Inflationary Models in String Cosmology

by

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Declaration

It is hereby declared that

1. The thesis submitted is my/our own original work while completing degree at Brac University.

2. The thesis does not contain material previously published or written by a third party, except where this is appropriately cited through full and accurate referencing.

3. The thesis does not contain material which has been accepted, or submitted, for any other degree or diploma at a university or other institution.

4. We have acknowledged all main sources of help.

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Abstract

Inflation is an acclaimed theory for uncovering how the Universe works and how it came to be. For years, physicists have attempted to not only cement a theory where the accelerated expansion of the Universe can help us to explore primordial states of the Universe, but also to incorporate other existing theories into inflation theory to produce even more ambitious models. Such models attempt to tie quantum field theories into those of inflationary cosmology in pursuit of a unified way of describing the phenoma in the Universe from the smallest of scales to the largest scales observable to man.

The goal of this thesis is to explore some of these theories and the steps that have been taken (both mathematically and in idea) to achieve inflationary models that take elements from field theories.

We begin from Effective Field theories and initial ideas of inflation, to understanding why inflation is important and why it is a theory so many physicists are invested in. We attempt to analyze some problems that arise in rationalizing inflation, the changes made to overcome them as well as the newer theories where further adjustments are made to make inflation as naturally derived as possible.

We venture into string theory and the ideas that lead to the construction of complex inflationary models such as the KKLMMT model. We analyze brane-antibrane interactions and the problems that these interactions bring. We explore how KKLMMT tries to account for such issues by stabilizing various contenders in the theory to possibly reach a final concrete model.

Finally, we explore racetrack inflation and reheating, how they arise from the ideas presented by such string cosmology models and what implications they bring to our understanding of the Universe.

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Chapter 1

Effective Field Theories

1.1 Kaluza-Klein Theory

It is a classical unified field theory which is constructed around the concept of the fifth dimension. The Kaluza-Klein theory was one of the first attempts to unite all fundamental forces under a single law.

It was constructed around considering one extra compactified spatial dimension, along with the existing three spatial and one time dimensions.[1]

The theory, however, had a few shortcomings:

- It never accounted for the strong and the weak nuclear forces, since they had not been theorized when the Kaluza-Klein theory was developed.
- Calculated mass and charge of the electron in accordance to Kaluza-Klein theory did not produce the experimentally-proven values.
- **Witten no-go theorem**: The standard model requires that the gauge fields within the Kaluza-Klein theory must chirally couple with massless fermions. However, this theory cannot easily produce these fermions - introducing extra gauge fields reduces the purity of the theory itself.

Kaluza-Klein theory is thus a forerunner to many such theories that attempt to construct a single theory applicable to all forces by considering the presence of compactified extra dimensions. String theory is one such theory.

However, where Kaluza-Klein theory opts for introducing only gravity to a (1+4)-dimensional spacetime, newer theories want to account for the later-discovered forces as well.

We need to take into consideration the symmetry groups of the forces we are unifying with gravity:

- Electromagnetism - $U(1)$
- Weak Interaction - $SU(2)$
- Strong Interaction - $SU(3)$

Hence, we obtain $SU(3)\times SU(2)\times U(1)$ which is the Standard Model (SM) symmetry group.
In order to describe all possible interactions, the theory has to be extended to higher dimensions. The topology of spacetime in a higher dimensional model needs to be preserved.

For example, if we consider a $d = (1 + (3 + D)$ - dimensional spacetime, it will have a topology of $M^4 \times B^D$. B^D has to contain the symmetry subgroups mentioned above. So, we consider $B^D = CP^2 \times S^2 \times S^1$ where the symmetry subgroups are preserved. CP^2 is a 4-dimensional complex space, S^2 is a 2-dimensional sphere and S^1 is a 1-dimensional circle. This gives us $D = 7$.

Thus, a theory comprising of all forces (by preserving their symmetry subgroups) would need to have atleast $d = (1 + 10)$ dimensions.

1.2 Effective Field Theory and Inflation

In order to work towards a unified theory, it is necessary to obtain a quantum field theory that accommodates inflation. There are two possible complementary methods to reach such a theory [2]:

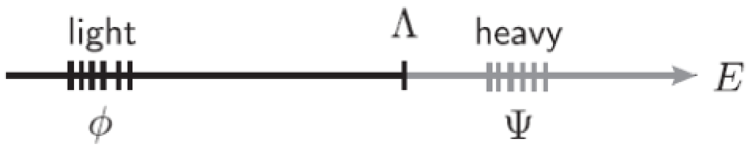

Figure 1.1: The light degrees of freedom exist below Λ which is the cutoff scale. Effective field theory is applied here, while the heavy fields are integrated over.(Baumann and McAllister, 2014, p.54)

Top-Down approach: A theory that begins from high-energy degrees of freedom in the ultraviolet (UV) region, such as string theory, and works downwards to derive inflation in the low-energy region.

We represent the interactions between light fields by the Lagrangian, $\mathcal{L}_l$, those between heavy fields by $\mathcal{L}_h$ and between both sets of fields by $\mathcal{L}_{lh}$. The Lagrangian of the UV theory is then given by:

$$\mathcal{L}[\phi, \Psi] = \mathcal{L}_l[\phi] + \mathcal{L}_h[\Psi] + \mathcal{L}_{lh}[\phi, \Psi] \tag{1.1}$$

By integrating out over the heavy modes, we obtain a path integral:

$$e^{iS_{\text{eff}}}[\phi] = \int [\mathcal{D}\Psi] e^{iS[\phi,\Psi]} \tag{1.2}$$

The S_{eff} in the above equation is called the **Wilsonian effective action**. This path integral sums over the heavy modes to try to remove the heavy fields, denoted by Ψ.

Toy Model: In the toy model, we take the UV theory Lagrangian as:

$$\mathcal{L}[\phi,\Psi] = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 - \frac{1}{2}(\partial\Psi)^2 - \frac{1}{2}M^2\Psi^2 - \frac{1}{4}g\phi^2\Psi^2 \quad (1.3)$$

$\mathbf{Z}_2$ symmetry $\phi \to -\phi$ is conserved by this model, where ϕ represents the light fields. We can thus obtain the Lagrangian for ϕ as:

$$\begin{aligned}\mathcal{L}_{\text{eff}}[\phi] = &-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_{\text{R}}^2\phi^2 - \frac{1}{4!}\lambda_{\text{R}}\phi^4 \\ &- \sum_{i=1}^{\infty}\left(\frac{c_i(g)}{M^{2i}}\phi^{4+2i} + \frac{d_i(g)}{M^{2i}}(\partial\phi)^2\phi^{2i} + ...\right)\end{aligned} \quad (1.4)$$

The heavy fields renormalize the mass and λ (the quartic coupling), which obtain loop corrections.

$$\begin{aligned} m_{\text{R}}^2 &= m^2 + \frac{g}{32\pi^2}(\Lambda^2 - \text{M}^2\ln(\Lambda^2/\mu^2)) + ... \\ \lambda_{\text{R}} &= \lambda - \frac{3g^2}{32\pi^2}\ln\left(\Lambda^2/\mu^2\right)\end{aligned} \quad (1.5)$$

We can consider the term $\ln(\Lambda^2/\mu^2) = L$ and after dimensional regularization, we can simply define L as:

$$L \to \frac{1}{\epsilon} + \gamma - \ln(4\pi) \quad (1.6)$$

Upon renormalizing the parameters of the Lagrangian (1.4), we notice that the quadratic divergences in (1.5) can be absorbed. For values of $M \to \infty$, decoupling renders the effects of the heavy particles ignorable.

Bottom-Up approach: A theory that begins from low-energy degrees of freedom in the infrared (IR) region and parameterizes upwards to high-energy degrees to derive inflation.

We model an effective action based on symmetries of the UV theory that sums over all operatos $\mathcal{O}_i[\phi]$:

$$\mathcal{L}_{\text{eff}}[\phi] = \mathcal{L}_l[\phi] + \sum_i c_i\frac{\mathcal{O}_i[\phi]}{\Lambda^{\delta_i-4}} \quad (1.7)$$

Λ is the cutoff scale, δ_i shows the dimensions of the operators and the prefactors c_i are called Wilson coefficients and are dimensionless. However, using symmetries we can still parameterize on the unknown heavy scale using the cutoff scale in order to obtain inflation through effective field theory, while accounting for the degrees of freedom from both the inflaton field and the gravitational field. The resulting action can be represented as:

$$\mathcal{S}_{\text{eff}}[\phi] = \int d^4x\sqrt{-g}\left[\frac{M_{\text{pl}}^2}{2}R + \mathcal{L}_l[\phi] + \sum_i c_i\frac{\mathcal{O}_i[\phi]}{\Lambda^{\delta_i-4}}\right] \quad (1.8)$$

If Wilsonian naturalness is assumed in this case, then the UV couplings as well as the Wilson coefficients should be of order-unity. This can make it seem as if certain coefficients are suppressed. However, it can be explained as necessary in our endeavor to achieve inflation due to the presence of internal symmetries.

A clear clue as to what the cutoff scale for this theory could be is the knowledge that it cannot be greater than the Planck scale. Violations to the perturbative unitarity due to graviton-graviton scattering is avoided by ensuring the cutoff scale is less than or near to equal the Planck scale.

Gravity in EFT:

We can attempt to model gravity as an effective theory by considering the spacetime metric $g_{\mu\nu}$ in flat space:

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + \frac{1}{M_{\text{pl}}^2} h_{\mu\nu} \tag{1.9}$$

As mentioned previously in the case of low-energy degrees of freedom that are below the cutoff scale, we can use this metric by expanding it in the form of small perturbations.

The Einstein-Hilbert action is given by:

$$S_{\text{EH}} = \frac{M_{\text{pl}}^2}{2} \int \mathrm{d}^4x \sqrt{-g} R \tag{1.10}$$

We now accommodate the perturbations in the Einstein-Hilbert action to get [2]:

$$S_{\text{EH}} = \int \mathrm{d}^4x \left[(\partial h)^2 + \frac{1}{M_{\text{pl}}} h(\partial h)^2 + \frac{1}{M_{\text{pl}}^2} h(\partial h)^2 + ... \right] \tag{1.11}$$

which in turn resembles the Yang-Mills action:

$$S_{\text{YM}} = \int \mathrm{d}^4x \left[(\partial A)^2 + gA^2 \partial A + g^2 A^4 \right] \tag{1.12}$$

The Einstein-Hilbert action is unlike the Yang-Mills action in that, due to the presence of the expansion of $\sqrt{-g}$ and $g_{\mu\nu}$, there can be an infinite number of terms. We can assume that all the relevant terms are within the low-energy effective action and represent it as:

$$S_g = \int \mathrm{d}^4x \sqrt{-g} \Big[M_\Lambda^4 + \frac{M_{\text{pl}}^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{1}{M^2}(d_1 R^3 + ...) + ... \Big] \tag{1.13}$$

Here also, c_i and d_i are dimensionless and can be assumed to be of order-unity. However, a point of interest lies in the renormalized M_Λ which is the cosmological constant. This gives rise to the cosmological constant problem since in this context, the value of M_Λ is not close to its natural value M_{pl}.

1.2.1 Inflation in EFT

As discussed above, we now apply EFT from bottom up to inflation. We take into account the inflaton field and the gravitational field - both of which are degrees of freedom for inflation. The Lagrangian for an EFT to model inflation takes the form:

$$\mathcal{L}_{\text{eff}}[\phi, \Psi, g] = \sqrt{-g}\left[\frac{M_{\text{pl}}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V_0(\phi) + \sum_i c_i \frac{O_i[\phi, \Psi]}{\Lambda^{\delta_i - 4}}\right] \tag{1.14}$$

The kinetic term $-1/2(\partial\phi)^2$ and renormalizations can be rewritten as $\mathcal{L}_l[\phi]$. Minimal coupling the Lagrangian (1.14) to gravity gives us the effective action for the inflaton field:

$$S_{\text{eff}}[\phi] = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\text{pl}}^2}{2}R + \mathcal{L}_l[\phi] + \sum_i c_i \frac{\mathcal{O}_i[\phi]}{\Lambda^{\delta_i - 4}}\right] \tag{1.15}$$

In this vicinity, we are presented with three scenarios due to ultraviolet sensitivity [3]:

Eta Problem

To construct a complete idea of inflation in EFT, the problems caused due to heavy fields beyond the cutoff scale need to be resolved. These problems include:

- renormalization of light-field couplings
- introduction of new interactions that cannot be normalized

A quantum correction to the inflaton mass, $\Delta m^2 \sim \Lambda^2$, needs to be made to reach the cutoff scale, which leads to a large renormalization of η, an inflationary parameter, given by:

$$\Delta\eta \sim \frac{\Lambda^2}{H^2}1 \tag{1.16}$$

in order to maintain consistency with $\Lambda > H$. However, this does not result in natural slow-roll inflation. This dilemma is called the **Eta Problem** and can be summarized as:
Dim-6 -

$$\Delta V = V_0\frac{\phi^2}{\Lambda^2} \xrightarrow{\Lambda < M_{\text{pl}}} \Delta\eta \equiv M_{\text{pl}}^2\frac{V''}{V} \approx \frac{M_{\text{pl}}^2}{\Lambda^2} > 1 \tag{1.17}$$

Eta problem can have possible solutions in the dimensional gauge coupling.

Non-Gaussianity

When constructing models of the primordial Universe, we encounter Gaussian-distributed density fluctuations which are responsible for the variation in temperature of the CMB [4].

Unlike slow-roll scenarios where the inflaton (a single scalar field) rolling down its potential causes density perturbations, there are instances where association with any other field such as the curvaton could be the cause of these fluctuations. In these cases, positive non-Gaussianity helps establish that the inflation was not driven by the slow-roll "ideal" of a single scalar field.

Dim-8 -

$$\Delta\mathcal{L} = \frac{(\partial\phi)^4}{\Lambda^4} \xrightarrow{\Lambda^2<\dot{\phi}} f_{\rm NL} \sim \frac{\dot{\phi}^2}{\Lambda^4} < 1 \tag{1.18}$$

Thus, understanding non-Gaussianity is important to correlate cosmological perturbations of the primordial Universe and to determine the exact method behind the generation of such perturbations.

Lyth Bound

The Lyth Bound is basically a predicted upper bound on the quantity of gravitational waves generated during slow-roll inflation with respect to the inhomogeneities of the CMB. It restricts how steep an inflationary potential can be.

To denote the **power spectrums** for two types of fluctuations, while including the de Sitter fluctiations and curvature perturbations, we obtain:

Scalar fluctuations given by:

$$P_s = \left(\frac{H}{\pi}\right)^2 \left(\frac{H}{\dot{\phi}}\right)^2 \tag{1.19}$$

Tensor fluctuations given by:

$$P_t = \left(\frac{2}{\pi^2}\right) \left(\frac{H}{M_{pl}}\right)^2 \tag{1.20}$$

Using the above, the ratio becomes:

$$\frac{P_t}{P_s} = 8\left(\frac{\dot{\phi}}{HM_{pl}}\right)^2 \tag{1.21}$$

We take $P_t/P_s =$ r and obtain:
Dim-∞ -

$$\Delta\phi \sim \left(\frac{r}{0.01}\right)^{1/2} M_{\rm pl} \xrightarrow{r>0.01} \Delta\phi > M_{\rm pl} \tag{1.22}$$

where $r > 0.01$ shows the Lyth Bound for large field inflation.

Chapter 2

Basics of Inflation

2.1 de Sitter Space

Inflation dictates that any two objects, not bound together by any force, will drift apart from each other due to the patch of space between them expanding exponentially with time.

This section of spacetime is modeled as a maximally symmetric Lorentzian manifold, called the **de Sitter space**, and it can be described by the metric [5]:

$$ds^2 = -(1 - \Lambda r^2)dt^2 + \frac{1}{1 - \Lambda r^2}dr^2 + r^2 d\Omega^2 \tag{2.1}$$

For the metric to hold, it needs to meet the following conditions:

- The vacuum energy density must be constant in space and time. It must also be proportional to Λ.
- The energy density of the space must be constant. This is denoted by Λ and is called the **Cosmological Constant**.

2.2 Why inflation?

Besides providing an insight into why CMB radiation is almost evenly distributed and in explaining why the Universe appears isotropic, cosmological inflation is also an intriguing solution to two problems - the Flatness Problem and the Horizon Problem.

2.3 The Scale Factor

The **scale factor** is an important parameter of the Friedmann equations, which characterizes the dynamics of the spacetime geometry. The normalized scale factor, $a(t)$, scales the energy densities of the cosmological parameters (matter, space and vacuum) differently.

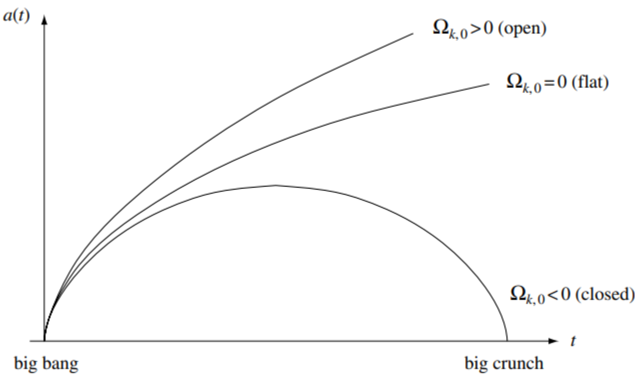

Figure 2.1: The effect of scale factor and density on the fate of the Universe

To relate the proper distance between two moving objects, the scale factor acts as:

$$d(t) = a(t)d_0 \tag{2.2}$$

Here, $d(t)$ is the proper distance at the current time t while d_0 is the distance at an initial reference time t_0. This gives us:

$$d_0 = d(t_0) \tag{2.3}$$

which results in $a(t_0) = 1$, according to 2.2.

However, if we consider the accelerating expansion of the Universe, it would mean that the first time-derivative of the scale factor, $\dot{a}(t)$, is increasing over time. This is because the space between the objects increase faster with time, so an object observed later would be moving with a smaller velocity than it had when it was observed earlier.

Thus, if an object emits light at a redshift of z, then the scale factor at the time the object was initially observed would become:

$$a = \frac{1}{1+z} \tag{2.4}$$

This is the scale factor considered when modeling an expanding Universe with the FLRW metric.

2.4 Flatness Problem

Fine-tuning is the precise adjustment of parameters in a given model to produce observational results, without a specific known mechanism.

The **Flatness problem** is an example of a fine-tuning problem, where certain parameters at the beginning of the Universe take very specific values. Changing these values, even slightly, alters the appearance of the Universe drastically.[6]

	ω_i	$\rho_i(t)$	Ω_i *(density parameter)*
Matter	$\omega_m = 0$	$\rho_m(t) = \rho_{m,0}\left[\frac{R_0}{R(t)}\right]^3$	$\Omega_m(t) = \frac{8\pi G}{3H^2(t)}\rho_m(t) = \rho_{m,0}\frac{8\pi G}{3H^2(t)}\left[\frac{1}{a(t)}\right]^3$
Radiation	$\omega_r = {}^1/_3$	$\rho_r(t) = \rho_{r,0}\left[\frac{R_0}{R(t)}\right]^4$	$\Omega_r(t) = \frac{8\pi G}{3H^2(t)}\rho_r(t) = \rho_{r,0}\frac{8\pi G}{3H^2(t)}\left[\frac{1}{a(t)}\right]^4$
Vacuum	$\omega_\Lambda = -1$	$\rho_\Lambda(t) = \rho_{\Lambda,0} = \frac{\Lambda c^2}{8\pi G}$	$\Omega_\Lambda(t) = \frac{8\pi G}{3H^2(t)}\rho_\Lambda(t) = \rho_{\Lambda,0}\frac{\Lambda c^2}{3H^2(t)}$

Figure 2.2: Density Parameters of the components of cosmological fluid

The dimensionless **scale factor**, $a(t)$, parameterizes the relative expansion of the Universe. As mentioned previously, these parameters are the mass densities of matter (m), radiation (r) and vacuum (Λ). Each component is modeled as a perfect fluid, given by the general equation of state:

$$p_i = \omega_i \rho_i c^2 \tag{2.5}$$

To satisfy the weak energy condition, the equation of state parameter ω_i must be constant. We obtain the **density parameters** (a function of the cosmic time t for all three components:
The **cosmological field equations** are as such:

$$\begin{aligned} \ddot{R} &= -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)R + \frac{1}{3}\Lambda c^2 R \\ \dot{R}^2 &= \frac{8\pi G^2}{3} + \frac{1}{3}\Lambda c^2 R^2 - c^2 k \end{aligned} \tag{2.6}$$

Once we substitute the densities into the second equation, we get:

$$\Omega_{\rm m} + \Omega_{\rm r} + \Omega_\Lambda - \frac{c^2 k}{H^2 R^2} = 1 \tag{2.7}$$

having taken the Hubble parameter, $H = \frac{\dot{R}}{R}$. We now define the **curvature density parameter** as:

$$\begin{aligned} \Omega_k(t) &= -\frac{c^2 k}{H^2(t) R^2(t)} \\ &= -\frac{k}{H^2 a^2} \end{aligned} \tag{2.8}$$

thus obtaining the relation:

$$\Omega_{\rm m} + \Omega_{\rm r} + \Omega_{\Lambda} + \Omega_k(t) = 1 \tag{2.9}$$

where all the quantities are based on cosmic time, t.

The spatial curvature, k, determines the value of Ω_k. Depending on the value of k, we face three scenarios:

$$\begin{gathered}
\text{Open case: if } k = -1\text{: } \Omega_{\rm m} + \Omega_{\rm r} + \Omega_{\Lambda} < 1 \\
\text{Flat case: if } k = 0\text{: } \Omega_{\rm m} + \Omega_{\rm r} + \Omega_{\Lambda} = 1 \\
\text{Closed case: if } k = +1\text{: } \Omega_{\rm m} + \Omega_{\rm r} + \Omega_{\Lambda} > 1
\end{gathered}$$

Thus, to understand the Flatness problem, we look into the curvature density. We obtain its time derivative:

$$\begin{aligned}
\dot{\Omega}_k(t) &= -\frac{2k\dot{H}}{H^3(t)a^2(t)} - \frac{2k\dot{a}}{H^2(t)a^3(t)} \\
&= -\frac{2k}{H^2a^2}\left(\frac{\dot{H}}{H} + \frac{\dot{a}}{a}\right) \\
&= -2\Omega_k(t)\left(\frac{\dot{H}}{H} + \frac{\dot{a}}{a}\right)
\end{aligned} \tag{2.10}$$

taking $H = \dot{a}/a$ now, where a is just the scale factor.
The evolution of this scale factor with respect to time is illustrated in the Friedmann equations:

$$\begin{aligned}
\left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi Gp}{3} - \frac{k}{a^2} \\
\left(\frac{\ddot{a}}{a}\right) &= \frac{4\pi G}{3}(p + 3P)
\end{aligned} \tag{2.11}$$

2.5 Horizon Problem

From the existence of the first singularity up to the formation of the first atoms, signals that result in such occurrences were initially thought to have a finite distance over which they can travel within a given time period. But the anisotropies that we can observe in the cosmic microwave background radiation far exceeds that distance limit.
The **Horizon Problem** is that these "signals" could not have traveled distances so large within the time of the creation of the first atoms. Thus a more favoured theory arose: that while the Universe had an initial state that was inhomogeneous, anisotropic and curved at the beginning, quasi-de Sitter evolution occured and resulted in the homogeneous, isotropic and relatively flat Universe we observe today.

The Friedmann-Robertson-Walker (FRW) metric is a spatially flat metric which describes the Universe on large scales [2]:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}x^2 \tag{2.12}$$

If the metric is written in terms of conformal time (τ), we obtain:

$$\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + dx^2\right] \tag{2.13}$$

$$\Delta\tau = \int_0^t \frac{\mathrm{d}t'}{a(t')} = \int_0^a \frac{\mathrm{d}\ln a}{aH} \tag{2.14}$$

Considering that the photons travel along null geodesics where $ds^2 = 0$, we obtain:

$$\mathrm{d}\chi^2 = \pm\frac{\mathrm{d}t}{a(t)} \tag{2.15}$$

We can now find the comoving distance a photon would have to travel to reach modern observers:

$$\chi(z) = \int_{t_{em}}^{t_0} \frac{dt}{a(t)} \tag{2.16}$$

$$\chi(z) = \int_{a_{em}}^{1} \frac{1}{a^2} H(a)\, da \tag{2.17}$$

This will help us find the comoving distance a photon emitted during the time of recombination would have to travel, where the Hubble Parameter was introduced into the above integral.

Now accounting for the scale factor $a = \frac{1}{1+z}$, we get:

$$\chi(z) = \int_0^z \frac{1}{H(z)}\, dz \tag{2.18}$$

From above, we will now discuss the Hubble parameter. It has been denoted as:

$$H = \frac{\dot{a}}{a} = \frac{da}{dt}\frac{1}{a} \tag{2.19}$$

after which, we account for an evolution of the parameter as such:

$$\left(\frac{H(z)}{H_0}\right)^2 = \Omega_{\mathrm{m},0}(1+z)^3 + \Omega_{\Lambda,0} \tag{2.20}$$

We consider the density values for the matter-dominated era since recombination epoch falls within that time:

$$\begin{gathered}\left(\frac{H(z)}{H_0}\right)^2 = \Omega_{\mathrm{m},0}(1+z)^3 \\ H(z) = H_0\sqrt{\Omega_{\mathrm{m},0}}(1+z)^{3/2}\end{gathered} \tag{2.21}$$

Introducing the above into our integral for the comoving distance, we get:

$$\chi(z) = \frac{1}{H_0\sqrt{\Omega_{\mathrm{m},0}}} \int_0^z \frac{1}{(1+z)^{3/2}}\, dz \tag{2.22}$$

When it is solved within the bounds, we obtain an expression for the comoving distance given by:

$$\chi(z) = \frac{2}{H_0\sqrt{\Omega_{\mathrm{m},0}}}\left(1 - \frac{1}{\sqrt{1+z}}\right) \tag{2.23}$$

and when we consider that the redshift $z \gg 1$, the solution simplifies to:

$$\chi(z) \approx \frac{2}{H_0\sqrt{\Omega_{\mathrm{m},0}}} \tag{2.24}$$

We want to show that the observable regions could not have been within the limits of the comoving distance. If these regions are too far for causal signals to travel within the time frame, then it would suggest that space was not static but was undergoing inflation.

We consider the angular diameter d_A the transverse distance across the Universe and express it in terms of the comoving distance:

$$d_A(z) = \frac{\chi(z)}{(1+z)} \tag{2.25}$$

$$d_A(z) \approx \frac{2}{H_0\sqrt{\Omega_{\mathrm{m},0}}}\frac{1}{(1+z)} \tag{2.26}$$

During recombination, the size of the Universe can be expressed as D, given below with respect to the transverse distance:

$$\begin{aligned} D &= \frac{1}{1+z}\int_z^\infty \frac{1}{H(z)}\mathrm{d}z \\ &\approx \frac{1}{1+z}\frac{2}{H_0\sqrt{\Omega_\mathrm{m}}}\left[\frac{1}{\sqrt{1+z}}\right]_z^\infty \\ &\approx \frac{2}{H_0\sqrt{\Omega_\mathrm{m}}}\frac{1}{(1+z)^{3/2}} \end{aligned} \tag{2.27}$$

Now we designate two observable regions in the night sky, and find the angle between them from our point of observation:

$$\delta\theta = \frac{D}{d_A} \tag{2.28}$$

Finally we account for the redshift and take the value of $z = 1100$ to obtain:

$$\delta\theta = \frac{1}{\sqrt{(1+z)}} = 0.03\mathrm{rad} \tag{2.29}$$

This value of 0.03 rad or approximately 1.7 suggests that any regions in the sky further than this could not have been close enough to allow exchange of information, i.e. the comoving distance would be too great at the recombination period for the CMB to have uniform temperature as we observe today. This problem leads us to the realization of inflation being a key occurrence even during the primordial phase of the Universe.

2.6 The Inflaton Field

Unlike vector fields such as the electric and magnetic fields, scalar fields do not have internal degrees of freedom. The **inflaton field**, $\phi(\mathbf{x}, t)$, is one such field, which is included to theorize how an inflationary phase arose in the early Universe.

The potential of the inflaton field is denoted by $V(\phi) = \frac{1}{2}m^2\phi^2$. m is the mass of the field, which basically implies that, when the field is quantised, m denotes the mass of particles produced.

For an expanding FRW metric in Minkowski spacetime, the action of this field is given by [7]:

$$S = \int d^3x\, dt \left[\frac{1}{2}\dot{\phi}^2 - \frac{c^2}{2}\nabla\phi \cdot \nabla\phi - V(\phi)\right] \tag{2.30}$$

It is important to find the equations of motion for the inflaton ϕ that are consistent with the principle of least action. We implement a transformation to the inflaton $\phi \to \phi + \delta\phi$ and the action varies as such:

$$\begin{aligned} \delta S &= \int d^3x\, dt \left[\dot{\phi}\delta\dot{\phi} - c^2\nabla\phi \cdot \nabla\delta\phi - \frac{\partial V}{\partial\phi}\delta\phi\right] \\ &= \int d^3x\, dt \left[-\ddot{\phi} + c^2\nabla^2\phi - \frac{\partial V}{\partial\phi}\right]\delta\phi \end{aligned} \tag{2.31}$$

The equation of motion is then obtained by taking $\delta S = 0$:

$$\ddot{\phi} - c^2\nabla^2\phi + \frac{\partial V}{\partial\phi} = 0 \tag{2.32}$$

which is basically the Klein-Gordon equation.
In order to generalize the inflaton action :

$$S = \int d^3x\, dt\, a^3(t) \left[\frac{1}{2}\dot{\phi}^2 - \frac{c^2}{2a^2(t)}\nabla\phi \cdot \nabla\phi - v(\phi)\right] \tag{2.33}$$

$$S = \int d^3x\, dt\, a^3(t) \left[\frac{1}{2}\dot{\phi}^2 - V(\phi)\right] \tag{2.34}$$

The variation of the action now becomes:

$$\begin{aligned} \delta S &= \int d^3x\, dt\, a^3(t) \left[\dot{\phi}\delta\dot{\phi} - \frac{\partial V}{\partial\phi}\delta\phi\right] \\ &= \int d^3x\, dt \left[-\frac{d}{dt}\left(a^3\dot{\phi}\right) - a^3\frac{\partial V}{\partial\phi}\right]\delta\phi \end{aligned} \tag{2.35}$$

Equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial\phi} = 0 \tag{2.36}$$

2.7 Slow-Roll Inflation

Slow-roll inflation couples an inflaton field ϕ to gravity, where $V(\phi)$ is an arbitrary inflaton potential. The action is given by [7]:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{pl}}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right] \tag{2.37}$$

In the equation of motion (2.36), the term $3H\dot{\phi}$ is the **Hubble drag**.We now consider this Friedman equation:

$$H^2 = \frac{8\pi G}{3c^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \tag{2.38}$$

We take the time derivative of this equation to get:

$$\begin{aligned} 2H\left(\frac{\ddot{a}}{a} - H^2\right) &= \frac{8\pi G}{3c^2}\left(\ddot{\phi} + \frac{\partial V}{\partial \phi}\right) \\ &= -\frac{8\pi G}{c^2}H\dot{\phi}^2 \\ &= -\frac{1}{M_{\mathrm{pl}}}H\dot{\phi}^2 \end{aligned} \tag{2.39}$$

After rearranging the terms in (2.39) we get the Raychaudhuri equation:

$$\frac{\ddot{a}}{a} = -\frac{1}{3M_{\mathrm{pl}}^2}\left(\dot{\phi}^2 - V(\phi)\right) \tag{2.40}$$

Now we can think of a model, where we begin with a scalar field with small $\dot{\phi}$ at the top of a potential. This scalar rolls down the potential, gains kinetic energy and is only held back from eternal oscillation due to the presence of the Hubble drag. As it gains the kinetic energy and stops at some point, it leaves the inflationary period. We now set $V(\phi) \gg \frac{1}{2}\dot{\phi}^2$ - this is our slow roll condition. For slow roll inflation to occur, the potential must have a value within this limit. Using this, we can now find that the Friedman equation becomes:

$$H^2 \approx \frac{8\pi G}{3c^2}V(\phi) \tag{2.41}$$

We want inflation to last for a longer period of time, which means the scalar must not accelerate. To ensure no resultant force drives the scalar to accelerate, the Hubble drag must have the larger value in equation (2.36). This is called the over-damped regime, where $\ddot{\phi} \ll H\dot{\phi}$. The equation of motion now takes the form:

$$3H\dot{\phi} \approx -\frac{\partial V}{\partial \phi} \tag{2.42}$$

Inputting a potential $V = \frac{1}{2}m^2\phi^2$, we can solve the adjusted Friedman equation as well as the new equation of motion to find:

$$\begin{aligned} H &= \alpha\phi \\ \dot{\phi} &= -\frac{m^2}{3\alpha} \\ \alpha^2 &= \frac{4\pi G m^2}{3c^2} \end{aligned} \tag{2.43}$$

Integrating $\dot{\phi}$ over time gives us:

$$\phi(t) = \phi_0 - \frac{m^2}{3\alpha}t \tag{2.44}$$

At $t = 0$, the scalar field begins rolling from an initial value of ϕ_0 and $H = \alpha\phi$ can be integrated to find the scale factor $a(t)$ during the phase of nearly exponential expansion (the quasi-de Sitter phase):

$$a(t) = a(0)exp\left[\frac{2\pi G}{c^2}(\phi_0^2 - \phi(t)^2)\right] \tag{2.45}$$

Long as $V(\phi) \gg \dot{\phi}^2$, (2.45) holds up and inflation proceeds. At the end of the inflationary period t_{end}, where $V(\phi)$ approaches the value of $\dot{\phi}$, inflation will halt. After the phase of inflation, the universe will be larger that before by a factor of:

$$\frac{a(t_{end})}{a(0)} \approx exp\left[\frac{2\pi G\phi_0^2}{c^2} - \frac{1}{3}\right] \tag{2.46}$$

Since the scalar field was positioned higher up the potential, it could roll for longer and the expansion was also substantially larger.

2.8 Duality and Inflation

When the symmetries of string cosmology are applied to the standard cosmological equations, we can get duality-based solutions that compare to the standard scenario solutions. An inflationary evolution is introduced to resolve the naturalness problem, which is shown by the relation [5]:

$$r(t) = (aH)^{-1} \tag{2.47}$$

This function decreases as time t increases. Thus, if we consider a scale factor where $a(t) \sim t^\beta$ over the period of inflation, we reach the condition:

$$r(t) = (\dot{a})^{(-1)} \sim t^{1-\beta} \to 0 \tag{2.48}$$

2.8.1 Class I

This class of inflationary solutions work over the range of cosmic-time coordinates where t is positive. It is parameterized by:

$$a \sim t^\beta \tag{2.49}$$

where t is cosmic time and a is the power law scale factor, given the conditions:

$\beta > 1$ $\quad$ $t > 0$ $\quad$ $t \to +\infty$

Power Inflation

Power inflation is characterized by:

- accelerated expansion

$$\dot{a} \sim \beta t^{\beta-1} > 0, \qquad \ddot{a} \sim \beta(\beta-1)t^{\beta-2} > 0$$

- decreasing curvature

$$H = \frac{\dot{a}}{a} = \frac{\beta}{t} > 0, \qquad \dot{H} = -\frac{\beta}{t^2} < 0$$

Therefore, this model illustrates the accelerated evolution from a region of higher curvature to that of a lower curvature, similar to slow-roll inflation.

de Sitter Inflation

In de Sitter inflation, a limiting case $\beta \to \infty$ is introduced:

$$a \sim e^{kt} \tag{2.50}$$

where $t > 0$ and $t \to +\infty$. de Sitter inflation shows:

- accelerated expansion

$$\dot{a} \sim ke^{kt} > 0, \qquad \ddot{a} \sim k^2 e^{kt)} > 0$$

  where curvature $k =$ constant.

- constant curvature, which implies that the horizon does not grow in size either.

Unlike the power inflation case, where the curvature can take different values, in the de Sitter case the curvature remains at a constant value. Thus, $\dot{H} = 0$
Both of the cases satisfy the condition (2.46) as they concur that accelerated expansion is occurring over time.

2.8.2 Class II

This class of inflationary solutions work over the range of cosmic-time coordinates where t is negative. It is parameterized by:

$$a \sim (-t)^{\beta} \tag{2.51}$$

where t is cosmic time and a is the power law scale factor, given the conditions:

$$\beta < 1 \qquad t < 0 \qquad t \to 0_{-}$$

Super Inflation

Also consistent with the idea of the background geometry undergoing accelerated expansion, we obtain:

- accelerated expansion

$$\dot{a} \sim -\beta(-t)^{\beta-1} > 0, \qquad \ddot{a} \sim \beta(\beta-1)(-t)^{\beta-2} > 0$$

- increasing curvature

$$H = -\frac{\beta}{(-t)} > 0, \qquad \dot{H} = -\frac{\beta}{t^2} > 0$$

We can once again realize that the condition (2.46) is fulfilled.

Accelerated Contraction

For values of β such that $0 < \beta < 1$, we get:

- accelerated contraction

$$\dot{a} \sim -\beta(-t)^{\beta-1} < 0, \qquad \ddot{a} \sim \beta(\beta-1)(-t)^{\beta-2} < 0$$

- increasing curvature

$$H = -\frac{\beta}{(-t)} < 0, \qquad \dot{H} = -\frac{\beta}{t^2} < 0$$

Since Class II backgrounds are defined such that t is negative, the evolution varies from low to high curvature which is reminiscent of going back in time from the Big Bang singularity.

Chapter 3

String Theory and Branes

String Theory: a theory where elementary particles are assumed to behave like rotating strings, instead of point particles. The ultimate goal of string theory is to unite the concepts of quantum field theory to those of general relativity in order to achieve a comprehensive, universal description of fundamental physics. The scope of string theory is broad, thus, in the context of this paper, only relevant topics shall be brought forward.

String theory models are based on the assuming the existence of segments of vibrating strings instead of elementary point-like particles like electrons or quarks.

Each vibration mode present on a string then corresponds to a different particle and determines its intrinsic features such as mass and charge.

These strings are described to be in a system constituting of 10 or 11 dimensions, where 6 or 7 dimensions are curled up. While we can generalize the larger dimensions in this scenario to just be space and time dimensions, the "extra dimensions" become crucial in characterizing the universe and everything in it as we know it. [8]

Strings may form a segment with two endpoints, where these endpoints do not meet: this is called an **open string**. If the end points meet and the string forms a closed loop, it is called a **closed string**.

From quantum field theory, we recall [8]:

- **Fermionic field**: a quantum field where the quanta are called fermions. They obey Fermi–Dirac statistics. Fermionic fields obey canonical anticommutation relations. e.g. the **Dirac field** which describes fermions with spin-1/2 (electrons, protons) is a fermionic field and it can be described as either a 4-component spinor or as a pair of 2-component Weyl spinors.

- **Bosonic field**: a quantum field where the quanta are called bosons. They obey Bose-Einstein statistics. Bosonic fields obey canonical commutation relations.

In superstring theory, when we consider closed strings, the fermionic fields can be periodic or anti-periodic on the path around the string. The periodicity depends on what coordinates have been picked on the worldsheet.

To start off, we consider a couple of **boundary conditions**.

- **Ramond boundary conditions** which accounts for periodic conditions in the w-frame,
- **Neveu–Schwarz boundary conditions** which account for anti-periodic conditions in the w-frame.

The conditions reverse in the case of the z-frame for closed strings.
The Neveu–Schwarz sector and Ramond sector are also defined in the open string. They now depend on the boundary conditions of the fermionic field at the edges of the open string [10].
Ramond (R) Boundary Condition:

$$\Psi^i{}_+\left(\frac{\pi}{2},\tau\right) = \Psi^i{}_-\left(\frac{\pi}{2},\tau\right) \tag{3.1}$$

yields the even-mode expansions:

$$\Psi^i{}_+(\sigma,\tau) = \sum_{n:even} d^i_n e^{-in(\tau+\sigma)} \tag{3.2}$$

$$\Psi^i{}_-(\sigma,\tau) = \sum_{n:even} d^i_n e^{-in(\tau-\sigma)} \tag{3.3}$$

Neveu-Schwarz (NS) Boundary Condition:

$$\Psi^i{}_+\left(\frac{\pi}{2},\tau\right) = -\Psi^i{}_-\left(\frac{\pi}{2},\tau\right) \tag{3.4}$$

yields the odd-mode expansions:

$$\Psi^i{}_+(\sigma,\tau) = \sum_{r:odd} b^i_r e^{-ir(\tau+\sigma)} \tag{3.5}$$

$$\Psi^i{}_-(\sigma,\tau) = \sum_{r:odd} b^i_r e^{-ir(\tau-\sigma)} \tag{3.6}$$

The above conditions restrict the oscillators to modes such that we end up with a massive ground state for R and a massless ground state for NS.

3.1 Calabi-Yau manifolds

Compact, complex Kähler manifolds that accomodate the tiny, hidden extra dimensions are called Calabi-Yau manifolds. They exhibit SU(d) holonomy and can be represented as having a Ricci-flat metric [11]:

$$R_{i\bar{j}} = -\frac{\partial}{\partial \bar{z}^{\bar{j}}}\Gamma^k_{ik} = 0 \tag{3.7}$$

The trace of $U(1)$ vanishes. This is responsible for the holonomy lying in $SU(d)$ and thus plays an important role in providing the Ricci-flat metric.

3.2 Branes

A **brane** is defined as a physical object that generalizes the idea of a point particle to higher dimensions and can propagate through spacetime.
An **antibrane** is an extended object which has the same tension as the corresponding brane, but with opposite Ramond-Ramond (R-R) charge. [12]

3.2.1 p-branes

"P-brane" is a term that was given by M. J. Duff et al. in 1988. The "brane", shortened from the word "membrane", now refers to a two-dimensional brane while a p-dimensional brane is generally called "p-brane". A p-brane sweeps out a (p+1)-dimensional volume in spacetime called its worldvolume. Physicists often study fields analogous to the electromagnetic field, which live on the worldvolume of a brane.

3.2.2 D-branes and Boundary Conditions

Dirichlet membranes, or D-branes, are a class of extended objects in ten dimensional string theory, where if open strings end, they obey the **Dirichlet boundary conditions**:

$$\delta X^M(\tau, \overline{\sigma}) = 0$$

D_p-branes depict branes of p spatial dimensions.

D_0 - brane is a point particle.

D_1 - brane is a string.

D_2 - brane is a membrane.

Endpoints of an open string propagating through spacetime must lie on the surfaces of D-branes. The dynamics of the worldvolume of a D-brane has $(p+1)$ dimensions including the temporal dimension, and can be described by a gauge theory. The Ramond-Ramond (R-R) potential C_{p+1} charges the D_p-brane; the electrical coupling is given by the Chern-Simons action:

$$S_{\text{CS}} = \mu_p \int_{\sum_{p+1}} C_{p+1} \tag{3.8}$$

Here, μ_p denotes the brane charge and $\sum_{p+1}$ is the worldvolume of the D_p-brane. In Type II string theory, there are two cases:

- Type IIA string theory features stable D_p-branes, where $p =$ even.
- Type IIB string theory also has stable D_p-branes, where $p =$ odd.

These branes can be pictured as a result of a Kaluza–Klein compactification of the eleven dimensional M-theory. Addition of a single $U(1)$ vector field accomodates this theory in the string action. The presence of more gauge fields (like in the case of $SU(2)$) would then theoretically link to higher dimensional theories that have been compactified.

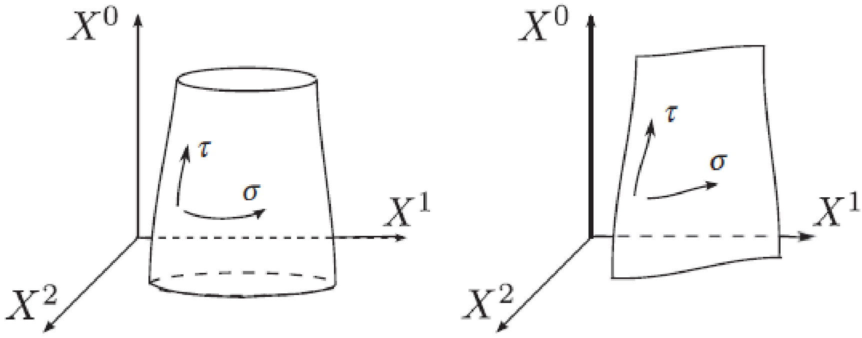

Figure 3.1: In a 3-dimensional target spacetime, the embedding of a closed string makes a cylindrical worldsheet, and an open string makes a strip-like worldsheet.

3.3 Bosonic String Theory

String Theory is based on considering one-dimensional extended objects (strings) as the fundamental objects. While a point-particle can only map a worldline, a string can map a (1+1) dimensional manifold called a **worldsheet**, Σ , in spacetime. [9]

Σ is parametrized by:

τ - proper time

σ - spatial length of the string

The worldsheet of the string is embedded into a D-dimensional Minkowski target spacetime, and is characterized by the function $X^M(\tau, \sigma)$.

Thus, we get the **Nambu-Goto string action**, which is parametrisation invariant:

$$S_{NG} = -\frac{1}{2\pi\alpha'}\int_\Sigma \mathrm{d}^2\sigma\sqrt{-\det(\partial_\alpha X^M \partial_\beta X^N \eta_{NM})} \tag{3.9}$$

where

η_{NM} is the Minkowski spacetime metric

$\mathrm{d}^2\sigma = \mathrm{d}\sigma^0\mathrm{d}\sigma^1$

l_s is string length. The relation used is $l_s^2 = \alpha'$

The tension τ of the string is $\tau_{F1} = 1/2\pi\alpha'$

Since the quantisation of the theory through Nambu-Goto action is very difficult, we bring in an auxiliary field $h_{\alpha\beta}(\sigma)$ which is a worldsheet metric. The dynamics of a bosonic string propagating through D-dimensional Minkowski spacetime is then shown by the **Polyakov string action**:

$$S_P = -\frac{1}{4\pi\alpha'}\int_\Sigma \mathrm{d}^2\sigma\sqrt{-h}h^{\alpha\beta}(\partial_\alpha X^M \partial_\beta X^N \eta_{NM}) \tag{3.10}$$

The existence of the energy-momentum tensor $T_{\alpha\beta}$ on the worldsheet puts Virasoro constraints on the dynamical fields X^M when $T_{\alpha\beta} = 0$. The Polyakov action also preserves a number of symmetries:

- **Poincaré transformations:** The Polyakov action (3.10) is invariant under Lorentz transformations and translations to the spacetime vector X^M in the target spacetime.

$$X^M \mapsto X'^M = \underbrace{\Lambda^M_N}_{\text{Lorentz Transformation}} X^N + \overbrace{a^M}^{\text{Spacetime Translation}}$$

- **Diffeomorphism:** Given a reparametrisation to the worldsheet: $\sigma^\alpha \mapsto \sigma'^\alpha = f^\alpha(\sigma)$, the following transformations occur:

$$h_{\alpha\beta}(\tau, \sigma) = \frac{\partial f^\gamma}{\partial \sigma^\alpha}\frac{\partial f^\delta}{\partial \sigma^\beta} h_{\gamma\delta}(\tau', \sigma')$$

$$X^M(\tau, \sigma) = X'^M(\tau', \sigma')$$

  The Polyakov action (3.10) is invariant under these reparametrisation transformations.

- **Weyl Transformation:** We consider a conformal gauge with a diagonal worldsheet metric:

$$h_{\alpha\beta} = e^{2\omega(\tau,\sigma)}\eta_{\alpha\beta} \tag{3.11}$$

  where the metric is $\eta = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. The Polyakov action (3.10) should be invariant under the transformations:

$$\begin{aligned} h_{\alpha\beta} &\mapsto e^{2\omega(\tau,\sigma)} h_{\alpha\beta}(\tau, \sigma) \\ X'^M(\tau, \sigma) &= X^M(\tau, \sigma) \end{aligned} \tag{3.12}$$

  Now that the gauge is considered, we can obtain the Polyakov action in a form consistent with the conformal gauge:

$$S_P = \frac{1}{4\pi\alpha'} \int \mathrm{d}^2\sigma (\partial_\tau X^M \partial_\tau X^N - \partial_\sigma X^M \partial_\sigma X^N)\eta_{MN} \tag{3.13}$$

  Upon including light cone coordinates ($\sigma^\pm = \tau \pm \sigma$) and derivatives of the form $\partial_\pm = \partial/\partial\sigma^\pm$, we obtain a relativistic wave equation:

$$(\partial_\tau^2 - \partial_\sigma^2)X^M = \partial_+\partial_- X^M = 0 \tag{3.14}$$

  A boundary condition, $\partial_\sigma X^M \delta X_M|_0^{\sigma_0} = 0$ supplements the equation of motion, and the Virasoro constraints are also included.

3.4 Superstrings

Superstring theory includes supersymmetry in string theory in order to model gravity. While bosonic string theory accounts only for bosons, superstring theory extends to both fermions and bosons. [13]
Type I is based on unoriented open and closed strings with one supersymmetry in the ten-dimensional sense (16 supercharges). We shall discuss type II in further detail below.

3.4.1 Type II Superstrings

In the ten-dimensional scenario, Type II string theories have two supersymmetries, and hence 32 supercharges. While IIA theory is parity conserving (non-chiral), the IIB theory is parity violating (chiral).
Type IIA and type IIB string theory are related by T-duality, a type of duality symmetry, that implies all five superstring theories are different versions of an eleven-dimensional theory; the M-theory.

Type IIA Superstrings

Eleven-dimensional supergravity has maximum Poincaré invariance as well as space-time supersymmetry. Type IIA supergravity arises after the compactification of 11-D SUGRA on S^1. It has two Majorana-Weyl spinors with opposing chiralities, and 32 real supercharges. This theory accomodates two bosonic fields, with 128 bosonic states. The bosonic action is [8]:

$$2\kappa^2{}_{11}S_{11} = \int d^{11}x\sqrt{-G}(R - \frac{1}{2}|F^4|^2) - \frac{1}{6}\int A_3 \wedge F_4 \wedge F4 \tag{3.15}$$

where G_{MN} is the metric and F_4 is the field strength of the 3-form potential A_{MNP}, which is denoted by A_3.
Since the algebra of this theory is ideal, we can obtain an action for type IIA theory by dimensional reduction of the action (3.15).
First, we reconfigure the metric.
The metric G_{MN} corresponds to the earlier eleven-dimensional theory, while we obtain a new metric, $G_{\mu\nu}$, which is ten-dimensional to fit the IIA theory. [8]

$$\begin{aligned} ds^2 &= G^{11}_{MN}(X^\mu)dx^M dx^N \\ &= G^{10}_{\mu\nu}(x^\mu)dx^\mu dx^\nu \\ &+ exp(2\sigma(x^\mu))[dx^{10} + A_\nu(x^\mu)dx^\nu]^2 \end{aligned} \tag{3.16}$$

The metric (3.16) has been reduced to a ten-dimensional metric, an A_1 gauge field and a scalar σ.
The type IIA action is given by:

$$S_{\text{IIA}} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}} \tag{3.17}$$

For fields in the Neveu-Schwarz sector:

$$S_{\text{NS}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left(R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}|H_3|^2\right) \tag{3.18}$$

In the above equation, Φ represents the dilaton while the H_3 is the field strength: $H_3 = \text{d}B_2$. The B_2 can be identified as a 2-form gauge potential.
For fields in the Ramond sector:

$$S_{\text{R}} = -\frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left(|F_2|^2 + |\tilde{F}_4|^2\right) \tag{3.19}$$

The Ramond fields have a 1-form gauge potential and the field strength is derived with respect to this gauge potential C_1. So, the field strength terms in (3.19) are basically $F_2 = \mathrm{d}C_1$ and $F_4 = \mathrm{d}C_3$.
The fields in both sectors, given by the Chern-Simons action:

$$S_{\text{CS}} = -\frac{1}{4\kappa_{10}^2} \int B_2 \wedge F_4 \wedge F_4 \tag{3.20}$$

Thus the type IIA action can be written as:

$$\begin{aligned} S_{\text{IIA}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} e^{-2\Phi} \left(R + 4(\partial\Phi)^2 - \frac{1}{2}|H_3|^2 \right) \\ - \frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left(|F_2|^2 + |\tilde{F}_4|^2 \right) - \frac{1}{4\kappa_{10}^2} \int B_2 \wedge F_4 \wedge F_4 \end{aligned} \tag{3.21}$$

Type IIB Superstrings

The type IIB action is given by:

$$S_{\text{IIB}} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}} \tag{3.22}$$

For fields in the Neveu-Schwarz sector (the same as in IIA):

$$S_{\text{NS}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} e^{-2\Phi} \left(R + 4\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2}|H_3|^2 \right) \tag{3.23}$$

For fields in the Ramond sector:

$$S_{\text{R}} = -\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2 \right) \tag{3.24}$$

The fields in both sectors, given by the Chern-Simons action:

$$S_{\text{CS}} = -\frac{1}{4\kappa_{10}^2} \int C_4 \wedge H_3 \wedge F_3 \tag{3.25}$$

Adding the terms gives us the type IIB action:

$$\begin{aligned} S_{\text{IIB}} = \frac{1}{2\kappa^2} \int d^{10}X \sqrt{-G} e^{-2\Phi} \left[R + 4(\partial\Phi)^2 - \frac{1}{2}|H_3|^2 \right] \\ - \frac{1}{4\kappa^2} \int d^{10}X \sqrt{-G} \left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2 \right) - \frac{1}{4\kappa^2} \int C_4 \wedge H_3 \wedge F_3 \end{aligned} \tag{3.26}$$

and here we define:

$$\begin{aligned} \tilde{F}_3 &= F_3 - C_0 \wedge H_3 \\ \tilde{F}_5 &= F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3 \end{aligned}$$

For $\tilde{F}_5$, the equation of motion and Bianchi identity are given by:

$$d * \tilde{F}_5 = d\tilde{F}_5 = H_3 \wedge F_3 \tag{3.27}$$

A constraint on the solution is:

$$*\tilde{F}_5 = \tilde{F}_5 \tag{3.28}$$

in order to keep them consisted with the spectrum of the IIB string (degrees of freedom must be self dual). To apply Weyl rescaling to the action (3.26), we choose the Einstein frame metric:

$$G_{\mathrm{E}_{\mu\nu}} = e^{-\Phi/2} G_{\mu\nu} \tag{3.29}$$

The axio-dilaton τ is defined as:

$$\tau = C_0 + ie^{-\Phi} \tag{3.30}$$

And we let:

$$\begin{aligned} \mathrm{M}\ _{ij} &= \frac{1}{\mathrm{Im}\,\tau}\begin{bmatrix} |\tau|^2 & -\mathrm{Re}\,\tau \\ -\mathrm{Re}\,\tau & 1 \end{bmatrix} \\ F_3^i &= \begin{bmatrix} H_3 \\ F_3 \end{bmatrix} \end{aligned} \tag{3.31}$$

We focus on $SL(2,\mathbf{R})$ symmetry, where the kinetic term for τ is invariant and the kinetic term for F_3 transforms as:

$$\mathrm{M}\ ' = (\Lambda^{-1})^T \mathrm{M}\ \Lambda^{-1} \tag{3.32}$$

We obtain the type IIB action [8]:

$$S_{\mathrm{IIB}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G_{\mathrm{E}}}\left(R_{\mathrm{E}} - \frac{\partial_\mu\bar{\tau}\partial^\mu\tau}{2(\mathrm{Im}\,\tau)^2} - \frac{\mathrm{M}\ _{ij}}{2}F_3^i\cdot F_3^j - \frac{1}{4}|\tilde{F}_5|^2\right) - \frac{\epsilon_{ij}}{8\kappa_{10}^2}\int C_4\wedge F_3^i\wedge F_3^j \tag{3.33}$$

3.5 de Sitter vacua

Compactified dimensions in string theory had to be stabilized in order for inflation to occur. In type IIB string theory, we include two canonically normalized fields, given by φ which is the dilaton field, and ρ whcih denotes the volume of the compactified space. Compactification would result in an effective potential within the effective 4D theory [14]:

$$V(\varphi,\rho,\phi) \sim \exp(\sqrt{2}\varphi - \sqrt{6}\rho)\tilde{V}(\phi) \tag{3.34}$$

$\tilde{V}(\phi)$ works towards inflation if the fields φ and ρ remain constant, but this is unlikely since the exponent in the **above** effective potential drives dilaton to $-\infty$ and ρ is consequently pushed to $+\infty$. [15]

Chapter 4

Inflation in String Theory

In the previous chapters, we covered inflation and the basic premise of how it came to be a sustaining theory and why it is important. However, we want to establish a model that is consistent from the microscopic to the macroscopic scales. In this scenario, we utilize the ideas of string theory and incorporate quantum fluctuations and superstrings into the pre-existing concepts to find such a model.

4.1 Two Problems

4.1.1 I

It is contradictory for a string to be breaking and bounding a domain wall simultaneously.
For a $D1$ brane that couples to a 4-D massless field $C_{(2)}$ while ending on a $D3$ brane, we can write the effective action in four dimensions [8]:

$$\frac{1}{2}\int |dC_{(2)}|^2 + |G_{(2)}|^2 + G_{(2)} \wedge C_{(2)} \tag{4.1}$$

where the field equation for $C_{(2)}$ is:

$$d *_4 dC_{(2)} = G_{(2)} \tag{4.2}$$

We are keeping only the terms that can affect the local dynamics. $B_{(2)}$ takes the place of the F-string in the scenario of a wrapped D-brane in higher dimensions. The axion field for $C_{(2)}$ is:

$$dC_{(2)} = *_4(d\phi + A_{(1)}) \tag{4.3}$$

Here, $A_{(1)}$ is a brane gauge field, and ϕ undergoes the following non-linear transformation under its gauge transformation:

$$\begin{aligned} \delta A_{(1)} &= d\lambda \\ \delta\phi &= -\lambda \end{aligned} \tag{4.4}$$

The Higgs mechanism removes the axion, thus no domain wall exists. When mass becomes 1, D-string breaking arises because, at infinity, the R-R field exponentially falls. The D-string is thus unstable, and dissolves to become a tube of flux.

4.1.2 II

To resolve the issue with the instability of the D-string, we think of a string with stable topology in association with a broken $U(1)$ gauge symmetry. This string, called " ϕ-string, would have newer degrees of freedom as well as a higher energy scale - the existence of the string would be reliant on the topology of the field space with higher energy.

To produce a ϕ-string, we use the Kibble argument in a vacuum manifold [8]:

$$\frac{U(2) \times U(1)}{U(1) \times U(1)} = S^3 \tag{4.5}$$

where the presence of a $D3 - \overline{D3}$ pair is included to model how it would have been during inflation. However, the problem begins when we account for the nonlinear Higgsing. It leads to breakage of the $U(1)$ denominator. There is once again no stable D-string in this situation because any strings produced in this way will simply fade away as inflation progresses.

4.2 Brane-World Scenario

In the early Universe, the attractive force between branes and antibranes arose as a result of potential energy. This energy was thought to drive inflation, while the annihilation of a brane and antibrane resulted in reheating.

After the realization that models with large extra dimensions could help resolve the heirarchy problems originating from the large discrepancy between the gravitational scale (M_{P}) and the electroweak interaction scale (M_{W}) In $D = (4 + n)$ dimensions, the gravi-dilaton action becomes [16]:

$$S = -\frac{1}{2\lambda_s^{d-2}} \int \mathrm{d}^D x \sqrt{|g_D|} e^{-\phi} [R_D + g^{AB} \partial_A \phi \partial_B \phi] \tag{4.6}$$

4-D Einstein action is given by:

$$-\frac{M_P^2}{2} \int d^4 x \sqrt{|g|} R_4(g) \tag{4.7}$$

Effective gravitational coupling and extra dimensional volume:

$$M_s^{2+n} V_n = g_s^2 M_P^2 \tag{4.8}$$

4.3 The KKLT Model

This model of string inflation was based on disconnecting a Klebanov-Strassler (KS) deformed connifold, then attaching a Calabi-Yau manifold. A $\overline{D3}$ anti-brane is then introduced at the tip of the connifold, which breaks supersymmetry and allows the AdS minimum to be lifted to a dS minimum.

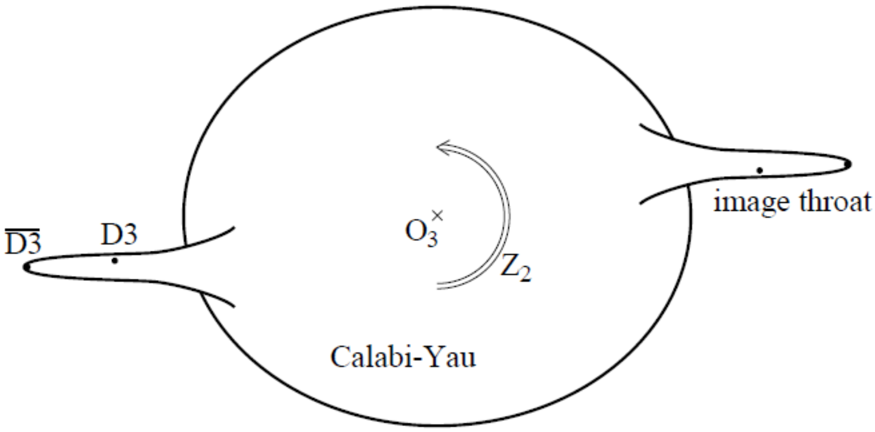

Figure 4.1: A warped Calabi-Yau manifold. $\mathbf{Z}_2$ denotes the orientifold.

The Tadpole condition is given by:

$$\frac{\chi(X)}{24} = N_{D3} + \frac{1}{2\kappa_{10}^2 T_3}\int_M H_3 \wedge F_3 \tag{4.9}$$

where T_3 is the tension of the $D3$ brane, $H3$ and $F3$ are 3-form fluxes arising in the NS and R sector in type IIB theory (as shown in (3.25)) and N_{D3} is the net number of $(D3-\overline{D3})$ branes within the noncompact dimensions. The deforming of this M-theory model to a locus allows it to be thought of as an orientifold of a type IIB CY compactification.

On the assumption that the model has one Kähler modulus, and another modulus frozen due to taking a limit for the F-theory, we confirm the fluxes are non-zero. Then, taking τ as a IIB axiodilaton, the Calabi-Yau moduli have a superpotential [16]:

$$\begin{aligned} W &= \int_M (F_3 - \tau H_3) \wedge \Omega \\ &= \int_M G_3 \wedge \Omega \end{aligned} \tag{4.10}$$

Introducing this into the tree-level Kähler potential gives:

$$V = e^K \left(\sum_{a,b} g^{a\bar{b}} D_a W \overline{D_b W} - 3|W|^2 \right) \to e^K \left(\sum_{i,j} g^{i\bar{j}} D_i W \overline{D_j W} \right) \tag{4.11}$$

Taking the potential over i,j is done to leave out moduli field running over ρ; it is then used to fix moduli at certain values which ensure G_3 to be imaginary self-dual. The presence of fluxes means a warp factor must exist, hence this model is a warped compactification.

4.4 The KKLMMT Model

The first idea was to consider a pair of branes: $D3$ and $\overline{D3}$ in warped geometry. The role of the inflaton field ϕ in this model, which is known as the Kachru-Kallosh-Linde-Maldacena-McAllister-Trivedi (KKLMMT) model, could be played by the

interbrane separation. The separation between the brane and the antibrane is the inflaton. There is a large gravitational redshift at the throat. Upon the annihilation of this inflaton, reheating occurs.
A description of this situation in terms of the effective 4d supergravity involved Kähler potential [17]:

$$k = -3 \log\left(\rho - \overline{\rho} - k(\phi, \overline{\phi})\right) \tag{4.12}$$

The KKLMMT model was derived from the idea of the type IIB string theory on a Calabi-Yau manifold, orientifolded by a $\mathbf{Z}_2$ symmetry. Its isolated fixed points become O3 - planes and the warped spacetime metric can be expressed as:

$$ds^2 = e^{2A(x\perp)}\eta_{\mu\nu}dx^\mu dx^\nu + ds^2 \tag{4.13}$$

Here, $e^{A(x\perp)}$ is normalized and turns into O(1) for most of the manifold.

The annihilation of a D3-brane and an anti-D3-brane, which takes place in the throat, leads to release of energy, some of which goes into reheating. The redshift is accounted for as $e^{A0} \ll 1$ which limits the measurable energy scales by factor of e^{A0}.

With regard to the production of strings, the KS throat in the KKLMMT model does not count since the S^2's topology causes it to collapse and the strings produced disintegrate quickly. We want to obtain strings which are more stable, so we are reliant on the $D1$ brane. Either the existence of branes which comply with the SM fields (i.e. the SM field can exist on the brane) is necessary, or the throat of the model must contain $\overline{D3}$ branes such that the modulus stabilization can allow for a situation where the produced strings can last.

Since we are only concerned with branes that will help in production of stable cosmic strings (i.e. they do not disintegrate throughout the inflationary phase), we want to look at a few instances where the branes are either at the throat or intersect the throat, and what results they may give. [8]

- **No branes located in the throat**: the branes in this situation are outside the throat where inflation occurs. We quickly realize branes within the throat must fluctuate while branes outside the throat cannot aid in production of the strings. However, in the case of $D3$ branes wrapping the S^3 inside the KS throat are baryons that are unsupressed by the warp factor. Since KKLMMT model puts a lower limit on M, it allows a $\overline{D3}$ brane to exist in the throat. Long as $M12$ the antibrane in the throat is stable and the baryon driven decay will not cause the (p, q) strings to become unstable. The baryon mass is given by:

$$\sigma = \overline{T}_{D3} V_{S^3} \tag{4.14}$$

  So that :

$$\overline{T}_{D3} = \frac{1}{(2\pi)^3 \alpha'^2 g_s} \tag{4.15}$$

$$\rho = \overline{\mu}_{p,q} - \overline{\mu}_{p-M,q} \tag{4.16}$$

Warp factor does not suppress this mechanism since the decay takes place in the throat and occurs rapidly over a wide range of parameters.

- **Stabilizing $\overline{D3}$ branes that exist in the throat**: in which case the N $D3$ branes and $\Delta + N$ $\overline{D3}$ branes which cause inflation, all exist in the same throat. For a tachyon vacuum manifold:

$$\frac{U(N) \times U(\Delta + N)}{U(N) \times U(\Delta)} \tag{4.17}$$

the excess antibranes must tend to zero to enable existence of a $U(1)$. KKLMMT allows for a system where stabilizing the branes in this throat does not immediately cause the (p, q) strings to break, thereby the stability of the strings is possible.

4.4.1 Brane - Antibrane Inflation

Type IIB string theory accomodates inflation in a process involving moduli stabilized by fluxes, which we will further explore in the next chapter.

To start off, we introduce a parallel pair of branes: a $D3$ brane and a $\overline{D3}$ brane (antibrane) where the separation is given by y^a. We want to obtain an equation of motion for F_5 while staying consistent with the form derived for charge-carrying branes as they couple with gauge fields:

$$\sum_i \mu_{D_3}^{(i)} \int_{M_4^{(i)}} A_4 \tag{4.18}$$

where $M_4^{(i)}$ denotes the world volume and the brane/antibrane $\mu_{D_3}^{(i)}$ has a charge of i. The Chern-Simons couplings have allowed the sourcing of F_5 and the equation of motion is [17]:

$$\partial_a(\sqrt{-g}F^{a\alpha\beta\gamma\delta}) + \sum_i \mu_{D3}^{(i)} \delta^{(6)}(y^a - y_i^a)\epsilon^{\alpha\beta\gamma\delta} \tag{4.19}$$

As mentioned above, y^a represents the separating vector between the branes and the values of $a = 5, ..., 9$. The permutation terms $\alpha\beta\gamma\delta$ can take any values from 0, 1, 2 and 3.

When we use this expression and integrate it over a compact manifold, we can obtain:

$$\int dF + \sum_i \mu D_3^{(i)} = 0 \tag{4.20}$$

We can observe clearly that this integral vanishes, as expected of gauge fields (since they abide by Gauss' Law). Now if we consider the distance between the two branes as r and the scaling of the gravitational potential to be $\frac{1}{r^4}$ with respect to the $D3$ brane, we can express the dual branes to have a total potential of:

$$V_{total} = \frac{4G_{10}}{\pi^2 r^4}\bigg(- \tau_3^2 \pm \frac{\mu_3^2}{g_s^2}\bigg) \tag{4.21}$$

The G_{10} is the Newton constant in 10 dimensions which can be written in terms of the string mass scale, M_s:

$$G_{10} = \frac{(2\pi)^6 g_s^2}{8M_s^8} \tag{4.22}$$

Also, the τ_3 in the potential stands for the brane tension of $D3$:

$$\tau_3 = \frac{M_s^{(3+1)}}{(2\pi)^3 g_s} \tag{4.23}$$

The interactions between the branes are basically that between the $D3 - D3$ and then between the $D3 - \overline{D3}$. The tension above is related to the charge such that:

$$\tau_3 = \frac{\mu_3}{g_s} \tag{4.24}$$

The "+" is applicable to the $D3 - D3$ interaction and the potential term cancels. However, for $D3 - \overline{D3}$, the "-" in the potential allows a potential to exist - which could possibly drive inflation.
An expression for r in this space becomes:

$$r(x^\mu) = \left(\sum_a (y^a(x^\mu) - \overline{y}^a(x^\mu))^2\right)^{\frac{1}{2}} \tag{4.25}$$

where we have considered the separation to be the inflaton field and x^μ is simply a coordinate that is parallel to the separation distance r.
We are now on course to obtain a Lagrangian for the brane-antibrane inflation. A kinetic term is required. The $D3$ brane in Minkowski space has a DBI action which is:

$$S_{D3/\overline{D3}} = \int d^4x |\eta_{\mu\nu} + \partial_\mu y^a \partial_\nu y^a|^{\frac{1}{2}} \tag{4.26}$$

To enable inflation, we must account for background to be homogenous and isotropic. For such a case, we obtain:

$$\left(\eta_{\mu\nu} + \partial_\mu y^a \partial_\nu y^a\right) = diag\left(-1 + \sum_a (\dot{y}^a)^2, +1, ..., +1\right) \tag{4.27}$$

We set $y = y(t)$ for time period t. Taking the determinant, we end up with the following:

$$\left|det\left(\eta_{\mu\nu} + \partial_\mu y^a \partial_\nu y^a\right)\right|^{\frac{1}{2}} = \sqrt{1 - \sum_a (\dot{y}^a)^2} \tag{4.28}$$

The kinetic term is then obtained upon Taylor expanding the term on the left, while limiting the velocities to only small values:

$$S = \frac{1}{2}\tau_3 \sum_a (\dot{y}^a)^2 + O(\dot{y}^a)^4 \tag{4.29}$$

Thus the inflaton field, canonically normalized, becomes:

$$\phi = \sqrt{\tau_3}|\overrightarrow{y} - \overrightarrow{\overline{y}}| \tag{4.30}$$

We define the 10-dimensional Planck scale $M_{10,pl}$ as:

$$M_{10,pl}^{-8} = 8\pi G_{10,N} \tag{4.31}$$

This is incorporated into the Lagrangian for the $D3 - \overline{D3}$ brane-antibrane inflation [18]:

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 + 2\tau_3\left(1 - \frac{1}{2\pi^3}\frac{\tau_3^3}{M_{10,Pl}^8\phi^4}\right) \tag{4.32}$$

or:

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 + 2\left(\tau_3 - \frac{c}{\phi^4}\right) \tag{4.33}$$

where we have taken $c = 4G_{10}\tau_3^4/\pi^2$.

To enable inflation, the energy density for the small fields cannot be negative. For such Coulomb-like potential, we can get inflation in regimes where $r \gg l_s$.

V must remain finite as r approaches zero. However, as r nears a certain value at y_c^2 - the **critical separation** - it gives rise to a tachyonic instability. The mass of a tachyon can be represented as:

$$m_T^2 = M_s^2\left(\frac{y^2}{y_c^2} - 1\right) \tag{4.34}$$

and at the critical separation, given by:

$$y_c^2 = \frac{2\pi^2}{M_s^2} \tag{4.35}$$

this tachyon can be taken as the ground state of the string stretching between the two branes $D3$ and $\overline{D3}$. Since the mass-squared is negative for a tachyon, it results in instability which causes the branes to annihilate.

Considering the canonically normalized potential $V(\phi)$:

$$V(\phi) = 2\tau_3\left(1 - \frac{1}{2\pi^3}\frac{\tau_3^3}{M_{10,Pl}^8\phi^4}\right) \tag{4.36}$$

we realize it cannot be flat enough unless r is sufficiently larger in size than that of the extra dimensions - which is impossible. Furthermore, introducing asymmetric compactifications also do not help, because there was no concrete method through which the moduli in this compactification could have been stabilized. Assuming stabilization to keep the potential V flat enough to allow inflation was not a good enough solution either.

4.4.2 Eta Parameter

The slow roll parameters are given by:

$$\begin{aligned} \epsilon &\equiv \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \\ \eta &\equiv M_{Pl}^2\left(\frac{V''}{V}\right) \end{aligned} \tag{4.37}$$

In order to find a relation between the $4D$ Planck mass and the ten dimensional space, we integrate out the extra dimensions in the 10D SUGRA action to obtain:

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R \tag{4.38}$$

The volume of X^6 is:

$$L^6 = \frac{G_{10}}{G_N} = \frac{M_{pl}^2}{M_{10,pl}^8} \tag{4.39}$$

We calculate the **eta parameter**:

$$\begin{aligned} \eta &= -\frac{10}{\pi^3}\left(\frac{L}{r}\right)^6 \\ &\sim -0.3\left(\frac{L}{r}\right)^6 \end{aligned} \tag{4.40}$$

This proves that the distance between the branes would have to be much larger than the value of L in order to attain a potential flat enough to allow inflation.

4.4.3 Flux Compactification

We now look into the compactifications with fluxes on the CY-orientifolds in further detail. These compactifications are in correspondence with type IIB string theory. We consider a general warped CY cone, and a 10 dimensional line element. We can write the metric with 4D Poincaré symmetry as such [2]:

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{mn} \mathrm{d}y^m \mathrm{d}y^n \tag{4.41}$$

The effect of the five-form contribution with respect to $\alpha(y)$ and a local warping factor must be considered. The five-form in question is variable throughout the compact dimension, and with the 10-D Hodge star it can be represented as:

$$\tilde{F}_5 = (1 + \star_{10}) \mathrm{d}\alpha(y) \wedge \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \tag{4.42}$$

Now we get the Einstein equation in trace-reversed form with T_{MN} standing for stress tensors of the SUGRA fields:

$$\mathcal{R}_{MN} = \kappa_{10}^2 \left(T_{MN} - \frac{1}{8} g_{MN} T \right) \tag{4.43}$$

After obtaining a modified Bianchi identity in terms of $\alpha(y)$, with proper assumptions we can discover the two conditions that will help satisfy the equations of motion in SUGRA. These conditions are:

$$\begin{aligned} \star_6 G_3 &= iG_3 \\ e^{4A} &= \alpha \end{aligned} \tag{4.44}$$

The G_3 in the above condition is a finite 3-form flux and it is imaginary self dual.

Now if we look at the action for type IIB theory in these fluxes:

$$V_{flux} = \frac{1}{2\kappa^2} \int d^{10}X \sqrt{-G_E} \left[\frac{-|G_3|^2}{2Im(\tau)} \right] \tag{4.45}$$

we can see that the G_3 flux creates potential for both the axio-dilaton as well as the complex structure moduli. Dimensional reduction will then allow us to find classical potentials which will allow the compact spaces to be stable at higher mass values.

Chapter 5

Brane-Antibrane Inflation in Flux Compactifications

In this section, we attempt to detail inflation in branes to understand the construction of inflationary models and find some solutions to the problems that arise.

In the early Universe, the attractive force between branes and antibranes arose as a result of potential energy. This energy was thought to drive inflation, while the annihilation of a brane and antibrane resulted in reheating. However, there are certain limitations to this idea:

- The potential was not flat enough by default for inflation to commence.
- There was no guarantee that choosing constant moduli would assure that the degree of flatness would be preserved.

5.1 Flatness of potential

The inflaton potential must be flat in order to let prolonged slow-roll inflation take place. Potential however is not naturally flat.

5.2 Modulus Stabilization

Moduli are zero energy deformations which form as a result of cycles of Calabi-Yau manifolds. These cycles are different in topology, and the generated moduli are characteristic of Calabi-Yau compactifications [19].

A **modulus field** is a massless scalar field, with a vacuum expectation value (vev) unconstrained by a scalar potential, which arises due to the fluctuations of metric in theories with extra dimensions. The vev of such a field is called a **modulus**.

We need to resort to modulus stabilization, a means to reign in any instabilities in the moduli. The mechanism for stabilization has to be known, for instance, by finding vacua where the moduli have positive masses-squared.

Some problems posed by unstable moduli include:

- Unspecified mechanism can lead to **arbitrary vev** which makes the outcomes of the theory harder to predict. For type II-B theory, this generates free parameters. Parameters that are time-dependent contradict observation.
- **Polonyi problem**: models where supersymmetry breaking is caused by the Polonyi-type field. Light scalar fields which usually undergo subsequent oscillations and when these fields decay, they create issues for processes like baryogenesis to occur.
- Obtaining Standard Model masses from a base theory which offers no free parameters does not feel plausible until modulus stabilization enables the introduction of a string landscape.

We observe type II-B string theory, where the moduli on the Calabi-Yau manifold can be stabilized using fluxes of the R-R and NS-NS gauge fields. The small hierarchies produced with the ratio of fluxes can then be used to generate a warped throat with an exponentially large hierarchy without causing the hierarchy problem.

However, this form of compactification does not stabilize the Kähler modulus. It only works to stabilize the dilaton moduli and complex structure moduli.
The case for complex structure modulus stabilization features a scalar potential:

$$V_F = e^K (K^{ab} D_a W \overline{D_b W} - 3|W|^2) \tag{5.1}$$

where K is the Kähler potential and W denotes the superpotential. We obtain the volume modulus field after checking it against the effective 4D theory.

$$V_{\bar{D}3} = \frac{k}{T^2} \tag{5.2}$$

$$V_{inf} = \frac{k'}{T^2|\psi - \psi_0|^4} \tag{5.3}$$

T represents the volume modulus field. Further instances of volume stabilization will come up later.

5.3 Warped Compactification

The **conifold** is a 6-dimensional Calabi-Yau manifold with a 5-dimensional base which can contain conical singularities. It can be described in terms of complex coordinates w_i with the following condition constraining it [17]:

$$\sum_{i=1}^{4} \omega_i^2 = 0 \tag{5.4}$$

The singularity occurs at $w_i = 0$ and the topology of the base is given by $S_2 \times S_3$.

$$\sum_{i=1}^{4} \omega_i^2 = z \tag{5.5}$$

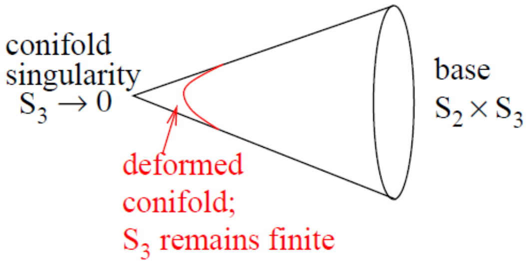

Figure 5.1: Deformed conifold

This condition deforms the conifold so that it is not singular, and z represents the complex structure modulus here. Upon setting the background values of the R-R field (type II-B) and NS-NS Kalb-Ramond field to non-zero values, this deformed conifold can yield the solution to Einstein's equations.

As mentioned previously, the fluxes of these gauge fields are used to obtain the warped throat. They can be thought of as having 3 lines of flux - 3 cycles which are consistent with Gauss' law: specifically the F_3 and H_3 background fluxes are associated with A cycle and the Poincaré dual B cycle and then they can be quantized.

$$\begin{aligned} \frac{1}{2\pi\alpha'}\int_A F_3 = 2\pi M \\ \frac{1}{2\pi\alpha'}\int_B H_3 = 2\pi K \end{aligned} \tag{5.6}$$

M and K are the integer values and α' is the slope parameter. By setting conditions upon the 3 cycles of fluxes, the subspace S_3's size can be stabilized. Now, the complex structure modulus simplifies to:

$$z = e^{-2\pi K/Mg_s} \equiv a^3(r_0) \tag{5.7}$$

The 3 cycle fluxes also affect the rest of the conifold by warping it when turned on. The 10-D line element is given by:

$$ds^2 = \frac{dx^\mu dx_\mu}{\sqrt{h(r)}} + \sqrt{h(r)}\left(dr^2 + r^2 ds^2_{T_{1,1}}\right) \tag{5.8}$$

where $T_{1,1}$ represents the Einstein-Sasaki space (a 5D base manifold). Incorporating the ideas of string landscape in the Randall-Sundrum model yields the Klebanov-Strassler throat:

$$\begin{aligned} h(r) &= \frac{R^4}{r^4}\left(1 + g_s\frac{M}{K} \times (\ln r\,\text{correction})\right) \\ &\cong \frac{R^4}{r^4} \end{aligned} \tag{5.9}$$

where $R^4 = 27/4\,\pi\, g_s N \alpha'^2$ and $N = MK$ from the quantization shown above. The $h(r)$ represents the **warp factor**. The conifold singularity or the tip is basically the base of this throat where:

$$\frac{r_0}{R} = z^{1/3} = a(r_0) = \text{warp factor} \tag{5.10}$$

While inflation on the AdS_5 space is not relevant, we can consider narrowing it in the range of values for r_0. Within $r_0 < r < r_{max}$, we can find that this space gives rise to a potentially applicable toy model to better understand the throats produced by the fluxes.

To explore the role of the Klebanov-Strassler throat, we consider a $D3$ brane and a $\overline{D3}$ brane. The gauge forces and gravitational forces act on both of these branes, however the cancelation of these forces on the $D3$ brane means that this brane does not feel any resultant force at the throat. On the other hand, the forces add for the $\overline{D3}$ brane and causes it to be pushed to the bottom. This is easily observed in the action for a brane at the location $r = r_1$:

$$S = -\tau_3 \int d^4x \frac{1}{h(r_1)} \sqrt{1 - h(r_1)(\partial r_1)^2} \pm \tau_3 \int d^4x (C_4)_{0123} \tag{5.11}$$

The forces cancel for $D3$ represented by the "+" and add for $\overline{D3}$ represented by the "-" sign. We can then write the DBI action as:

$$S_{DBI} = -\tau_3 \int d^4x \sqrt{-G} \tag{5.12}$$

The resulting induced metric appearing on the $D3$ brane can be expressed as:

$$G_{\mu\nu} = G_{AB} \frac{\partial X^A}{\partial x^\mu} \frac{\partial X^B}{\partial x^\nu} = \frac{1}{\sqrt{h}} \eta_{\mu\nu} - \sqrt{h} \partial_\mu r \partial_\nu r \tag{5.13}$$

The equation of motion for the R-R field, in terms of the C_4 gauge potential and the warp factor, becomes:

$$(C_4)_{\alpha\beta\gamma\delta} = \frac{1}{h(r_1)} \epsilon_{\alpha\beta\gamma\delta} \tag{5.14}$$

If we take into consideration $\partial r_1 = 0$, a point where the transverse fluctuations vanish, then the above equation simplifies further - owing to the two extra terms in the action to also vanish for $D3$ and add for $\overline{D3}$:

$$S = -2\tau_3 \left(\frac{r_1}{R}\right)^4 \int d^4x = -2\tau_3 a^4(r_1) \int d^4x \tag{5.15}$$

The term $\tau_3 a^4$ is the 4 dimensional potential that arises as a result of the warped brane tension. This potential is reduced to a minimum at the bottom of the throat and the $\overline{D3}$ sinks.

Just because $D3$ can stay afloat without "sinking" does not imply it can be implemented anywhere within a compact space. Branes have charge so the insertion of one requires adjustment of background, accounted for through the Tadpole Condition:

$$\frac{\chi}{24} = N_{D3} - N_{\overline{D3}} + \frac{1}{\kappa_{10}^2 T_3} \int_{C-Y} H_3 \wedge F_3 \tag{5.16}$$

where, within a certain topology, the net change in charge due to introduction of branes must result in a similar adjustment of the background fluxes. The Euler number χ of the Calabi-Yau imposes a limit upon how many branes can be added within a topology in this way.

5.4 Randall - Sundrum Model

The idea of this model was to dissociate hierarchies from a a higher-dimensional theory, and then generate different coupling constants as well as masses from this theory, as a means of solving the Hierarchy problem.
The Randall-Sundrum model aims to be an improved version of a previous 5D theory, the Kaluza-Klein Theory. Unlike KK theory, this model includes a compact dimension which is basically a circle with a marked upper and lower half (instead of a manifold). This is an orbifold.

To define, an **orbifold** is a generalized manifold that can be described as $\mathcal{O} = S^1/Z_2$, in terms of a group, Z acting on S^1 [20]:

$$\begin{aligned} &S^1 \to S^1 \\ &\mathrm{e}^{\mathrm{i}\phi} \mapsto \mathrm{e}^{-\mathrm{i}\phi} \end{aligned} \tag{5.17}$$

$\mathcal{O}$ can be thought of as an interval whose functions are even and determined by the functions that are on the circle.

Gravity in five dimensions can be expressed in terms of the cosmological constant (Lambda), the Ricci scalar (Ri), a 5D mass M and the length (L) of interval $\mathcal{O}$:

$$S = \int_{R^{1,3}} \int_{-L}^{L} \sqrt{-g}(M^3 \mathrm{Ri} - \Lambda) \mathrm{d}y d^4x \tag{5.18}$$

We incorporate this idea into the Minkowski metric to obtain vacuum solutions to Einstein equations.
The warp factor decays as $A(y) = k|y|$ in the Randall-Sundrum case, and the k is real even when comological constant is negative.
The 4D part in Einstein equation can then be expressed as:

$$(6A'^2 - A''^2) g_{\mu\nu} = 6k^2 g_{\mu\nu} \tag{5.19}$$

Locating a brane at $y = L$ now allows us to concoct a 4D theory where matter can live on a brane. The goal is to produce a weak scale, so we consider a Higgs like theory with action:

$$S_{\text{Higgs}} = \int \sqrt{-g_L}(g_L((Dh^\dagger), (Dh)) - \kappa(h^\dagger h - v^2)^2)\mathrm{d}^4x \tag{5.20}$$

The physical Higgs term can be found using the Randall-Sundrum metric:

$$S_{\text{Higgs}} = \int \left(\eta((D\hat{h})^\dagger, (D\hat{h})) - \kappa(\hat{h} \dagger \hat{h} - e^{-2kL}v^2)^2\right)\mathrm{d}^4x \tag{5.21}$$

Since there is no hierarchy affiliation in this case, the large value of ν can be narrowed down by kL, to a value ν_{eff} that would work with the weak scale.

In 4D Planck scale, after the metric is perturbed, we can integrate the action over compact dimension as such:

$$\int_{-L}^{L} e^{-2k|y|}\mathrm{d}y = 2\int_0^L e^{-2ky}\mathrm{d}y = -\frac{1}{k}(e^{-2kL} - 1) \tag{5.22}$$

It leads us to the solution:

$$M_{Pl}^2 = \frac{M^3}{k}(1 - \mathrm{e}^{-2kL}) \tag{5.23}$$

which implies that the weak scale is dependent on L, and for large enough values of L the dependence of Planck scale on extra dimensions is almost negligible.
Randall-Sundrum model is important in explaining reheating in different warped throats, which will be covered in a later chapter.

5.5 Warped case of Brane-Antibrane Inflation

We want to move to the KKLMMT idea of brane-antibrane inflation occuring within the Klebanov-Strassler throat. When trying to accomodate both branes at the same time, we need to account for the perturbations caused in the background by the $D3$ brane at $r = r_1$. We want to find the interaction energy of the two branes, for which we assess the background perturbation as [17]:

$$\begin{aligned} h(r) &\to h(r) + \delta h(r) \\ C_4(r) &\to C_4(r) + \delta C_4(r) \end{aligned} \tag{5.24}$$

We include the perturbations in the C_4 field caused by the $D3$ brane too. The Poisson equation in 6D (since none of the 6 dimensions other than r are reliant on x^μ) helps us obtain:

$$\nabla^2 \delta h = C\delta^{(6)}(\overrightarrow{r} - \overrightarrow{r}_1) \tag{5.25}$$

We take $r \ll r_1$ so that the perturbed field is now:

$$\delta h = \frac{R^4}{N r_1^4}, \quad \delta C_4 = \frac{\delta h}{h^2} \tag{5.26}$$

The solution we obtain is simply:

$$h(r) = R^4 \left[\frac{1}{r^4} + \frac{1}{N r_1} \right] \tag{5.27}$$

For the $\overline{D3}$ brane, we use the perturbed field forms and inflaton $\phi = \sqrt{\tau_3 r_1}$ to determine the Lagrangian:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\tau_3(\partial r_1)^2 - 2\tau_3 \left(\frac{r_0}{R}\right)^4 \left(1 - \left(\frac{r_0}{R}\right)^4 \frac{R^4}{N r_1^4}\right) \\ &= \frac{1}{2}(\partial\phi)^2 - 2\tau_3 a_0^4 \left(1 - a_0^4 \frac{R^4 \tau_3^2}{N\phi^4}\right) \end{aligned} \tag{5.28}$$

We also take $a_0 = a(r_0)$. At the end of the KS throat, the $\overline{D3}$ brane appears to be exponentially light, which is suggested by the presence of warping in the DBI action. We can now find the eta parameter:

$$\eta = a_0^4 \eta_{unwarped} \tag{5.29}$$

We see that $a \ll 1$ which gives us $\eta \ll 1$ and, while this exhibits slow roll to be possible due to warping, it does not account for the role of the Kähler modulus which has not been stabilized. The Káhler modulus affects the η-parameter and thus, it must also be stabilized.

Possible solutions to the eta problem:

- Superpotential corrections
- Tuning the length of the KS throat
- Multibrane inflation
- DBI inflation
- Shift Symmetry
- Racetrack Inflation

5.6 Volume Stabilization

While warped geometries can help us to construct viable models for inflation, it is important for the compactification volume to first undergo stabilization, lest it results in decompactification [21].

For a no-scale 4D ($\mathcal{N} = 1$) SUGRA, we have a Kähler potential with ϕ representing fields of the D-brane:

$$K(\rho, \overline{\rho}, \phi,) = -3log(\rho + \overline{\rho} - k(\phi, \overline{\phi}) \tag{5.30}$$

The massless fields that are significant here are those of the volume, the position (ϕ) and the axion of the branes. The axion here represents a circle over the moduli space ϕ with the metric:

$$ds^2 = \frac{3}{2r^2}(dr^2 + \left(d\chi + \frac{1}{2}ik_{,j}\, d\phi^j - \frac{1}{2}ik_{,\overline{j}}\, d\overline{\phi}^{\overline{j}}\right)^2) + \frac{3}{r}k_{,i\overline{j}}\, d\phi^i d\phi^{\overline{j}} \tag{5.31}$$

The moduli space defined above arises from the four form potential coupled to the worldvolume of the mobile $D3$. ρ is a complex variable with the axion being imaginary and the real part can be expressed as:

$$2r = \rho + \overline{\rho} - k(\phi, \overline{\phi}) \tag{5.32}$$

Thus the real part of ρ helps bring the Kähler potential to the metric form. If we now look at a superpotential W where $W = W_0$, we note that W_0 is constant. For variables that run over ρ and ϕ, we have $g^{ab}\partial_a K \partial_b K = 3$ so the cancelation in the potential gives us:

$$V = e^K \left(g^{ab}K_{,a}\, K_{,\overline{b}}\, |W|^2 - 3|W|^2\right) = 0 \tag{5.33}$$

Finally, we want to allow mobility of the $D3$ branes without compromising volume stabilization. We take advantage of the fact that the Káhler potential functions unconventionally when k is undergoing Kähler transformations - thus we use the transformations:

$$\begin{aligned} k(\phi, \overline{\phi}) &\to k + f(\phi) + \overline{f(\phi)} \\ \rho &\to \rho + f \\ \overline{\rho} &\to \overline{\rho} + \overline{f} \end{aligned} \tag{5.34}$$

which allows the volume (of internal dimensions) to be invariant while the circle constructed by the axion is fibered over ϕ. Thus the volume stays stable.

5.6.1 Stabilizing the Superpotential

We can attempt to stabilize the volume by introducting a nonperturbative superpotential for ρ [21]:

$$W(\rho) = W_0 + Ae^{-a\rho} \tag{5.35}$$

This can be considered a source for the superpotential. For our model to work, the superpotential must meet certain requirements. We have already established that, for the invariance of the volume under Kähler transformations to hold, the superpotential must rely on ϕ to some extent. For a scenario where $W = W(\rho)$ and the energy $V > 0$:

$$\begin{aligned} V(r, \phi) &= \frac{X(\rho)}{r^{\alpha}} \\ &= \frac{X(\rho)}{\left(\rho - \frac{\phi\overline{\phi}}{2}\right)^{\alpha}} \end{aligned} \tag{5.36}$$

$X(\rho)$ relies on the energy source, which will tend to zero for compact manifolds that are large. Since the energy for inflation must come from brane tension or the flux energy, the above is consistent. The system we are using to stabilize will work on ρ and not r so the potential changes as such:

$$V = V_0\left(1 + \alpha\frac{\phi\overline{\phi}}{2r} + ...\right) \tag{5.37}$$

Basically, this change in potential occurs due to the change in ϕ as the brane moves. For a $D3$ brane, the 4D effective potential is:

$$V = \frac{1}{6r}\left(\partial_\rho W\overline{\partial_\rho W}\left(1 + \frac{1}{2r}\frac{k_{,\phi}\,k_{,\overline{\phi}}}{k_{,\phi\overline{\phi}}}\right) - \frac{3}{2r}\left(\overline{W}\partial_\rho W + W\overline{\partial_\rho W}\right)\right) \tag{5.38}$$

We now deal with the $\overline{D3}$ brane so that the above idea works for both types of branes. The $\overline{D3}$ brane will cause an extra term to be added owing to its positive tension. The effective potential for the $\overline{D3}$ becomes:

$$V = \frac{1}{6r}\left(\partial_\rho W\overline{\partial_\rho W}\left(1 + \frac{1}{2r}\frac{k_{,\phi}\,k_{,\overline{\phi}}}{k_{,\phi\overline{\phi}}}\right) - \frac{3}{2r}\left(\overline{W}\partial_\rho W + W\overline{\partial_\rho W}\right)\right) + \frac{D}{(2r)^2} \tag{5.39}$$

Instead of ρ, we have a new scalar φ which is responsible for the movement of $D3$. We want to take (5.40) and say it has a de Sitter minimum, consider the minimum ρ as well as the potential at the minimum, $W(\rho)$ to be real. The new scalar is set as:

$$\varphi = \rho\sqrt{\frac{3}{(\rho + \overline{\rho})}} \tag{5.40}$$

We modify the potential (5.40) with the derivatives of ρ as follows:

$$V = \left(W'(\rho)^2\rho - 3W(\rho)W'(\rho) + \frac{D}{4}\right)\left(\rho - \frac{\phi\overline{\phi}}{2}\right)^{-2} \tag{5.41}$$

V_0 is expressed in a way that we can include the new scalar:

$$V_0(\rho_c) = \frac{1}{\rho_c^2}\left(W'(\rho_c)^2\rho_c - 3W(\rho_c)W'(\rho_c) + \frac{D}{4}\right) \tag{5.42}$$

which results in:

$$\begin{aligned} V &= \frac{V_0(\rho_c)}{\left(1 - \frac{\varphi\overline{\varphi}}{3}\right)^2} \\ &\approx V_0(\rho_c)\left(1 + \frac{2}{3}\varphi\overline{\varphi}\right) \end{aligned} \tag{5.43}$$

The field defined by φ has gained the mass which, in terms of φ turns out to be:

$$m_\varphi^2 = \frac{2}{3}V_{dS} = 2H^2 \tag{5.44}$$

This mass is in alighnment with the coupling:

$$\delta V = \left(\frac{1}{6}R_{AdS}\right)\varphi\overline{\varphi} \tag{5.45}$$

and result is consistent with what one would get for a conformally coupled scalar.

Chapter 6

Reheating after Inflation

Inflationary theories suggest that the matter created in the Universe was as a result of a period of reheating after inflation. **Reheating** can be defined as the production of Standard Model particles (matter) after accelerated expansion results in massive energy density storage within oscillations of the inflaton field ϕ.

There are aspects of the weak nuclear force and gravity that present large differences between them, giving rise to a type of **Hierarchy problem** [23].

6.1 Chaotic Inflation

If we consider a scalar field, denoted by ϕ, which has a mass m, the potential energy density is given by:

$$V(\phi) = \frac{1}{2}m^2\phi^2 \tag{6.1}$$

With a minimum at $\phi = 0$, the equation of motion for this scalar field should be the same as that of the harmonic oscillator:

$$\ddot{\phi} = m^2\phi \tag{6.2}$$

This would have been the case if the universe was not undergoing inflation. Due to the expansion of space, the Hubble constant introduces an additional term. The equation of motion for the scalar field ϕ now becomes:

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi \tag{6.3}$$

which is the Klein-Gordon equation. Thus, for a homogeneous universe, the Einstein equation becomes:

$$H^2 + \frac{k}{a^2} = \frac{1}{6}\left(\dot{\phi}^2 + m^2\phi^2\right) \tag{6.4}$$

6.2 Hybrid Inflation

In a model with two scalar fields, σ and ϕ, that interact, the end of inflation is brought about by the slow-roll of ϕ that in turn causes a rapid-roll of σ - this is called the 'waterfall' regime, which can be interpolated to either reach slow-roll or first order phase transition, depending on the set parameters. [23]

The effective potential of the hybrid inflation model to permit a waterfall regime is given by:

$$V(\sigma, \phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2 \tag{6.5}$$

where σ denotes the Higgs field that takes only positive values. This is to ensure no domain walls form in order for symmetry breaking to occur spontaneously. [5]

The effective potential can have a single minimum at $\sigma = 0$, provided that $\phi > \phi_c = M/g$ where ϕ_c is the critical value of the inflaton. If this condition is not met, and the inflaton field starts to become smaller than the critical value, phase transition happens which leads to the breaking of symmetry.

$$m^2\phi_c^2 = m^2\frac{M^2}{g^2} \ll \frac{M^4}{\lambda} \tag{6.6}$$

Then, the Hubble constant during the phase transition period becomes:

$$H^2 = \frac{2\pi M^4}{3\lambda M_p^2} \tag{6.7}$$

To ensure the inflaton field value is always above critical value (to facilitate inflation), we make assumptions for both m and M:

$$M^2 \gg \frac{\lambda m^2}{g^2}, \;\; m^2 \ll H^2 \tag{6.8}$$

This gives us the condition:

$$M^2 \gg mM_p\sqrt{\frac{3\lambda}{2\pi}} \tag{6.9}$$

which asserts that the Universe undergoes inflation when the inflaton $\phi > \phi_c$. Since this type of inflation occurs through the efforts of vacuum energy density instead of the inflaton field's energy density, it was named **Hybrid inflation**.

6.3 Reheating

Reheating is defined as the production of SM particles through coupling of the inflaton field ϕ, to SM matter, after a period of accelerated expansion. Reheating involves [23]:

- Degrees of freedom within the Standard Model must be heated to a temperature that allows for a process called **baryogenesis** to occur. Baryogenesis is essentially the physical process that led to the production of baryonic asymmetry, which results in the presence of a larger number of baryons than antibaryons in the Universe.

- The dynamics of the tachyon field determines what ultimately happens to the false vacuum energy that exists because of the tension between a pair of branes undergoing annihilation. Thus, when brane-antibrane inflation comes to an end, this complex field influences the production of **topological defects**.

Topological defects occur as a result of phase transitions that violate symmetry. These defects are sorted on the basis of the topology of the vacuum manifold:

- **Monopoles**: magnetic monopoles are zero-dimensional defects formed as a result of breaking spherical symmetry, that can be diluted and hence left out.
- **Domain Walls**: two-dimensional defects occuring in the form of a singularity that can be localized to a space.
- **Cosmic Strings**: these one-dimensional defects are naturally-occuring within string theory

When discussing the reheating problem in the scenario of brane-antibrane inflation, it is important to recognize that models of hybrid inflation relate more closely than the usual inflationary models. [18]

At the end of inflation in either type of model, the field is orthogonal to a tachyonic inflaton. However, when we consider hybrid inflation, the second field ends up at a minimum which is stable. It then oscillates near this minimum which results in reheating.

The tachyonic field can be described by Sen's effective action:

$$S = -\int d^{p+1}x e^{-|T|^2/a^2}\left(\sqrt{1-|\partial T|^2}\right) \tag{6.10}$$

The mode of an open string which exists between a brane and an antibrane will become tachyonic when the distance between the branes is not large enough, i.e. there exists a critical value for the separation, and for distances smaller than that, the mode becomes tachyonic. At both zero and non-zero string coupling scenarios, we cannot obtain reheating from the model because of the decaying closed string states.

To resolve this issue, we relocate inflation to a different throat. It cannot be present at the same throat as the SM. For this seperate throat situation, the geometry resembles the one in Randall-Sundrum model:

$$ds^2 = a^2(y)dx^2 + dy^2 + y^2 d\Omega_5^2 \tag{6.11}$$

The decaying of the tachyon condensate causes closed string states to attain the correct quantum numbers which would turn them into Kaluza-Klein gravitons.

Along the y-direction, there is exponential increase within the throats. So when subjected to strongly warped throats, the decay of these Kaluza-Klein gravitons results in most of it ending up as particles on branes. Therefore, for the SM brane to initiate reheating, it will have to exist in the deepest throat where warping is strong.

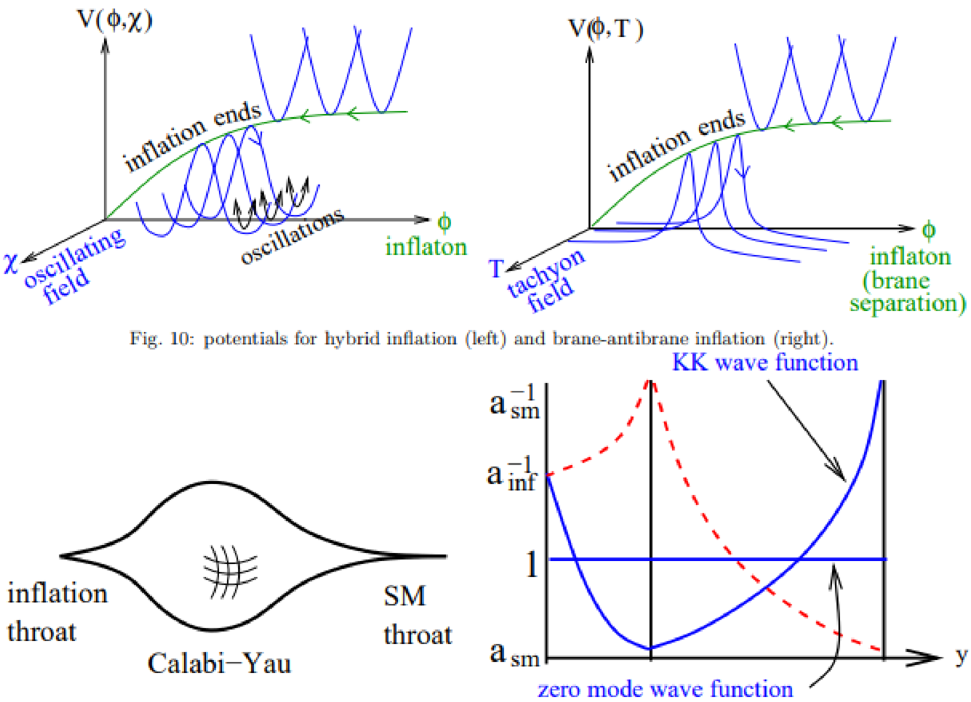

Figure 6.1: A representation of the two-throat scenario, and a graphical representation of the KK graviton wave function

Chapter 7

Racetrack Inflation

In a previous section, we mention the problems arising due to modulus stabilization, and how an undefined criteria for stabilization and setting these moduli can lead to inflation not happening.

The initial assumption was to fix all moduli, other than the inflaton, by an unknown mechanism. Since the scales chosen were much larger than those considered in inflation, the fixing was thought to not affect the inflationary dynamics. [25]

However, with the recent study of brane/antibrane inflation, the KKLT scenario for moduli fixing is a better replacement for the fixed unknown modulus fixing.

At its core, **racetrack inflation** is essentially a form of topological inflation which is concordant to the KKLT model and occurs in string moduli space. It is driven by the racetrack potential, which emerges with respect to type IIB string theory.

7.0.1 The Effective Theory

Gauginos: In supersymmetry theories of particle physics, a gaugino is the hypothetical fermionic supersymmetric field quantum (superpartner) of a gauge field, as predicted by gauge theory combined with supersymmetry. All gauginos have spin 1/2, except for gravitino (spin 3/2).

Conventionally, in KKLT the nonperturbative superpotential would be obtained through gaugino condensation in the gauge theories on $D7$ branes. The classical Kähler potential is written as:

$$k = -3\,ln\,(T + \overline{T}) \tag{7.1}$$

However, in this case, the gauginos condense to become a product, specifically SU(N)×SU(M) when we enlarge the group. With $a = 2\pi/N$ and $b = 2\pi/M$ and considering a compactification in KKLT form with a single Kähler modulus T, we obtain the **racetrack superpotential**:

$$W = W_0 + \mathcal{A}e^{-aT} + \mathcal{B}e^{-bT} \tag{7.2}$$

where W_0 denotes the flux superpotential that remains constant, while a and b are also constants. Here, $\mathcal{A}$ and $\mathcal{B}$ are prefactors. These prefactors are dependent on the vacuum expectation values (vevs) of the stabilized moduli. It is to be noted that W_0 is applicable only in cases of fields like the dilaton and complex structure moduli since they have already been fixed. This route essentially functions on using only moduli compactification to obtain inflation instead of introducing the $D3$ brane.
The scalar potential can be written as:

$$V = V_F + \delta V \tag{7.3}$$

where the two terms in the total potential arise from KKLT.
For a Kähler potential, K, and with i, j that covers all moduli fields, the first term of the potential is:

$$V_F = e^K \left(\sum_{i,j} K^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2 \right) \tag{7.4}$$

This term comes from the $\mathcal{N} = 1$ SUGRA formula, which is taken from the KKLT scenario. The Kähler potential used here is the result obtained from CY compactification, thus giving it the weak coupling form: $K = -3\log(T + T^*)$.

The second term is the non-supersymmetric potential which arises when there is tension amidst the $\overline{D3}$ branes. The energy density contribution relies on CY volume and no extra moduli are added due to the fluxes which fix the position of the $\overline{D3}$ brane. With E being the function of tension of the brane and X being the warp factor, this term is written as:

$$\delta V = \frac{E}{X^\alpha} \tag{7.5}$$

In the case of $\overline{D3}$ branes being present at the end of the CY throat, then the α in the above potential term takes the value $\alpha = 2$. Since warped regions are favoured by the $\overline{D3}$ brane, only for unwarped regions, the value of α changes to $\alpha = 3$.
Thus the potential depends on the type of model in this scenario - for example, it depends on how many Kähler moduli are present from the CY manifold or even on the type of superpotential induced as a result of these moduli.

7.0.2 The Scalar Potential

After understanding the model reliance, we move to the shape of the potential itself. The goal is to try and find any regions within the model where slow roll inflation could possibly emerge.

For a field T:

$$T \equiv X + iY \tag{7.6}$$

we express it as two separate terms, one real and the other imaginary. Any fields that roll slowly in the Y direction help account for the eta problem which is present in F-term inflation.

In the previous section, we obtained (7.4). We bring it back along with the weak-coupling Kähler modulus, and use the derivatives with respect to the field T, to obtain a potential in this form:

$$V_F = \frac{1}{8X^3}\left[\frac{1}{3}|2XW' - 3W|^2 - 3|W|^2\right] \tag{7.7}$$

By setting $2XW' - 3W = 0$, the solutions we get are the supersymmetric configurations which are then used in V_F. This will generate values of V_F such that $V_F \leq 0$ - such values align with the Minkowski vacua. Including the supersymmetry breaking terms into the scalar potential, we get a potential with many de-Sitter minima:

$$\begin{aligned} V &= \frac{E}{X^\alpha} + \frac{e^{-a}X}{6X^2}\left[aA^2(aX+3)e^{-aX} + 3W_0 aAcos(aY)\right] + \\ &= \frac{e^{-b}X}{6X^2}\left[bB^2(bX+3)e^{-bX} + 3W_0 bBcos(bY)\right] + \\ &= \frac{e^{-(a+b)}X}{6X^2}\left[AB^2(2abX + 3a + 3b) + cos((a-b)Y)\right] \end{aligned} \tag{7.8}$$

The values of these minima are dependent on the values of all the parameters denoted by letters as well as W_0.

In a standard racetrack model, the value of $(a - b)$ is usually quite small, by picking favourably large values for M and N to influence the values of a and b. For a situation where W_0 is zero and the limit $(a - b) \to 0$, the Y direction in the potential in (7.6) turns completely flat. These parameters can be tuned again to obtain regions flat enough to enable inflation.

Also, a large Calabi-Yau volume, given by $X = ReT$, should have extrema under circumstances that both M and N have a discrete fine tuning. This is advantagenous and unlike the KKLT scenario in that, it does not rely on W_0. Even for models where W_0 is a non zero value, there can exist several local minima, and it allows us to pick a value for E so that the potential's minimum can be tuned to vacuum energy in the present.

Such a model must have two degenerate minima so that there may be causally disconnected regions. with domain walls between them, located in separate vacua. At the point between them, eternal inflation can take place long as the field is close to saddle point and conditions to enable slow-roll are met. Then we can obtain regions that are near to the saddle, where inflation happens while sparing a large enough time period to explain the homogeniety and flatness of the universe.

Chapter 8

Conclusion

We have now arrived at the conclusion that string cosmology models such as KKLMMT may be difficult to establish, but through stabilization of potentials and various adjustments, it can be possible to produce inflation naturally.

Inflationary theories have attempted to explain phenomena in the Universe in ways that csnnot be rationalized in most other ways. Regardless, there are many problems that arise. Whether it be dependence on certain parameters, ultraviolet sensitivity or inconsistencies in particular scenarios, the KKLMMT model is still a promising theory in its attempts to resolve a lot of theories while staying consistent with the existing ideas in both cosmology and string theory. In the future, this model might shed light upon even more aspects of the Universe that we do not understand as of now. Showing that inflation can arise in string theory may just be the first step.

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