Mode interactions in Photonic Crystal nano-cavity arrays

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August 2014

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Declaration

We do hereby declare that the thesis titled “Mode Interactions in Photonic Crystal nano-cavity arrays” is submitted to the Department of Electrical and Electronics Engineering of BRAC University in partial fulfillment of the Bachelor of Science in Electrical and Electronics Engineering. This is our original work and was not submitted elsewhere for the award of any other degree or any other publication.

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Acknowledgement

We are extremely grateful to our supervisor, Dr. Md. Belal Hossain Bhuian, Assistant Professor, Department of Electrical and Electronics Engineering, BRAC University, for his continuous support and guidance that lead to the completion of this thesis. We are also thankful to Pratik Al Islam, Lecturer, Department of Electrical and Electronics Engineering, BRAC University, for his guidance.

Abstract

Over the last few decades Photonic Crystals (PhCs) have enjoyed immense attention from engineers, physicists and other interested parties due to its unique characteristics and its promise to revolutionize optical communication technologies. PhCs are made of dielectric materials periodic in one, two, or three dimensions and can exhibit photonic band gaps to certain frequencies. Therefore, it can be used to trap or guide "light" of certain frequencies within it. This thesis uses this very idea to construct a Photonic Crystal Slab (PCS) of finite thickness using photonic band gap maps, where air holes are introduced to create the periodicity. Cavity arrays are formed by manipulating the properties of certain air holes and the interactions between the modes within the cavity are analyzed. Two types of cavities are formed- one consisting of similar defects and one consisting of dissimilar defects. The software used for this thesis is Rsoft CAD Suit and the simulation tools used are BandSOLVE and FullWAVE.

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1. Motivation

In the present world, there is a constantly increasing necessity for efficient and resilient systems in all aspects of technology. No matter how much systems are developed, there still seems to be an insatiable need for further improvement. As such, deviating from conventional practice, experimentation with new materials and elements has begun in hopes of achieving the high demands of performance. One such discovery by using new materials, or rather by using older materials in a new way, has lead to the advent of what we know as photonic crystals.

Photonic crystals (PhC) are structures consisting of periodic dielectric materials that have the ability to control the propagation of light that passes through them. It was originally discovered in 1887 by an English physicist named Lord Rayleigh, who experimented with periodic multi-layer dielectric stacks and showed they exhibited a band gap in one dimension [1]. However, the name "photonic crystal" was coined much later in 1987 when Eli Yablonovitch and Sajeev John published their revolutionary papers on the topic [2] [3]. Since then number of research on photonic crystals sky rocketed mainly because of its never-seen-before abilities to control the flow of light which lead to the belief that its possibilities are limitless.

Since then photonic crystals have come a long way seeing extensive research in all possible fields where applicable. The existence of band gaps within the structure means that some frequencies of light can be trapped within the structure [4] [5]. Due to this reason there has been high concentration on experimentation and fabrication of optical devices such as waveguides [6], lasers [7], fibers [8], etc using PhCs. At the heart of a laser there lies a cavity, which essentially is a defect within the perfectly symmetrical structure and allows the localization of light. The walls around the cavity acts as mirrors and thus light can trapped and built up inside the cavity with growing intensity much needed for the lasers.

In this thesis, we have studied the properties of a cavity that is composed of several defects- in other words known as cavity array. We have designed the cavity in a photonic crystal slab (PCS) where air holes are punched to create the periodicity. The thickness of the slab is taken to be close to half of the lattice constant for which allows for maximum confinement [9]. The design of the slab is done by use of photonic band gaps (PBG) and the band gap map.

The second chapter of this thesis describes in brief the fundamental aspects of photonic crystals including their types, applications, different key components of the structure and the necessary details needed to learn about photonic crystals in a basic level. This part intends to elucidate the key ideas about photonic crystals that are necessary to fully understand this thesis easily.

The third chapter is a detailed explanation of the computation methods used during the simulations of our structure. The chapter discusses the detailed theoretical ideas behind the Plane Wave Expansion (PWE) Method used in the calculation of the photonic band gaps and also the Finite Difference Time Domain (FDTD) method used to simulate and excite the structure and acquire the Intensity vs Wavelength graphs.

Chapter four illustrates the prime objective of this thesis detailing the set up of the initial structure to the design of the various forms of cavity that we have used. This chapter also consists of the results that we have found by using the simulation methods mentioned in chapter three.

Chapter five summarizes all the results with discussions about future research possibilities and also tries to propose a new use for PhC cavities.

2. About photonic crystals

2.1 The Basic Properties of Photonic Crystals

A crystal structure is a unique arrangement of atoms or molecules in a crystalline liquid or solid. A crystal structure describes a highly ordered structure, occurring due to the intrinsic nature of molecules to form symmetric patterns. An electron passing through a crystal lattice experiences a periodic potential. Both the elements of the crystal and the arrangement of the lattice indicate the conduction properties of the crystal.

Since the electron propagates as wave and the energy of an electron in an atom is quantized, they can have only certain discrete values. This concept is applied to the electron energy in a molecule or a crystal with several atoms. When atoms are brought close together to form a crystal, their inter-atomic interactions result in the formation of electron energy bands, mainly two distinct bands (Kasap, et.al 2001). It is because of this band gap, only the waves with certain criteria are capable of travelling through a periodic potential without scattering and also propagate through it while others are restricted. There are no allowed electron energies in the band gap- it represents the forbidden electron energies in the crystal. The gap can extend to cover all possible directions of propagation of an electron if the lattice potential of the crystal is strong. Hence, this would result in a complete band gap. An example of such a material would be a semiconductor.

In semiconductor, the energy of the electron in the crystal falls into two distinct energy level called the valence band and the conduction band that are separated by the energy band gap or the forbidden energy, as shown in the figure 2.1.1.

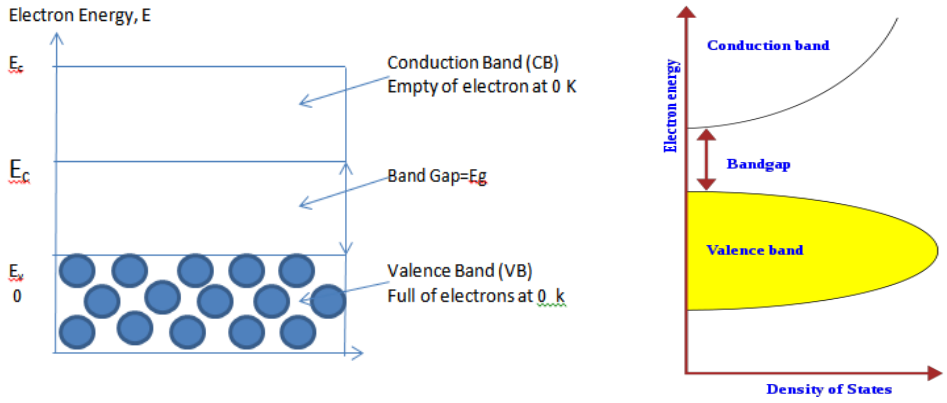

Figure 2.1.1 The energy band diagram of electrons in a semiconductor (Source: Kasap, et.al 2001)

In case of photonic crystals, atoms, molecules or ions are replaced by materials having different dielectric constants and the periodic dielectric function would replace the periodic potential of a crystal (Joannopoulos, et.al 1998). It has been found that if the material used has high efficiency and if the dielectric constant of the various medium used, varies

considerably from each other, then the refraction and reflection of light by the interfaces of photonic crystal allows photons to produce different phenomenon similar to the ones that can be obtained due to the periodic potential created by electrons within a crystal. Here the most important property that determines the significance of photonic crystal is the photonic band gap.

2.2 Photonic Band gap

The photonic band gap (PBG) are those frequency or energy of light for which propagation is prohibited inside the photonic crystal, just like the band gap found in semiconductors. As a result photons with energy inside the PBG when incident on the structure are reflected back. On the other hand if defects can be introduced into the periodic structure it will change the way photon interact just like the effect of introducing defect to the crystal structure of a semiconductor. As a result the radiation within the defect frequency will propagate inside the structure and for multiple defects, radiation will be guided like a waveguide. (Joannopoulos, et.al 1998). Details about photonic band gaps will be discussed in the following chapters.

2.3 Photonic crystals in Nature:

Figure 2.3.1 periodic structure appears in butterfly from South America

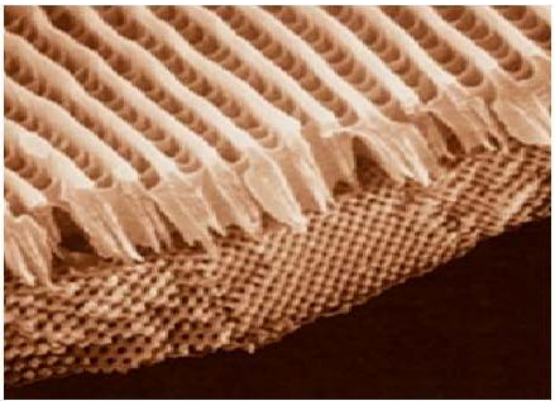

Figure 2.3.2 The periodic structure appears to be diffraction grating

Photonic structures are found in nature for millions of years. It has been used for the purpose of communication and reproduction between individuals in some species. We can see natural photonic crystal-type structures occurring in insects, spiders, birds, fishes and other marine animals, in plants and more. An example of a certain species of butterfly is shown in figure 2.3.1 and figure 2.3.2

2.4 Differences between semiconductors and photonic crystals:

	Semiconductor	Photonic crystal
Particle	Electron	Photon
Periodicity provided by	Atomic lattice	Dielectric media
Band Gap	Electronics band gap	Photonic band gap
Forbidden in particular	Energy	Frequency(cannot propagate through periodicity dielectric structure)
Band Gap	Band gap exists naturally	Fabrication is a challenge, precision need be the order of 0.5um, micro-lithography

Table: 2.1 Differences between photonic crystals and semiconductors

2.5 Different Dimensions of Photonic Crystal

Photonic crystals can be divided into three categories based on the arrangement of the elements in the lattice. The categories are:

- One dimensional (1D)
- Two dimensional (2D)
- Three dimensional (3D)

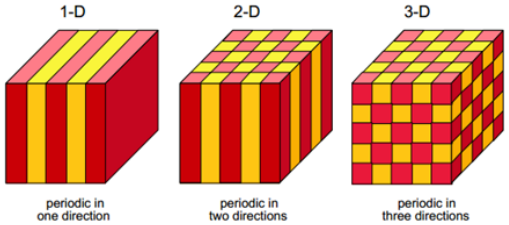

Figure 2.5.1 Examples of a) 1D b) 2D c) 3D photonic crystal

2.5.1 One dimensional Photonic Crystal

The simplest photonic crystal is the one dimensional photonic crystal. Here, the periodicity exists in one direction only, whereas in the other two directions it is uniform. 1D photonic crystals has alternating layers of materials with different dielectric constants.

Example of this is the Bragg mirror or the multilayer dielectric mirror such as the quarter wave stack. In these devices, light wave at each interface is partially reflected and if the reflections from multiple interfaces interfere destructively, it would eliminate the forward propagating wave. On the other hand, if the interference is constructive, then a large percentage of the incident light will undergo reflection (Kasap, et.al 2001).

It is also used as anti-reflecting coatings in order to improve the quality of optical devices. A 1D photonic crystal have very few variations in its photonic structure, since it has a layered structure, as a result only the number of layers, refractive index of each layer and the thickness of layers could be varied to bring out the changes.

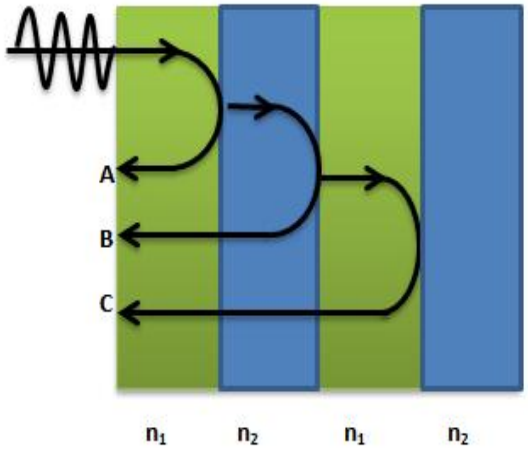

Figure 2.5.1.1 The principle of reflecting coating works

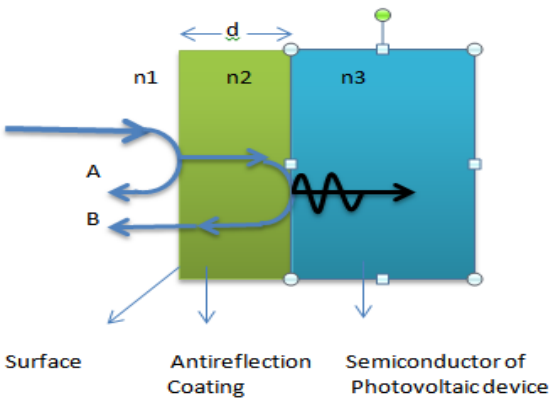

Figure 2.5.1.2 Illustration of how an anti-dielectric works

2.5.2 Two dimensional Photonic Crystal

For a two dimensional photonic crystal it has periodicity in two directions while remaining uniform in the third direction. Photonic band gap appears in the planes of periodicity. A further detail about the 2D photonic crystal will be discussed later in the following chapters.

2.5.3 Three dimensional Photonic Crystal

For three dimensional photonic crystal the periodicity exists in all three planes as a result changes can be made much more readily compared to 1D or 2D.

New form of application can be made by changing the arrangement of elements of 3D photonic crystals. The example of naturally occurring 3D photonic crystal is the stone opal, which is famous for its unique optical properties such as showing off different colors when turned around. This is because of its reflectance property which depends on the incident angle of light, which is then reflected at different wavelengths creating the phenomenon.

2.6 Applications of photonic crystals

Due to its radical ability to manipulate the flow of light, photonic, crystals have already seen much practical use. Three main structures of interest are mirrors, wave guides and cavities. Using these three structures, much research has been done to find practical uses of photonic crystals. For example, one dimensional photonic crystal is already being used as high reflection coatings in mirrors and lens.

Two dimensional photonic crystals have seen the most research lately. With theoretical applications ranging from waveguides and filters to laser cavities and chemical detectors, use of two dimensional photonic crystals seems to be limited only by imagination. One practical use of such crystal already on the verge of being commercialized is the photonic crystal fibers and may, in the near future, replace optical fibers in use, due to the crystal's superiority in guiding light.

Three dimensional photonic crystals are still in their infancy. Research on three dimensional photonic crystals are yet not as aggressive as with the lower dimensions, but its uses too may be vast. In fact, with improvement in the fabrication technology, three

dimensional photonic crystals may be used to implement optical transistors in optical computers. In other words, photonic crystals may one day replace semiconductors completely.

2.7 Reciprocal lattice and First Brillouin zone

For understanding the operation of optical devices, it is very important to know how the electromagnetic field would interact with the photonic crystal device. As a result we need to consider the distribution of electromagnetic field inside the lattice. Therefore to design photonic crystal based devices the use of reflectance spectrum, transmittance spectrum or the band structure of the crystal can be considered. It is best suited if band structures are used, since they provide complete information about electromagnetic waves propagating in the crystal.

The first step in this process would be to find out the parameters which would describe the structure of a photonic crystal. This includes terms such as unit cell, lattice vector, reciprocal lattice, reciprocal lattice vectors and the first Brillouin zone. The above terms will be explained in this chapter using the 1D or 2D structures for simplicity.

2.8 Unit Cell

Since Photonic crystal is made up of an infinite periodic structure if the information of a single unit cell is understood than the information can be used to understand the entire structure. A unit cell is any region in space that gives us the idea of the entire function of the crystal. A base point is selected and each point within the unit cell is closer to this base point rather than the neighboring base points. The photonic crystal lattice is determined by its unit cell, its shape and permittivity (Sukhoivanov, et.al 2009).

The following method is used to determine a unit cell of a photonic crystal (Sukhoivanov, et.al 2009)

1. At first the base element of the photonic crystal has been selected. If the photonic crystal is comprised of rods in air then a single rod can be chosen as the base element.
2. Line segments are drawn to connect the base point with corresponding points of neighboring elements.
3. Straight lines are drawn through the center of each of the previously drawn line segments and perpendicular to them.

4. The figure formed by these lines form the unit cell of the photonic crystal.

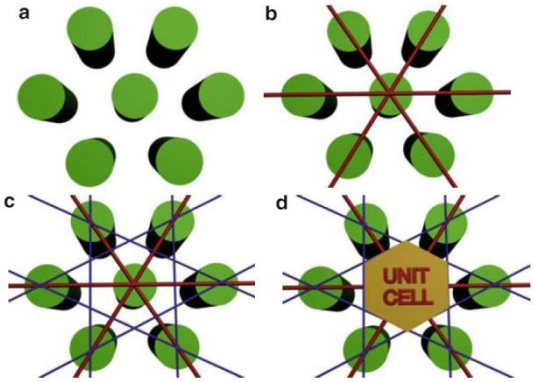

Figure 2.8.1 Process of unit cell determination

2.9 Lattice Vector

As mentioned before, the unit cell is translated to map out the entire function of the photonic crystal. This translation depends on the lattice vector. A lattice is a set of discrete points in space that repeats periodically. While choosing, it must fills out the entire unit cell. As photonic crystal has an infinite periodic structure, it is not possible to define an infinite number of lattice vectors. So a set of basis vectors, i.e. primitive lattice vector is defined. Its number is same as the number of dimension of the crystal (Sukhoivanov, et.al 2009). The lattice vector can be written in the general form **R** = l**a1** + m**a2** + n**a3**, for some integers l, m and n. The point **R** is known as lattice vector, while the basis vector, a1, a2 and a3 are termed primitive lattice vectors.

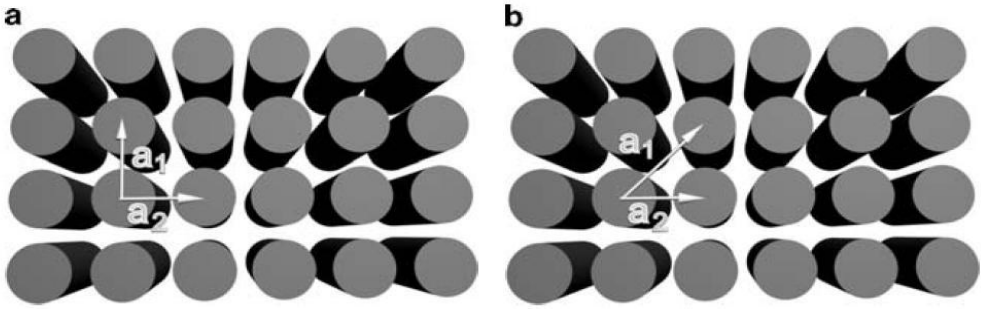

Figure 2.9.1 Possible choices of primitive lattice vectors (Source: Sukhoivanov, et.al 2009)

2.10 The Reciprocal lattice

For every lattice there is a second lattice associated called reciprocal lattice, which are inverse of each other. The lattice vectors have the dimensions of length therefore the reciprocal lattice vectors have dimensions of inverse length. The lattice and reciprocal lattice are related by the equation:

$$a_i \cdot b_i = \delta_{ij} 2\pi$$

The reciprocal lattice vector is defined in terms of the basis primitive lattice vectors as done previously for establishing lattice vectors. From the primitive lattice vectors of the photonic crystal, the primitive reciprocal lattice vectors can be obtained.

2.11 The Brillouin Zone

The unit cell of reciprocal lattice is the Brillouin Zone. It is determined in a similar manner as it has been done previously for the unit cell of the lattice vector in space. The process involved in establishing the Brillouin zone involves several steps (Sukhoivanov, et.al 2009):

- Unit cell of the photonic crystal is determined.
- Primitive lattice vector is set.
- Primitive reciprocal lattice vectors are computed.
- The base element of the photonic crystal, reciprocal lattice, is selected.
- Line segments are joined to the point of the base element with corresponding points of neighbor elements.
- Straight lines are drawn through the center of each of the previously drawn line segments and perpendicular to them.
- The region that developed is the Brillouin Zone.

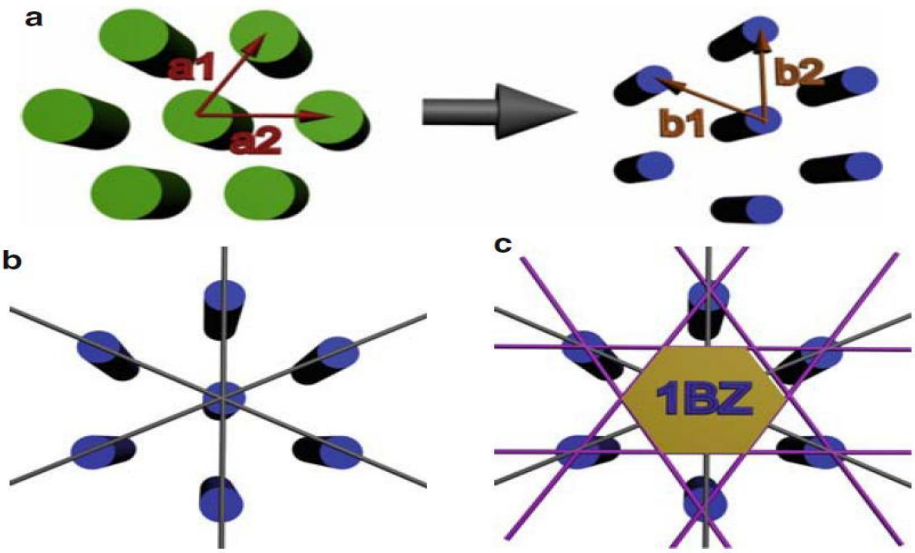

Figure: 2.11.1 The Brillouin Zone for a 2D photonic crystal (Source: Sukhoivanov, et.al 2009)

For the computation process of photonic crystal band structure, the reciprocal lattice and the brillouin zone play a major role, achieving the limits of the variation of the wave vectors where computation of the eigen-states will be carried out. Within a Brillouin zone, there exist points, known as the high symmetry points, which could be translated into themselves only when rotating the Brillouin zone by 90°,180°,30° *or* 60° depending on the photonic crystal lattice. Computation of the band structure starts from the centre of the Brillouin zone and is denoted by" Γ" At this point the wave vector, k is zero. The computation then follows through all the high symmetry points and then returns to this point. This forms the k-path.

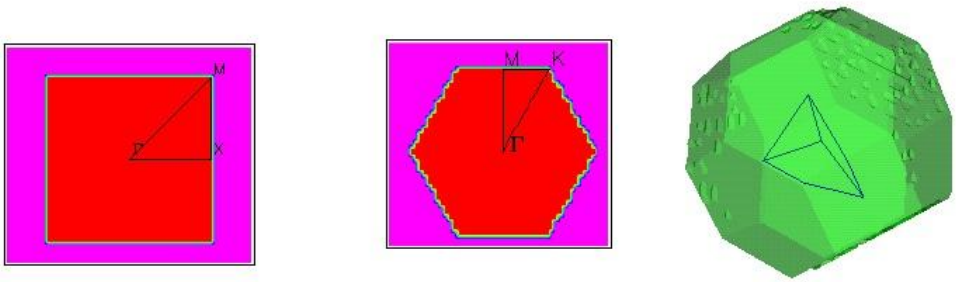

Figure: 2.11.2: K-path of photonic crystal with square lattice, hexagonal lattice and 3D hexagonal lattice

2.12 Two Dimensional Photonic crystals:

For a two dimensional photonic crystal, its periodicity appears in two of its axes and being uniform along the third, and the band gap appears in the plane of periodicity. If light propagates in this plane, then it is possible to divide the harmonic modes into two independent polarizations, each of them having their own band structure. It can also prevent the propagation of light in any direction within the plane, unlike the 1D crystal.

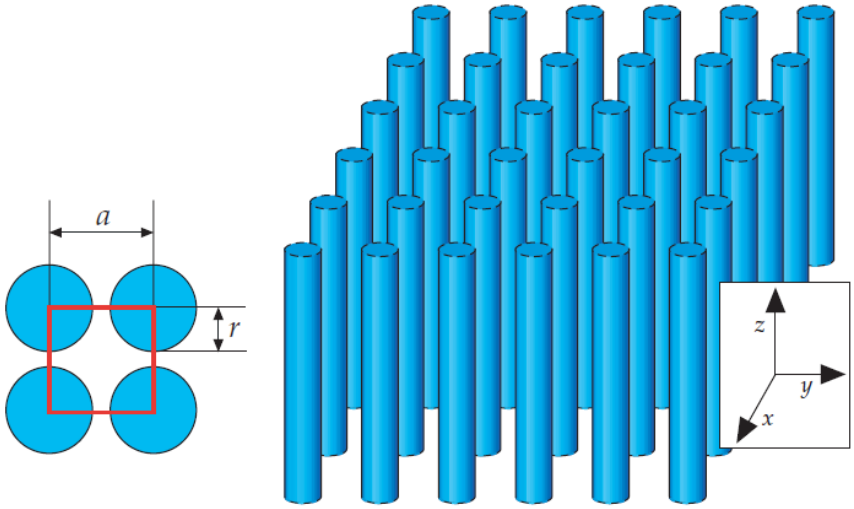

Figure 2.12.1 A square lattice of dielectric column, with 'r' and 'a' shown on left.

According to the figure 2.11, the columns are considered to be infinitely long. For certain values of the lattice constant, "a" (column spacing), the crystal can have a photonic band gap along the x and y axis. Within this gap, no states are permitted and the incident light is reflected.

There are an infinite number of lattice types available for a 2D photonic crystal due to the variation of shape of the elements and their placement. However, in general, there are two commonly used lattice types of the 2D photonic crystal, the square and the hexagonal type. The unit cell of a square lattice 2D photonic crystal has the shape of a square and in case of hexagonal lattice, the unit cell has the shape of a regular hexagon.

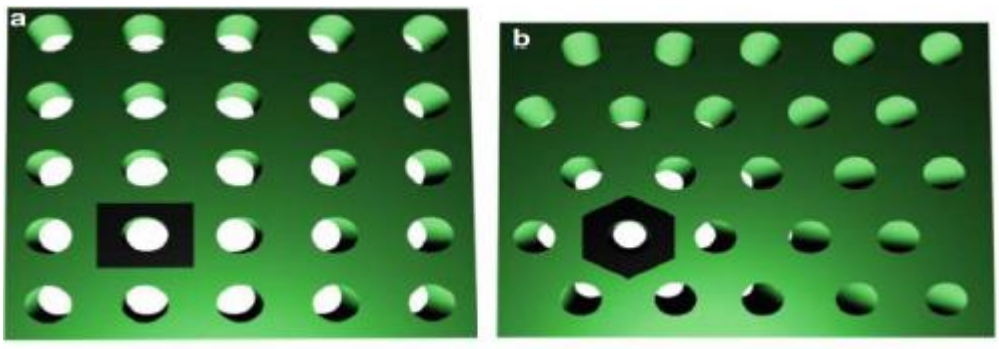

Figure 2.12.2 Examples of 2D photonic crystal lattice types

2.13 The Photonic band structure and band gap

Since the most important feature of the photonic crystal is the band gap as it is mentioned earlier, it corresponds to the range of light frequency or energy that is not allowed to propagate through the photonic crystal structure. However, by the help of a defects or irregularities in the structure it can be possible to guide these forbidden energies by means of a waveguide.

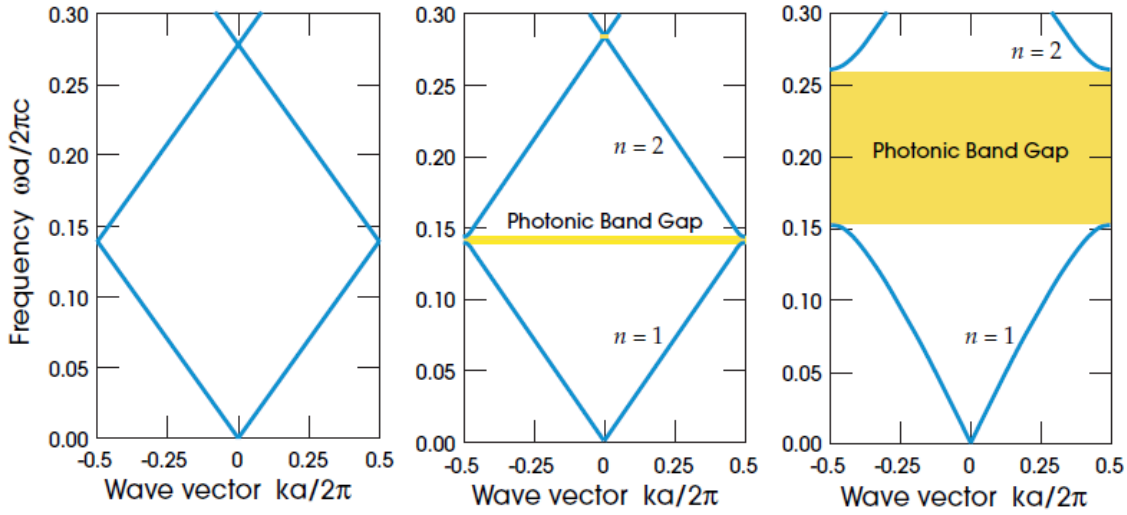

Figure 2.13.1 The photonic band structure for on axis propagation as computed for 3 different multilayer films.

The left diagram in figure 2.13 is for a homogenous medium, i.e. all the layers have the same dielectric constant. The middle one is for a structure where alternate layers have dielectric constants of 13 and 12. The rightmost one is for a multilayer whose alternate layers have dielectric constants in the ratio 13:1.

It is found that if other parameters are kept constant, then the band gap increases with increase in dielectric constant.

The problem associated with band structure computation of a photonic crystal is to find the dispersion relation, that is, the dependence of resonant frequencies of the photonic crystal on the wave vector of the radiation passing through it. In order to obtain the dispersion relation, it is necessary to solve the eigen problem equation inside infinite periodic structure.

2.14 The band gap and its size

For determining the extent of its band gap the scale of the crystal is an important factor. For this reason, the extent of the band gap cannot be characterized by only considering

its frequency width. Thus, the gap-mid gap ratio is used to determine the extent, since it is independent of the scale factor of a crystal. Let the frequency width of the band gap be $\Delta\omega$ and ωm be the frequency at the middle of the gap, and then gap-mid gap ratio is $\Delta\omega/\omega m$. If the system undergoes scaling all the frequencies undergo scaling by the same factor and the gap- mid gap ratio remains the same. For this reason, in band diagrams the frequency and wave vector are plotted in dimensionless units, $\omega a/2\pi c$ and $ka/2$, where frequency is equal to a/λ and λ is wavelength in vacuum as shown in Fig 2.13

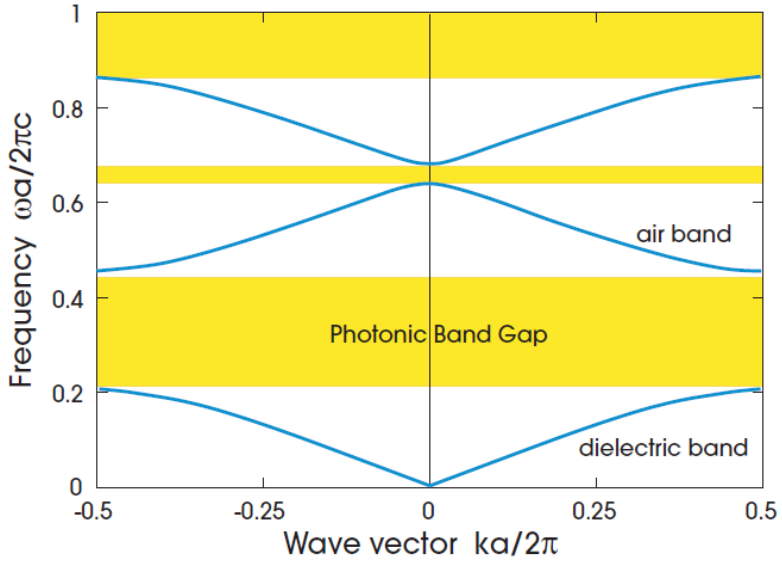

Figure 2.14.1 The photonic band structure of a multilayer film.

2.15 Analysis of the band structure

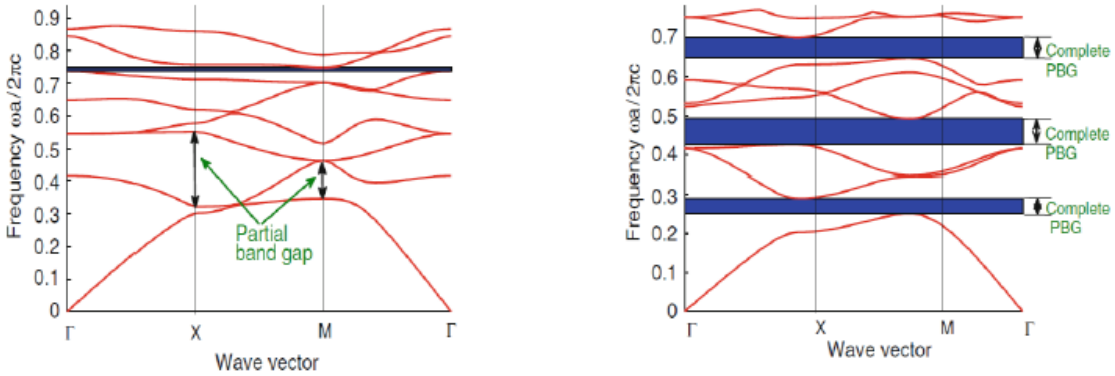

Figure 2.15.1 Complete and partial photonic band gaps

As mentioned earlier, the wave vector space could be connected to the coordinate space by connecting some points of the Brillouin Zone with the radiation propagation direction within the photonic crystal. Hence it could be inferred that within each direction of the photonic crystal there exists a band gap, and these specific areas are referred to as the partial photonic band gaps. At some points these band gaps exist in all possible angles by overlapping with each other and hence creating the complete photonic band gap. Therefore, any one of the mentioned ways, by calculating the gap- mid gap ratio or by specifying the frequencies where photonic band gap appears, it is possible to locate the band gap in a photonic crystal.

2.16 The Photonic Band Gap Maps

While designing a phonic crystal it is important to determine the photonic crystal parameter from known frequency characteristics. For example, in order to design the device with the band gap at the required frequency, it is important to be aware of the permittivity of the materials, the structure and size of elements being used or about the crystal lattice. To solve these problems, the photonic band gap map or the reduced band structure is used. The map is obtained by projecting the complete band gap, obtained at different values of the photonic crystal parameters such as refractive index, elements size, frequency etc. The band gap map is drawn by gathering information about the complete band gap from the band structure and then plotting it against the ratio: radius/lattice constant.

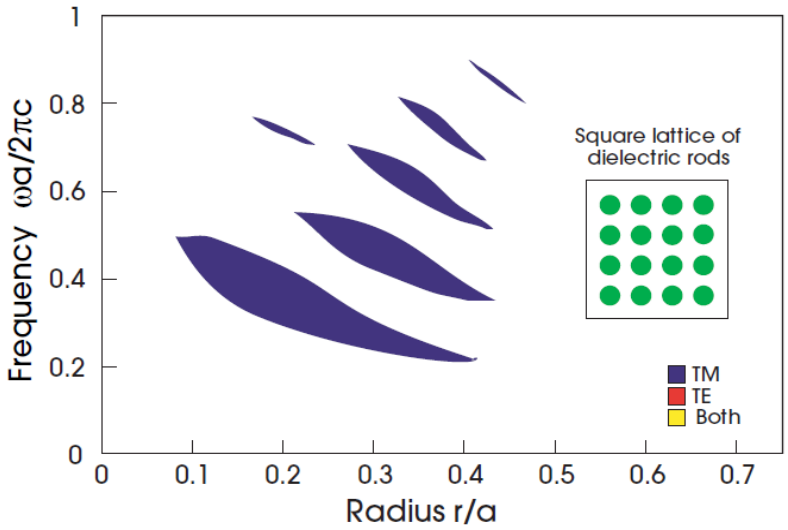

Figure 2.16.1: Gap map for a square lattice of dielectric rods (Source: Joannopoulos, et. al 1998).

For overcoming the problems in the designing process of photonic crystal device, photonic band gap map can be used. In addition to that these maps can also be used to determine the parameters at which maximum reflection occur.

3. Computational methods used: Plane Wave Expansion (PWE) and Finite Difference Time Domain (FDTD)

3.1 The Plane Wave Expansion Method (PWE Method)

This is a technique which uses an eigen formulation of the Maxwell's equation and solves the eigen frequencies for each of the propagation directions of the wave vectors in a photonic crystal. (Danner et.al 2011).

The PWE method is used for calculating electromagnetic band gaps (Satpathy, et al. 1990). It can also be used to simulate the field distribution in photonic crystals but the FDTD method is preferred to calculate the field distribution because the FDTD algorithm does not need to solve integral equations and it is easier to obtain frequency domain data from time domain results rather than the converse. It would be easy to obtain the frequency domain from the time domain when many frequencies are involved.

If we want to get the evaluation of the Plane Wave Expansion Method, it starts with the Maxwell's equations. The following derivations of equations are for 2D photonic crystals. The common symbols with their meanings are:

- ∇. : Divergence operator
- ∇×: Curl operator
- B: Magnetic flux density or magnetic field density or magnetic induction measured in tesla (T)
- H: Magnetic field strength or magnetic field measured in ampere per meter (A/m)
- D: Electric displacement field or electric flux density measured in coulomb per square meter (C/m2)
- E: Electric field measured in newton per coulomb (N/C)
- J: Current density measured in ampere per meter square (A/m2)
- ρ: Electric charge density measured in coulomb per meter square (C/m2)

$$\nabla . D = \rho \tag{3.1.1}$$

$$\nabla . B = 0 \tag{3.1.2}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{3.1.3}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \tag{3.1.4}$$

We considered that there is no charge and current density in a 2D photonic crystal.

$$\nabla . E = 0 \tag{3.1.5}$$

$$\nabla . H = 0 \tag{3.1.6}$$

$$\nabla \times E = -\mu\mu_0 \frac{\partial H}{\partial t} \tag{3.1.7}$$

$$\nabla \times | H = \varepsilon\varepsilon_0 \frac{\partial E}{\partial t} \tag{3.1.8}$$

Where, $D = \varepsilon\varepsilon_0 E$ and $B = \mu\mu_0 H$.

μ= Permeability in Am^2 and μ_0= Permeability of free space in Am^2

ε= Permittivity in F/m and ε_0= Permittivity of free space in F/m

With equation 3.1.7 and equation 3.1.8 it can be written as

$$\nabla \times n\nabla \times H + \frac{\mu}{c^2}\frac{\partial^2 H}{\partial t^2} = 0 \tag{3.1.9}$$

Here $n = \frac{1}{\varepsilon}$ *and* $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

H is written as inverse Fourier transform in time and space,

$$H(r,t) = \iint H(k',\omega)\, e^{i(k'.r-\omega t)} dk' d\omega \tag{3.1.10}$$

k' is the wave vector in the material.
A plane wave of frequency ω is,

$$E(r,t) = E_0 e^{i(k.r-\omega t)} \tag{3.1.11}$$

So it can be said that above equation is summation of plane waves with infinite frequencies and wave vectors. . This is the basic equation for plane wave expansion. Equation 3.1.10 is used in equation 3.1.11 and the new equation formed is,

$$\nabla \times n\nabla \times H(r,\omega) - \frac{\omega^2\mu}{c^2}H(r,\omega) = 0 \tag{3.1.12}$$

$$\text{Where, } H(r,\omega) = \int H\left(k',\omega\right)e^{ik'.r}\,dk' \tag{3.1.13}$$

k' and r are in the y-z plane. Next the equations defining the reciprocal lattice of a 2D photonic crystal are required for further development of the plane wave expansion method.

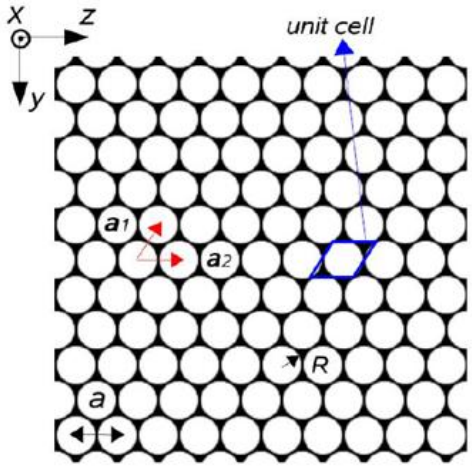

Figure 3.1.1 Lattice of a 2D hexagonal Photonic crystal. R represents radius.

The figure 3.1.1 shows the lattice of a 2D hexagonal photonic crystal where a1 and a2 are the basis vectors of the lattice. The blue enclosure defined by the basis vectors is the unit cell.

The translational vector in the lattice is,

$$R=n1a1+n2a2 \tag{3.1.14}$$

n1, n2∈ integers.

Because of the periodicity of the lattice,

$$(r) = n(r+R) \tag{3.1.15}$$

$$\mu(r) = \mu(r+R) \tag{3.1.16}$$

G is called the reciprocal vector,

$$G=m1b1+m2b2 \tag{3.1.17}$$

m1, m2∈ integers.

b1 and b2 are the basis vectors of the reciprocal lattice (shown in the figure 3.1.2).

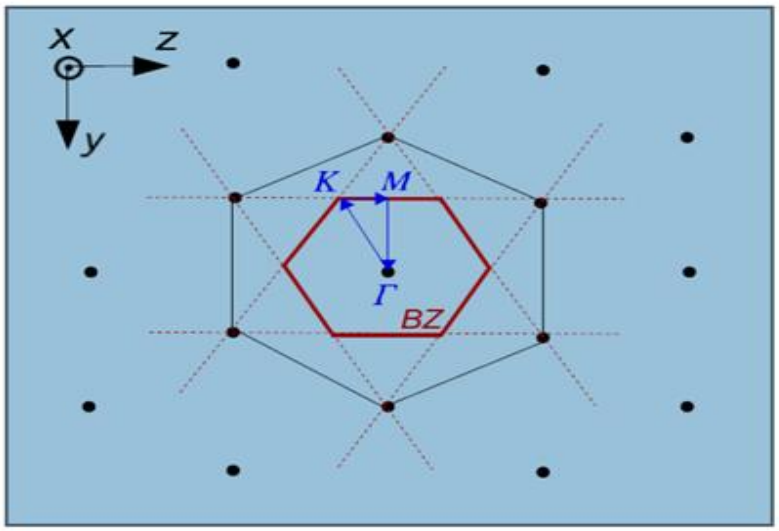

Figure 3.1.2 Reciprocal lattice of a 2D hexagonal photonic crystal. BZ stands for Brillouin zone.

$$b_1 = 2\pi \frac{a_2 \times \hat{x}}{a_1 . a_2 \times \hat{x}} \tag{3.1.18}$$

$$b_2 = 2\pi \frac{\hat{x} \times a_1}{a_1 . a_2 \times \hat{x}} \tag{3.1.19}$$

$$a_i . b_j = 2\pi \delta_{ij} \tag{3.1.20}$$

n(r) and u(r) can be now written as fourier series in terms of G,

$$n(r) = \Sigma_{G'} n(G') e^{iG' . r} \tag{3.1.21}$$

$$\mu(r) = \sum_{G''} \mu(G'') e^{iG'' . r} \tag{3.1.22}$$

K=k+g is substituted in equation (3.1.6)

$$H(r, \omega) = \Sigma_G \int_{BZ} H(K + G, \omega) e^{i(k+G) . r} \, dk \tag{3.1.23}$$

This is similar to Bloch's theorem which states that the energy eigen-function for a system where the wave function of a particle is placed in a periodic potential is written as the product of a plane wave envelope function and periodic function ((r)) that has the same periodicity as the potential. The equation takes the following form,

$$\varphi_{nk\,(r)} = e^{ik.r} u_{nk\,(r)} \tag{3.1.24}$$

The energy eigen-values are $\epsilon nk=(k+G)$ which is periodic with periodicity G of the reciprocal lattice vector. The energies associated with the index n differ with wave number k and form an energy band defined by band index n. The eigen-values for n are periodic in k and all values of ϵnk occur for k values within the first Brillouin zone.

Equations 3.1.23 to 3.1.24 are used to get a generalized eigen-value equation in terms of magnetic field H,

$$\sum_G n(G'-G)(k+G') \times [(k+G) \times H(k+G,\omega)] + \frac{\omega^2}{c^2}\mu(G'-G)H(k+G,\omega) = 0$$

Similarly the generalized eigen-value equation for electric field E is,

$$\sum_G \frac{1}{\mu}(G'-G)(k+G') \times [(k+G) \times E(k+G,\omega)] + \frac{\omega^2}{c^2}\varepsilon(G'-G)E(k+G,\omega) = 0$$

We used PWE method for computations to prove to that it is advantageous because as it follows the Fourier method it suffers from Gibb's phenomenon (the unusual manner in which the Fourier series of a piecewise continuous differentiable periodic function behaves at a discontinuity) and converges slowly to the solution. Though it has some demerits PWE is commonly used for calculating band gaps and dispersion relations.

3.2 Band Gap Calculation using Plane Wave Expansion

The Band Gap of a photonic crystal states the behavior of the incident light which propagates in a specific direction inside the photonic crystal. In this section it is explained how the Band Gap is calculated using the Plane Wave Expansion Method. For simplicity the process to calculate the Band Gap for a 1D photonic crystal is shown. 2D photonic crystal band gap calculation follows a similar process.
It is required to solve the eigen-problem for Helmholtz equation in order to calculate the band gap. The Helmholtz equation in terms of magnetic field is,

$$\frac{\partial}{\partial x}\frac{1}{\varepsilon(x)}\frac{\partial}{\partial x}H(x)+\frac{\omega^2}{c^2}H(x)=0$$

(3.2.1)

The eigen-function of an infinite periodic structure such as a photonic crystal is also infinitely periodic. Thus, the Bloch theorem is used to represent the eigen-function of the photonic crystal.

$$H(x)=h_{k,n(x)}.e^{(j.k.x)}$$

(3.2.2)

Here hk,(x) is a periodic function and n is the state number.

Equation (3.2.2) is expanded to Fourier series by reciprocal lattice vectors,

$$H(x)=\sum_{G}h_{k,n}\,(G).e^{j.(k+G).x)}$$

(3.2.3)

hk, (G) is a periodic function in terms of wave vectors also known as Fourier expansion coefficient.

The dielectric function is also periodic, thus for convenience its reverse is taken as a Fourier series function,

$$\frac{1}{\varepsilon(x)}=\sum_{G"\in G}\aleph(G")e^{(j.G".x)}$$

(3.2.4).

Equation (3.2.4) is not dependent on co-ordinates but only on the reciprocal lattice vector. The operator in equation (3.2.4) can be represented in the form of a matrix, whose element can be found from the following equation,

$$\hat{\theta}_{G,G'}=\aleph(G-G')\big((k+G').(k+G)\big)$$

(3.2.5)

Set of solutions of system of equations (3.2.5) can be found as the eigen-value of matrix differential which is acting as an operator

$$\hat{\theta}=\begin{vmatrix}\hat{\theta}_{G_1G_1'} & \cdots & \hat{\theta}_{G_NG_1'}\\ \vdots & \ddots & \vdots\\ \hat{\theta}_{G_1G_N'} & \cdots & \hat{\theta}_{G_NG_N'}\end{vmatrix}$$

(3.2.6)

The set of G and G’ should be the same so that the matrix is a square one. Equation (3.2.6) is a Hermitian matrix. A Hermitian matrix has the following condition,

$$\hat{\theta}_{G,G'} = \hat{\theta}^{*}_{G',G} \tag{3.2.7}$$

Equation (3.2.7) means that it has a complex conjugate. The eigen-value of the matrix is ω 2 c 2 which gives the eigen frequency of the structure. The result of the eigen-problem solution is represented as a band structure which is shown in Figure 19,

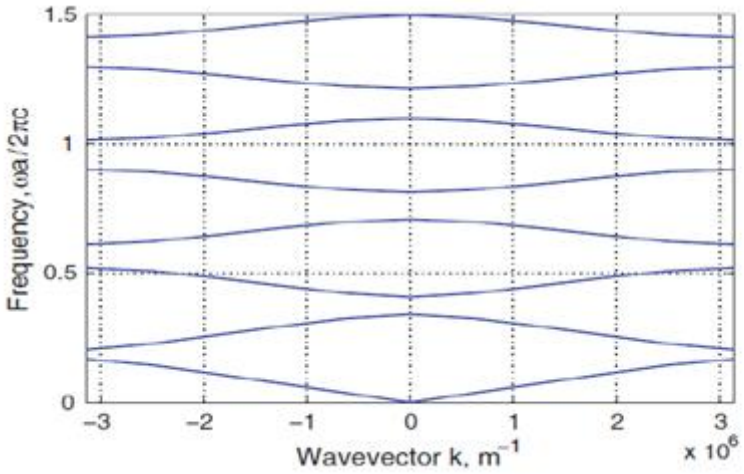

Figure 3.2.1 Band structure of a 1D Photonic Crystal

The x-axis of the band structure represents the wave-vector and the y-axis represents the normalized frequency which is $\omega a2$. The frequency is normalized by the period of the photonic crystal, so the scale of the structure does not matter. The period of the photonic crystal can be of micrometers of millimeters but the band structure remains unchanged regardless of the size of the period due to the normalized frequency. The layer thickness ratio and permittivity is kept constant in this case.

Analyzing the band structure of figure 3.2.1 in detail, the eigen-frequencies of the photonic crystal start from zero frequency at k=0. Higher on the frequency axis exists the photonic band gap. The photonic crystal does not have eigen-states within the photonic band gap. The bands and photonic band gaps appear consecutively on the frequency axis except when the photonic band gap width is almost equal to zero.

3.3 Finite-Difference Time-Domain Method (FDTD)

It is Plane Wave Expansion which is used for eigen-states calculations for an infinite periodic structure in addition to calculations of field distributions which correspond to those states but it is not suitable when the need is to find field distribution in complicated structures or examine dynamic characteristics.

In this method there is one of the most advanced numerical analysis technique today for computation of the field distribution inside Photonic crystal devices. In this thesis the software Rsoft Photonic CAD is used for the simulation.

This method consists of discretization of space. The derivatives in Maxwell's equations are replaced by finite differences that results in a system of algebraic equations which are linear on coordinates. The system is solved starting from initial and boundary conditions. (Sukhoivanov, et al. 2009).

This method helps us to find the field distribution by solving the system of Maxwell's equations on the discrete cell. The solution is based on the permittivity distribution function which determines radiation propagation conditions, initial conditions which contains radiation parameters such as the wavelength or the radiation spectrum for non-monochromatic wave, the amplitude and the initial phase, and boundary conditions which determine the radiation behavior at the boundary of the computation region (Sukhoivanov, et al. 2009). When these conditions are set, the field distribution is computed one by one.

Here are the symbols of common terms with meaning:

- B: Magnetic flux density or magnetic field density or magnetic induction measured in tesla (T)
- D: Electric displacement field or electric flux density measured in coulomb per square meter (C/m2)
- E: Electric field measured in newton per coulomb (N/C)
- H: Magnetic field strength or magnetic field measured in ampere per meter (A/m)
- c: Speed of light in vacuum measured in meters per second (m/s)
- μ: Permeability measured in ampere square meter (Am2)
- ε: Permittivity measured in farad per meter (F/m)

Maxwell's Equations are taken to derive in terms of finite differences. The equations for the medium are considered neglecting dispersion, absorption or light generation:

$$\nabla \times E(r,t) = -\frac{1}{c}\frac{\partial B(r,t)}{\partial t}$$

$$\nabla \times H(r,t) = \frac{1}{c}\frac{\partial D(r,t)}{\partial t}$$

The curl operators are evaluated and the equations are written separately by their vector components. All equations are in single direction.

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{1}{c}\frac{\partial B_x}{\partial t}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{1}{c}\frac{\partial B_y}{\partial t}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{1}{c}\frac{\partial B_z}{\partial t}$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \frac{1}{c}\frac{\partial D_x}{\partial t}$$

$$\frac{\partial H_x}{\partial y} - \frac{\partial H_z}{\partial z} = \frac{1}{c}\frac{\partial D_y}{\partial t}$$

$$\frac{\partial H_y}{\partial y} - \frac{\partial H_x}{\partial z} = \frac{1}{c}\frac{\partial D_z}{\partial t}$$

FDTD is mainly based on discretization of space. Thus all partial derivatives are interchanged with differences:

$$\frac{\partial}{\partial x} \approx \frac{\Delta}{\Delta x}$$

$$\frac{\partial}{\partial y} \approx \frac{\Delta}{\Delta y}$$

$$\frac{\partial}{\partial z} \approx \frac{\Delta}{\Delta z}$$

The derivative is represented by its approximate value which is the function variation divided by the argument variations taken at different cell nodes as shown in figure 3.3.1.

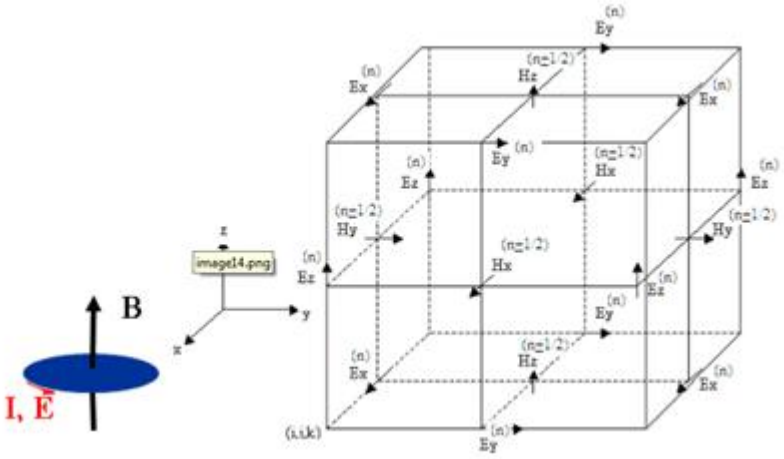

Figure 3.3.1 Yee Cell used for FDTD method computation

In the figure 3.3.1, a single cell is shown with all corresponding components of the field vectors. By solving the system of Maxwell's equations by the FDTD method, values of the field intensity in each cell node is obtained.

Replacing the derivatives, the recurrent expression is obtained which gives the value of the field component in a node of the cell using known values of the field components in adjacent nodes. The recurrent equations are:

$$H_{x(i,j,k)}^{n+1/2} = H_{x(i,j,k)}^{n-1/2} + \frac{c\Delta t}{\mu\Delta z}\left(E_{y\left(i,j,k+\frac{1}{2}\right)}^{n} - E_{y\left(i,j,k-\frac{1}{2}\right)}^{n}\right) - \frac{c\Delta t}{\mu\Delta y}\left(E_{z\left(i,j+\frac{1}{2},k\right)}^{n} - E_{z\left(i,j-\frac{1}{2},k\right)}^{n}\right)$$

$$H_{y(i,j,k)}^{n+1/2} = H_{y(i,j,k)}^{n-1/2} + \frac{c\Delta t}{\mu\Delta x}\left(E_{z(i+1/2,j,k)}^{n} - E_{z(i-1/2,j,k)}^{n}\right) - \frac{c\Delta t}{\mu\Delta z}\left(E_{x(i,j,k+1/2)}^{n} - E_{x(i,j,k-1/2)}^{n}\right)$$

$$H_{z(i,j,k)}^{n+1/2} = H_{z(i,j,k)}^{n-1/2} + \frac{c\Delta t}{\mu\Delta y}\left(E_{x(i,j+1/2,k)}^{n} - E_{x(i,j-1/2,k)}^{n}\right) - \frac{c\Delta t}{\mu\Delta x}\left(E_{y(i+1/2,j,k)}^{n} - E_{y(i-1/2,j,k)}^{n}\right)$$

$$E_{x(i,j,k)}^{n+1} = E_{x(i,j,k)}^{n} + \frac{c\Delta t}{\varepsilon\Delta y}\left(H_{z\left(i,j+\frac{1}{2},k\right)}^{n+\frac{1}{2}} - H_{z\left(i,j-\frac{1}{2},k\right)}^{n+\frac{1}{2}}\right) - \frac{c\Delta t}{\varepsilon\Delta z}\left(H_{y\left(i,j,k+\frac{1}{2}\right)}^{n+\frac{1}{2}} - H_{y\left(i,j,k-\frac{1}{2}\right)}^{n+\frac{1}{2}}\right)$$

$$E_{y(i,j,k)}^{n+1} = E_{y(i,j,k)}^{n} + \frac{c\Delta t}{\varepsilon\Delta z}\left(H_{z\left(i,j,k+\frac{1}{2}\right)}^{n+\frac{1}{2}} - H_{z\left(i,j,k-\frac{1}{2}\right)}^{n+\frac{1}{2}}\right) - \frac{c\Delta t}{\varepsilon\Delta x}\left(H_{y\left(i+\frac{1}{2},j,k\right)}^{n+\frac{1}{2}} - H_{y\left(i-\frac{1}{2},j,k\right)}^{n+\frac{1}{2}}\right)$$

$$E_{z(i,j,k)}^{n+1} = E_{z(i,j,k)}^{n} + \frac{c\Delta t}{\varepsilon\Delta x}\left(H_{z\left(i+\frac{1}{2},j,k\right)}^{n+\frac{1}{2}} - H_{z\left(i-\frac{1}{2},j,k\right)}^{n+\frac{1}{2}}\right) - \frac{c\Delta t}{\varepsilon\Delta y}\left(H_{y\left(i,j+\frac{1}{2},k\right)}^{n+\frac{1}{2}} - H_{y\left(i,j-\frac{1}{2},k\right)}^{n+\frac{1}{2}}\right)$$

From the above equations it shows the step-by-step recurrent computation of the electric field and the magnetic field starting from one side of the computation cell and moving to the other side. The values of the electric and magnetic field components are taken at the nodes (i+1/2, j+1/2, k+1/2) and (i−1/2, j−1/2, k−1/2) in order to take central differences

approximation. Computation is carried out over and over again for different times till required computation time is achieved.

FDTD is not a process of having any approximation or theoretical restrictions but it has a complex solution of Maxwell's equation. With this process we get a wide range of frequencies converted in a simple and single simulation. It calculates the electric field and magnetic field all over the computational process. It also helps to provide displays of the electromagnetic field movement. This display helps to understand what is happening in the whole model and whether the model is correct or not.

4. Photonic crystal cavity array design and defect interactions

4.1 Designing a photonic crystal

So far we have learned much about photonic crystals, its behaviors, applications and ways to perform simulation. We shall now design a photonic crystal cavity whose loss characteristics are to be analyzed. We do so by first starting with a basic array layout and then using it to simulate and plot the band gap map of the structure, which will allow us to fashion the crystal to our desire. Figure 4.1.1 shows the basic array layout of a photonic crystal made by punching air holes in a Gallium Arsenite (GaAs) slab. The thickness of the slab is set to be 0.6 x lattice constant for maximum confinement [14] [15]. As seen from the figure, we have used a hexagonal array of air columns that are punched in the GaAs photonic crystal slab (PCS). We did however have a choice of a cubic array of holes. However, we chose the hexagonal array the reasons for which will be elucidated later in the following section.

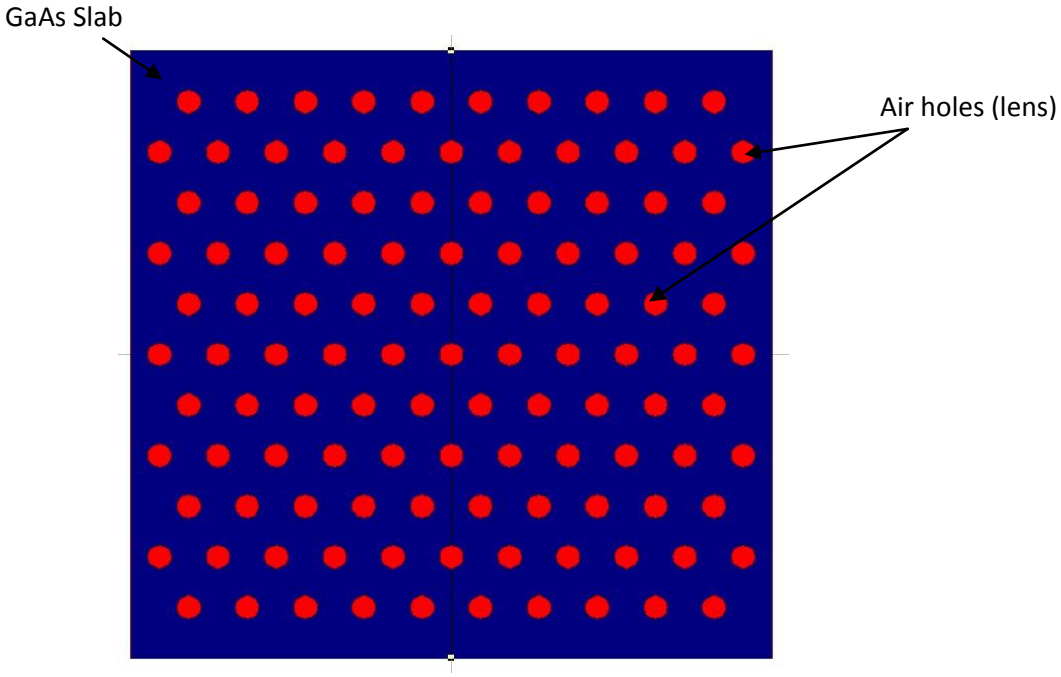

Figure 4.1.1 Basic crystal layout

Now that the basic layout is created, we need to determine whether a band gap exists within this structure. It must be noted that since we intend to create a cavity using this structure, we must make sure that a band gap exists for light confinement to occur. To do this we use the BandSOLVE simulation tool of the RSOFT CAD suit. Before starting the simulation we need to check two things. First we need to check whether the first brillouin zone is set up as expected and second we need to check whether the domain of the structure itself is set up properly. We also need to set a kind of filter known as light lines at this point. The light line of a PCS defines a distinguishing feature of a PCS band diagram: the light cone. Modes which lie within the light cone are radiation modes, and extend infinitely within the cladding. Modes which lie below the light cone are guided in the plane of the slab [16].

Figure 4.1.2 and figure 4.1.3 shows the first brillouin zone and the domain respectively. Figure 4.1.4 shows the band structure within this PCS. It is to be noted the band structure shows only odd- parity because we are only concerned with the TE modes.

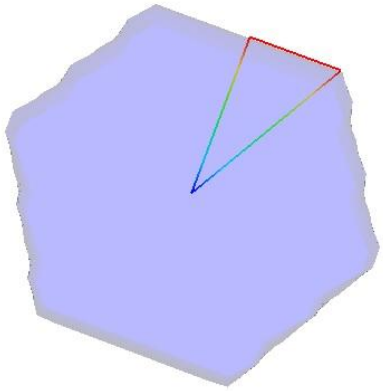

Figure 4.1.2 First Brillouin Zone

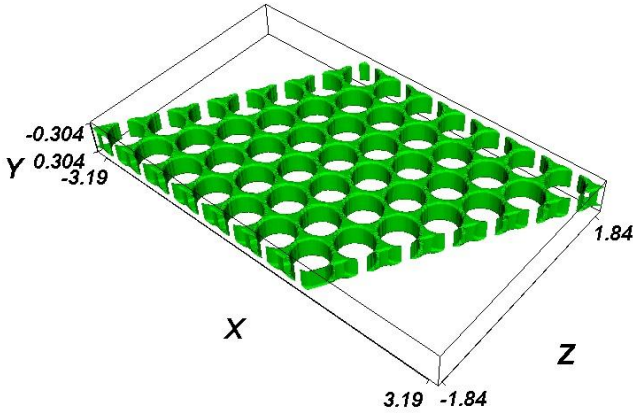

Figure 4.1.3 Domain of the PCS

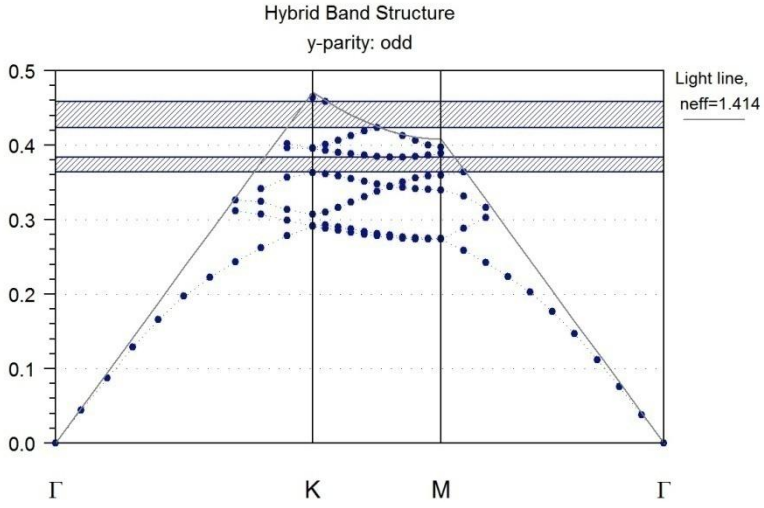

Figure 4.1.4 Realization of band gap in the structure

From the figures we can see that the structure is set up as per our requirement. With all the parameters set accordingly, we can generate the band gap map of this crystal which will allow us to calculate the lattice constant and radius for our desired structure. Here we use the plot graphs utility to plot a graph of Radius vs Frequency for this structure. It must be noted that the x-axis is indeed a ratio of the radius and lattice constant (r/a) for this structure. Figure 4.1.5 shows the generated band gap map.

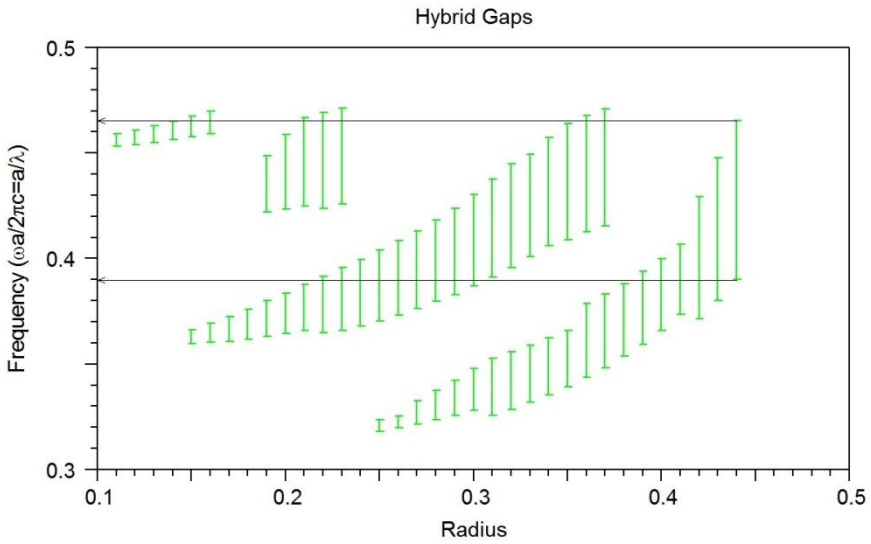

Figure 4.1.5 Band Gap Map

From the figure we see the largest gap is found when the ratio r/a is 0.44 and thus this value is a very suitable choice for us. From there we measure the corresponding higher frequency W_H and lower frequency W_L to be 0.465 and 0.389 respectively. Thus we find the mid-gap frequency W_M to be 0.427. We can see that y-axis really is a ratio of the lattice constant and the wavelength. We chose the wavelength in this case to be that used in communications (1.5µm). Thus we can easily calculate the lattice constant from the ratio $a=\lambda W_M$, yielding a= 0.64. Now using this value of a, we can calculate the value of the radius from the ratio r=0.44a, which yields r= 0.282.

Using these values, we edit the parameters of the PCS and create our desired structure. Figure 4.1.6 shows the modified PCS ready to be tailored according to our desire to create cavities.

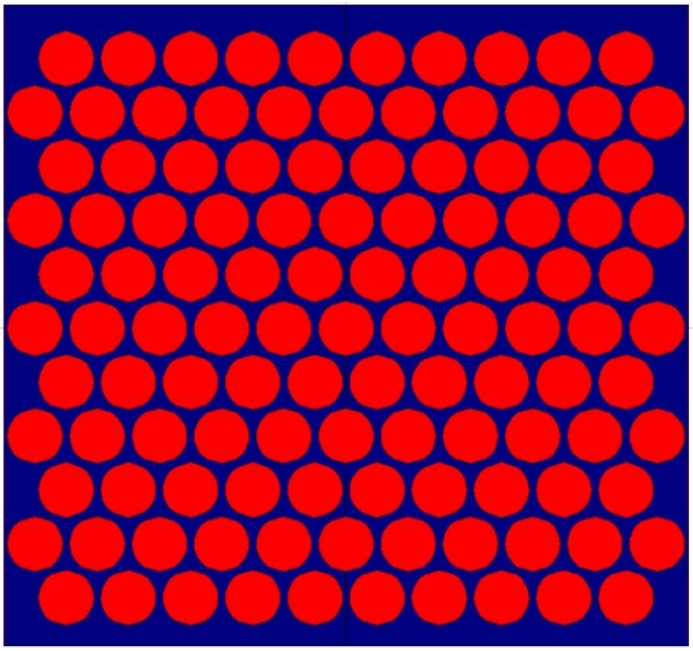

Figure 4.6 Modified PCS

4.2 Designing an cavity array with similar defects

As we have seen earlier, two-dimensional photonic crystals of finite depth can exhibit a photonic band gap for electromagnetic waves propagating in the plane of the crystal. However, by lattice defects, one can permit localized modes that have frequencies within the photonic band gap. Such modes decay exponentially away from the defect. In other words, the defect behaves as an optical cavity, and the surrounding photonic crystal represents mirrors surrounding the cavity. Therefore, the defects introduce peaks into the density of optical states inside the photonic band gap. Moreover, the defects break the discrete translational symmetry of the photonic crystal [16].

The simplest way of forming a microcavity starting from the unperturbed hexagonal photonic crystal lattice of air holes is by removing a hole, changing the radius of a single hole, or by changing its refractive index (Jelena Vuckovic 2002). However, we deviate from this conventional method and create an array of defects surrounding a central defect and try to study the interactions between the defect modes.

4.2.1 Cavity array with holes removed

In order to create the defects, we completely remove the air columns instead of changing the radius or the refractive index. Figure 4.2.1.1 shows the cavity array formed by removing the holes in the same 2D PCS that we designed earlier.

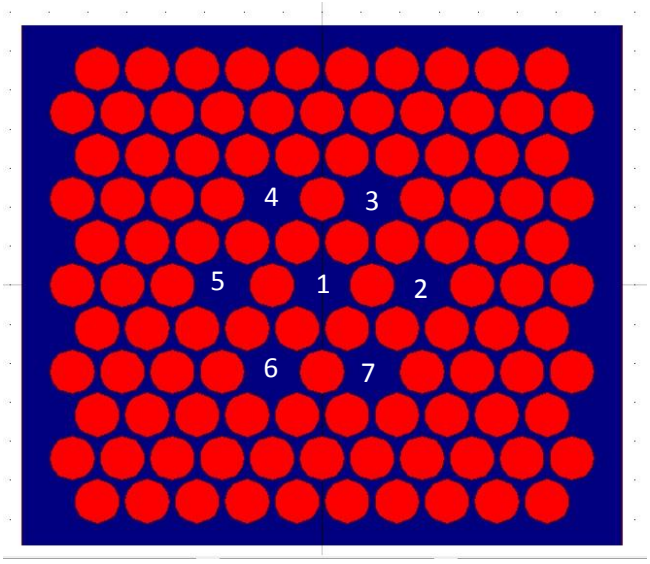

Figure 4.2.1.1 Cavity array with similar defects. For simplicity of understanding, the defects are numbered as shown.

Now that the cavity is set up, we can start our FDTD simulation using the FullWAVE simulation tool in the Rsoft CAD Suit. Since we are interested in the interaction between the defect modes, all we need to do is excite the central defect (defect 1) with an impulse. Doing so will not only show us which mode is most confined in the central defect but also show which wavelengths have been lost and also which "losses" are captured by the surrounding defects. In order to find the wavelengths, we place a time monitor in the central defect which gives us the intensity vs wavelength for the simulation. Similarly the time monitor is placed in each defect to yield the intensity vs wavelength graphs. Figure 4.2.1.2 shows the placement of the time monitor and figure 4.2.1.3 shows the intensity vs wavelength graph found for defect 1.

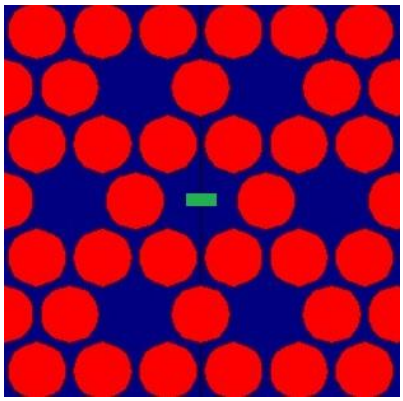

Figure 4.2.1.2 Time monitor placement

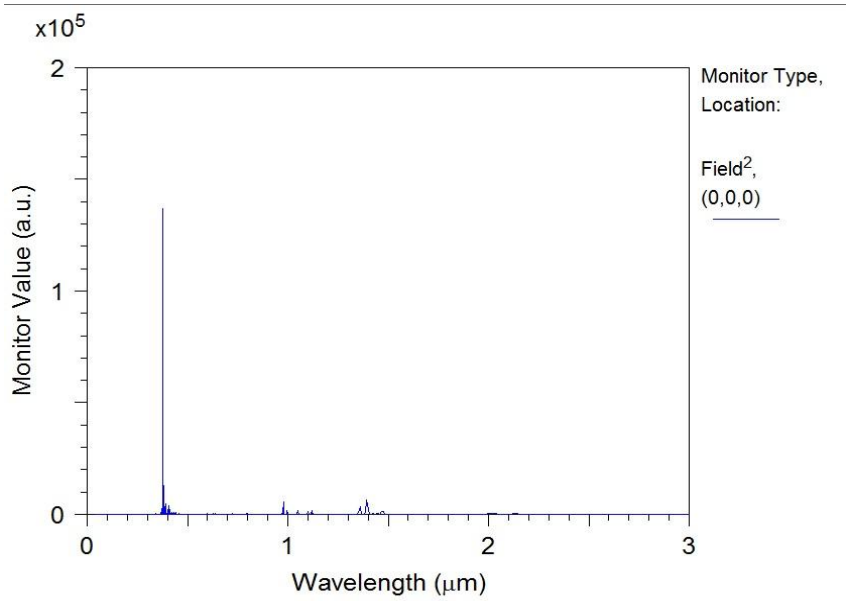

Figure 4.2.1.3 Intensity vs Wavelength graph for defect 1

From the graph we see that even though we are not concerned confinement yet there is a very high peak present indicating a significant quality factor, Q, at a wavelength of 0.37µm. However, we are not concerned about the quality factor so details about it are not going to be discussed.

We shall now take a look at how the modes interact with each other. To do so, we place the time monitors in each defect and extract the intensity vs wavelength graphs. Figure 4.2.1.4 shows the graphs in the same order as the cavities.

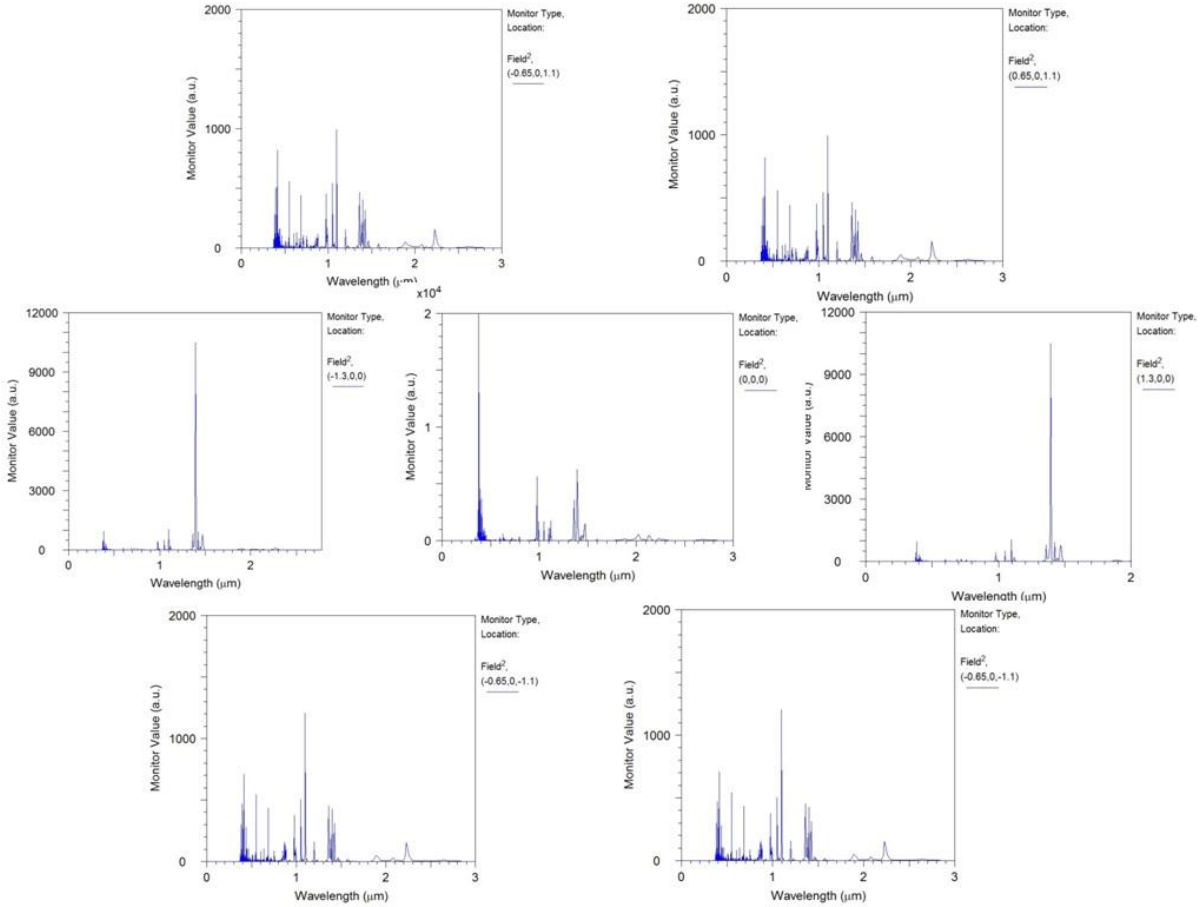

Figure 4.2.1.4 Intensity vs Wavelength graphs for defect modes 1 to 7 in the same order as in the crystal

From the first glace one very interesting phenomenon is immediately noticed. We find that the losses in the horizontal axis are virtually identical on both sides. In other words defects 2 and 5 interact with the same wavelengths. We also find that diagonal interaction (between defects 3, 4, 6, 7) is rather weak for the all the wavelengths seen in the graphs and in fact is much noisy compared to the horizontal graphs. It can also be noted here that these two kinds of losses are also distinct from each other.

From the above results, we can conclude that for an cavity array with similar defects, created by removing the holes, the horizontal losses are almost identical on both sides and the diagonal losses are identical in the four corners.

4.2.2 Cavity array with radius of holes changed

Let us now study the interaction between modes in a cavity array formed by defects consisting of holes whose radius has been reduced. In this case the radius of the defect holes were reduced by an arbitrary factor of 2. Figure 4.2.2.1 shows the layout of the structure.

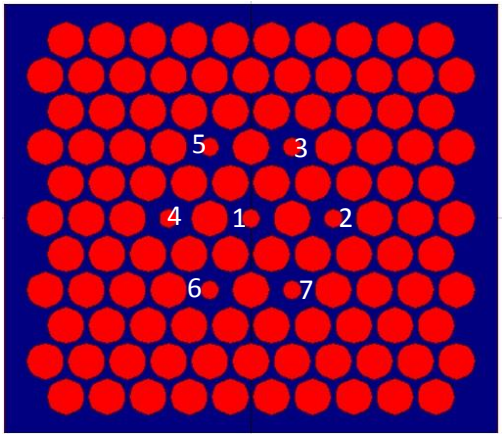

Figure 4.2.2.1 Layout of cavity array with the defect radius reduced

We use the same simulation techniques used before to yield the intensity vs wavelength for the individual defects and compile them as shown in figure 4.2.2.2.

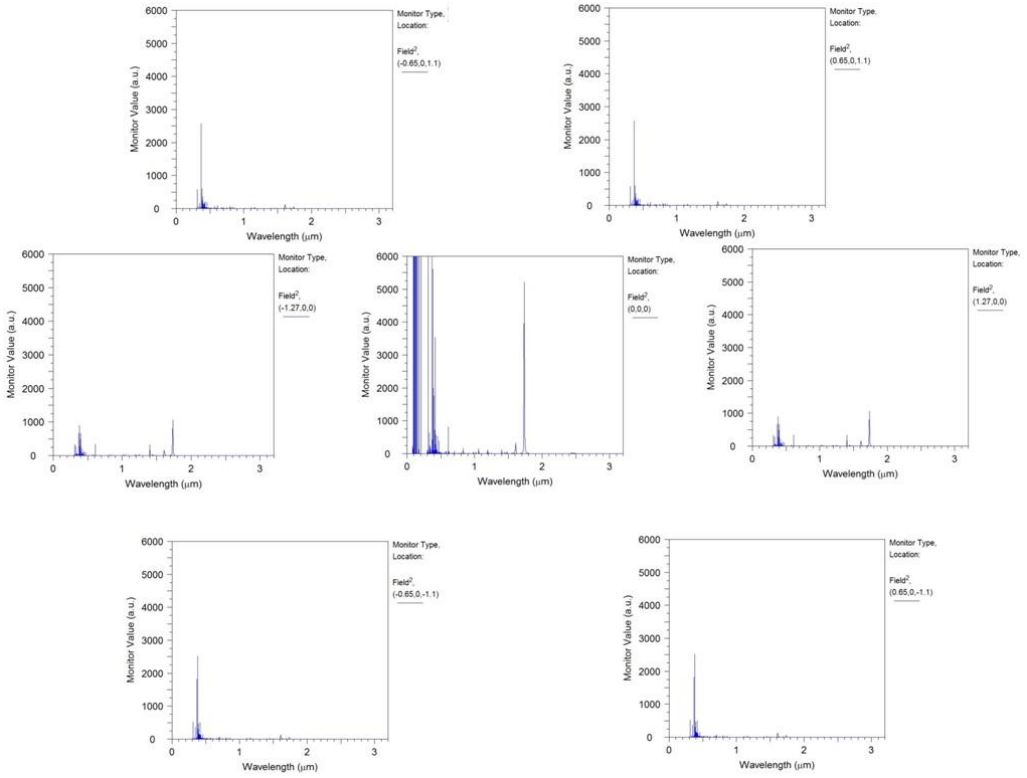

Figure 4.2.2.2 Intensity vs Wavelength graphs for defect modes 1 to 7 in the same order as in the crystal

From the graph it is again obvious, as before that, the horizontal interaction is similar in both sides and the diagonal interaction are similar in the corners. However, it must be noted that the peak wavelengths has changed due to the overall changes in the structure. In fact due to reduction of the holes, a donor defect state is excited and pulled into the band gap from the air band which causes the changes in peak wavelengths.

4.2.3 Cavity array with refractive index of holes changed

In order to create the defects by changing the refractive index, we start with the original PCS structure, and then instead of changing the radius or removing the holes, we simply change the refractive index of the holes. For convenience, we take the value of the refractive index to be an arbitrary value of 1.5. Figure 4.2.3.1 shows the layout of the structure. The yellow holes represent defects with their refractive index changed.

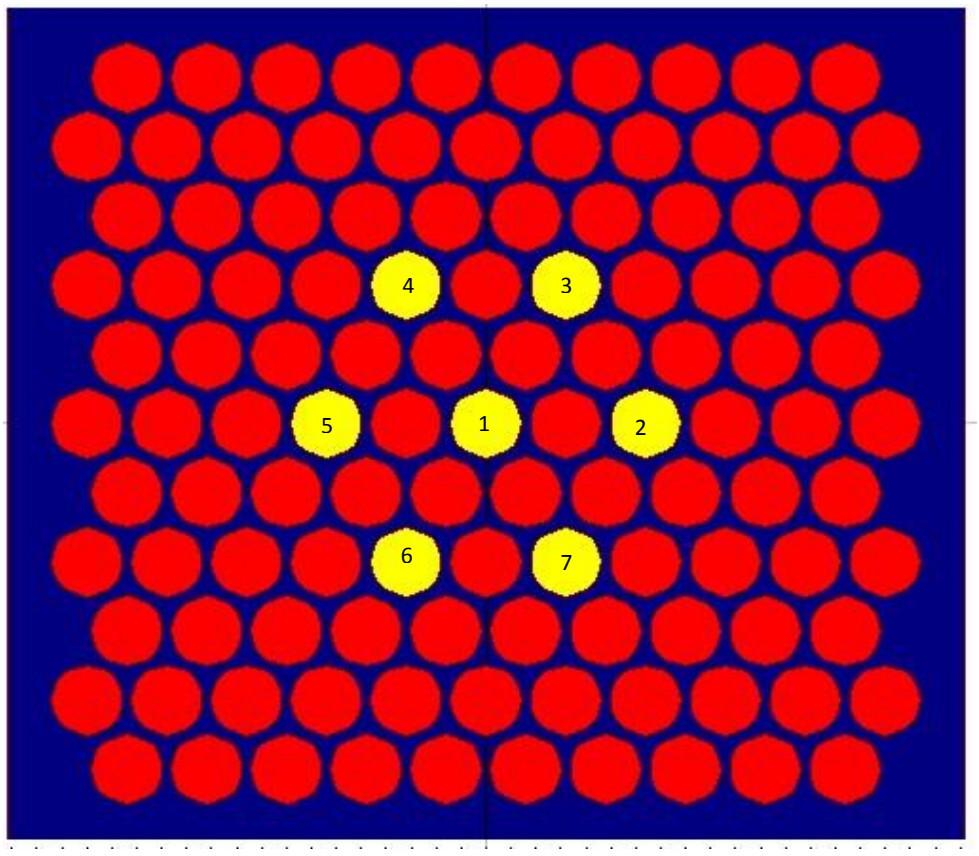

Figure 4.2.3.1 Cavity array with defects formed by changing the refractive index

For this structure, the intensity vs wavelength graphs are plotted following the same techniques as before. The results are shown in figure 4.2.3.2.

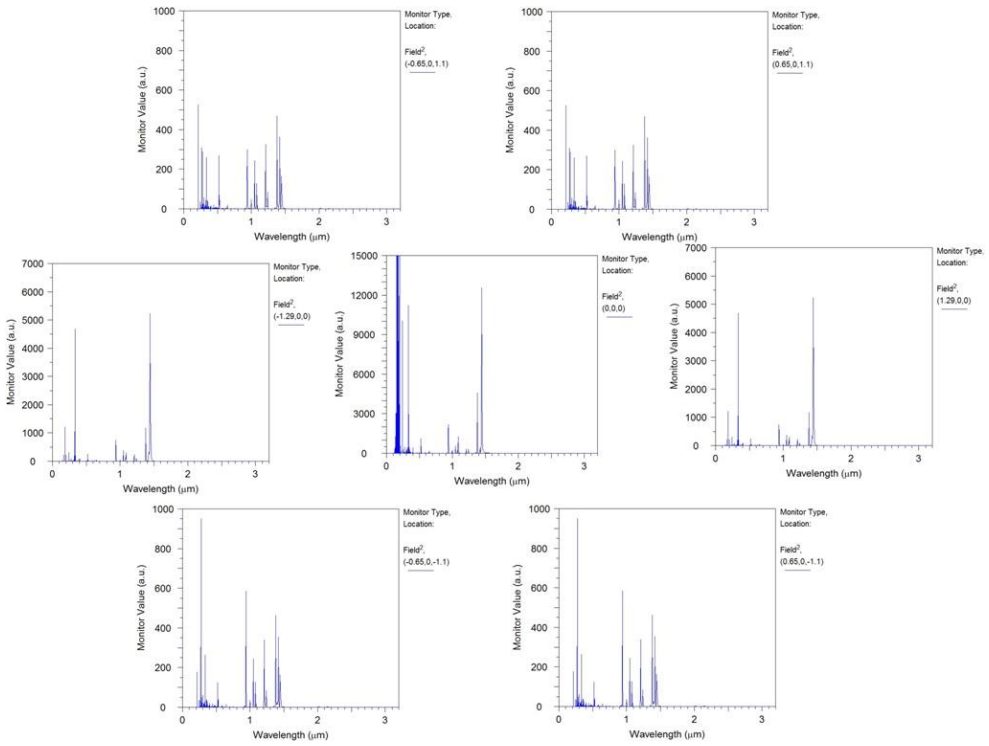

Figure 4.2.3.2 Intensity vs Wavelength for the defect modes in the same order as in the crystal

From the figure, we again find that the horizontal defects interact with each other in the same manner on both sides and the diagonal defects interact with each other in the same manner at the corners. Also it can be noted here that there is significant changes in the peak wavelengths as well. This is again for the same reason as before. By tuning its refractive index between 1 and the refractive index of the slab (GaAs refractive index 3.46), a donor defect state is excited and pulled into the band gap from the air band.

From the above results, it can be concluded that for cavity arrays formed by same defects, irrespective of how the defects are formed, the interaction between the horizontal defects will be the similar and the diagonal defects will also be similar. However, these two interactions are distinct from each other as the graphs show, the diagonal interactions are weaker and in all cases noisy. Therefore, by experimenting and optimizing, the interactions can be tuned to requirement. Doing so will not only enable us to use the cavity as a resonator, but also allow us to control the manner in which light, or information, is lost from the cavity which truly will open new prospects for the uses of photonic crystal cavities.

4.3 Designing cavity arrays with different defects

In the previous we designed a cavity array where the defects were made by removing the air holes, as seen in figure 4.2.1, meaning that the defects were identical. Now we are interested to analyze the interaction between the defect modes of a cavity array which is

made of defects that are different from each other. In this case, we create the defects by reducing the radius of the air holes. The central air hole (defect 1) radius is reduced arbitrarily by a factor of 2. The surrounding defect radii are reduced by a factor of 2.25, 2.50, 2.75, 3, 3.25 and 3.50 for the defects 2, 3, 4, 5, 6 and 7 respectively. Figure 4.3.1 shows the crystal layout after these modifications are made.

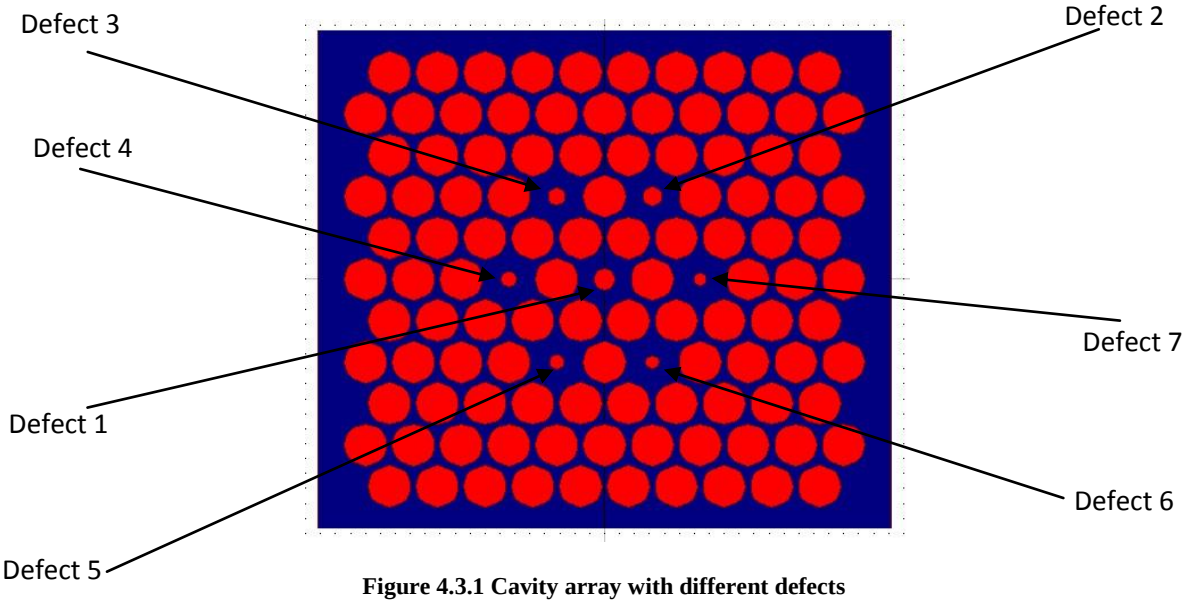

Figure 4.3.1 Cavity array with different defects

With the PCS set up correctly we now carry out the FDTD simulation again using the same procedures used as before, exciting the central defect, and yield the intensity vs wavelength graphs. Figure 4.3.2 shows the graph for the central hole (defect 1), which indicates that there is high confinement however at an even lower wavelength. Figure 4.3.2 shows the graphs for all the defects in the same order as in the cavity and we shall now use it for analyzing the losses.

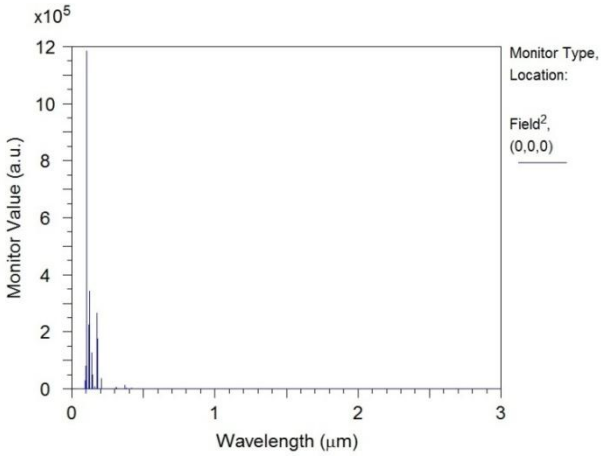

Figure 4.3.2 Intensity vs Wavelength graph for defect 1

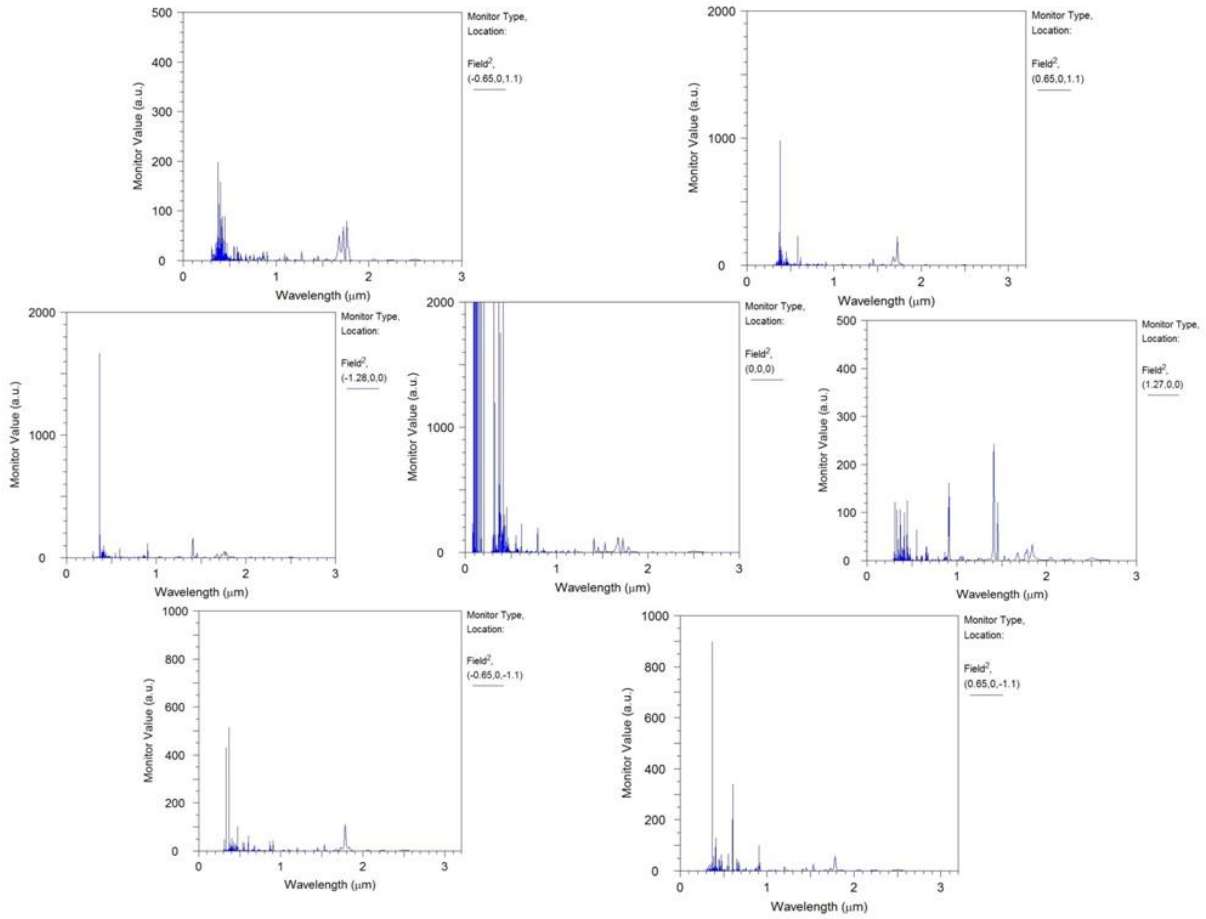

Figure 4.3.3 Intensity vs Wavelength graphs for defect modes 1 to 7 in the same order as in the crystal

From the graphs we unfortunately could not find any conclusive evidence that indicates there is some form of interaction between the defect modes. Also from the graphs, it is evident that the horizontal losses are not same in both directions and there are no perceivable similarities in the diagonal losses either. In fact, no conclusive information was found by experimenting with different values of the radii. This failure goes on to buttress our findings in the previous section by indeed proving that our findings are only valid for similar defects. One interesting observation is made however. It can be seen that for different radii, the modes in the surrounding defects are different. This phenomenon indicates that modes confined in each defect can be tuned to requirement by changing the property of the defects.

5. Conclusion and future research

This thesis attempts to investigate the interaction between defect modes in a cavity array tailored in a PCS, created by using the band gap map. The band gap map was used to initially calculate the dimensions of the PhC in which the cavities were tailored and analyzed. It was successfully shown that a cavity array composed of similar defects in a hexagonal lattice will have virtually similar losses in the horizontal direction and also similar losses in the diagonal corners. However, these two losses are distinct from each other. The findings also allowed us to conclude that the PhC structured can be tailored according to our wish by using the band gap map to manipulate the interacting wavelengths. The theory is quite practical considering we have used photonic crystal slab of finite thickness instead of considering infinite columns.

There is much room to further investigate this theory. Due to software limitations we were not able to simulate an active device which could generate its own lights. By studying an active device's intensity vs wavelength graphs, this theory may be further scrutinized as to whether it holds for all kinds of PhC cavities. Further investigation may also be carried out by introducing wafers on top and at the bottom of the PCS which will challenge the validity of the theory. This theory can be investigated using different kind of array cavities (honey comb lattice for example) as well given that the initial crystal layout consists of a hexagonal structure.

Optimizing PhC cavities have seen extensive research in the last couple of decades. One of the prime objectives when designing a PhC cavity is to reduce the losses; in other words to increase the quality factor, Q. By using the knowledge found in this thesis, such losses in PhC cavities may be reduced especially when designing a hexagonal cavity. Also the cavity may be designed (by introduction of non linear materials within the crystal) to store light, or information, in different defects within itself, or it can also be designed to act as ademultiplexer which can filter at least six wavelengths by using the six surrounding defects.

In the near future photonic crystal based components will see extensive deployment in the technological world due to its radically different characteristics. PhC cavities offer a very attractive use of the 2D PhC because of its ability to store information for significantly long periods of time given that quality factor is high. PhC cavities are already being used to experimentally fabricate high efficiency lasers, LEDs, and other optical devices for its high confinement abilities. This paper may lead to creating array cavities that are much more efficient than the ones presently researched upon leading to very effective information storage cells that can be used in optical RAMs or other optical memory devices.

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