the obstacle, giving rise to a bending of waves around the obstacle, even when a Line of Sight path does not exist between transmitter and receiver. At high frequencies, diffraction, like reflection, depends on the geometry of the object, as well as the amplitude, phase, and polarization of the incident wave at the point of diffraction.

2.3.4 Scattering

Scattering occurs when the medium through which the wave travels consists of objects with dimensions that are small compared to the wavelength, and where the number of obstacles per unit volume is large. Scattered waves are produced by rough surfaces, small objects, or by other irregularities in the channel. In practice, foliage, street signs, and lamp posts induce scattering in a mobile communications system.

2.4 Types of Fading

2.4.1 According to Mutipath

There are two types of fading according to the effect of Multipath. These are Large Scale Effect and Small Scale Effect [4].

2.4.1.1 Large Scale Effect

In Large Scale Fading, the received signal power varies gradually due to signal attenuation determined by the geometry of the path profile.

2.4.1.2 Small Scale Effect

Multipath propagation leads to rapid fluctuation of the phase and amplitude of the signal if it moves over a distance in the order of wave length. This is know as Small Scale Effects.

2.4.2 According to Delay Spread

There are two types of fading according to the effect of Delay Spread. These are Flat Fading and Frequency Selective Fading [4].

2.4.2.1 Flat Fading

Flat Fading is one in which all frequency components of a received radio signal vary in the same proportion simultaneously. If the bandwidth of the mobile channel is greater than the bandwidth of the transmitted channel, it is called flat fading.

2.4.2.2 Frequency Selective Fading

Selective fading or frequency selective fading is a radio propagation anomaly caused by partial cancellation of a radio signal by itself — the signal arrives at the receiver by two different paths, and at least one of the paths is changing (lengthening or shortening). This typically happens in the early evening or early morning as the various layers in the ionosphere move, separate, and combine. The two paths can both be skywave or one be groundwave.

Selective fading manifests as a slow, cyclic disturbance; the cancellation effect, or "null", is deepest at one particular frequency, which changes constantly, sweeping through the received audio.

2.4.3 According to Doppler Spread.

There are two types of fading according to the effect of Doppler Spread [4]. These are Fast Fading and Slow Fading. To understand fast fading and slow fading the coherence time must be understood. The terms slow and fast fading refer to the rate at which the magnitude and phase change imposed by the channel on the signal changes. The coherence time is a measure of the minimum time required for the magnitude change of the channel to become decorrelated from its previous value. In other words it can be said that coherence time is the time duration over which two received signals have a strong potential for amplitude correlation.

2.4.3.1 Slow fading

Slow fading arises when the coherence time of the channel is large relative to the delay constraint of the channel. In this regime, the amplitude and phase change imposed by the channel can be considered roughly constant over the period of use. Slow fading can be caused by events such as shadowing, where a large obstruction such as a hill or large building obscures the main signal path between the transmitter and the receiver.

2.4.3.2 Fast fading

Fast fading occurs when the coherence time of the channel is small relative to the delay constraint of the channel. In this regime, the amplitude and phase change imposed by the channel varies considerably over the period of use. In a fast-fading channel, the transmitter may take advantage of the variations in the channel conditions using time diversity to help increase robustness of the communication to a temporary deep fade. Although a deep fade may temporarily erase some of the information transmitted, use of an error-correcting code coupled with successfully transmitted bits during other time instances can allow for the erased bits to be recovered.

2.5 Types of Small Scale Fading

There are many models that describe the phenomenon of small scale fading [4]. Out of these models, Rayleigh fading, Ricean fading and Nakagami fading models are most widely used.

2.5.1. Rayleigh fading model

Rayleigh fading is primarily caused by multipath reception. Rayleigh fading is a statistical model for the effect of a propagation environment on a radio signal [3]. It is a reasonable model for tropospheric and ionospheric signal propagation as well as the effect of heavily built-up urban environments on radio signals. Rayleigh fading is most applicable when there is no line of sight between the transmitter and receiver. If there is at least one line of sight component, Ricean fading is more applicable.

In a wireless system, a signal transmitted into the channel interacts with the environment in a very complex way, bouncing off various surfaces along the way to the receiver. There are reflections from large objects, diffraction of electromagnetic waves around objects. The result of these complex interactions is the presence of many signal components, or multipath signals, at the receiver. In addition to this, if the transmitter, receiver or the objects in the path of the signal are in motion, Doppler shift is introduced. As a result of these two phenomenon, the received signal is time varying and may be highly attenuated. This is a major impairment in a wireless communication system. At any given time instance, a number of plane waves will be incident on the mobile antenna. Assuming that the career frequency is *fc*, and the mobile station is moving at a velocity of *v*, if the *nth* wave is incident on the mobile antenna at an angle of $\theta_n(T)$ relative to the direction of motion of the mobile, the Doppler shift introduced in the incident wave is given by

$$f_{D,n}(t) = f_d \cos\theta(t) \qquad (2.2)$$

where $f_d = {}^{v}\!/_{f_c}$ and *c* is the wavelength of the transmitted signal. If the transmitted Signal s(t) is given by $\mathrm{Re} = \{u(t)e^{j2\pi f_c t}\}$ where u(t) is the complex lowpass signal, v(t), the received complex low pass signal is given by

$$v(t) = \sum_{n=1}^{N_0} \alpha_n(t) e^{-j2\pi[(f_c + f_{D,n(t)})\tau_{n(t)} - f_{D,n(t)t}]} u(t - \tau_n(t)) \qquad (2.3)$$

where N is the total number of incident waves and $\alpha_n(t)$ and $\tau_n(t)$ are the amplitude and time delay.

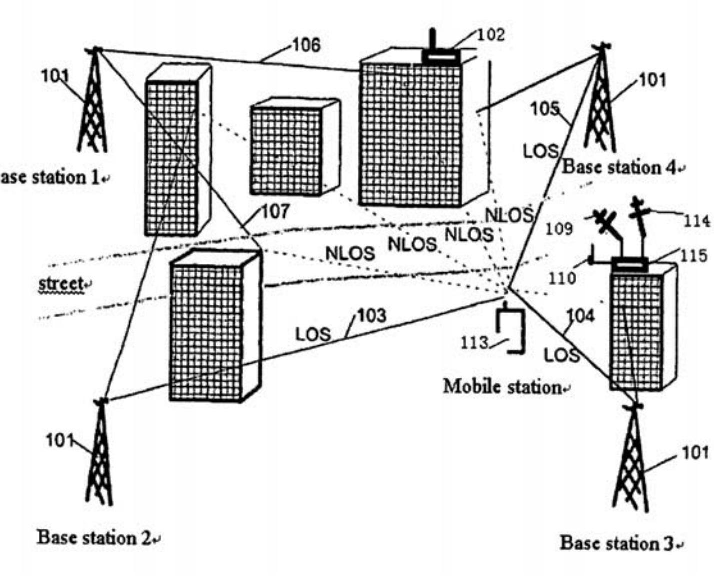

Fig 2.2. Rayleigh Fading environment in a typical urban area

In most applications no complete direct LOS propagation exists between the base-station antenna and the mobile antenna because of the natural and constructed obstacles. This is especially true for urban areas. In the urban areas, every time a signal is reflected, deflected and scattered, new multi path signals are created thus leading to more distortion in the received signal.

2.5.2.Ricean fading model

The Ricean fading model is similar to the Rayleigh fading model, except that in Ricean fading, a strong dominant component is present [4]. This dominant component is a stationary (nonfading) signal and is commonly known as the LOS (Line of Sight Component)

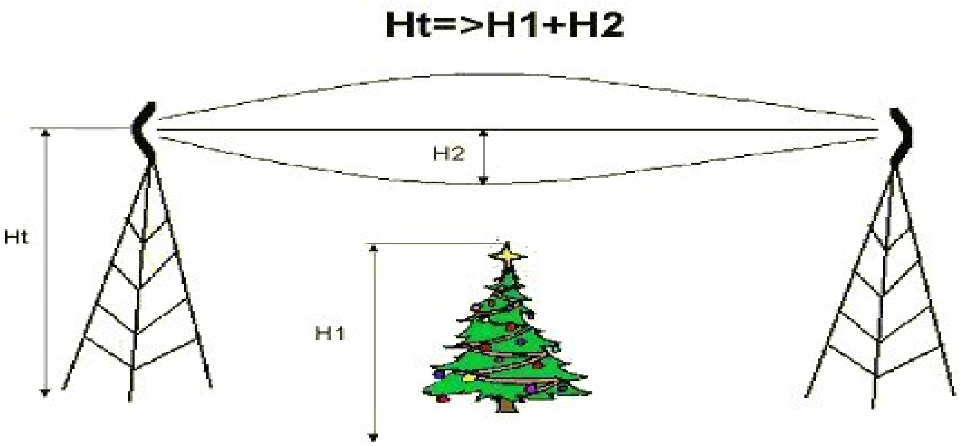

Fig 2.3. Illustration of the LOS (Line of Sight) Component

2.5.3.Nakagami fading model

Nakagami fading occurs for instances of multipath scattering with relatively large delay-time spreads, with different clusters of reflected waves. Within any one cluster, the phases of individual reflected waves are random, but the delay times are approximately equal for all waves [4].

2.6 Autocorrelation

The Autocorrelation of a signal is the expected value of the product of a random variable or signal realization with a time-shifted version of itself. With a simple calculation and analysis of the autocorrelation function, we can discover a few important characteristics about a random process. These include:

* How quickly the random signal or processes changes with respect to the time function

* Whether the process has a periodic component and what the expected frequency might be.

2.6.1 In-phase and quadrature component

There are two components in the Autocorrelation Component, the In-Phase Component and the Quadrature Component. Every sinusoid can be expressed as the sum of a sine function (phase zero) and a cosine function (phase $\pi/2$). If the sine part is called the ‘in-phase'' component’, the cosine part can be called the `phase-quadrature' or ‘quadrature’ component. In general, `phase quadrature' means `90 degrees out of phase’, i.e., a relative phase shift of $+\pi/2$.

It is also the case that every sum of an in-phase and quadrature component can be expressed as a single sinusoid at some amplitude and phase. Figure 2.4 illustrates in-phase and quadrature components overlaid.

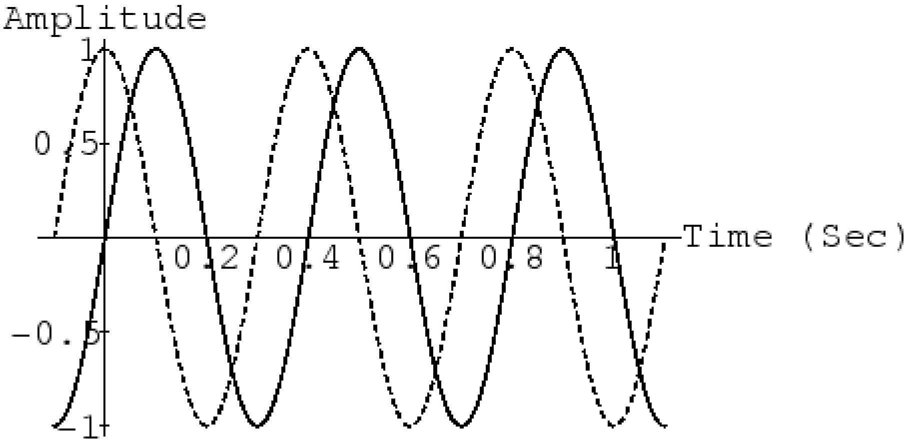

Figure 2.4: In-phase and quadrature sinusoidal components.

2.7 Cross-correlation

If two processes are wide sense stationary, the cross-correlation is defined as the expected value of the product of a random variable from one random process with a time-shifted, random variable from a different random process

2.7.1 Intra-fader

The cross-correlation functions *R(t)* between in-phase and quadrature components of any single fader (intra-fader) should be close to zero.

2.7.2 Inter-fader

Also, the cross-correlation functions between any pair of different faders (inter-fader) should be as close to zero as possible.

CHAPTER III

BER (BIT ERROR RATE) ANALYSIS

3.1. Varying the Diversity Order

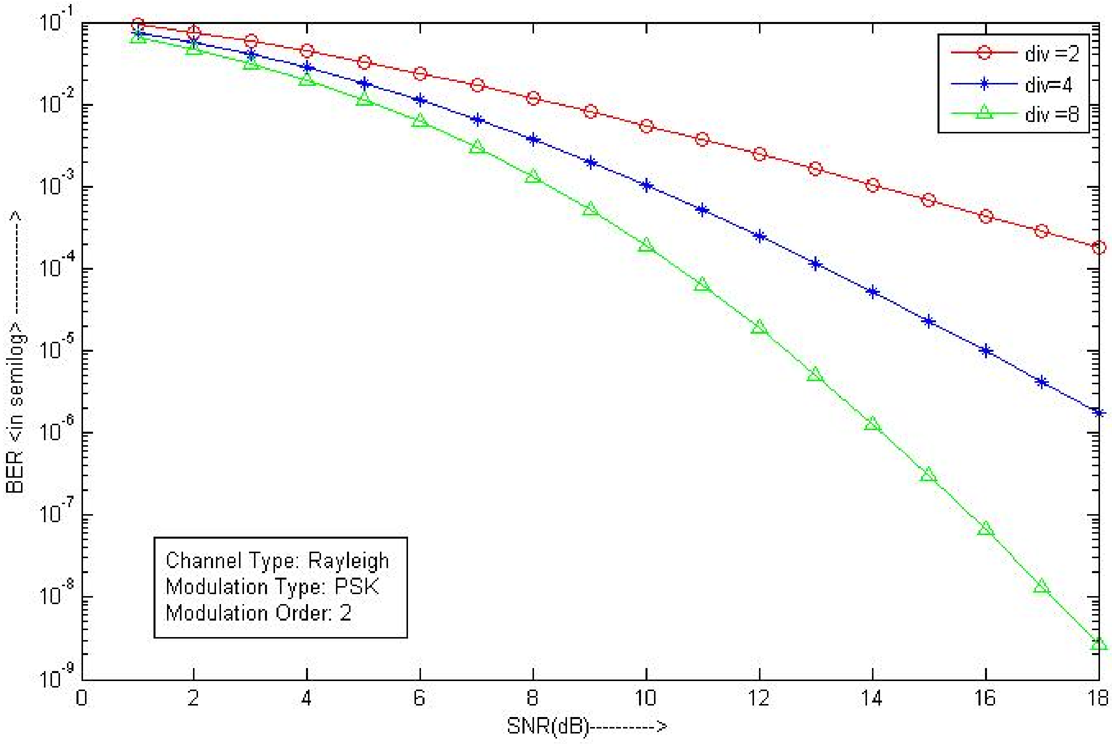

Fig 3.1. BER analysis with different orders of diversity

The effects of varying different parameters were observed while doing BER (Bit Error) Analysis for a Rayleigh Fading Channel. At first, the diversity order was varied, keeping the channel type (Rayleigh), Modulation type (PSK) and Modulation order (2) fixed. When the diversity order was increased, it was seen that the BER decreases faster with increasing SNR (Sound to Noise Ratio). Since diversity essentially means the number of independent fading propagation paths, it is good to have a higher order of diversity so that the same signal can be sent a number of times which would lead to a better reception at the receiving end.

3.2. Varying the Modulation Order

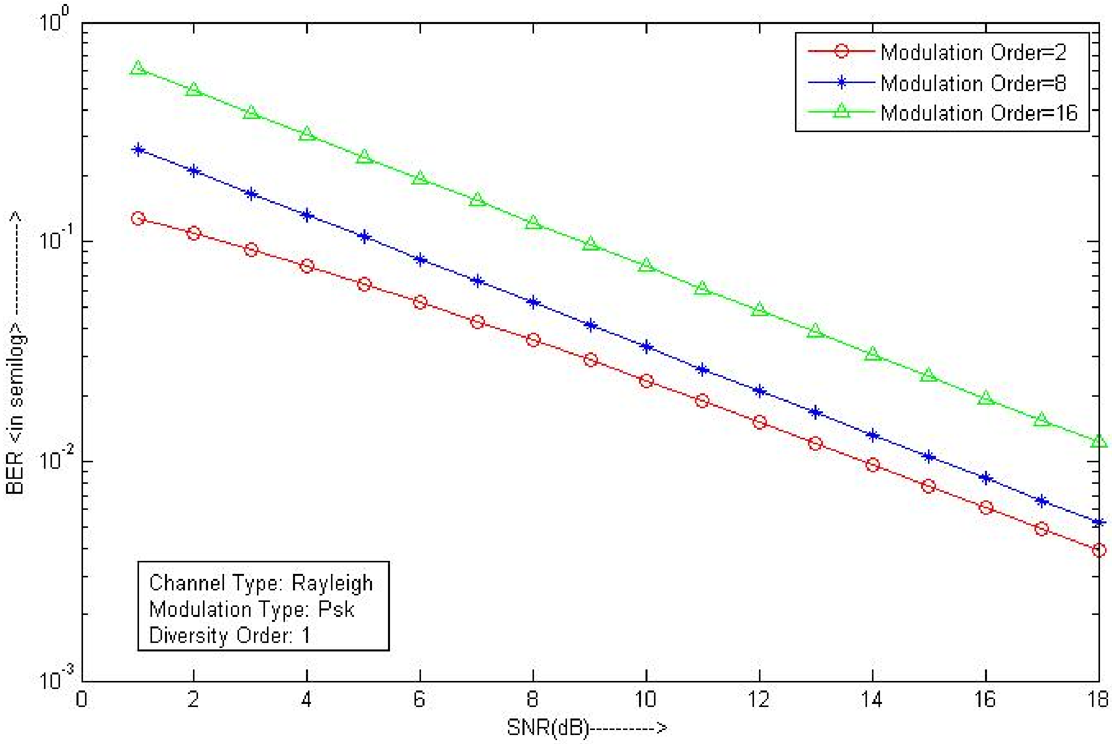

Fig 3.2. BER analysis with different orders of modulation

Secondly, the Modulation Order was varied keeping the other parameters fixed. A Diversity Order of 1 and a Modulation Type of PSK were used for the purpose. Modulation Order essentially means the number of bits that can be transmitted in the same signal. So a Modulation Order of 16 means that 16 bits of data can be transmitted in the same frequency. That is likely to lead to higher error rates. So, for any given value of the SNR, it is seen that the BER values is higher for higher orders of Modulation.

3.3.Varying Modulation Type

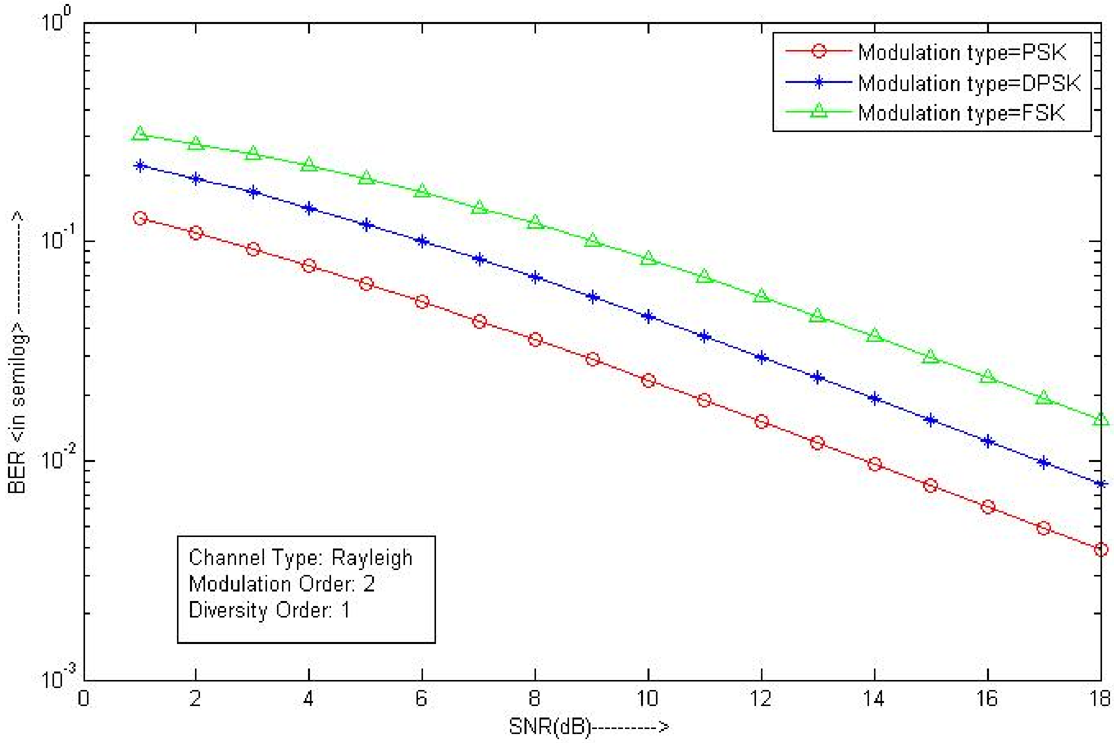

Fig 3.3. BER analysis with different types of modulation

The Modulation Type was varied next, keeping the other parameters fixed. A Diversity Order of 1 and a Modulation Order of 2 were used for the purpose. It was seen that the values of BER for PSK(Phase Shift Keying) is less than that of DPSK(Differential Phase Shift keying) which is less than FSK(Frequency Shift Keying). PSK essentially means BPSK(Binary Phase Shift Keying) which is the simplest from of PSK using 180 degrees Phase Shift. BPSK modulates at 1 bit per symbol which reduces the chance of producing errors. Hence, for any given value of the SNR, PSK produces the lowest BER.

3.4. Comparison Between AWGN and Rayleigh

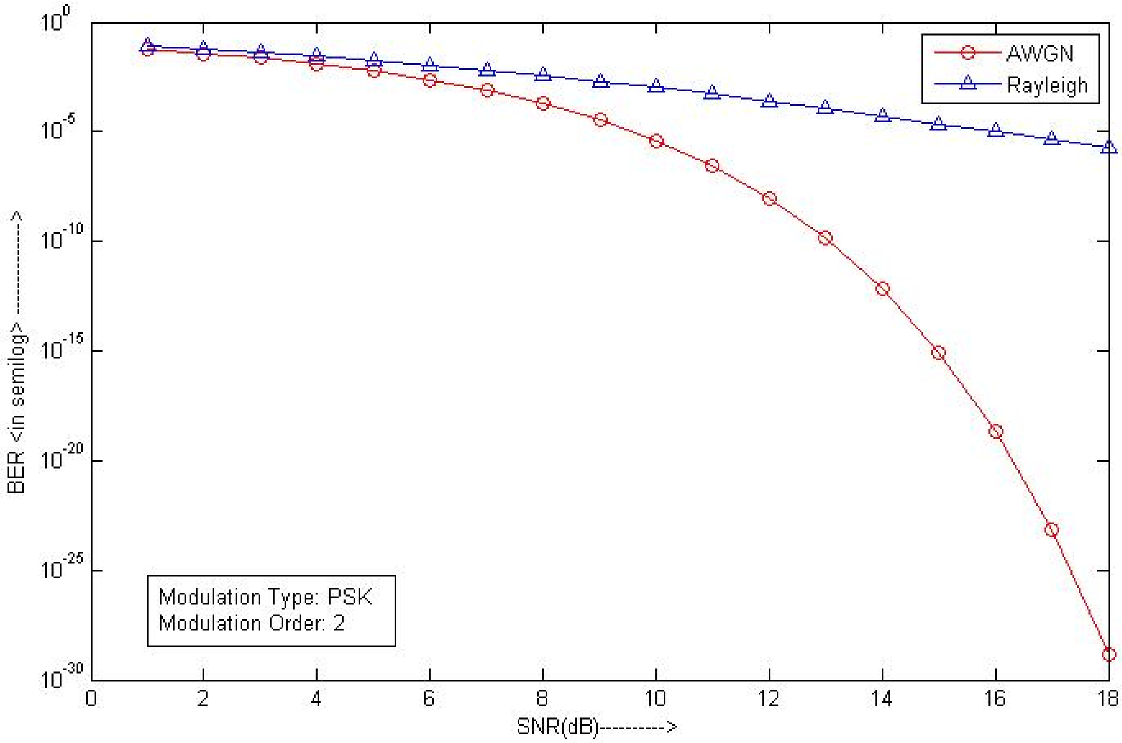

Fig 3.4. BER analysis AWGN and Rayleigh fading

AWGN (Additive White Gaussian Noise) channel model is one in which the only impairment is the linear addition of wideband or white noise with a constant spectral density. It is present even in complete vacuum or free space and does not account for the phenomena of fading, frequency selectivity or any other form of interference.

A comparison was done between AWGN and Rayleigh Fading in terms of the BER. Initially, when the SNR is zero, the corresponding values of BER for Rayleigh Fading and AWGN are same. With increasing SNR, the gap between Rayleigh and AWGN gradually keeps increasing. Rayleigh Fading always takes into account the phenomenon of AWGN, so the AWGN is partially responsible for the BER value in Rayleigh Fading. It is not possible to avoid AWGN in any fading channel which is evident from the figure above.

CHAPTER IV

SIMULATION OF DIFFERENT RAYLEIGH FADING MODELS

4.1 Jakes Method

The Jakes fading model, also known as the Sum of Sinusoids model, is a deterministic method for simulating time-correlated Rayleigh fading waveforms and is still widely used today. The model assumes that N equal-strength rays arrive at a moving receiver with uniformly distributed arrival angles α_n, such that ray n experiences a Doppler shift $\omega_{n,} = \omega_m \cos(\alpha_{n)}$, where $\omega_m = 2\pi fv/c$ is the maximum Doppler frequency shift, v is the vehicle speed, f is the carrier frequency, and c is the speed of light [1].

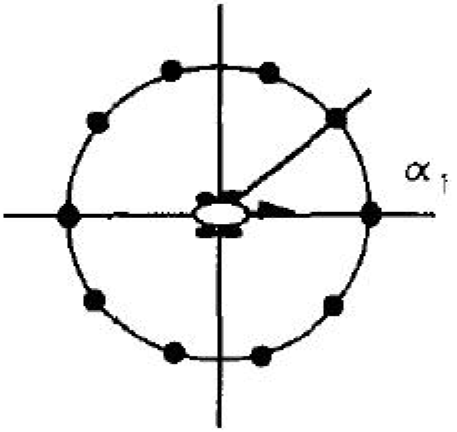

Fig 4.1. Jakes fading model with N=10

Using $\alpha_n = 2\pi n/N$, there is quadrantal symmetry in the magnitude of the Doppler shift, except for angles 0 and π. As a result, the fading waveform can be modeled with $No + 1$ complex oscillators, where $No = (N/2 - 1)/2$.

This leads to the equation,

$$T_k(t) = \sqrt{\frac{1}{2N_o+1}}\{2\sum_{n=1}^{N_0}(\cos\beta_n + j\sin\beta_n)\cos(\omega_m \cos\alpha_n .t + \theta_{nk}) + \sqrt{2}\cos(\omega_m t + \theta_{0k})\}$$

(4.1)

where, k is the waveform index, $k=1,2,....N_0$ and λ is the wavelength of the transmitted carrier frequency. Here, $\beta_n = \pi n/(N_0+1)$. To generate the multiple waveform, Jakes suggests using

$$\theta_{nk} = \frac{\pi n}{N_0+1} + \frac{2\pi(k-1)}{N_0+1}$$

(4.2)

A software simulation was carried out using the Jakes Model described above and the inputs included the Vehicle Speed *v* (in kmph), the Central Carrier Frequency *fc* in MHz, the Symbol Frequency *fs* in kbps, the Number of sub-channels *U* and the Number of Channel Coefficients to Generate *M.* The output was shown as a power spectrum, with the variation of the signal power in the y axis and the sampling time (or the sample number) on the x axis.

The Rayleigh Envelope that results for inputs of *v* = 100 kmph, *fc* = 2000 MHz, *fs* =10 kbps, *U* =3 and *M* = 100000 is shown below:

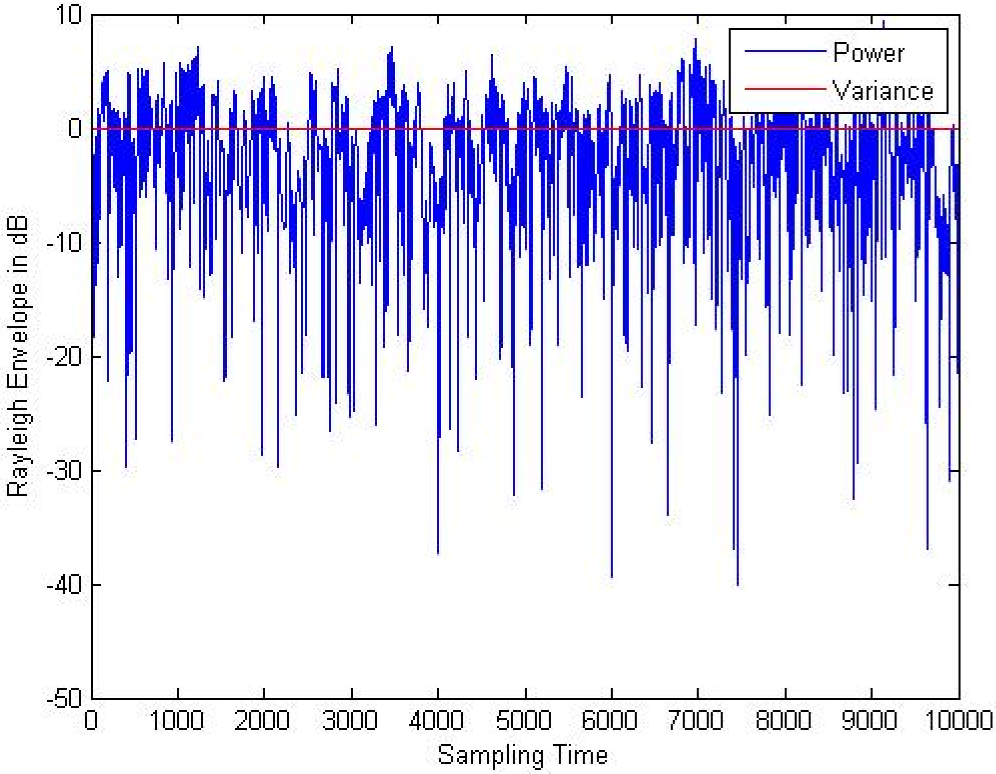

Fig 4.2. Simulation of Jakes fading model with typical input values

Afterwards, several input parameters were varied to see how it affects the obtained Rayleigh Envelope. The inputs which were varied include: Speed *v*, the Central Carrier Frequency *fc*, the Symbol Frequency *fs* and the Number of Channel Coefficients to Generate *M.*

4.1.1 Varying the speed

Sets of data for three different speeds (*v*) were used and the corresponding Rayleigh Envelopes that resulted are shown below:

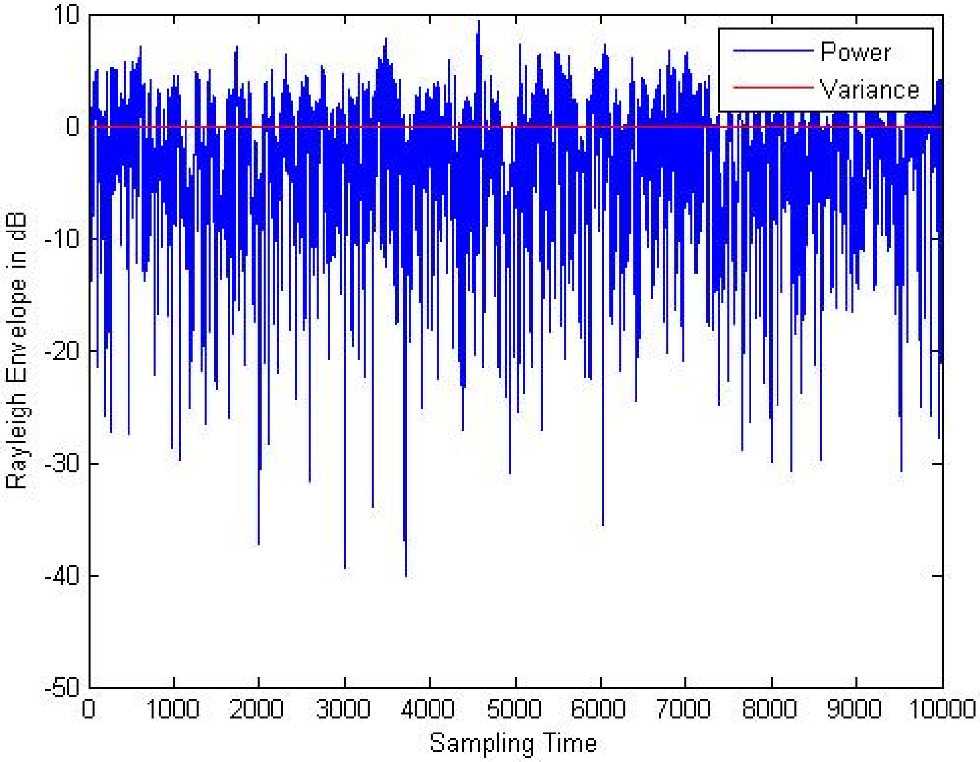

Fig 4.3. Simulation of Jakes fading model with *v* = 200 km/h

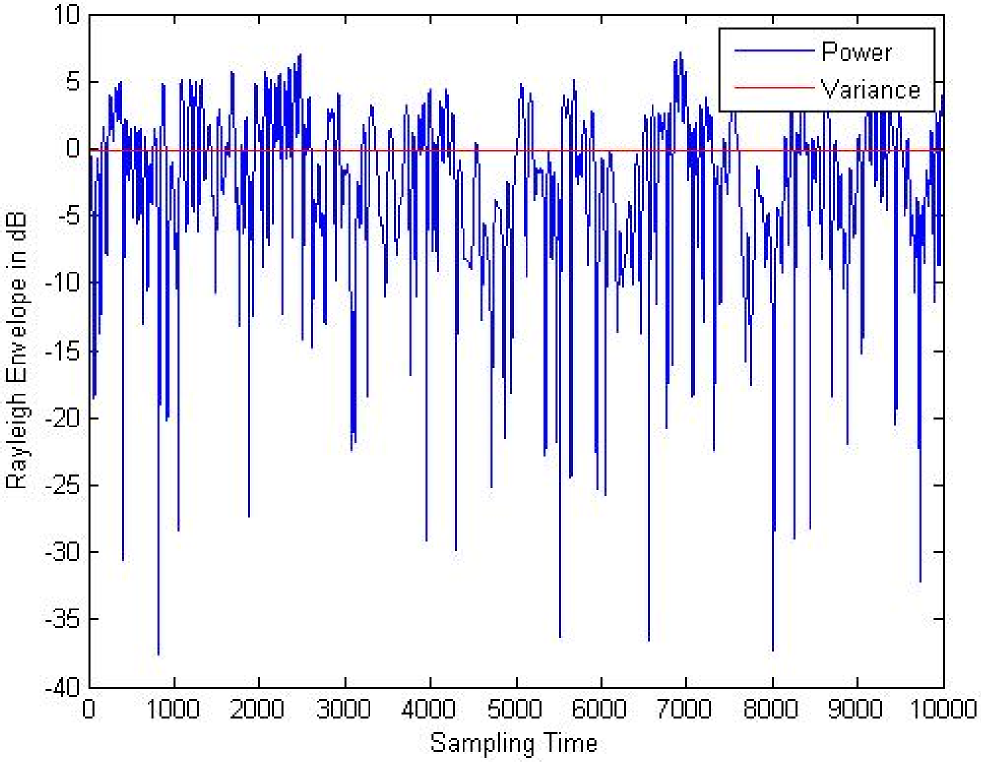

Fig 4.4. Simulation of Jakes fading model with *v* = 50 km/h

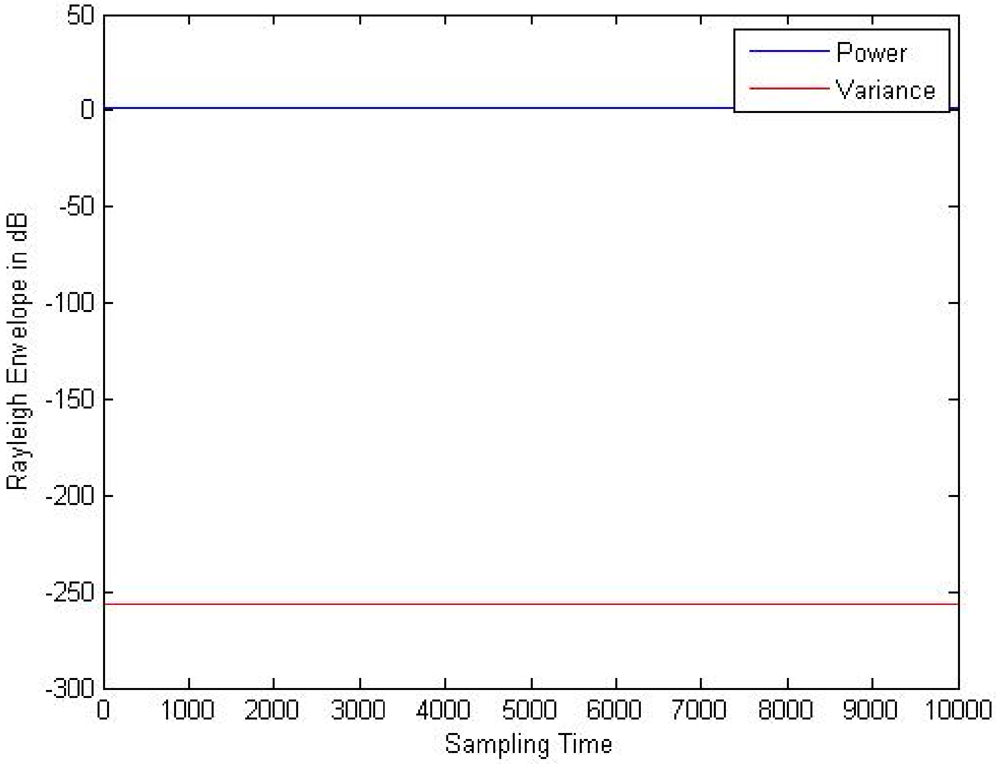

Fig 4.5. Simulation of Jakes fading model with *v* = 0 km/h

It can be clearly seen that increasing speeds increase the level of fluctuation of the Rayleigh Envelope. With a speed of 0 km/h, that is, the transmitter and receiver both stationary, the transmitted signal has no variation and the level of fluctuation is zero.

4.1.2 Varying the central carrier frequency

Sets of data for two different Central Carrier Frequencies (*fc*) were used and the corresponding Rayleigh Envelopes that resulted are shown below:

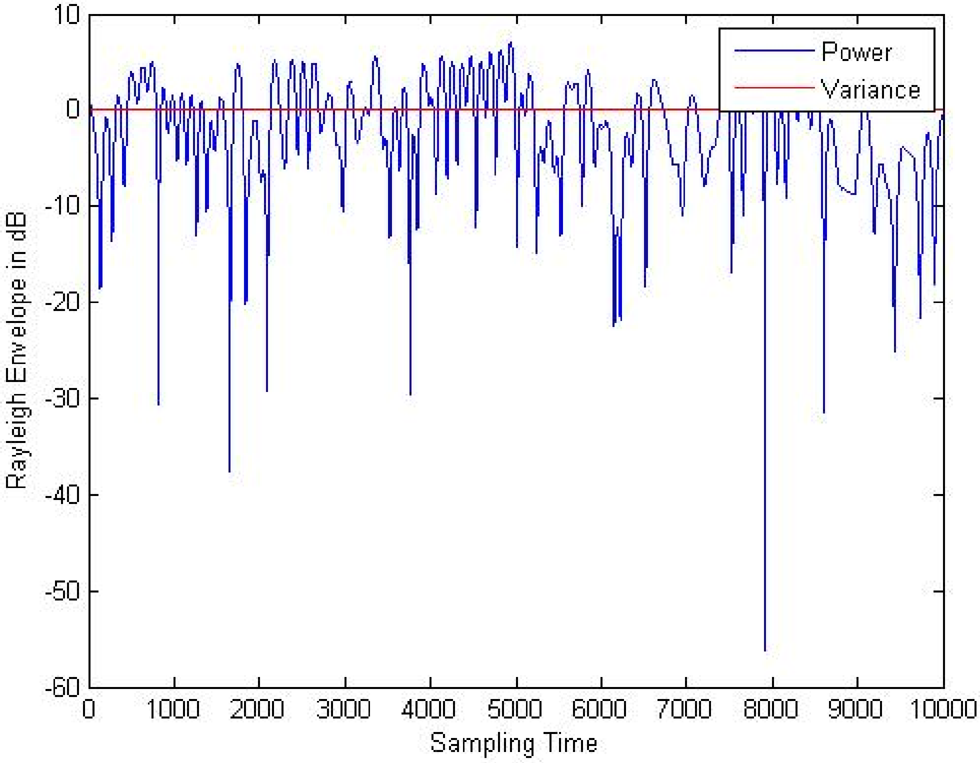

Fig 4.6. Simulation of Jakes fading model with *fc* = 1000 MHz

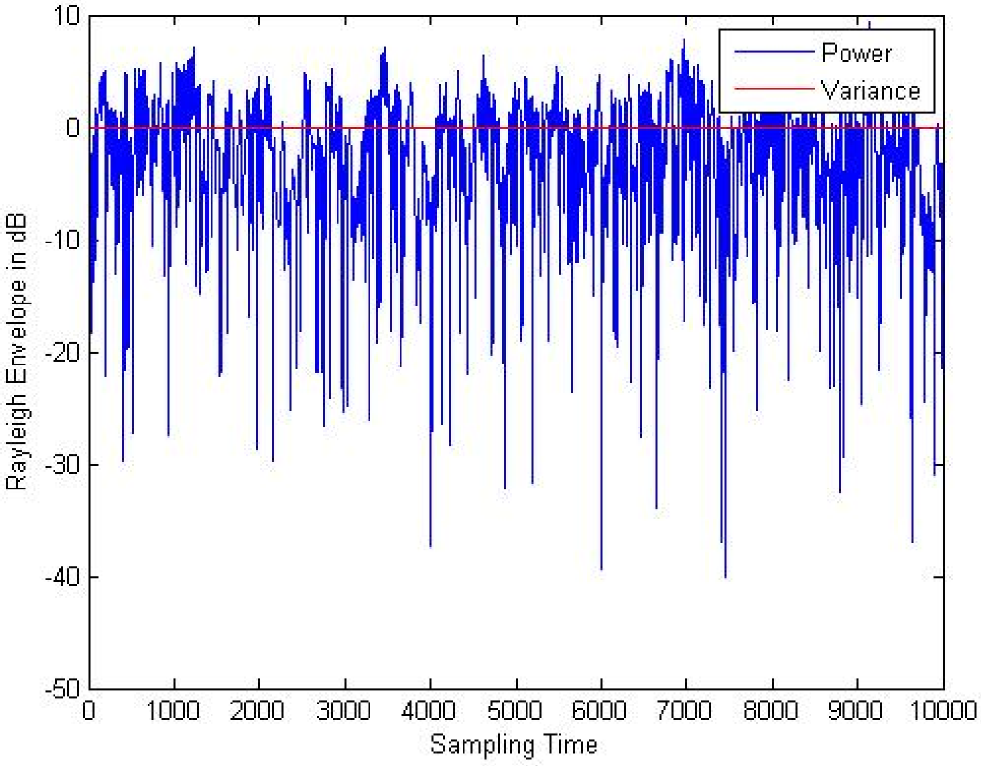

Fig 4.7. Simulation of Jakes fading model with *fc* = 3000 MHz

Therefore it is seen that the fluctuation and variance increases with higher central carrier frequency. This happens because, with a given bandwidth, and all other factors remaining constant, a higher frequency means a higher ISI and hence higher fluctuations in the signal.

4.1.3 Varying the symbol frequency

Sets of data for two different Symbol Frequencies (*fs*) were used and the corresponding Rayleigh Envelopes that resulted are shown below:

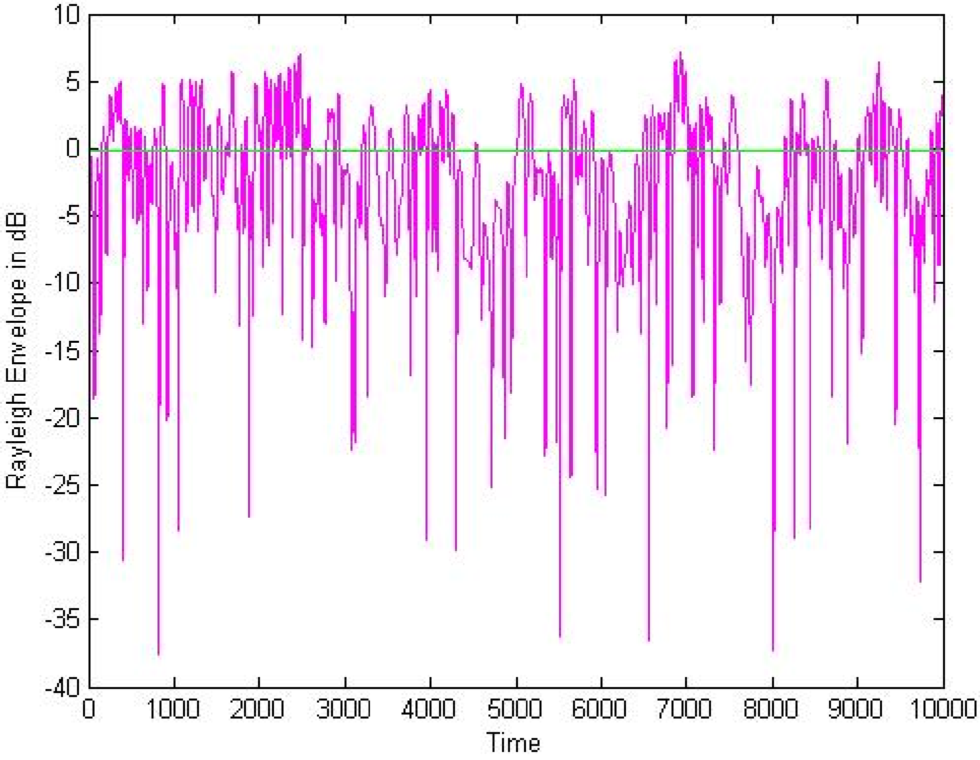

Fig 4.8. Simulation of Jakes fading model with *fs* = 10 ksps

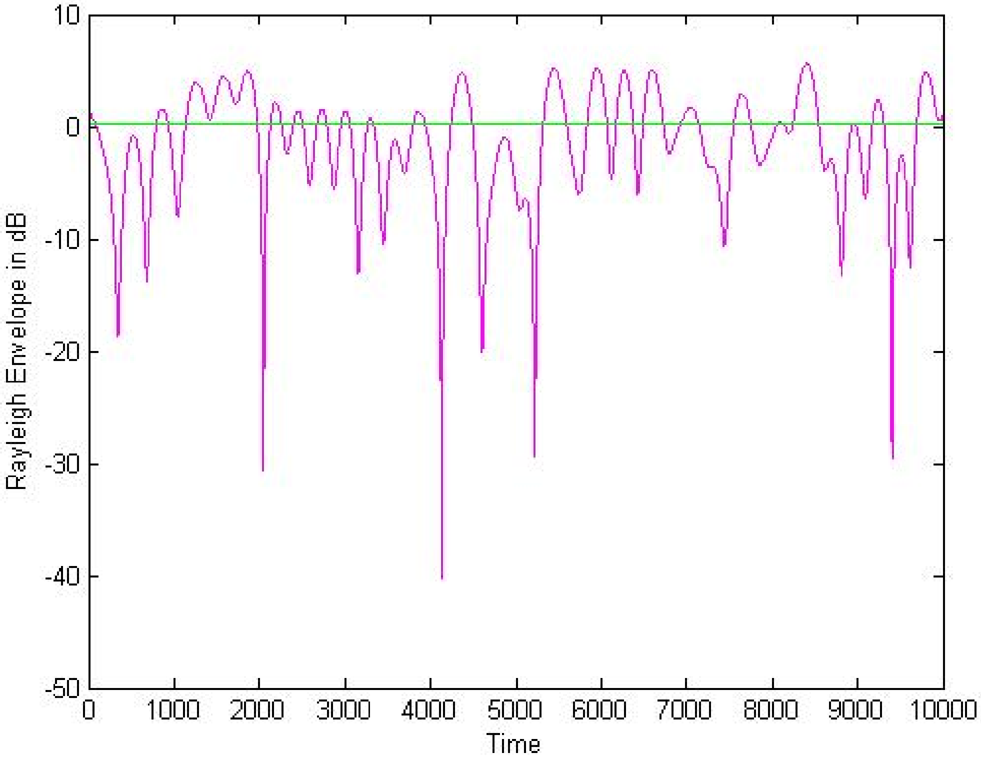

Fig 4.9. Simulation of Jakes fading model with *fs* = 100 ksps

Therefore it is clearly seen that with a higher Symbol Frequency, the fluctuation and variance of the Rayleigh Envelope is lower. This is logically true since a higher amount of resources gets used for the same amount of transmitted data so the chance of an error happening gets significantly lower.

4.1.4 Varying the number of channel coefficients

Sets of data for three different Number of Channel Coefficients (*M*) were used and the corresponding Rayleigh Envelopes that resulted are shown below:

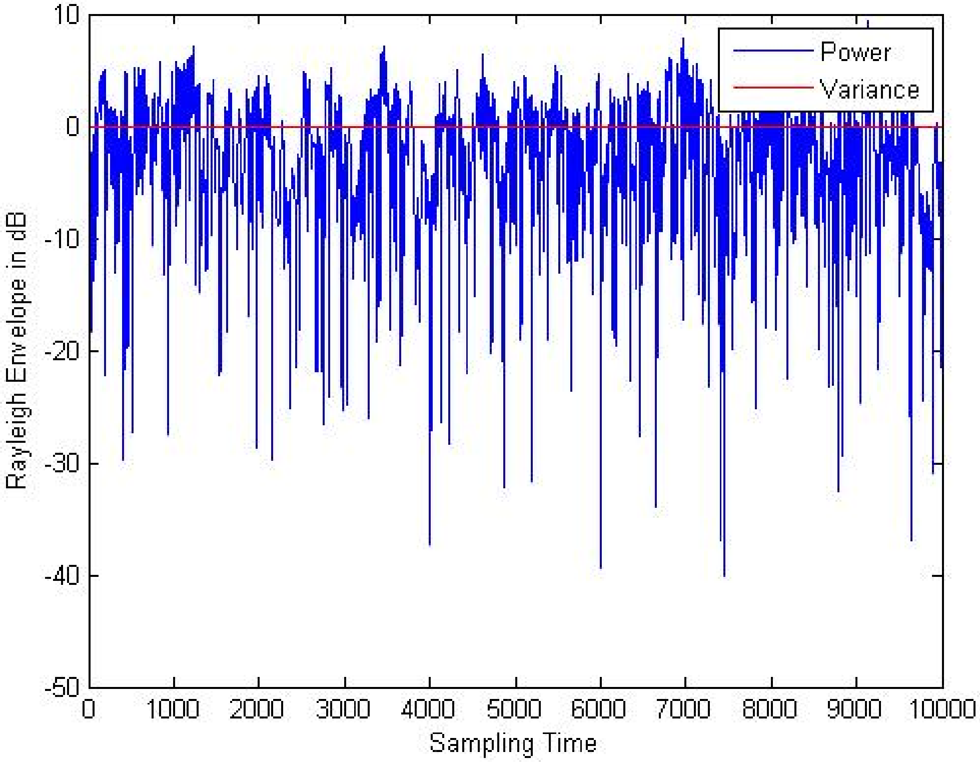

Fig 4.10. Simulation of Jakes fading model with *M* = 10000

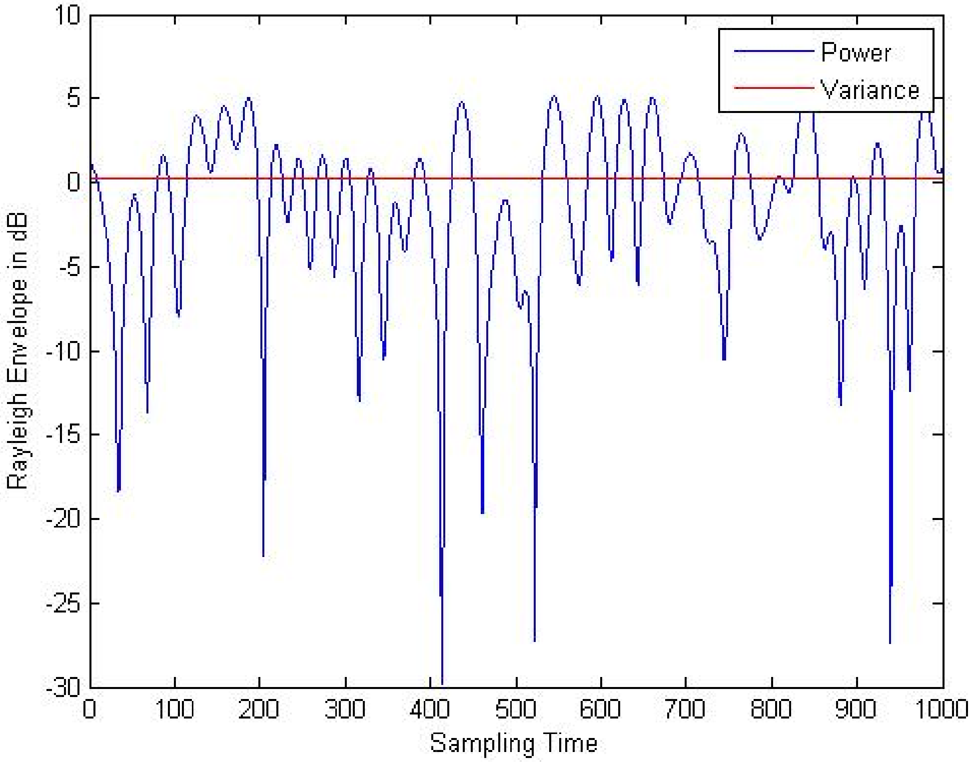

Fig 4.11. Simulation of Jakes fading model with *M* = 1000

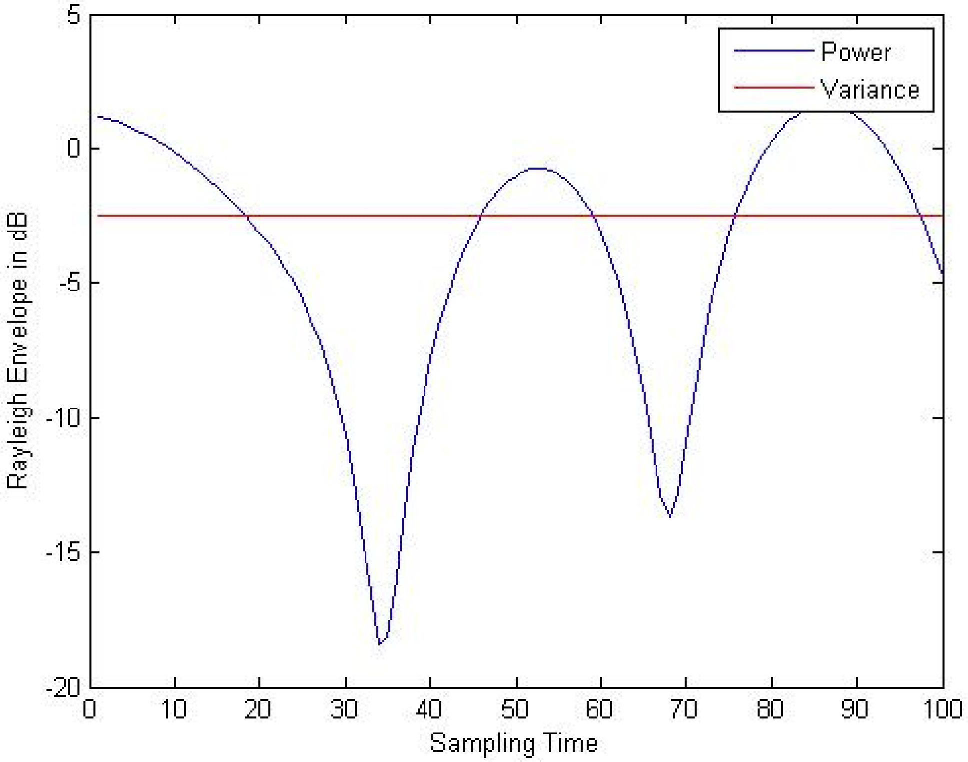

Fig 4.12. Simulation of Jakes fading model with *M* = 100

The above diagrams lead us to conclude that the higher the number of channels it is required to generate, higher will be the fluctuation in a given time period. For a requirement of 100 Channel Coefficients only, fluctuation is very low over the entire period of time.

4.2 Dent Method

The Jakes Model has shortcomings in the correlation functions in the way that it must satisfy the following constraint

$$\theta_{n,j} - \theta_{n,k} = i\pi + \pi / 2 \qquad (4.3)$$

If this condition is not satisfied, the correlation between certain waveform pairs can be quite significant which is not desirable. To overcome this problem, Dent suggested the usage of one type of orthogonal functions (Walsh Hadamard Code) [1]. To completely eliminate correlation, the oscillators must have equal power. This is achieved by reformulating the Jakes model in terms of slightly different arrival angles.

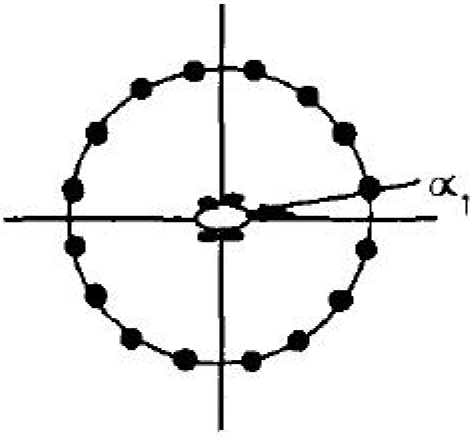

Fig 4.13. Dent fading model with N=12

Using $\alpha_n=(2\pi n/N)-(\pi/N)$ and $\beta_n=(\pi n/N_0)$, the waveform can be described by the following equation:

$$T_k(t)=\sqrt{\frac{2}{N_0}}\sum_{n=1}^{N_0}A_k(n)(\cos\beta_n+j\sin\beta_n)\cos(\omega_m\cos\alpha_n.t+\theta_n) \tag{4.4}$$

where $k=1,2\ldots N_0$, $N_0=N/4$, α_n , β_n and θn are the independent random phases, each of which is uniformly distributed in [0, 2π). $A_k(n)$ is the kth Walsh-Hadamard codeword in n which satisfies the following condition:

$$\frac{1}{N_0}\sum_{n=1}^{N_0}A_k^{\ *}(n)A_1(n)=\begin{cases}1,k=l\\0,k\neq l\end{cases} \tag{4.5}$$

Just as was the case in Jakes Method, several input parameters were varied in the Dent method and the corresponding Rayleigh Envelope found. The inputs which were varied include: Speed *v*, the Central Carrier Frequency *fc*, the Symbol Frequency *fs* and the Number of Channel Coefficients to Generate *M.*

4.2.1 Varying the speed

Sets of data for three different speeds (*v*) (the same speeds that were used in the Jakes Method) were used and the corresponding Rayleigh Envelopes that resulted are shown below:

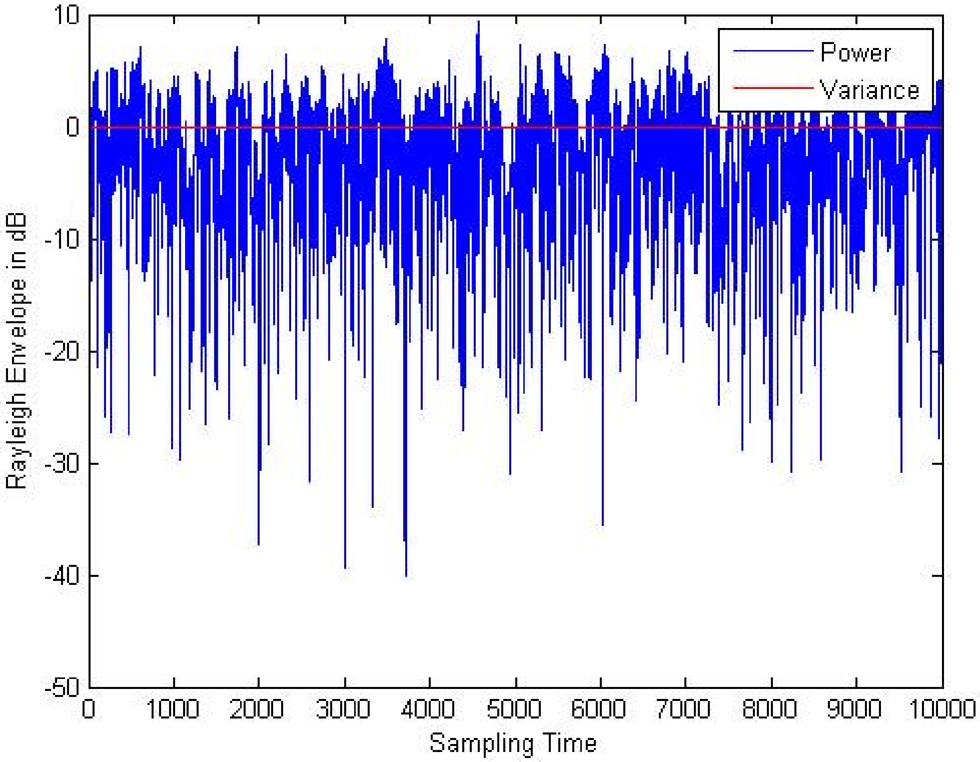

Fig 4.14. Simulation of Dent fading model with *v* = 200 km/h

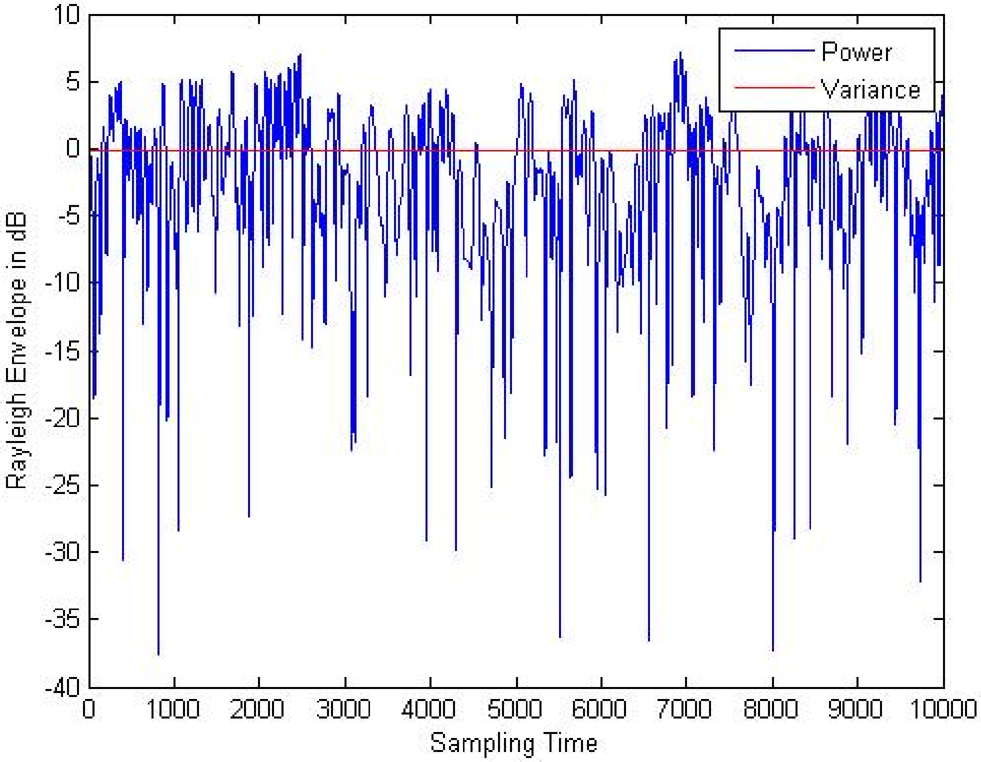

Fig 4.15. Simulation of Dent fading model with *v* = 50 km/h

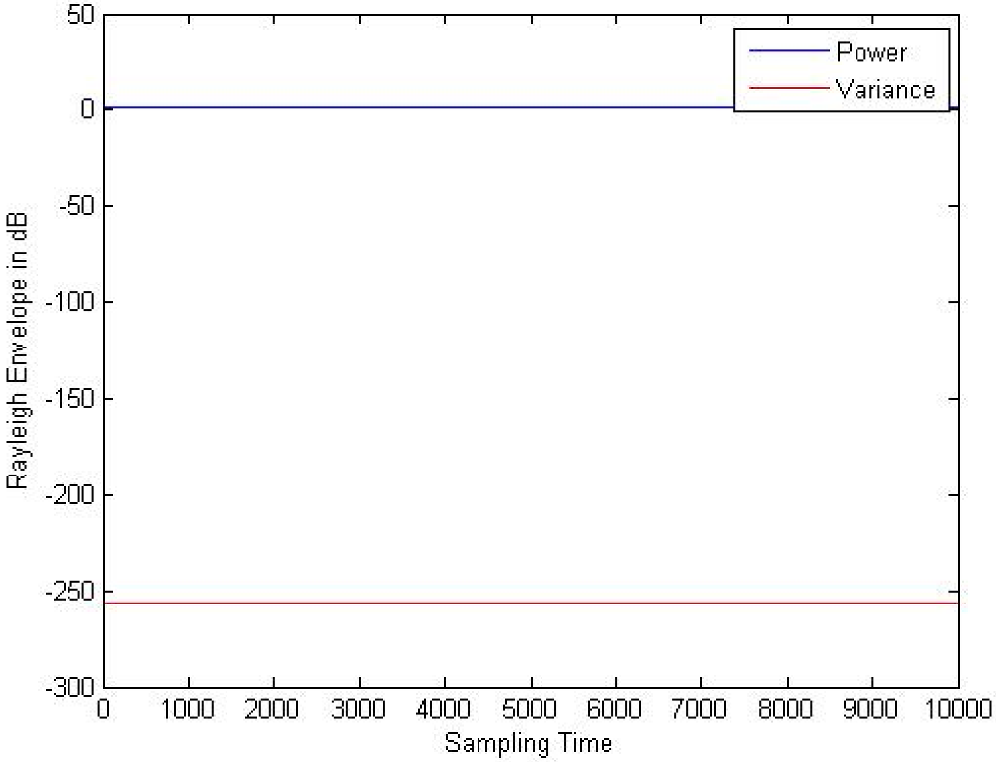

Fig 4.16. Simulation of Dent fading model with v = 0 km/h

It can be clearly seen that increasing speeds increase the level of fluctuation of the Rayleigh Envelope jus t like it happened with the Jakes Method. With a speed of 0 km/h, that is, the transmitter and receiver both stationary, the transmitted signal has no variation and the level of fluctuation is zero.

4.2.2 Varying the central carrier frequency

Sets of data for two different Central Carrier Frequencies (*fc*) (the same frequencies that were used in the Jakes Method) were used and the corresponding Rayleigh Envelopes that resulted are shown below:

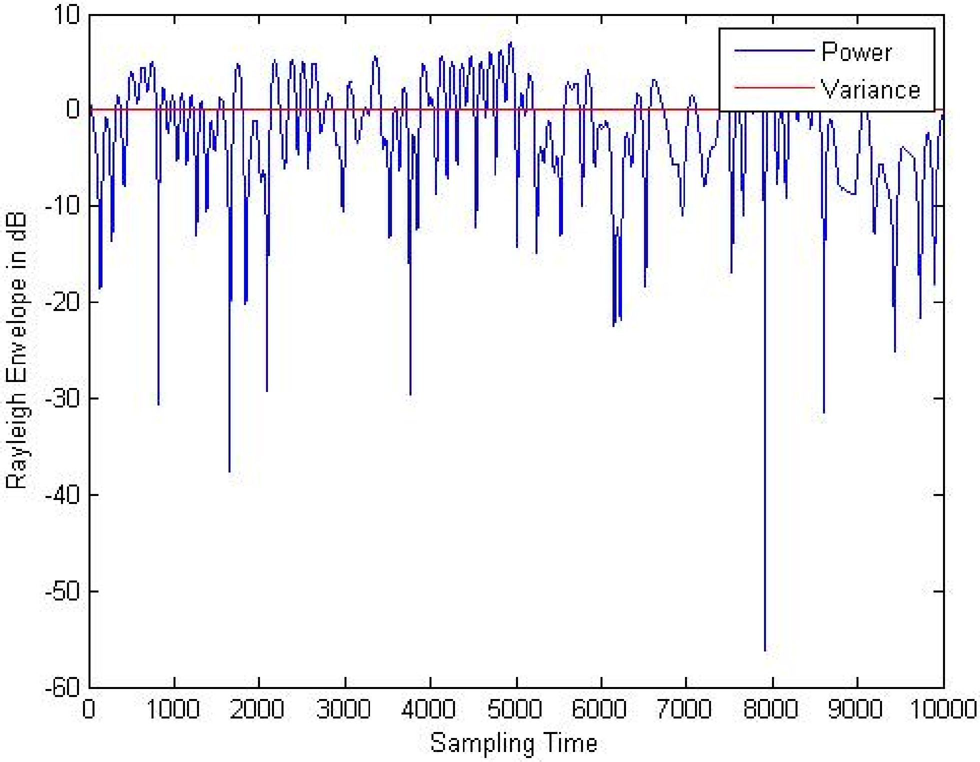

Fig 4.17. Simulation of Dent fading model with *fc* = 1000 MHz

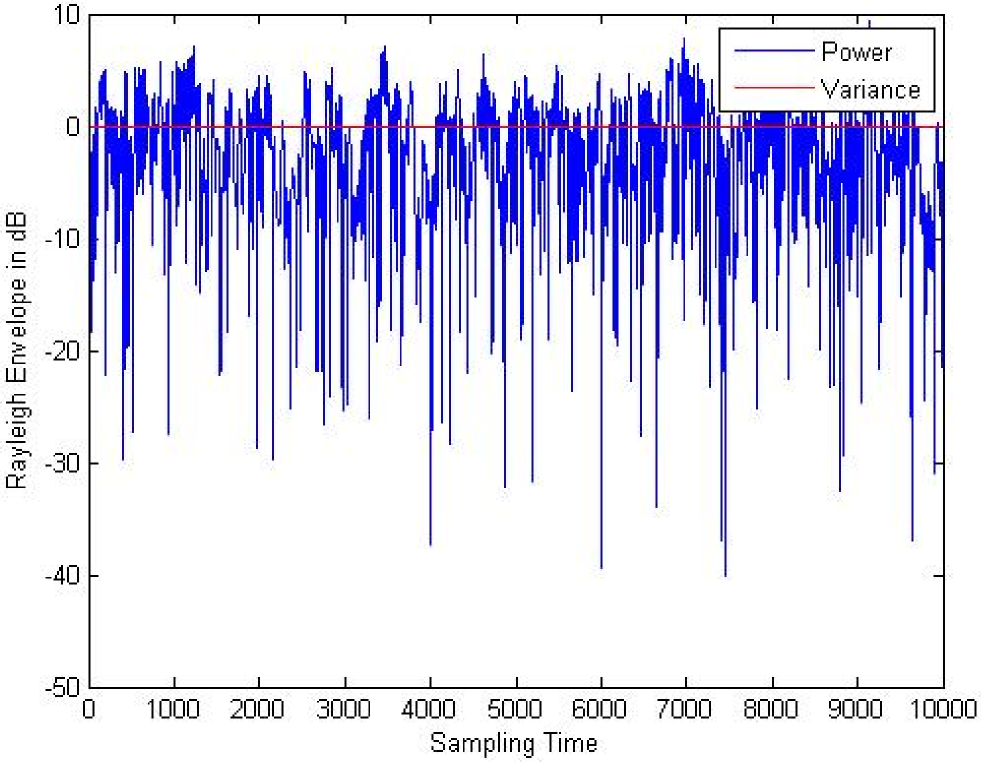

Fig 4.18. Simulation of Dent fading model with *fc* = 3000 MHz

Therefore it is seen that the fluctuation and variance increases with higher central carrier frequency (just as it was seen in the Jakes Model). This happens because, with a given bandwidth, and all other factors remaining constant, a higher frequency means a higher ISI and hence higher fluctuations in the signal.

4.2.3 Varying the symbol frequency

Sets of data for two different Symbol Frequencies (*fs*) (the same frequencies that were used in the Jakes Method) were used and the corresponding Rayleigh Envelopes that resulted are shown below:

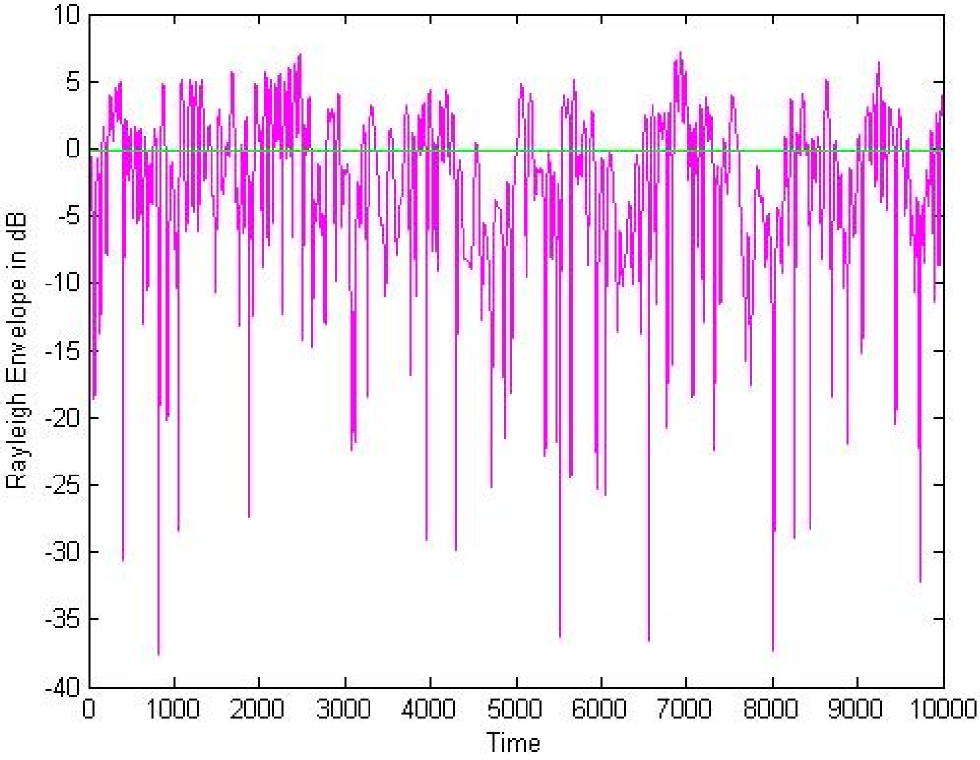

Fig 4.19. Simulation of Dent fading model with *fs* = 10 ksps

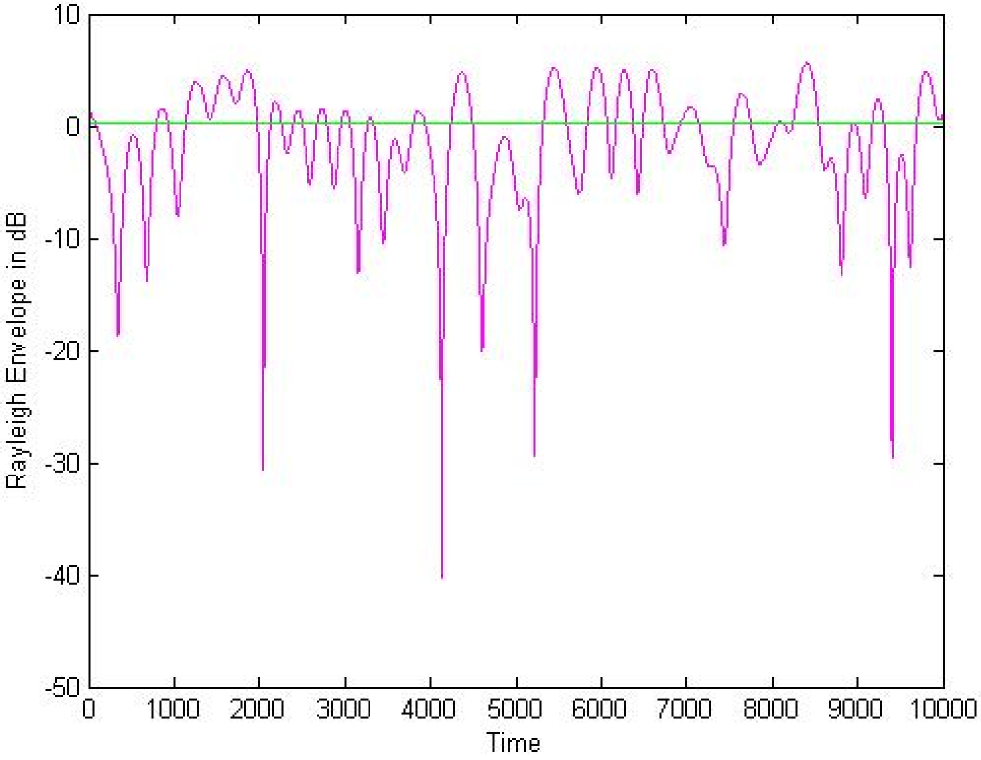

Fig 4.20. Simulation of Dent fading model with *fs* = 100 ksps

Therefore it is clearly seen that with a higher Symbol Frequency, the fluctuation and variance of the Rayleigh Envelope is lower. Similar results were found in the Jakes Method.

4.2.4 Varying the number of channel coefficients

Sets of data for three different Number of Channel Coefficients (*M*) (the same values of M that were used in the Jakes Method) were used and the corresponding Rayleigh Envelopes that resulted are shown below:

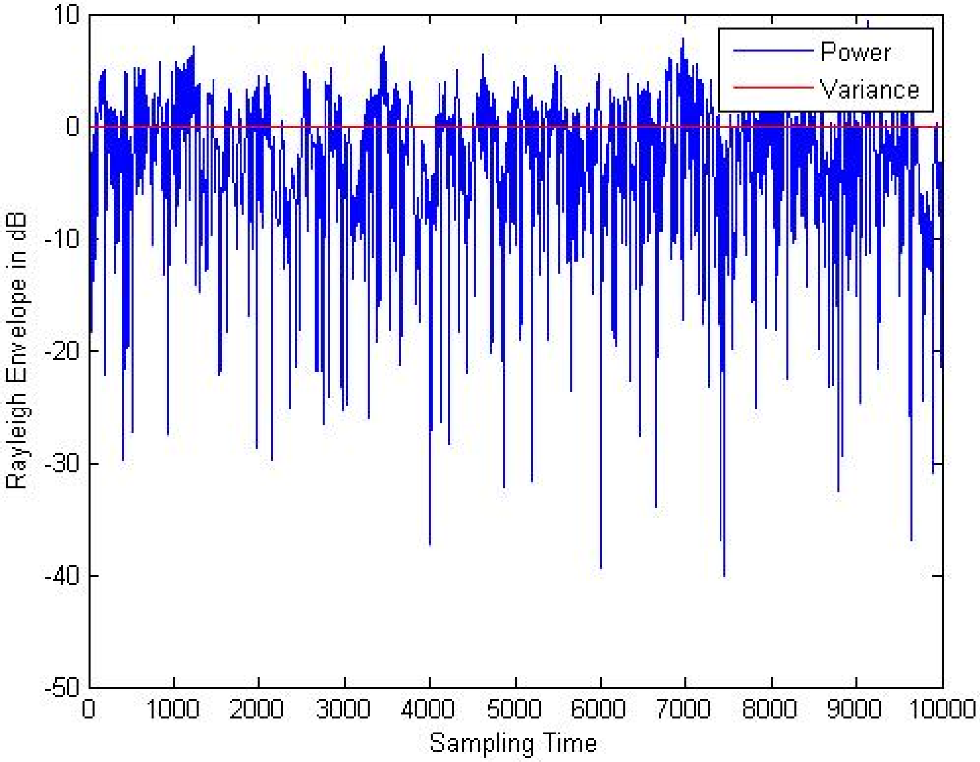

Fig 4.21. Simulation of Dent fading model with *M* = 10000

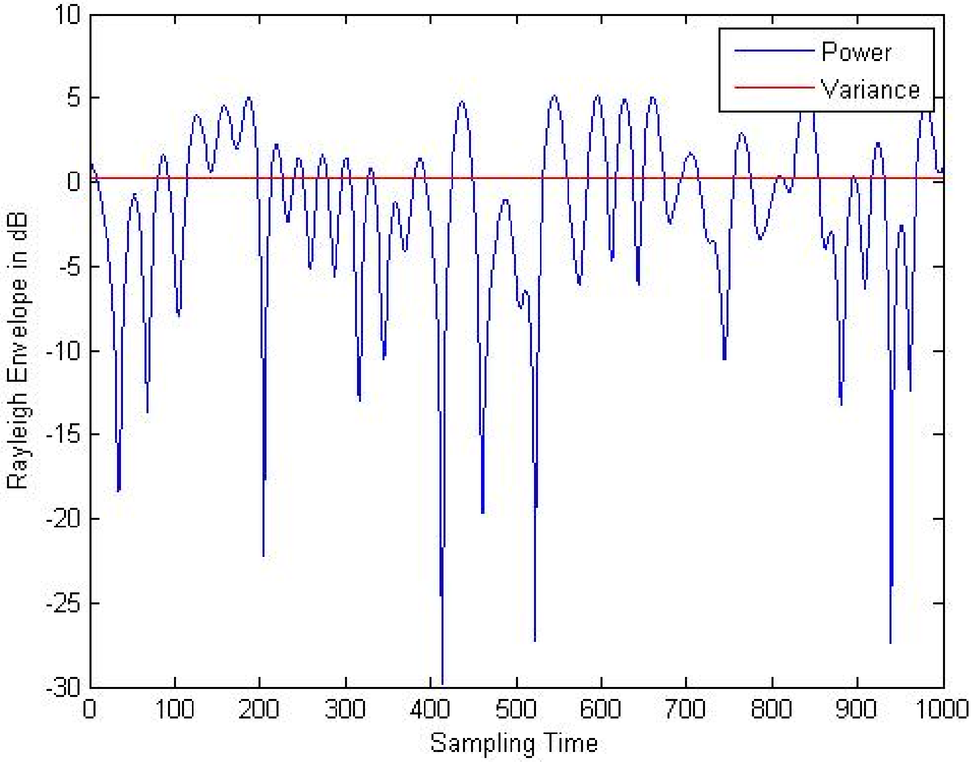

Fig 4.22. Simulation of Dent fading model with *M* = 1000

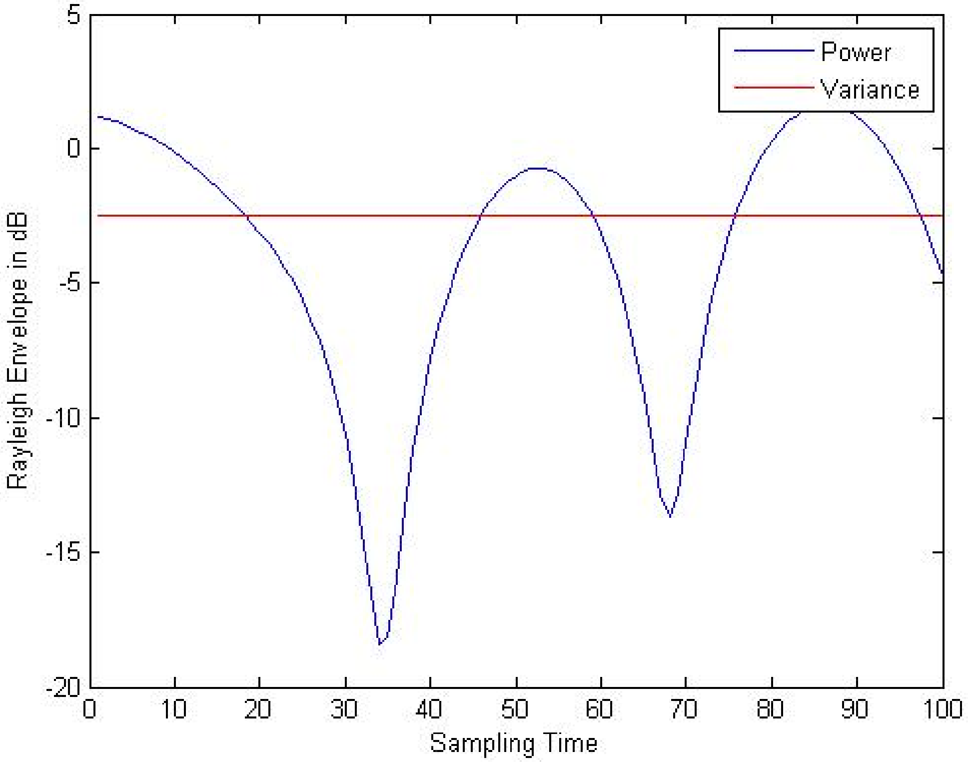

Fig 4.23. Simulation of Dent fading model with *M* = 100

The above diagrams lead us to conclude that the higher the number of channels it is required to generate, higher will be the fluctuation in a given time period. Similar results were found in the Jakes Method.

CHAPTER V

COMPARISON OF THE DIFFERENT MODELS

5.1 Using Mean-Square Errors of the Autocorrelation

To compare between the different models used to describe Rayleigh Fading, the Mean Square Errors of the Autocorrelation and Cross-correlation has been chosen as a parameter to measure the ‘correctness’ of the model which describes its applicability to real life situations [2] . For the purpose, a simulation was run to generate four uncorrelated complex baseband fading waveforms using N_0=8 oscillators. With a Doppler frequency $\omega_m=2\pi(20)$, 384 000 samples of each fading waveform were generated using a sampling frequency f_s=3840 Hz. The Jakes and Dent models were then compared and evaluated in the correlation properties. The autocorrelation functions $R(\tau)$ of in-phase and quadrature components should be close to their theoretical Bessel function $(J_0(\omega_m\tau))$=2. So the quality of the fade’s autocorrelation function can be measured by the mean-square-error defined by

$$\varepsilon^2 = \frac{\sum_{i=0}^{N-1}(R(i/f_s)) - (J_0(\omega_m i/f_s)/2)^2}{N} \tag{5.1}$$

where N denotes the time interval over which the mean-square-error is evaluated, and we choose N=1000. The results are given in Table 5.1.

	Jakes Model	Dent Model
In-phase component	6.0 x 10-3	5.0 x 10-3
Quadrature component	6.7 x 10-3	6.1 x 10-3

Table 5.1 Autocorrelations of the two models

5.1 Using Mean-Square Errors of the Cross-correlation

The cross-correlation functions $R(\tau)$ between in-phase and quadrature components of any single fader (intra-fader) should be close to zero, as also should the cross-correlation functions between any pair of different faders (inter-fader). Therefore the quality of the cross-correlation function can be measured by the mean-square-error defined by

$$\varepsilon^2 = \frac{\sum_{i=0}^{N-1} (R(i/f_s))^2}{N} \tag{5.2}$$

where N=1000.The results are presented in Table 5.2. In general, the results obtained from the autocorrelation and the cross-correlation results indicate good model performance for the Dent model compared to the Jakes Model.

	Jakes Model	Dent Model
Intra-fader	7.3 x 10-3	7.0 x 10-3
Inter-fader	4.94 x 10-2	2.59 x 10-2

Table 5.2 Cross-correlations of the two models

CHAPTER VI

CONCLUSION

The two fading models described here, Jakes and Dent (which is basically an improvement of the Jakes Model) have very similar properties in terms of the physical parameters that are taken into account when designing the models. Also, it was seen, that both of them behaved in the same pattern in the simulation models when subject to change in the input parameters of Speed *v*, the Central Carrier Frequency *fc*, the Symbol Frequency *fs* and the Number of Channel Coefficients to Generate *M.*

But differences arose when the MSE (Mean-Square Error) was calculated for the Autocorrelation and Cross-correlation of the signals using the Jakes and Dent Methods. It was seen that the values of the MSE for the In-phase Component and Quadrature Component were both lower for the Dent Method. Also, the MSE values for the Cross-correlation functions, both Intra-fader and Inter-fader, were lower for Dent compared to the Jakes Model.

And of course, as mentioned before, the scope of signals not getting correlated in the Jakes method were very limited and hence the Dent method was designed with an orthogonal function, namely the Walsh-Hadamard Code. This ensured that signals were much less correlated to each other.

Furthermore, two other new models have been designed in recent times, the Li and Huang Model and the Wu model. Both have built up on the initial Jakes Model, trying to reduce the computational complexities and the Mean Square Error values. In terms of correlation parameters, the Wu model claims to be almost as good as that of the Li and Huang model, and to be better than that of the Jakes and Dent models. However, the computational complexity of the Wu model is said to be only half of the Li and Huang model, which is almost the same as that of the Jakes and Dent models. On a concluding note, it is well worth repeating that none of the models can be said to be ‘perfect’, nor do they claim to be so. It has remained and will always remain a strive towards perfection.

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[1] ‘Microwave Mobile Communications’ edited by W Jakes

‘Jakes Fading Model Revisited’ by P Dent, G E Bottomley & T Croft

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[3] ‘A Comparative Study of Rayleigh Fading Wireless Channel Simulators’ by VRS Ramaswamy

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[6] ‘The Performance of Deterministic Rayleigh Fading Channel Simulators with respect to the BER’ – by M Patzold & F Laue

[7] ‘Fading’ – Online Article in Wikipedia
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[8] ‘Doppler Effect’ – Online Article in Wikipedia
en.wikipedia.org/wiki/Doppler_effect

VITA

Mahdin Mahboob was born on August 22nd in 1985 in Dinajpur, Bangladesh. He received his BSc. in Electronics and Communications Engineering from BRAC University, Dhaka, Bangladesh in 2007 along with a Minor in Computer Science.

Mahdin has held the position of President of the Proposed IEEE Student Branch at BRAC University during his senior year of undergraduate study. He can be reached by e-mail at mahdin.mahboob@gmail.com.

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Sittul Muna received her BSc. in Electronics and Communications Engineering from BRAC, University, Dhaka, Bangladesh in 2007.

A. MATLAB Code for BER Analysis (Varying the Order of Diversity)

```
%Thesis Topic: Comparison Of different models for the analysis of rayleigh
fading channels
%Done by:
%Mahdin Mahboob(ID: 05310051)
%and Sajidul Alam(ID: 06310054)

X1=1:1:18;

k=18;
for n = 1:k
   EbNo = 1:1:18;
   M = 2;
   divorder = 2;
   B = berfading(EbNo,'psk',M,divorder);

end

semilogy(X1,B,'ro-');

xlabel('SNR(dB)---------->');
ylabel('BER <in semilog> -------------->');

hold on

for n = 1:k
   EbNo = 1:1:18;
   M = 2; %
   divorder = 4;
   B = berfading(EbNo,'psk',M,divorder);

end

semilogy(X1,B,'b*-');
xlabel('SNR(dB)---------->');
ylabel('BER <in semilog> -------------->');


hold on

for n = 1:k
   EbNo = 1:1:18;
   M = 2; %
   divorder = 8;
   B = berfading(EbNo,'psk',M,divorder);
```

```
end

semilogy(X1,B,'g^-');
xlabel('SNR(dB)---------->');
ylabel('BER <in semilog> -------------->');

legend('div =2','div=4','div =8' );
```

B. MATLAB Code for BER Analysis (Varying the Modulation Order)

```
%Thesis Topic: Comparison Of different models for the analysis of Rayleigh
fading channels
%Done by:
%Sajidul Alam(ID: 06310054)
%and Mahdin Mahboob(ID: 05310051)


X1=1:1:18;

k=18;


for n = 1:k
   EbNo = 1:1:18;
   M = 2; %
   divorder = 1;
   B = berfading(EbNo,'psk',M,divorder);

end

semilogy(X1,B,'ro-');

xlabel('SNR(dB)---------->');
ylabel('BER <in semilog> -------------->');

hold on

for n = 1:k
   EbNo = 1:1:18;
   M = 8;
   divorder = 1;
```

```
   B = berfading(EbNo,'psk',M,divorder);

end

semilogy(X1,B,'b*-');
xlabel('SNR(dB)---------->');
ylabel('BER <in semilog> -------------->');


hold on


for n = 1:k
   EbNo = 1:1:18;
   M = 16;
   divorder = 1;

   B = berfading(EbNo,'psk',M,divorder);

end

semilogy(X1,B,'g^-');
xlabel('SNR(dB)---------->');
ylabel('BER <in semilog> -------------->');


legend('Modulation Order=2','Modulation Order=8','Modulation Order=16' );
```

C.MATLAB Code for BER Analysis (Varying the Modulation type)

```
%Thesis Topic: Comparison Of different models for the analysis of Rayleigh fading channels
%Done by:
%Mahdin Mahboob(ID: 05310051)
%and Sajidul Alam(ID: 06310054)


X1=1:1:18;

k=18;
```

```
for n = 1:k
   EbNo = 1:1:18;
   M = 2; %
   divorder = 1;
   B = berfading(EbNo,'psk',M,divorder);

end


semilogy(X1,B,'ro-');

xlabel('SNR(dB)---------->');
ylabel('BER <in semilog> -------------->');

hold on

for n = 1:k
   EbNo = 1:1:18;
   M = 2;
   divorder = 1;
   B = berfading(EbNo,'dpsk',M,divorder);

end

semilogy(X1,B,'b*-');
xlabel('SNR(dB)---------->');
ylabel('BER <in semilog> -------------->');


hold on

for n = 1:k
   EbNo = 1:1:18;
   M = 2;
   divorder = 1;

   B = berfading(EbNo,'fsk',M,divorder,'noncoherence');

end

semilogy(X1,B,'g^-');
xlabel('SNR(dB)---------->');
ylabel('BER <in semilog> -------------->');


legend('Modulation type=PSK','Modulation type=DPSK','Modulation type=FSK' );
```

D.MATLAB Code for Comparison Between AWGN and Rayleigh

```
%Thesis Topic: Comparison Of different models for the analysis of Rayleigh
fading channels
%Done by:
%Mahdin Mahboob(ID: 05310051)
%and Sajidul Alam(ID: 06310054)


X1=1:1:18;

k=18;


   for n = 1:k

      B(n) = berawgn(n,'psk',2,'nondiff');

   end

   semilogy(X1,B,'ro-');

xlabel('SNR(dB)---------->');
ylabel('BER <in semilog> -------------->');

hold on


X1=1:1:18;

k=18;


for n = 1:k
   EbNo = 1:1:18;
   M = 2;
   divorder = 4;
   B = berfading(EbNo,'psk',M,divorder);

end


semilogy(X1,B,'b^-');
xlabel('SNR(dB)---------->');
ylabel('BER <in semilog> -------------->');
```

```
 legend('AWGN','Rayleigh');
```

E. MATLAB Code for Jakes Model

```
%Thesis Topic: Comparison Of different models for the analysis of rayleigh
fading channels
%Done by:
%Mahdin Mahboob(ID: 05310051)
%and Sajidul Alam(ID: 06310054)

function
[chann,fm,doppler_rate]=modified_jakes_convert_2_jakes_1(v,fc,fs,U,M,segma)

% v:  Vehicle speed in kmph
% fc: Central carrier frequency in MHz
% fs: Symbol frequency in ksps
% U:  Number of sub-carriers or sub-channels
% M:  Number of channel coefficients to generate
% segma: Variance of the channel coefficients
% chann: channels coefficients matrix of size U by M
% fm: Maximum doppler frequency in Hz
% doppler_rate: Doppler rate or fading rate



% Example:
% --------
%
[chann,fm,doppler_rate]=modified_jakes_convert_2_jakes_1(100,2000,10,3,1000
0,0);
```

```
fm=(1e3/3600)*v*fc/3e2; % maximum doppler frequency in Hz
doppler_rate=fm/(fs*1000);
No=24; % number of distinct oscilators
NN=4*No; % total number of osillators
omega_m=2*pi*fm; % maximum doppler frequency

for u=1:U
   sum=0;
   t=1/(fs*1000):1/(fs*1000):(1/(fs*1000))*M; % sampling frequency
   for n=1:No
      omega_n=omega_m*cos(2*pi*(n)/NN); % modified line
      sum=sum+(cos(pi*n/No)+j*sin(pi*n/No))*cos(omega_n*t+theta(n)); % sum of No distinct oscillators

   end
   T=sqrt(2/No)*sum;

   T=repmat(10.^(segma/20),1,M).*T; % Define the variance of the channel (segma) in dB.

   chann(u,:)=T; % the u'th sub-channel
end

% Plot the Rayleigh envelope for the first channel
figure(1)
plot(10*log10(abs(chann(1,:).^2)),'r')
hold on
plot(repmat(10*log10(var(chann(1,:))),M,1),'g')
ylabel('Rayleigh Envelope in dB');
xlabel('Time');
```

```
    hold on
```

F. MATLAB code for Dent Model

```
%Thesis Topic: Comparison Of different models for the analysis of rayleigh fading channels
%Done by:
%Mahdin Mahboob(ID: 05310051)
%and Sajidul Alam(ID: 06310054)

function [chann,fm,doppler_rate]=modified_jakes_1(v,fc,fs,U,M,segma)

% v:  Vehicle speed in kmph
% fc: Central carrier frequency in MHz
% fs: Symbol frequency in ksps
% U:  Number of sub-carriers or sub-channels
% M:  Number of channel coefficients to generate
% segma: Variance of the channel coefficients
% chann: channels coefficients matrix of size U by M
% fm: Maximum doppler frequency in Hz
% doppler_rate: Doppler rate or fading rate

% Example:
% --------
% [chann,fm,doppler_rate]=modified_jakes_1(100,2000,10,3,10000,0);

    fm=(1e3/3600)*v*fc/3e2; % maximum doppler frequency in Hz
    doppler_rate=fm/(fs*1000);
    No=24; % number of distinct oscilators
    NN=4*No; % total number of osillators
```

```
    omega_m=2*pi*fm; % maximum doppler frequency
    H=hadamard(No); % Walsh_Hadamard codes, to generate uncorrelated
channels
            for n=1:No
         theta(n)=rand*2*pi;  % Randomly generated initial phases
     end

    for u=1:U
        sum=0;
        t=1/(fs*1000):1/(fs*1000):(1/(fs*1000))*M; % sampling frequency
        for n=1:No
            omega_n=omega_m*cos(2*pi*(n-0.5)/NN);
            sum=sum+H(u,n)*(cos(pi*n/No)+j*sin(pi*n/No))*cos(omega_n*t+theta(n));
% sum of No distinct oscillators
        end
        T=sqrt(2/No)*sum;
        T=repmat(10.^(segma/20),1,M).*T; % Define the variance of the channel
(segma) in dB.

        chann(u,:)=T; % the u'th sub-channel
    end

    % Plot the Rayleigh envelope for the first channel
    figure(1)
    plot(10*log10(abs(chann(1,:).^2)),'b')
    hold on
    plot(repmat(10*log10(var(chann(1,:))),M,1),'r')
    ylabel('Rayleigh Envelope in dB');
    xlabel('Sampling Time');

    legend('Power','Variance',2,'Location','NorthEast');
```