# OPTIMAL POWER FLOW CONSIDERING LINE LOSS

A Thesis submitted to the Department of Electrical & Electronic Engineering, BRAC University in partial fulfillment of the requirements for the Bachelor of Science degree in Electrical & Electronic Engineering

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# <u>Declaration</u>

We do hereby declare that the thesis titled "OPTIMAL POWER FLOW CONSDERING LINE LOSS" is submitted to the Department of Electrical and Electronics Engineering of BRAC University in partial fulfillment of the Bachelor of Science in Electrical and Electronics Engineering. This is our original work and has not been submitted elsewhere for the award of any other degree or diploma.


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# Acknowledgments

# Thesis Abstract:

The main objectives of the optimization of electric energy systems are to meet load demands with adequacy and reliability and to keep it at the same time economical, meaning to keep the prices as low as possible. Electric energy demand has been shown to be an exponential function doubling its rate over every decade. This ever-increasing load has led to larger and more complex systems. Interconnections throughout the whole country is growing and expanding. The main advantages of such interconnections are continuity of service and economy of power production. Power interchanges between interconnected systems are scheduled to take advantage of hour apart peak demand periods or available lower cost capacity. During emergencies, spinning reserve capacity is shared, contributing to the continuity of service. This extensive interconnection of large scale power systems has resulted in the formulation of many new concepts in power system planning and operation. The gradient and Newton methods of solving an OPF suffer from the difficulty in handling inequality constraints. Linear programming, however, is very adept at handling inequality constraints, as long as the problem to be solved is such that it can be linearized without loss of accuracy. We will include transmission losses in our OPF analysis and also implement it in MATLAB.

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# **Chapter 1**
## Introduction

## **1.1History of evolution:**

The optimal power flow or OPF has had a long history in its development. It was first discussed by Carpentier in 1962 and took a long time to become a successful algorithm that could be applied in everyday use. Current interest in the OPF centers around its ability to solve for the optimal solution that takes account of the security of the system. In the economic dispatch we have a single constraint which will hold the total generation equal to the total load plus losses. Thus, the statement of the economic dispatch problem results in a Lagrangian with just one constraint:

$$L = \sum F(P_i) + \lambda \left(P_{load} + P_{losses} - \sum P_i\right)$$

If we think about the single "generation equals load plus losses" constraint:

$$P_{load} + P_{losses} - \sum P_i = 0$$

We realize that what it is actually saying is that the generation must obey the same conditions as expressed in a power flow-with the condition that the entire power flow is reduced to one simple equality constraint [1]. There is good reason, as we shall see shortly, to state the economic dispatch calculation in terms of the generation costs, and the entire set of equations needed for the power flow itself as constraints [1]. This formulation is called an optimal power flow. We can solve the OPF for the minimum generation cost and require that the optimization calculation also balance the entire power flow-at the same time. Note also that the objective function can take different forms other than minimizing the generation cost. It is common to express the OPF as a minimization of the electrical losses in the transmission system, or to express it as the minimum shift of generation and other controls from an optimum operating point. We could even allow the adjustment of loads in order to determine the minimum load shedding schedule under emergency

conditions. Regardless of the objective function, however, an OPF must solve so that the entire set of power constraints is present and satisfied at the solution.

## 1.2 Why Linear programming:

Now, as we all know the optimal power flow is a very large and very difficult mathematical programming problem. Almost every mathematical programming approach that can be applied to this problem has been attempted and it has taken developers many decades to develop computer codes that will solve the OPF problem reliably. The attributes of these methods are summarized next:

**Lambda iteration method**:  Losses may be represented by a *[B]* matrix, or the penalty factors may be calculated outside by a power flow. This forms the basis of many standard on-line economic dispatch programs.

 **Gradient methods:** Gradient methods are slow in convergence and are difficult to solve in the presence of inequality constraints.

 **Newton's method:** Very fast convergence, but may give problems with inequality constraints.

 **Linear programming method (LPOPF):** One of the fully developed methods now in common use. Easily handles inequality constraints. Nonlinear objective functions and constraints handled by linearization.

 So we can see from the above mentioned points that all the other methods for the calculation of OPF has some drawbacks except the Linear Programming method because it is the only process where we can take various linear inequalities relating to some situation, and finding the "best" value obtainable under those condition.

# Chapter 2

# LINEAR OPTIMAL LOAD FLOW

## 2.1 Introduction

The term Optimal Load Flow refers to an operating state or load flow solution where some power system quantity is optimized subject to constraints on the problem variables and on some functions of these variables. The constraints are usually classified under two categories:
1) load constraints and
2) operating constraints.
The load constraints require that the load demands be met by the system and can be expressed in the form of the familiar load flow equations. The operating constraints impose minimum or maximum operating limits on system variables and are associated with both steady-state land transient stability limitations. These restrictions are imposed on various power system quantities such as equipment loadings (mainly for_ transmission lines and transformers), bus voltages, phase angle differences, real and reactive injected powers, etc. In this chapter, an optimal linear load flow is one in which the objective function to be optimized and the constraints are linear „functions of the system variables. These linear programs usually have several drawbacks and yield only approximate results to the exact solution. Also, many operating constraints cannot be handled by these programs and in most cases a general nonlinear formulation is needed to represent the model adequately. Several methods have been devised to solve nonlinear programs but none exhibit the efficiency and reliability of the Simplex method.

## 2.2 System Description

In this section some of the notations and terminology that will be used throughout this paper are presented below. The mathematical description of a power system is given by linear circuit theory and the complex power relations for the loads and the generating plants.

N = number of busses in the system excluding the voltage the voltage reference node

$|V_i|$ = voltage magnitude of bus i

$\delta_I$ = voltage angle of bus i

$V_i$ : complex voltage of bus I given by $|Vi|\ e^{j\delta i}$

$P_i$ : net real power injected into bus i

$Q_i$ : net reactive power imjected into bus i

$P_{ij}$ : real power flow from bus I to bus j at bus j

$Q_{jj}$ : reactive power flow from bus I to bus j at bus j

$\theta_i, \theta_j$= the phase angles at buses *I* and **k,** respectively;

$|E_i|, |E_k|$ = the bus voltage magnitudes, respectively

$G_{ik} + jB_{ik}, =Y_{ik}$ is the ik term in the *Y* matrix of the power system.

We shall use the LPOPF reduced model method to solve an OPF problem. An LP and an AC power flow will be used to solve a series of dispatch problems. The transmission system will be the six bus system showed in figure 2.1, the MW limits on the transmission lines will be those given in figure 2.2 and figure2.3. The generator cost functions are those found in section **2.3** and linearized as shown below.

**Bus 3**

246.1kv∠-4.3°

**Bus2** → 2.9    2.9 →

241.5kV∠-3.7°

← 12.3    5.7    ←

← 60

← 89.6

→ 19.1

→ 23.2

→ 43.8

60.7    → 42.8

57.9

**Bus 6**

→ 26.2

12.4

25.7

16.0

→ 15.5    15.4

← 27.8

12.8

→ 15

18

**Bus 1**

**Bus 5**

241.5kV∠0⁻

1.6

9.7

231kV∠-5.9    70    70

→ 28.7

15.4

→ 34.5

13.5

→ 35.6

11.3

→ 4.0

2.8

← 18.0

← 26.1

→ 43.6

20.1

**Bus 4**

← 42.5

← 19.9

→ 4.1

← 4.9

70    70

226.7kV∠-5.3°

where → MW

MVAR

Generator

Load

70    70

227.6kV∠-4.2°
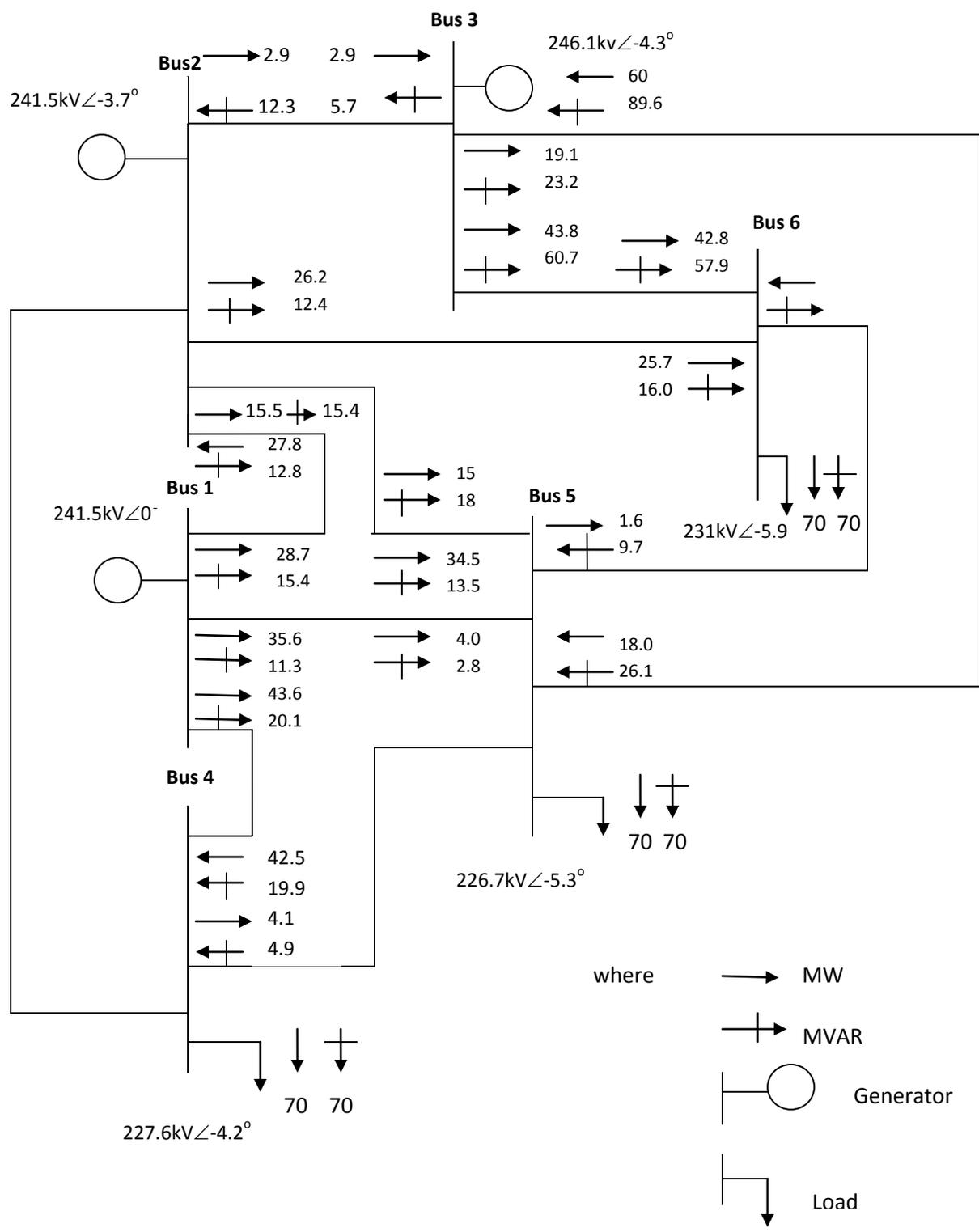
**FIG. 2.1 Six-bus network** base **case AC power** flow.

| Line | MW Limit |
|------|----------|
| 1-2 | 30 |
| 1-4 | 50 |
| 1-5 | 40 |
| 2-3 | 20 |
| 2-4 | 40 |

| | |
|---|---|
| 2-5 | 20 |
| 2-6 | 30 |
| 3-5 | 20 |
| 3-6 | 60 |
| 4-5 | 20 |
| 5-6 | 20 |

**FIG.2.2 MW Limits for Six-bus network**

| Line | Limit | MW Flow |
|---|---|---|
| 1-2 | 30 | 28.69 |
| 1-4 | 50 | 43.58 |
| 1-5 | 40 | 35.60 |
| 2-3 | 20 | 2.93 |
| 2-4 | 40 | 33.09 |
| 2-5 | 20 | 15.51 |
| 2-6 | 30 | 26.25 |
| 3-5 | 20 | 19.12 |
| 3-6 | 60 | 43.77 |
| 4-5 | 20 | 4.08 |
| 5-6 | 20 | 1.61 |

**FIG.2.3 Line Flows: Power Flow 0**

## 2.3 Methodology

Figure 2.4 shows the type of strategy used to create an OPF using linear programming. The power flow equations could be for the DC representation, the decoupled set of AC equations, or the full **AC** power flow equations. The choice will affect the difficulty of obtaining the linearized sensitivity coefficients and the convergence test used. In the formulation below, we show how the OPF can be structured as an LP. First, we tackle the problem of expressing the nonlinear input-output or cost functions as a set of linear functions.

Let the cost function be $F_i(p_i)$ as shown in Figure 2.5. We can approximate this nonlinear function as a series of straight-line

segments as shown in Figure 2.6. The three segments shown will be represented as $P_{i1}, P_{i2}, P_{i3}$ , and each segment will have a slope designated:

$$S_{i1}, S_{l2}, S_{i3}$$

then the cost function itself is

$$F_i(P_i) = F_i(P_i^{min}) + S_{i1}P_{i1} + S_{i2}P_{i2} + S_{i3}P_{i3}$$
$$0 \leq P_{ik} \leq P_{ik}^{+} \text{ For k=1, 2, 3}$$

And finally

$$P_i = P_i^{min} + P_{i2} + P_{i3}$$

Initial power flow condition

Solve AC power flow equations

Create linear objective function

Obtain linearized constraint
sensitivity coefficients

Set up and solve LP for new control
variable setting

Not
converged

converged

Convergence test

Significant
adjustment for one
or more control
variables

No significant movement
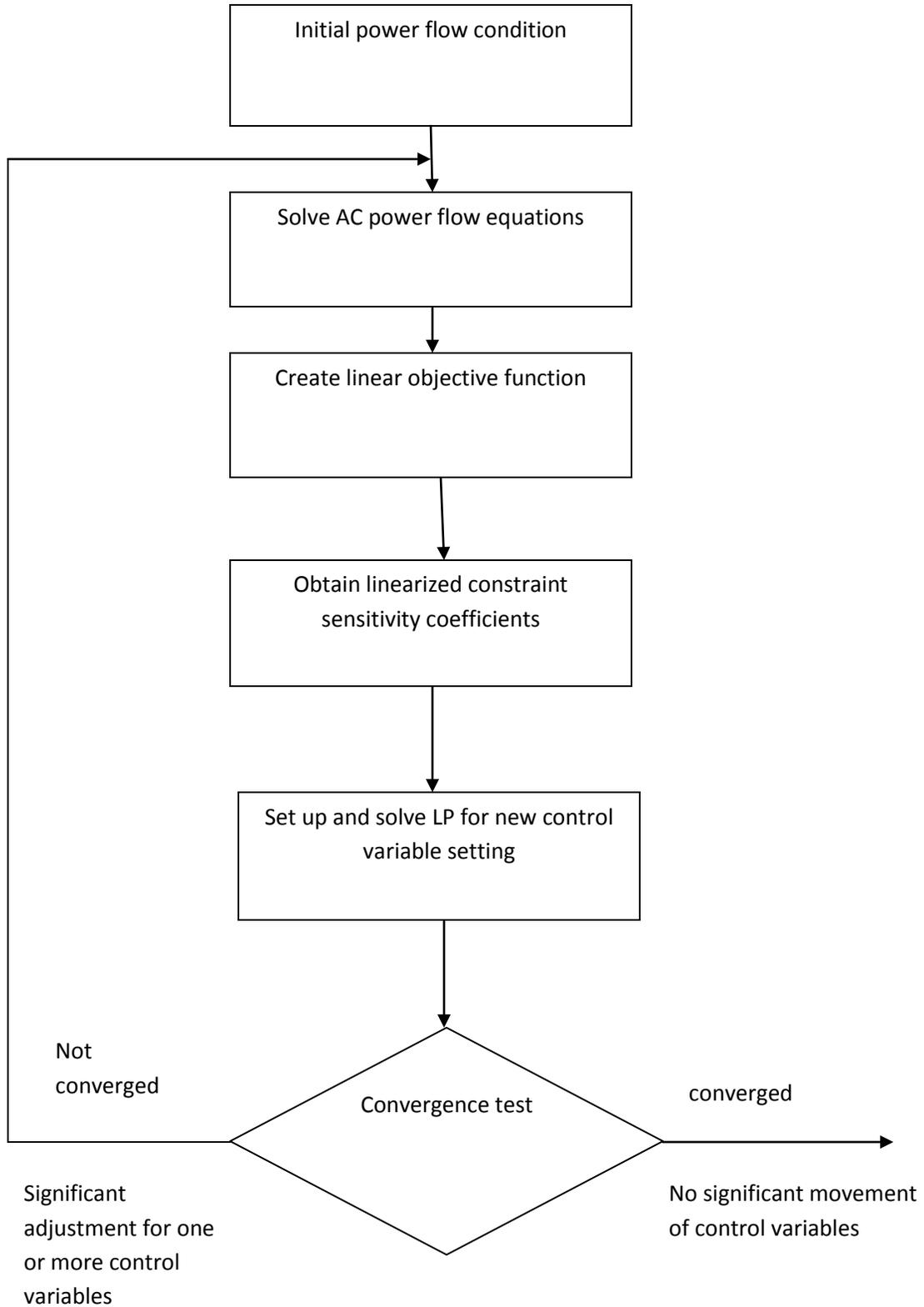of control variables

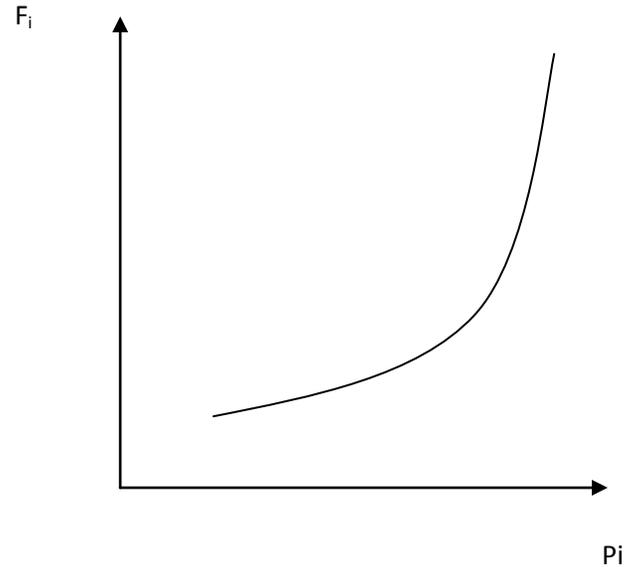**FIG.2.4** Strategy for solution of the LPOPF

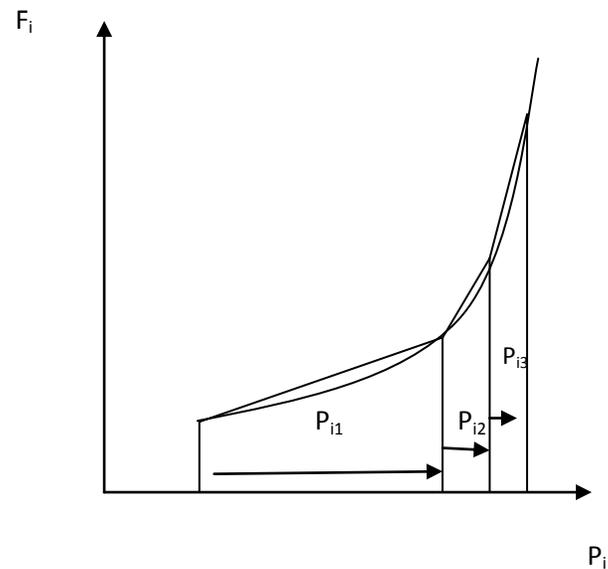FIG. 2.5 A nonlinear cost function characteristic



FIG. 2.6 A linearized cost function

The cost function is now made up of a linear expression in the $P_{ik}$ values. In the formulation of the OPF using linear programming, we only have the control variables in the problem. We do not attempt to place the state variables into the LP,

nor all the power flow equations [1]. Rather, constraints are set up in the LP that reflect the influence of changes in the control variables only. In the examples we present here, the control variables will be limited to generator real power, generator voltage magnitude, and transformer taps. The control variables will be designated as the *u* variables [1]. The next constraint to consider in an LPOPF are the constraints that represent the power balance between real and reactive power generated, and that consumed in the loads and losses.

The real power balance equation **is:**

$$\mathbf{P_{gen} - P_{load} - P_{loss} = 0}$$

The loss term here represents the $I^2R$ losses in the transmission lines and transformers. We can take derivatives with respect to the control variables, **u,** and this result in:

$$\Sigma_u \left( \frac{\partial P_{gen}}{\partial u} \right) \Delta u - \Sigma_u \left( \frac{\partial P_{load}}{\partial u} \right) \Delta u - \Sigma_u \left( \frac{\partial P_{loss}}{\partial u} \right) \Delta u = 0$$

If we make the following substitution:

$$\Delta u = u - u^0$$

then, the power balance equation becomes

$$\Sigma_u \left( \frac{\partial P_{gen}}{\partial u} \right) u - \Sigma_u \left( \frac{\partial P_{load}}{\partial u} \right) u - \Sigma_u \left( \frac{\partial P_{loss}}{\partial u} \right) u = K_p$$

Where

$$K_p = \Sigma_u \left( \frac{\partial P_{gen}}{\partial u} \right) u_0 - \Sigma_u \left( \frac{\partial P_{load}}{\partial u} \right) u_0 - \Sigma_u \left( \frac{\partial P_{loss}}{\partial u} \right) u_0$$

**A** similar equation can be written for the reactive power balance:

$$\Sigma_u \left( \frac{\partial Q_{gen}}{\partial u} \right) \Delta u - \Sigma_u \left( \frac{\partial Q_{load}}{\partial u} \right) \Delta u - \Sigma_u \left( \frac{\partial Q_{loss}}{\partial u} \right) \Delta u$$

where the loss term is understood to include $I^2X$ as well as the charging from line capacitors and shunt reactors. **A** substitution using $\Delta \mathbf{u} = \mathbf{u} - \mathbf{u}^o$, as above, can also

be done here. The LP formulation, so far, would need to restrict control variables to move only within their respective limits, but it does not yet constrain the OPF to optimize cost within the limits of transmission flows and load bus voltages. To add the latter type constraints, we must add a new constraint to the **LP** [1]**.**

For example, say we wish to constrain the MVA flow on line *nm* to fall within an upper limit:

$$MVA\,flow_{nm} \leq MVA\,flow_{nm}^{max}$$

We model this constraint by forming a Taylor's series expansion of this flow and only retaining the linear terms:

$$MVA\,flow_{nm} = MVA\,flow_{nm}^0 + \sum_u \left( \frac{\partial}{\partial u} MVA\,flow_{nm} \right) \Delta u \leq MVA\,flow_{nm}^{max}$$

Again, we can substitute $\Delta u = u - u^o$ so we get

$$\sum_u \left( \frac{\partial}{\partial u} MVA\,flow_{nm} \right) u \leq MVA\,flow_{nm}^{max} - K_f$$

Where

$$K_f = MVA\,flow_{nm}^0 + \sum_u \left( \frac{\partial}{\partial u} MVA\,flow_{nm} \right) u^0$$

Other constraints such as voltage magnitude limits, branch **MW** limits, etc., can be added in a similar manner. We add as many constraints as necessary to constrain the power system to remain within its prescribed limits. Note, of course, that the derivatives of $P_{loss}$, and **MVA** flow$_{nm}$ are obtained from the linear sensitivity coefficient calculations presented in the next section.

## 2.4 Linear Programming Method with Only Real Power Variables

As an introduction to the LPOPF, we will set up and solve a power system example which only has generator real powers as control variables. Further, the model for the power system power balance constraint will assume that load is constant and that the losses are constant. Finally, since the entire model used in the LP is based on a MW-only formulation, we shall use the *"a"* factor derived in Chapter 3 to model the effect of changes in controls on the constraints. **As** indicated in Figure 2.4, we shall solve the LP and then make the adjustments to the control variables and solve a power flow in each main iteration. This guarantees that the total generation equals load plus losses, and that the MW flows are updated properly. The cost functions can be treated as before using multiple segmented "piecewise linear" approximations.

The "power balance" equation for this case is as follows:

$$\mathbf{P_1 + P_2 + \dots\dots + P_{ref} = P_{load} + P_{losses} = Constant}$$

To constrain the power system, we need the expansion of the constraints, such as MW flows, bus voltages, etc., as linear functions of the control variables. In this case, the linear control variables will be represented as a vector **u:**

$$u = \begin{bmatrix} P_1 \\ \vdots \\ P_{ref} \end{bmatrix}$$

This is done with the linear sensitivity approach, as derived in the previous section. The result is a set of constraints:

$$h(\mathrm{u}) = \mathrm{h}^+$$

Which is written as

$$h(u) = h(u^0) + \frac{\partial h}{\partial u}(u - u^0) \leq h^+$$

However, we shall observe that the derivatives **δh/δu** can be replaced with the *"a"* sensitivity coefficients which will be developed n the next section.

Thus, for a MW flow constraint on line *rs* we have:

$$MW_{rs} = MW_{rs}^0 + \sum_u a_{rs-u}(u - u^0) \leq MVA_{rs}^{max}$$

$$MW_{rs} = \sum_u a_{rs-u} \leq MW_{rs}^{max} - \left(MW_{rs}^0 - \sum_u a_{rs-u} u^0\right)$$

| Line | Limit | MW Flow |
|------|-------|---------|
| 1-2 | 30 | 28.69 |
| 1-4 | 50 | 43.58 |
| 1-5 | 40 | 35.60 |
| 2-3 | 20 | 2.93 |
| 2-4 | 40 | 33.09 |
| 2-5 | 20 | 15.51 |
| 2-6 | 30 | 26.25 |
| 3-5 | 20 | 19.12 |
| 3-6 | 60 | 43.77 |
| 4-5 | 20 | 4.08 |
| 5-6 | 20 | 1.61 |

**FIG. 2.7 Line Flows : Power flow Zero**

# Chapter 3
# LPOPF Formulation

## 3.1 Step 0:

First we have to run a base AC power flow (this will be the AC power flow shown in Figure **2.1** and it will be designated as POWER Flow 0 in numbering the various power flow calculations in this example)[3]. Looking at Figure 2.1 and the limit set we are using from Figure 2.2 , shown above, we note that there are no overloads. The generation values for this power flow are:

$P1$ = 107.87 MW, $P2$ = 50 MW, and $P3$ = 69 MW *powerflow* 0: *result*

The total cost for this initial dispatch is 3189.4 $/h.

## 3.2 Step 1

We now set **up** the LP to solve for the optimum cost with only the power balance equation in the LP constraint set. By the nature of the cost curve

| Unit | Break Point 1 (Unit min) | Break Point 2 | Break Point 3 | Break Point 4 (Unit max) |
|---|---|---|---|---|
| 1 | 50 | 100 | 160 | 200 |
| 2 | 37.5 | 70 | 130 | 150 |
| 3 | 45 | 90 | 140 | 180 |

**FIG. 3.1** Generator Unit Break Point MWs

| Generator | $S_{i\,1}$ | $S_{i\,2}$ | $S_{i\,3}$ |
|---|---|---|---|
| 1 | 12.4685 | 13.0548 | 13.5875 |
| 2 | 11.2887 | 12.1110 | 12.8222 |
| 3 | 11.8333 | 12.5373 | 13.2042 |

FIG. 3.2 Generator Cost Curve Segment Slope

Segments, we also incorporate the limits on the generators. The generator cost functions are as follows:

$$Generator\ on\ bus\ 1: F_1\ (P_1) =\ 213.1 + 11.669P_1 +\ 0.00533P_1^2\ \ R/h$$

$$with\ limits\ of: 50.0MW\ \leq\ P_1\ \leq 200.0\ MW$$

$$Generator\ on\ bus\ 2: F_2\ (P_2) = 200.0 + 10.333P_2 +\ 0.00889P_2^2\ \ R/h$$

$$with\ limits\ of: 37.5\ MW\ \leq\ P_2\ \leq 150.0\ MW$$

$$Generator\ on\ bus\ 3: F_3\ (P_3) =\ 240.0 + 10.833P_3 +\ 0.00741P_3^2\ \ R/h$$

$$with\ limits\ of: 45.0MW\ \leq\ P_3\ \leq 180.0\ MW$$

The LP will be run with the unit cost functions broken into three straight-line segments such that the break points are located as shown in Figure 3.1. The generator cost function segment slopes are computed as follows:

$$S_{ij}\ =\ \frac{F_i(P_{ij}^+) -\ F_i(P_{ij}^-)}{P_{ij}^+ -\ P_{ij}^-}$$

where $P_{ij}^+$ and $P_{ij}^-$ are the values of $P_i$ at the end of the $j^{ih}$ cost curve segment. The values are shown in Figure 3.2. The segment limits are shown in Table 3.3. The LP cost function is:

$$\left[F1\left(P_1^{min}\right) +\ 12.4685P_{11} +\ 13.0548\ P_{12} +\ 13.5878\ P_{13}\right] + \left[F_2\left(P^{min}2\right) + 11.2887P_{21} + 12.111P_{22} +\ 13.5878P_{23}\right] + \left[F_3\left(P_3^{min}\right) + 11.8333P_{31} +\ 12.5373P_{32} +\ 13.2042P_{33}\right]$$

| Segment | Min MW | Max MW |
|---------|--------|--------|
| $P_{11}$ | 0 | 50 |
| $P_{12}$ | 0 | 60 |
| $P_{13}$ | 0 | 40 |
| $P_{21}$ | 0 | 32.5 |
| $P_{22}$ | 0 | 60 |
| $P_{23}$ | 0 | 20 |
| $P_{31}$ | 0 | 45 |
| $P_{32}$ | 0 | 50 |
| $P_{33}$ | 0 | 40 |

Fig 3.3 Segment limits

Since the $F_i(P_i^{min})$ terms are constant, we can drop them in the LP. Then, the cost function becomes:

$$[12.4685P_{11} + 13.0548\,P_{12} + 13.5878\,P_{13}] + [11.2887P_{21} + 12.111P_{22} + 12.8222P_{23}]$$
$$+ [11.8333P_{31} + 12.5373P_{32} + 13.2042P_{33}]$$

The generation, load, and losses equality constraint is:

$$P_1 + P_2 + P_3 = P_{load} + P_{losses}$$

The load is 210 MW and the losses from the initial power flow are 7.87 MW. Substituting the equivalent expression for each generator's output in terms of its three linear segments, we obtain:

$$[(P_1^{min}\,) + P_{11} + P_{12} + P_{13}] + [(P^{min}{}_2) + P_{21} + P_{22} + P_{23}] + [(P_3^{min}\,) + P_{31} + P_{32} + P_{33}]$$
$$= P_{load} + P_{losses}$$

This results in the following after the $P_i^{min}$ , $P_{load}$ and $P_{loss}$ values are substituted:

$$[P_{11} + P_{12} + P_{13}] + [(P_{21} + P_{22} + P_{23}] + [P_{31} + P_{32} + P_{33}] = 210 + 7.87 - 50 - 37.5 - 45$$
$$= 85.37$$

We now solve the LP with the cost function and equality constraint given above, and with the six variables representing the generator outputs. The solution to this LP is shown in Figure 3.4.

| Segment | Min MW | Solution | Max MW |
|---------|--------|----------|--------|
| $P_{11}$ | 0 | 0 | 50 |
| $P_{12}$ | 0 | 0 | 60 |
| $P_{13}$ | 0 | 0 | 40 |
| $P_{21}$ | 0 | 32.5 | 32.5 |
| $P_{22}$ | 0 | 7.87 | 60 |
| $P_{23}$ | 0 | 0 | 20 |
| $P_{31}$ | 0 | 45 | 45 |
| $P_{32}$ | 0 | 0 | 50 |
| $P_{33}$ | 0 | 0 | 40 |

Figure 3.4: First LP solution

The total generation on each generator is :

$$P_i = P_i^{min} + P_{i1} + P_{i2} + P_{i3}$$

Note that this solution of necessity will have only one of the variables not at a break point while the others will be at a break point. Note also that the output on bus 1 is at its low limit. When we substitute these values for the generation at buses 1, **2,** and **3,** and run the power flow, we get the following:

**$P_1$ = 50 MW, $P_2$ = 77.87 MW, and $P_3$ = 90 MW** *powerflow 1: result*

This solution was done manually to verify the result of our program. The manual calculations are given below:

| Combination | Load(85.37) MW | Cost | Comment |
|---|---|---|---|
| $P_{11}$, $P_{12}$ | 50+35.37 | 1085.2 | |
| $P_{11}$,$P_{21}$,$P_{22}$ | 50+32.5+2.87 | 1025.1 | |
| $P_{11}$,$P_{31}$ | 50+35.37 | 1042 | |
| $P_{31}$,$P_{32}$ | 45+40.37 | 1038.6 | |
| $P_{21}$,$P_{31}$,$P_{22}$ | 32.5+45+7.87 | 994.69 | Lowest cost |
| $P_{11}$,$P_{21}$,$P_{31}$ | 50+32.5+2.87 | 1024.3 | |
| $P_{21}$,$P_{31}$,$P_{32}$ | 32.5+45+7.87 | 998.05 | |

The total cost for this dispatch is 3129.1 $/h. This illustrates the fact that the LP uses a linear model of the power system and when we put its results into a nonlinear model, such as the power flow, there are bound to be differences. Since the losses have changed (to 6.70 MW), the power output of the reference bus must decrease to balance the power flow. However, the solution to the optimal LPOPF has the reference-bus power output below its minimum of 50 MW. To correct this condition we set up another LP solution with the same cost function but with a slightly different equality constraint that reflects the new value of losses. The result of this LP is:

**$P_1$ = 50 MW, $P_2$ = 76.7 MW, and $P_3$ = 90 MW** *LP 1.1: result*

Once again, we enter these results into the power flow and obtain:

**$P_1$ = 49.99 MW, $P_2$ = 76.7 MW and $P_3$ = 90 MW** *powerflow 1.1 result*

| Line | Limit | MW Flow |
|------|-------|---------|
| 1-2 | 30 | 4.28 |
| 1-4 | 50 | 25.60 |
| 1-5 | 40 | 20.11 |
| 2-3 | 20 | -6.42 |
| 2-4 | 40 | 48.75[a] |
| 2-5 | 20 | 17.75 |
| 2-6 | 30 | 20.88 |
| 3-5 | 20 | 28.91[a] |
| 3-6 | 60 | 54.63 |
| 4-5 | 20 | 1.84 |
| 5-6 | 20 | 3.87 |

[a]Overloaded line

Fig 3.5: Line Flows: Power flow 1.1

The total cost for this dispatch is 3129.6 $/h and the losses are 6.7 MW.
This represents the least cost dispatch that we shall obtain in this example. As constraints are added later to meet the flow limits, the cost will increase. Note also that we have two overloads on the optimum cost dispatch as shown in Figure 3.5.

## 3.3 Step 2

The LP and power flow executions in step 1 resulted in a less-costly dispatch than the original power flow, but in doing so we have overloaded two transmission lines. We shall refer to these overloads as *(n − 0)* overloads. This notation means that there are *n* lines minus zero outages in the network at the time of the overload.

We must redispatch the power system at this point to remove the *(n − 0)* overloads. To do this, we add two constraints to the LP, one for each overloaded line. The power flow constraint on line 2-4 is modeled as:

$$f_{2-4} = f_{2-4}^0 + a_{2-4,1}(P_1 - P_1^0) + a_{2-4,2}(P_2 - P_2^0) + a_{2-4,3}(P_3 - P_3^9) \leq 40$$

Before gong any further with our calculation we shall talk about the term Called **SENSITIVITY COEFFICIENT" a"** introduced in the above equation.

## 3.4 Linear sensitivity analysis:

Before continuing with the discussion of the linear programming and interior OPF methods, we shall develop the concept of linear sensitivity analysis. Linear sensitivity coefficients give an indication of the change in one system quantity (e.g., MW flow, MVA flow, bus voltage, etc.) as another quantity is varied (e.g., generator MW output, transformer tap position, etc.) These linear relationships are essential for the application of linear programming. Note that as the adjustable variable is changed, we assume that the power system reacts so as to keep all of the power flow equations solved. As such, linear sensitivity coefficients can be expressed as partial derivatives for example:

$$\frac{\delta MVAflowij}{\delta MWgenk}$$

shows the sensitivity of the flow (MVA) on line (I to j) with respect to the power generated at bus **k.** Some sensitivity coefficients may change rapidly as the adjustment is made and the power flow conditions are updated. This is because some system quantities vary in a nonlinear relationship with the adjustment and resolution of the power flow equations. This is especially true for quantities that have to do with voltage and MVAR flows. Sensitivities such as the variation of MW flow with respect to a change in generator MW output are rather linear across a wide range of adjustments and lead to the usefulness of the DC power flow equations and the *"a"* factor. For this reason, the value represented by a 23ensitivity coefficient is only good for small adjustments and the sensitivities must be recalculated often.

## 3.5 Sensitivity Coefficients of an AC Network Model:

The following procedure is used to linearize the **AC** transmission system model for a power system. To start, we shall define two general equations giving the power injection at a bus. That is, the net power flowing into a transmission system from the bus. This function represents the power flowing into transmission lines and shunts at the bus:

$$P_i(|E|, \theta) = Re\left[\left(\sum_f E_i\left[(E_i - t_{ij}\, E_j\,)y_{ij}\,\right]\right) + E_i\left(E_i \sum_l y_{shunt}\right)*\right]$$

$$Q_i(|E|,\theta) = Im\left[\left(\sum_f E_i\left[(E_i - t_{ij}\,E_j\,)y_{ij}\,\right]\right) + E_i\left(E_i\sum_l y_{shunt}\right)*\right]$$

Where

$E_i = |Ei|\angle\theta i$

$T_i$ = the transformer tap in branch ij

$y_{ij}$ = the branch admittance

$y$ = the sum of the branch and bus shunt admittance at bus i

Then at each bus:

$$P_i(|E|,\theta) = P_{gen} - P_{load}$$

The set of equations that represents the first-order approximation of the **AC** network around the initial point is the same as generally used in the Newton power flow algorithm. That is:

$$\sum \frac{\partial P_i}{\partial|E_j|}\,\Delta|E_j| + \sum\frac{\partial P_i}{\partial\theta j}\,\Delta|\theta_j| = \Delta P_{gen\,i}$$

This can be placed in matrix form for easier manipulation:

$$\begin{bmatrix} \delta P1/\delta E1 & \delta P1/\delta\theta 1 \\ \vdots & \vdots \end{bmatrix}\begin{bmatrix} \Delta|E1| \\ \Delta\theta 1 \\ \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \Delta Pgeni \\ \vdots \end{bmatrix}$$

This equation will be placed into a more compact format that uses the vectors **x** and **u,** where **x** is the state vector of voltages and phase angles, and **u** is the vector of control variables. The control variables are the generator MW, transformer taps, and generator voltage magnitudes (or generator MVAR). Note that at any given generator bus we can control a voltage magnitude only within the limits of the unit VAR capacity. Therefore, there are times when the role of the state and control are reversed. Note that other controls can easily be added to this formulation. The compact form of this equation then is written:

$$\lfloor Jpx\rfloor\,\Delta X = \lfloor Jpu\rfloor\,\Delta u$$

Now, we will assume that there are several transmission system dependent variables, **h,** that represent, for example, MVA flows, load bus voltages, line amperes, etc., and we wish to find their sensitivity with respect to changes in the control variables. Each of these quantities can be expressed as a function of the state and control variables; that is, for example:

$$h = \begin{bmatrix} MVAflownm(|E|, \theta) \\ Ek \end{bmatrix}$$

Where I $E$ | represents only load bus voltage magnitude.
**As** before, we can write a linear version of these variables around the
Operating point

$$\Delta h = \begin{bmatrix} \dfrac{\delta h1}{\delta|E1|} & \dfrac{\delta h1}{\delta\theta1} & \cdots \\ \dfrac{\delta h2}{\delta|E1|} & \dfrac{\delta h2}{\delta\theta1} & \cdots \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} |\Delta E1| \\ \Delta\theta1 \end{bmatrix}$$

h1 = line nm flow
h2= bus k voltage magnitude

This last equation gives the linear sensitivity coefficients between the transmission System quantities, **h,** and the control variables, **u.**

## 3.6 Step 2(cont..)

Substituting 48.75 for $f^0_{2\text{-}4}$, 76.7 for $P^0_2$ and 90 for $P^0_3$ we get the following for

the constraint for line 2-4 (note that $a_{2-4} = 0$) and, finally, we expand $P2$ and $P3$ in terms of the segments:

$$48.75 + 0.31(37.5 + P_{21} + P_{22} + P_{23} - 76.7)$$
$$+ 0.22(45 + P_{31} + P_{32} + P_{33} - 90) \leq 40$$

Or

$$O.31P_2 + 0.31P_{22} + o.31P23 + 0.22P_{31} + 0.22P_{32} + O.22P_{33} \leq 13.302$$

The constraint for line 3-5 is built similarly and results in:

$$0.06P_{21} + 0.06P_{22} + 0.06P_{23} + o.29P_{31} + O.29P_{32} + 0.29P_{33} \leq 6.492$$

The solution to the LP gives:

$$P_1 = 87.02 \text{ MW}, P_2 = 70.0 \text{ MW and } P_3 = 59.66 \text{ MW} \quad \text{LP 2: result}$$

Also note that only the first transmission line constraint is binding in the LP, the remaining constraint is "slack," that is, it is not being forced up against its limit. When these values are put into the power flow we obtain:

$$P_1 = 87.54 \text{ MW}, P_2 = 70.0 \text{ MW and } P_3 = 59.66 \text{ MW power flow 2: result}$$

The flows on the two constrained lines are:

$$f_{2\text{-}4} = 39.40 \text{ MW}, \text{ and } f_{3\text{-}5} = 20.36 \text{ MW}$$

The total operating cost has now increased to 3155.0 R/h.

We now run another complete LP-power flow iteration to account for changes in losses and to bring the constraints closer to their limits. The solution to the second-iteration LP gives:

$P_1 = 86.16$ MW, $P_2 = 73.3$ MW and $P_3 = 57.73$ MW  *LP 2.1:* result

Both transmission line constraints are binding in the second LP. When these values are put into the power flow we obtain:

$P_1 = 86.16$ MW, $P_2 = 73.3$ MW and $P_3 = 57.73$ MW power flow 2.1: result

The flows on the two constrained lines are:

$f_{2-4} = 39.99$MW     and     $f_{3-5} = 20.06$MW

The total operating cost has now decreased slightly to 3153.3 $/h. There are no more (n − 0) line overloads.

## 3.7 $Y_{bus}$ Matrix for the system

| Bus no. | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | $4.01 + j11.89$ | $-2 - j4$ | $0$ | $-1.18 - j4.71$ | $-0.83 - j3.11$ | $0$ |
| 2 | $-2 - j4$ | $7.33 + j19.385$ | $-0.77 - j3.85$ | $-4 - j8$ | $-1 - j3$ | $-1.56 - j4.45$ |
| 3 | $0$ | $-0.77 - j3.85$ | $4.15 + j16.705$ | $0$ | $-1.46 - j3.17$ | $-1.92 - j9.62$ |
| 4 | $-1.18 - j4.71$ | $-4 - j8$ | $0$ | $6.18 + j14.78$ | $-1 - j2$ | $0$ |
| 5 | $-0.83 - j3.11$ | $-1 - j3$ | $-1.46 - j3.17$ | $-1 - j2$ | $5.29 + j14.425$ | $-1 - j3$ |
| 6 | $0$ | $-1.56 - j4.45$ | $-1.92 - j9.62$ | $0$ | $-1 - j3$ | $4.48 + j17.135$ |

# Chapter 4
# Validation of the result

The solution of the step 0 and step 1 found by our program exactly matches with the answer of the book. However, the value of the sensitivity coefficient does not exactly match with the book. The probable reasons for this mismatch may be the exclusion of the transformer tap changes from the matrix and also not taking the reactive power in account. A comparison between our results and the results of the existing method is given below:

| Step 1 | Result of the book | Result of our method | Deviation(%) |
| --- | --- | --- | --- |
| P1 | 50 | 50 | 0 |
| P2 | 77.87 | 77.87 | 0 |
| P3 | 90 | 90 | 0 |
| Step 2 | | | |
| $a_{2-4,2}$ | 0.31 | 0.33 | 6.4 |
| $a_{2-4,2}$ | 0.22 | 0.235 | 6.8 |
| $a_{3-5,2}$ | 0.06 | 0.062 | 3.3 |
| $a_{3-5,3}$ | 0.29 | 0.305 | 5.17 |

# Chapter 5
# Conclusion

## 5.1 Conclusion

Maximum interchange Capability and Maximum Supply Capacity were studied in this thesis in the context of optimal load flows. It was shown that these problems could be formulated as linear programs using incremental network models. The numerous advantages of linear programming over other optimization techniques makes it enviable for the implementation of many optimization problems that arise in power systems operation. The Maximum lnterchange Capability problem was undertaken first. The controversial issue raised was the degree of validity of the power transfer capability numbers which represented the "possible emergency help" that a system in distress could receive from its neighbors. A new approach to give more meaningful numbers was considered by solving a related problem, that of Maximum Supply Capacity. An iterative solution using linear programming and based on an incremental network model lead to the exact optimal solution. The N numerical results indicated that for the sample power system used, the linear approximation of LPOF Formulation was accurate enough for practical purposes. However the method described in this thesis should be used for networks for which that linearization is not valid.

## 5.2 Recommendations for future work

The proposed method can be easily extended to find the Maximum Supply Capacity of a lossy network. The method was shown to handle almost any type of operating constraint. Additional features are computational reliability, fast speed of calculation, and ability to handle large systems. These features and others make linear programming suitable for many optimization problems that arise in power systems operation.

# References

1) Power Generation Operation and control (**Allen J. Wood & Bruce F, Wollenberg**)

2) OPTIMAL LOAD FLOWS USING LINEAR PROGRAMING (**SAID AHMED-ZAID**)

3) Power System Analysis (**John j. Grainger & William D. Stevenson, JR** )

4) **S.Sivanagaraju, B.V.Rami Reddy**, "Electrical Power System Analysis", (Firewall Media, 2007)

5) **D.P.Kothari, I.J.Nagarath**, "Modern Power System Analysis", (McGraw- Hill, 2006)

6) **LakeneshwarPrakash Singh**, "Advanced Power System Analysis and Dynamics", (New Age International, 2008)

7) NPTEL online program on Load Flow Analysis

8) Linear Programming for Optimization (**Mark A. Schulze**, Ph.D.Perceptive Scientific Instruments, Inc.)

9) LINEAR PROGRAMMING A Concise Introduction ( **Thomas S. Ferguson**)

# Appendixes

## MATLAB Code Developed for the Proposed Method
### This is our program and result

```matlab
function [value, take1] = getOptimumNumber(totalLoad)

 MW = [50 60 40 32.5 60 20 45 50 40]
 take = [1 0 0 1 0 0 1 0 0];
 take1 = [0 0 0 0.37 0 0 0.51 0 0];
 coeff = [12.4685 13.0548 13.5878 11.6587 12.481 13.1922 12.3433 13.0473 13.7142]
value = 0;

while(totalLoad>0)

minCoeff = Inf;
minIndex = 0;
for(i = 1:length(take))
 if(take(i)==1)
  if(coeff(i)<minCoeff)
    minCoeff = coeff(i);
    minIndex = i;
  end
 end
end

minCoeff;
minIndex

if(minIndex== 0)
   disp(['Total Load is more than total generation capacity'])
 return;
end

if(totalLoad >= MW(minIndex))
 value = value + MW(minIndex)*coeff(minIndex);
 totalLoad = totalLoad - MW(minIndex);
else
 value = value + totalLoad*coeff(minIndex);
 totalLoad = 0;
end

take(minIndex) = 2;
if(minIndex<length(take) && take(minIndex + 1) == 0)
 take(minIndex + 1) = 1;
end
take1(minIndex) = 1;

end

end
>> getOptimumNumber(85.37)
```

minIndex =

   4

minIndex =

   7

minIndex =

   5

ans =

   1.032483345000000e+003

**Inerpretation of the result:**

| minIndex | Generation Unit | Limit MW Min-Max | Solution | Comment |
|---|---|---|---|---|
| 1 | $P_{11}$ | 0-50 | | |
| 2 | $P_{12}$ | 0-60 | | |
| 3 | $P_{13}$ | 0-40 | | |
| 4 | $P_{21}$ | 0-32.5 | 32.5 | This is first solution. As the load(85.37MW) is greater than the max limit so load remaining after taking max limit is:<br>85.37-32.5=52.87 |
| 5 | $P_{22}$ | 0-60 | 7.87 | This is the third solution. As the remaining load(7.87MW) is less than max limit only 7.87 is taken. |
| 6 | $P_{23}$ | 0-20 | | |
| 7 | $P_{31}$ | 0-45 | 45 | This is the second solution As the remaining load(52.87MW) is greater than the max limit so remaining after taking  max limit is:<br>52.37-45=7.87 |
| 8 | $P_{32}$ | 0-50 | | |
| 9 | $P_{33}$ | 0-40 | | |

Load flow Data for the network:

```
function mpc = case6ww
%CASE6WW  Power flow data for 6 bus, 3 gen case from Wood & Wollenberg.
%   Please see CASEFORMAT for details on the case file format.
%
%   This is the 6 bus example from pp. 104, 112, 119, 123-124, 549 of
%   "Power Generation, Operation, and Control, 2nd Edition",
%   by Allen. J. Wood and Bruce F. Wollenberg, John Wiley & Sons, NY, Jan 1996.

%   MATPOWER
%   $Id: case6ww.m,v 1.7 2010/03/10 18:08:15 ray Exp $

%% MATPOWER Case Format : Version 2
mpc.version = '2';

%%-----  Power Flow Data  -----%%
%% system MVA base
mpc.baseMVA = 100;

%% bus data
% bus_i type  Pd Qd Gs Bs area  Vm Va  baseKV zone   Vmax    Vmin
mpc.bus = [
   1  3  0  0  0  0  1  1.05   0  230 1  1.05   1.05;
   2  2  0  0  0  0  1  1.05   0  230 1  1.05   1.05;
   3  2  0  0  0  0  1  1.07   0  230 1  1.07   1.07;
   4  1  70 70 0  0  1  1   0  230 1  1.05   0.95;
   5  1  70 70 0  0  1  1   0  230 1  1.05   0.95;
   6  1  70 70 0  0  1  1   0  230 1  1.05   0.95;
];

%% generator data
% bus Pg Qg Qmax  Qmin   Vg mBase  status Pmax   Pmin   Pc1 Pc2 Qc1min Qc1max Qc2min Qc2max
ramp_agc   ramp_10 ramp_30 ramp_q  apf
mpc.gen = [
   1  0  0  100 -100   1.05   100 1  200 50 0  0  0  0  0  0  0  0  0  0;
   2  50 0  100 -100   1.05   100 1  150 37.5  0  0  0  0  0  0  0  0  0  0;
   3  60 0  100 -100   1.07   100 1  180 45 0  0  0  0  0  0  0  0  0  0;
];

%% branch data
% fbus  tbus  r  x  b  rateA  rateB  rateC  ratio  angle  status angmin angmax
mpc.branch = [
   1  2  0.1 0.2 0.04   40 40 40 0  0  1  -360   360;
   1  4  0.05   0.2 0.04   60 60 60 0  0  1  -360   360;
   1  5  0.08   0.3 0.06   40 40 40 0  0  1  -360   360;
   2  3  0.05   0.25   0.06   40 40 40 0  0  1  -360   360;
   2  4  0.05   0.1 0.02   60 60 60 0  0  1  -360   360;
   2  5  0.1 0.3 0.04   30 30 30 0  0  1  -360   360;
   2  6  0.07   0.2 0.05   90 90 90 0  0  1  -360   360;
   3  5  0.12   0.26   0.05   70 70 70 0  0  1  -360   360;
   3  6  0.02   0.1 0.02   80 80 80 0  0  1  -360   360;
   4  5  0.2 0.4 0.08   20 20 20 0  0  1  -360   360;
   5  6  0.1 0.3 0.06   40 40 40 0  0  1  -360   360;
];
```

```
%%-----  OPF Data  -----%%
%% generator cost data
%   1   startup shutdown   n   x1  y1  ... xn  yn
%   2   startup shutdown   n   c(n-1)  ... c0
mpc.gencost = [
    2   0   0   3   0.00533 11.669  213.1;
    2   0   0   3   0.00889 10.333  200;
    2   0   0   3   0.00741 10.833  240;
];
```