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THROUGHPUT OPTIMIZATION OF RAYLEIGH CHANNEL USING VARIOUS PARAMETERS

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ABSTRACT

In wireless communication throughput is a key factor to measure the quality of wireless data link. Throughput is defined as the number of information bits received without error per second and we would naturally like this quantity as to be high as possible. This research work looks at the problem of optimizing throughput for a packet based wireless data transmission scheme from a general point of view. The purpose of this work is to show the very nature of throughput and how it can be maximized that means how we can optimize the throughput by observing its certain changing parameter such as transmission rate, packet length, signal-to-noise ratio (SNR) etc. Our main result is a mathematical technique for determining the optimum transmission rate and packet size as a function of the other variables. The key to maximizing the throughput rate is maintaining the signal-to-noise ratio at an optimum level determined by the nature of the channel. Have We tried to take a more a more general look at throughput by considering its definition for a packet-based scheme and how it can be maximized based on the channel model being used. In this research work we have used Rayleigh Fading Channel.

Key words: Throughput, Optimization, Rayleigh Fading Channel, Wireless Communication, Signal-to-Noise Ratio (SNR).

I. INTRODUCTION

Throughput is defined [1] as the number of information bits received without error per second and we would naturally like this quantity as to be high as possible. In a wireless data system many variables affect the throughput such as the packet size, the transmission rate, the number of overhead bits in each packet, the received signal power, the received noise power spectral density, the modulation technique, and the channel conditions. From these variables, we can calculate other important quantities such as the signal-to-noise ratio, the binary error rate, and the packet success rate. Throughput depends on all of these quantities.

In this paper we discuss the general look at throughput by considering its definition for a packet-based scheme and how it is maximized based on the channel model being used. Here we have used Rayleigh Fading channel model. Rayleigh fading [2] is a statistical model for the effect of a propagation environment on a radio signal, such as that used by wireless devices. Rayleigh fading models assume that the magnitude of a signal that has passed through such a transmission medium will vary randomly, or fade, according to a Rayleigh distribution — the radial component of the sum of two uncorrelated Gaussian random variables. Rayleigh fading is most applicable when there is no dominant propagation along a line of sight between the transmitter and receiver.

As an initial step in a theoretical study, we examine the influence of transmission rate and packet size in a noise-limited transmission environment. The transmitter, operating at R b/s, sends data in packets. Each packet contains L bits including a payload of K bits and a cyclic redundancy check error-detecting code with C bits. A forward error correction encoder produces the remaining L-K-C bits in each packet. The channel adds white noise with power spectral density watts/Hz and the signal arrives at the receiver at a power level of P watts. In this research work we assume to be the sum of all noise and interference, which can be modeled as Gaussian white noise. The CRC decoder detects transmission errors and generates acknowledgments that cause packets with errors to be retransmitted. Table 1 displays a summary of the variables in our analysis and their notation.

Table 1: Variables in Analysis

Quantity	Notation	Value
Signal to Noise Ratio	γ	10
Received signal power	P (watts)	$5*10^{-9}$ W
Receiver noise power spectral density	N0 (W/Hz)	10^{-15} W/Hz
Binary transmission rate	R bits/s	Varied
Packet size	L bits	Varied
Cyclic Redundancy Check	C bits	16 bits

An important objective [2] of data communications systems design and operation is to match the transmission rate to the quality of the channel. A good channel supports a high data rate, and conversely. For a given channel, there is a transmission rate that maximizes throughput. At low rates, transmitted data arrives without error with high probability and an increase in the data rate increases throughput. Above the optimum rate, the error probability is high and it is possible to increase throughput by decreasing the data rate, thereby increasing the probability of correct reception. Recognizing this fact, practical communications systems including facsimile, telephone modems, wireless local area networks, and cellular data systems incorporate rate adaptation to match the transmission rate to the quality of the channel. In some systems (facsimile and telephone modems), the adaptation is static, occurring at the beginning of a communication session only. In others, the adaptation is more dynamic with the rate rising and falling in response to changes in channel conditions.

The research work begins with the analysis by looking at throughput optimization as a function of the packet length with a fixed transmission rate followed by an analysis of throughput as a function of transmission rate with a fixed packet length. Using the optimization equations obtained, the throughput can be jointly optimized with respect to both the packet length and transmission rate, both written in terms of the SNR. These equations can be used to find the optimal signal-to-noise ratio (SNR) that the system should be operated at to achieve the maximum throughput. We have used these equations and implement them using MATLAB. To understand the throughput variation we have used different values of transmission rate, packet length, signal-to-noise ratio (SNR). We have simulated those in MATLAB and then observed the results in graphical representation in MATLAB window. We have talked about different variables and how changing certain parameters can yield better throughput performance.

II. THROUGHPUT ANALYSIS

A. Throughput Analysis

The amount of data [4] transferred from one place to another or processed in a specified amount of time. Data transfer rates for disk drives and networks are measured in terms of throughput. Typically, throughputs are measured in kbps, Mbps and Gbps, the speed with which data can be transmitted from one device to another. Data rates are often measured in megabits (million bits) or megabytes (million bytes) per second. These are usually abbreviated as Mbps and MBps, respectively.

B. Assumptions and Definitions

Our analysis includes the following simplifying assumptions: [2]

1. The CRC decoder detects all errors in the output of the sending decoder channel. That means no matter what kind of data is transmitting and received by the receiving channel we are assuming that the receiving channel decoder will be able to get all the data most accurately. If there is any error in the bit stream then the CRC(Cyclic Redundancy Check) decoder will be able to correct all the errors in the received data.
2. Transmission of acknowledgments from the receiver to the transmitter is error free and instantaneous.
3. In the presence of errors, the system performs selective repeat ARQ (Adaptive Retransmission Query) retransmissions.

4. The received signal power is P watts, either a constant or a random variable with a Rayleigh probability density function, representative of fading wireless channels. In this paper, we consider "fast fading" in which the values of P for the different bits in a packet are independent, identically distributed Rayleigh random variables.

System throughput (T) is the number of payload bits per second received correctly:

$$T = \frac{K}{L} Rf(\gamma) \tag{1}$$

where (KR/L) b/s is the payload transmission rate and f(γ) is the packet success rate defined as the probability of receiving a packet correctly. This probability is a function of the signal-to-noise ratio

$$\gamma = \frac{E_b}{N_0} = \frac{P}{N_0 R} \tag{2}$$

In which E_b = P/R joules is the received energy per bit. We will now look at maximizing the throughput in a Gaussian white noise channel with respect to the transmission rate and packet length.

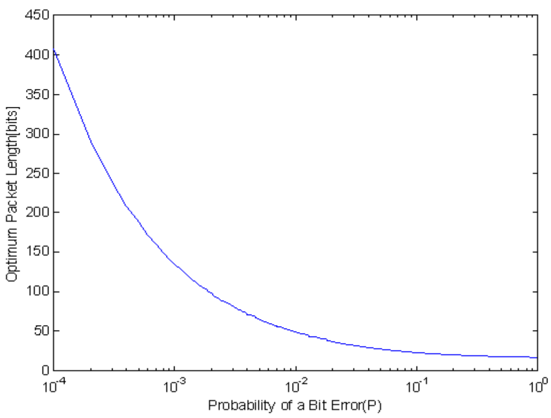

Figure 1: Optimum packet length as a function of P

III. SIMULATION AND RESULTS

A. Throughput vs. Transmission Rate: Fixed Packet Length

1. Equation simulation

To find the transmission rate, R=R* b/s, that maximizes the throughput, we differentiate Equation (1) with respect to R to obtain:

$$\frac{dT}{dR} = (K/L)\, f(\gamma) + (K/L)\, R \, \frac{df(\gamma)}{d\gamma}\frac{d\gamma}{dR} = (K/L) \left(f(\gamma) + R\frac{df(\gamma)}{d\gamma}(-P/N_0R^2) \right) \tag{3}$$

Next we set the derivative to zero:

$$f(\gamma) - (P/N_0R)\, \frac{df(\gamma)}{d\gamma} = \tag{4}$$

$$f(\gamma)\gamma\, \frac{df(\gamma)}{d\gamma} = 0,$$

$$f(\gamma) = \gamma\, \frac{df(\gamma)}{d\gamma}. \tag{5}$$

We adopt the notation $\gamma = \gamma^*$ for a signal-to-noise ratio that satisfies Equation (5). The corresponding transmission rate is

$$R^* = \frac{P}{\gamma^* N_0}. \tag{6}$$

A sufficient condition for a locally maximum throughput at R=R* is:

$$\frac{d^2T}{dR^2}\bigg|_{R=R^*} < 0 \tag{7}$$

The solution to Equation (5), γ*, is the key to maximizing the throughput of a packet data transmission. To operate with maximum throughput, the system should set the transmission rate to R* in Equation (6). γ* is a property of the function, $f(\gamma)$, which is the relationship between packet success rate and signal to interference ratio. This function is a property of the transmission system including the modem, codecs, receiver structure and antennas. Each system has its own ideal signal-to-noise ratio, γ*. Depending on the channel quality, reflected in the ratio P/N0, the optimum transmission rate is R* in Equation (6).

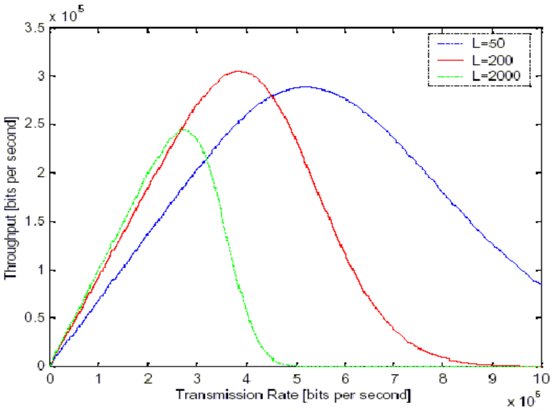

Figure 2: Throughput vs rate for fixed packet length

2. Graphical Analysis

In the figure 2 we have take three Readings. The first one was packet length of 50 bits. We have got the maximum throughput at transmission rate of 0.58 Mbps and the throughput was .27 Mbps. As we increase the transmission rate the throughput was seen to be fallen down and at a certain period it went to at the value zero. In our second assumption we have seen that for packet length of 200 bits the throughput was 0.30 Mbps and at the transmission rate of 0.4 Mbps it has gone its highest pick. After then it has also fallen down to zero. The third assumptions also showed the same. We have noticed that when the packet length size was small then the throughput has reached its highest pick with higher transmission rate and also has fallen in a wide range. But as soon as the packet length has kept higher then the curve of throughput is stepper rather than flat. When we have increased our packet length size then the throughput has reached the maximum pick at a lower transmission rate and also has fallen down quite quickly. So at the end we have come to some several decisions.

We have seen that if we keep our packet length less than 400 bits and greater than 50 bits, then we will be able to get the maximum throughput and the transmission rate shouldn't be so high. It has to be in a range of 0.3 Mbps to 0.8 mbps. So using the general equations for calculating throughput in respect of transmission rate and keeping the packet length fixed the throughput can be optimized in a certain range.

B. Throughput vs. Packet Length: Fixed Transmission Rate

1. Equation simulation

Each packet, of length L bits, is a combination of a payload (K bits) and overhead (L-K bits). Because the packet success rate, $f(\gamma)$ is a decreasing function of L, there is an optimum packet length, L*. When L<L*, excessive overhead in each packet limits the throughput. When L>L*, packet errors limit the throughput. When there is no forward error correction coding, which we shall assume for the entirety of this chapter, (K=L-C, where C bits is the length of the cyclic redundancy check), there is a simple derivation of L*. In this case,

$$f(\gamma) = (1 - P(\gamma))^L \qquad (8)$$

Where $P_e(\gamma)$ is the binary error rate of the modem. Therefore, in a system without FEC, the throughput as a function of L is

$$T = f(\gamma) = \frac{L-C}{C} R(1 - P_e(\gamma))^L \qquad (9)$$

To maximize T with respect to L, we consider L to be a continuous variable and differentiate Equation (9) to obtain

$$\frac{dT}{dL} = R\frac{L-C}{L}(1-P_e(\gamma))^L \ln(1-P_e(\gamma)) + R\frac{C}{L^2}(1-P(\gamma))^L \qquad (10)$$

Setting the derivative equal to zero produces a quadratic equation in L with the positive root:

$$L^* = \frac{1}{2}C + \frac{1}{2}\sqrt{C^2 - \frac{4C}{\ln(1-P_e(\gamma))}} \qquad (11)$$

As shown in Figure 3, 4 and 5 (in which C=16), the optimum packet size is a decreasing function of $P_e(\gamma)$.

As the binary error rate goes to zero, the packet error rate also approaches zero and the optimum packet size increases without bound. Because $P_e(\gamma)$ decreases with γ, L* increases monotonically with signal-to-noise ratio. Better channels support longer packets. Of course, in practice L is an integer and the optimum number of bits in a packet is either the greatest integer less than L* or the smallest integer greater than L*.

Equations (5) and (11) can be viewed as a pair of simultaneous equations in variables L and γ . Their simultaneous solution produces the jointly optimum packet size and Signal - to- noise ratio of a particular transmission system. We will use the notation, L** and γ **, respectively for the jointly optimized variables.

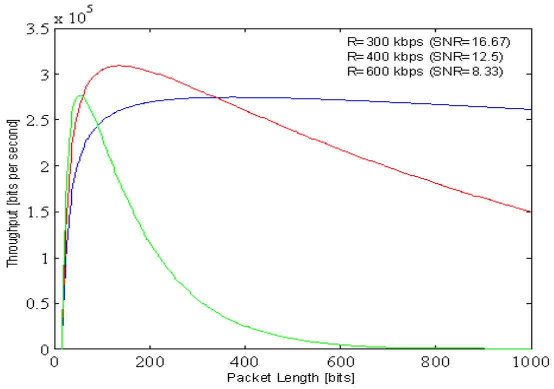

Figure 3: Throughput vs L for a fixed transmission rate (1)

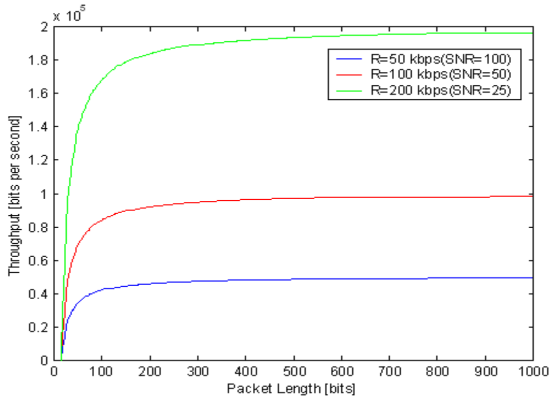

Figure 4: Throughput vs L for a fixed transmission rate (2)

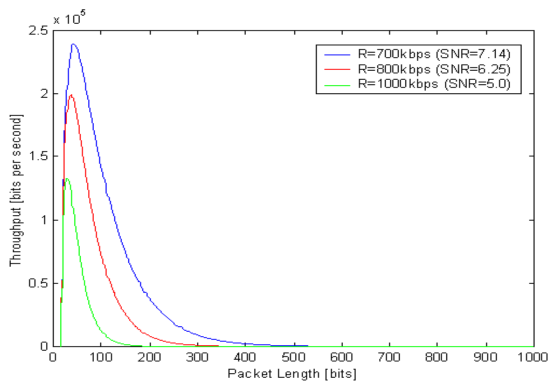

Figure 5: Throughput vs. L for a fixed transmission rate (3)

2. Graphical Analysis

Figure 3 shows and throughput optimization for fixed transmission rate varying the value of the packet length. We have taken three different assumptions for figure 3. In the first assumption was we have taken the transmission rate as 300 kbps and for this value the SNR came as 16.67.We have got the value of SNR from the Equation 2. Where P (Received Signal Power) is Watts, (Received noise power spectral density) is W/Hz. Those values are constant here. In the second assumption we have taken the value of transmission rate as 400kbps and SNR as 12.5. In the third we have taken the transmission rate as 600kbps and SNR as 8.33. We have always kept the value of C (cyclic redundancy check) as 16bits.

Figure 4 shows the same work as 3. The only difference is we have changed the value of transmission rates. The first value is 50 kbps, SNR is 100. Second one is 100kbps, SNR is 50 and the third one is 200kbps, SNR is 25.

In the figure 5 we have increased the transmission rate like 700, 800 and 1000 kbps. From those rates we have got the value of SNR 7.14, 6.25 and 5.0 respectively. In every assumption we have also kept the value of C as 16bits as a constant.

One very important thing has also been observed. We have observed that if we keep our transmission rate in the range of 0.2mbps to 0.4 mbps we will be able to get the maximum throughput. And for the maximum throughput the packet has come in the range of 200bits to 400 bits. So, this observation has proved our decision when we observed throughput in terms of transmission rate for certain fixed packet length. In this observation we have also seen that when we have taken the transmission rate higher then the throughput curve is going to be more stepper rather than flatter.

C. Rayleigh Fading Channel

For a model that corresponds to mobile radio communications, we can perform the same analysis for a fast fading Rayleigh channel. For non-coherent FSK in a Rayleigh fading channel, the probability of a bit error is given by:

$$\overline{P_e}(G) = \frac{1}{2+G} \qquad (12)$$

We can see how a changing packet length affects the throughput by choosing a fixed transmission rate and graphing (4), with Pe(G) replacing Pe(γ), as a function of the packet length. To illustrate the effects of changing the transmission rate on the throughput graph, we have three plots on Figure 6. The solid line uses a transmission rate of 10 kbps corresponding to G = 500 from (6) which from (11) yields a packet length of L*(500) = 98 bits to maximize the throughput. The small dotted line uses a transmission rate of 100 kbps corresponding to G = 50 which yields a packet length of L*(50) = 38 bits to maximize the throughput. The large dotted line uses a transmission rate of 500 kbps corresponding to G = 10 which yields a packet length of L*(10) = 24 bits to maximize the throughput. The same conclusions and observations can be made from Figure 6, 7 and 8 as those made from Figure 3, 4 and 5. The only real difference is the scale of the numbers used. Because a fading channel imposes more rigorous conditions on a transmission system, the achievable throughput will be lower than a Gaussian channel. Consequently, the system will have to operate at higher average SNR values and smaller average packet lengths.

From (9) the bit rate to maximize throughput is found to be:

$$R^* = \frac{P}{N_0}\left[\frac{L-3-\sqrt{L^2-6L+1}}{4}\right] \qquad (13)$$

This solution results from substituting (11) for $f(G)$ and (12) for $\overline{P_e}(G)$. To see how throughput changes as a function of the transmission rate we graph the throughput as a function of R with L fixed. To illustrate the effects of changing the packet length we have three plots on Figure 9. The solid line uses a packet length of 20 bits. We can use this value in (13) to tell us that the transmission rate to maximize the throughput is R* = 296.2 kbps (G* = 16.88). The small dotted line uses a packet length of 40 bits. From (13), the transmission rate to maximize throughput is R* = 135.3 kbps (G* = 36.95). The large dotted line uses a packet length of 100 bits. From (13), the transmission rate to maximize throughput is R* = 51.6 kbps (G* = 96.98). The same conclusions and observations can be made from Figure 9, 10 and 11 as those made from Figure 2, 3 and 4. Again, the only real difference is the numbers used. The transmission rate and throughput values are much smaller and the G values are much larger. An interesting result that follows from (13) is:

$$G^* = \frac{4}{L-3-\sqrt{L^2-6L+1}} = \frac{1}{2}(L-3+\sqrt{L^2-6L+1} \qquad (14)$$

This allows us to determine the value of the SNR to achieve maximum throughput for a given packet length in a Rayleigh fading channel.

To maximize the throughput with respect to both the packet length and transmission rate we can write the throughput as a function of SNR by using equation (9) and substituting L* (12) for the length, and R* (6) for the rate. The result is in Figure 10, 11. The same changes are made in P/No as were made in Figure 6 and 7 and the same conclusions can be drawn. The SNR value that maximizes throughput for a Rayleigh fading channel is G** = 28.12 and is independent of the value of P/No. This can be seen by the vertical line in Figure 10. We can now use this value in (11) to find that the packet length to achieve maximum throughput is L**(G**) = 31 bits. This value is also independent of P/No. The rate to maximize throughput R** is dependent on P/No from (6).

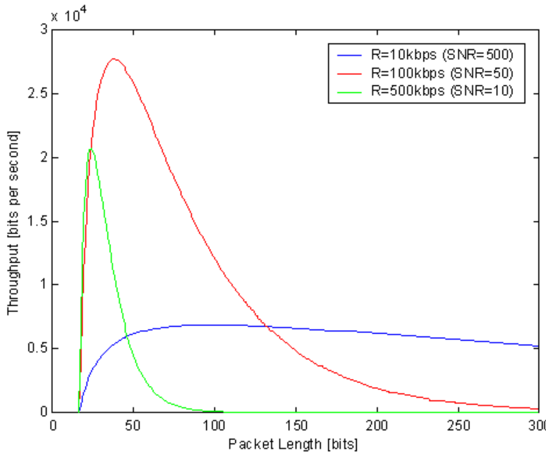

Figure 6:Throughput vs L for a Fixed Transmission Rate(Rayleigh Fading Channel) (1)

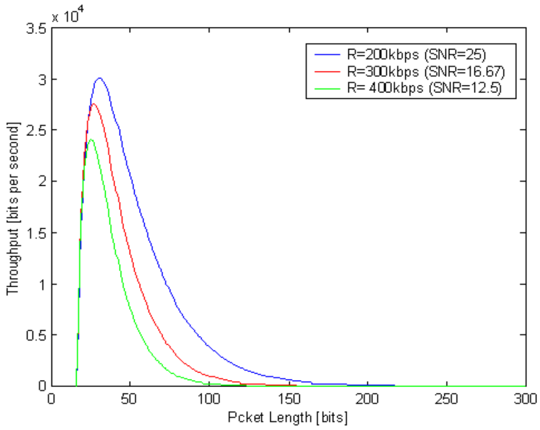

Figure 7: Throughput vs L for a Fixed Transmission Rate(Rayleigh Fading Channel) (2)

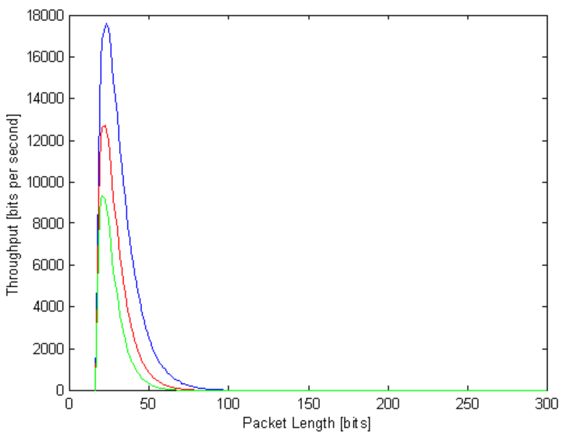

Figure 8: Throughput vs L for a Fixed Transmission Rate(Rayleigh Fading Channel) (3)

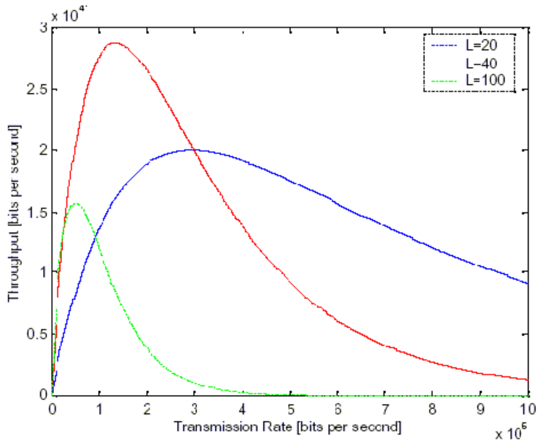

Figure 9: Throughput vs Rate for a Fixed Packet Length(Rayleigh Fading Channel)

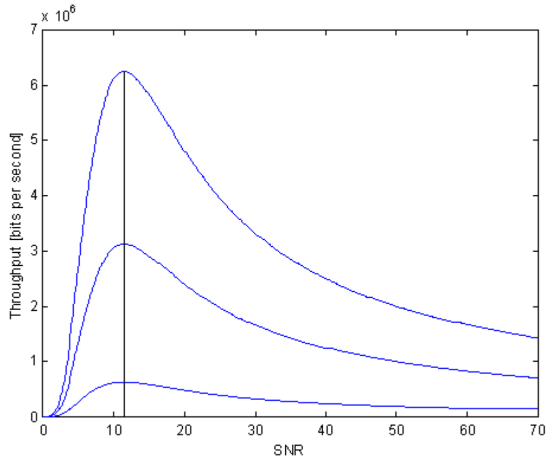

Figure 10: Throughput vs SNR Using Joint Optimization (Rayleigh Fading Channel) (1)

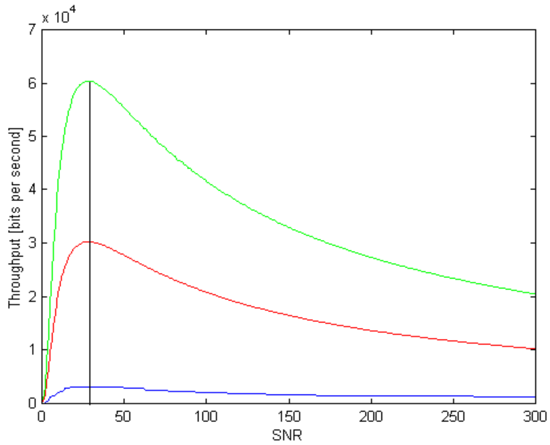

Figure 11: Throughput vs SNR Using Joint Optimization(Rayleigh Fading Channel) (2)

1. Graphical analysis

In Rayleigh fading channel we have observed all the possibilities that we have done in the previous section. That means in this section we have analyzed throughput in terms of transmission rate keeping packet length fixed, packet length keeping the transmission rate fixed. Also in this section we have observed throughput in terms SNR using joint optimization under the Rayleigh fading channel.

Figure 6, 7 and 8 is the analysis of throughput in terms of packet length where the transmission rate is kept fixed. From those graphs we have observed that for transmission rate of 150 Kbps to 300 kbps we have got the maximum throughput of 300Kbps. If we go further then the throughput has dropped toward zero.

Figure 9 is the representation of throughput with the function of transmission rate and fixed packet length. We have also observed that for transmission rate of 100 to 300 Kbps we have got the highest peak of throughput and the packet size was within 100 to 200 bits, which has matched with our previous observations.

Figure 10 and 11 has done with the throughput analysis in terms of SNR where we have used joint optimization. In our observations we have noticed that the throughput has no effect on the value of SNR in rayleigh fading channel. For different assumption the throughput is different but the highest pick of each throughput is at the same value of SNR. In our observation we have got the value of SNR is 38 db.

IV. CONCLUSION

A. Conclusion

Maximizing throughput in a wireless channel is a very important aspect in the quality of a voice or data transmission. In this research, we have shown that factors such as the optimum packet length and optimum transmission rate are all functions of the signal to noise ratio. These equations can be used to find the optimum signal to noise ratio that the system should be operated at to achieve the maximum throughput. The key concept behind this research is that for each particular channel (Rayleigh Fading) and transmission scheme, there exists a specific value for the signal to noise ratio to maximize the throughput. Once the probability of error, is known, this optimal SNR value can be obtained.

B. Limitation

We have talked about many different variables and how changing certain parameters can yield better throughput performance. Implementing these concepts in real systems is not as easy as one might think. In order for a system to make parameter changes it must be able to make simple measurements in the system. For example, if rate adaptation is to be employed, the receiver cannot easily determine its distance from the transmitter, but it can easily measure the ratio of the received signal power to the noise power spectral density. As distance increases, this ratio decreases. When its value reaches a certain point, the receiver can tell the transmitter to lower the transmission rate.

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