Ion acoustic wave in relativistic magnetized quantum plasma with positive ions, negative ions, electrons and stationary dust particles

by

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A thesis submitted to the Department of Mathematics and Natural Sciences
in partial fulfillment of the requirements for the degree of
B.Sc. in Applied Physics and Electronics

Department of Mathematics and Natural Sciences
Brac University
July 2022

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Approval

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Abstract

The propagation of the ion acoustic (IA) solitary waves (SWs) in a degenerate quantum plasma with external magnetic field (containing magnetized positive and negative ions, unmagnetized quantum electrons with stationary charged dust particles) have been theoretically investigated. The Reductive Perturbation method is adopted to derive the Korteweg-de Varies (K-dV), the modified K-dV (mK-dV) and mixed mK-dV (mmK-dV) equations. The stationary solutions of K-dV, mK-dV and mmK-dV are shown and analyzed numerically to study the basic characteristics of IASWs that are commonly formed in degenerate quantum plasmas. Main process of the propagation of waves through oscillation in plasma is investigated graphically. The basic properties (phase speed, width, amplitude) of IASWs are found to be significantly modified by the effects of number densities of the plasma particles, mass of the ions, relativistically degenerate electrons , external magnetic field and charge state of the dust particles. The outcomes from this conceptual analysis can be significant for understanding the layout and prominence of the solitary structures in astrophysical compact objects. Besides that the data from this research can be used for developing advanced concepts related to plasma physics in future.

Keywords: Ion acoustic solitary waves, Magnetic Effect, Higher order nonlinearity, Degenerate pressure, Reductive perturbation method, Compact objects

Dedication

"To my parents with their unconditional sacrifices and efforts for the prosperity of my life"

Acknowledgement

All praise to the Great Allah for whom my thesis work have been completed without any major interruption.

My reserach works were solely possible due to my supervisor ***M M Hasan*** sir with his kind support and advice. He helped me whenever I needed. Without his constant supports and guides, it won't be possible for me to complete the work. Thanking him throughout my life would still not reflect what he has done for me.

I would like to thank **Md. Mosaddidur Rahman** sir for his guideline and inspiration.

I would love to thank Shangida Nasrin for her mental support which was crucial for me to complete the work.

And finally to my parents without their throughout support it may not be possible. With their kind support and prayer I am now on the verge of my graduation.

Md. Nasim Sarker

List of Graphs

List of Abbreviation

K-dV= Korteweg-de Vries
mK-dV= modified K-dV
mmK-dV= mixed mK-dV
SWs= solitary waves
IA= ion-acoustic
ω_p= Positive ion cyclotron frequency
ω_n= Negative ion cyclotron frequency
a= Average distance between plasma particle
λ_{Ds}= Debye length of the species
ε_c= Coulomb potential energy
Γ_c= Coulomb coupling parameter
k_B= Boltzmann constant
$M_{\bigodot}$=M_{sun}= Solar mass
T_{Fe}= The Fermi temperature of electron
e= Magnitude of charge of electron and ion
q_s= Magnitude of charge of plasma species
V_p= Wave phase speed
c= Light speed in vacuum
$\hbar$= $\frac{h}{2\pi}$= Reduced Planck's constant
ϕ_m= Amplitude of wave
Δ= Width of wave
k= Wave number in perturbation mode
Ψ= Electrostatic potential
σ= Ratio to positive and negative ion number density
μ= Ratio to electron and negative ion number density
η= Ratio to dust and negative ion number density
β= Ratio to positive and negative ion mass
α= Quantum parameter

Table of Contents

Nomenclature

The next list describes several symbols & abbreviation that will be later used within the body of the document

α	Alpha
β	Beta
Δ	Delta
ϵ	Epsilon
η	Eta
γ	Gamma
λ	Lambda
μ	Mu
ω	Omega
Φ	Phi
Ψ	Psi
ρ	Rho
σ	Sigma
τ	Tau
ξ	Xi
ζ	Zeta

Chapter 1

Introduction

1.1 Plasma (Matter in Extreme Condition)

The plasma is an urgent stage in the process of formation of matter from elementary particles up to condensed matter. It is the higher energy "fourth state of matter". The word plasma is referred to as a statistical system of charged particles, for instance, electrons and different ions, exhibiting collective behaviour due to the long range Coulomb forces. Since the particles in plasma are electrically charged (generally by being stripped of electrons), it is frequently described as an "ionized gas". Plasma can be characterized by regimes of high temperature and low density commonly found in space (e.g., interplanetary and interstellar media) as well as in laboratory (e.g., gas discharges and thermonuclear fusion experiments). The charged particle systems with sufficiently high density and low temperature also exhibit plasma effects. The dynamics of a plasma is governed by internal fields produced by the plasma particles and the externally applied fields [1]. The word 'Plasma' comes from a Greek word means "something molded or created". But according to the physicists plasma is an 'ionized gas'. Sir William Crookes in 1879, was first identified plasma in a Crookes tube, and he named it "radiant matter". In 1897, Sir J. J. Thomson identified the nature of the matter and Irving Langmuir, the Nobel prize winning American scientist first used the term 'Plasma' to describe an ionized gas in 1927. The term plasma represents a macroscopically neutral gas

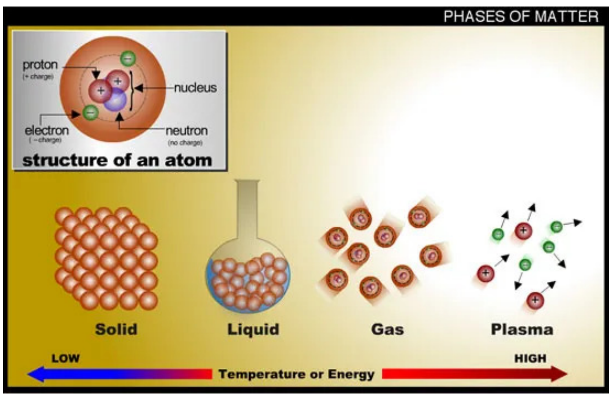

Figure 1.1: Various states of matter (Photo: UC Regents).

containing many interacting charged particles (electrons and ions) and neutrals [2]. It is estimated that up to 99 % of matter in the entire visible universe is plasma

[3]. In the science of astrophysics the origin and evolution of the stars, planets, etc. are inquired by the physical properties of plasma found in space and interstellar medium. Plasma consists of a collection of free-moving electrons and ions - atoms that have lost electrons. Energy is needed to strip electrons from atoms to make plasma. The energy can be of various origins: thermal, electrical, or light (ultraviolet light or intense visible light from a laser). With insufficient sustaining power, plasmas recombine into neutral gas. Due to the presence of a non-negligible number of charge carriers makes the plasma electrically conductive and it can be accelerated and steered by electric and magnetic fields which allows it to be controlled and applied [4]. Plasma research is yielding a greater understanding of the universe. It also provides many practical uses: new manufacturing techniques, consumer products, and the prospect of abundant energy. A plasma is defined as a partial or

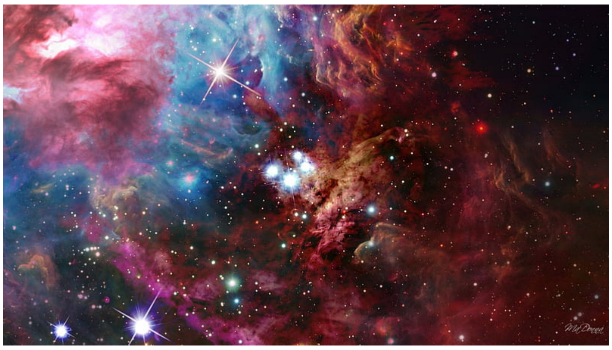

Figure 1.2: The visible Universe is 99.999 % plasma(Photo: https://www.livescience.com/).

fully ionized gas usually containing macroscopically neutral and charged particles exhibiting collective behaviour under the influence of electrostatic force due to long range Coulomb force (which are also subject to magnetic and other forces). It is

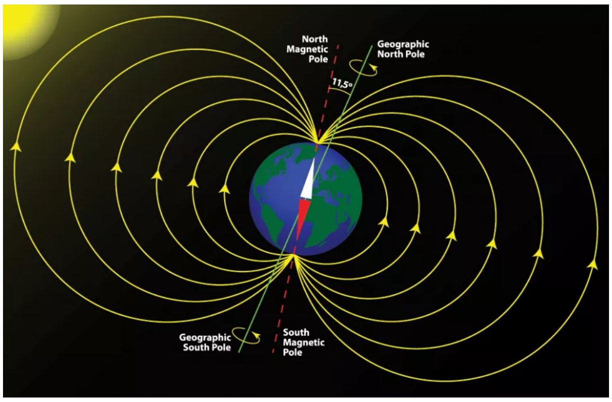

Figure 1.3: Drawing of earths magnetic field. Earth's magnetic field holds the Van Allen Belts in place (Photo :https://www.livescience.com/).

predicted that Kristian Birkeland a Norwegian explorer and physicist was the first who told that space is filled with plasma and in 1913 he wrote: "It seems to be a natural consequence of our points of view to assume that the whole of space is filled with electrons and flying electric ions of all kinds. We have assumed that each stellar system in evolutions throws off electric corpuscles into space. It does not seem unreasonable therefore to think that the greater part of the material masses in the universe is found, not in the solar systems or nebulae, but in empty space" [5]. In 1937, when interstellar space was thought to be a vacuum, plasma physicist Hannes Alfvén argued that if plasma spread through the universe, then it could generate a galactic magnetic field. During the 1940s and 50s, Alfvén developed magnetohydro-dynamics (MHD) which enables plasmas to be modelled as waves in a fluid, for which he won the 1970 Nobel Prize for physics.

1.1.1 Plasma Properties

Plasma exists in many forms and have some basic properties. All ionized gas are not plasma. They called so if they fulfill the following characteristics.

Density of Plasma Particles

Plasma can react to electromagnetic fields, conducts electrical current and possesses a well-defined space potential due to the presence of free charge carriers. Positive ions may be singly charged or multiply charged. For a plasma containing only singly charged ions, the ion population is adequately described by the ion density n_i,

$$n_i\text{=number of particles/volume, } [n_i] = cm^{-3} \text{ or } [n_i] = m^{-3} .$$

Besides the ion density, we characterize a plasma by its electron density n_e and the neutral density n_a. Strongly coupled plasmas tend to be cold and dense, whereas weakly coupled tend to be diffuse and hot.

Macroscopic Neutrality

Plasma is macroscopically neutral if there is no perturbation. If there is no external disturbance is present in a plasma system then the net resulting electric charge must be zero. First of all, plasmas are macroscopically neutral i.e. [plasma dimension] > [Debye radius]. Macroscopical neutrality or quasi-neutrality describes the apparent charge neutrality of a plasma overall, while at smaller scales, the positive and negative charges making up the plasma, may give rise to charged regions and electric fields. The equilibrium charge neutrality condition in a plasma reads $\Sigma_s q_s n_{s0} = 0$, where n_{s0} is the unperturbed number density of the plasma species s (s equals e for electrons, i for ions, etc.), $q_i = Z_i e$ is the ion charge (we note that the ion charge state $Z_i = \pm 1$ will be used in our present article) and e is the magnitude of the electron charge. Thus, a plasma is loosely described as an electrically neutral medium of positive and negative particles, i.e. the overall charge of a plasma is roughly zero [6].

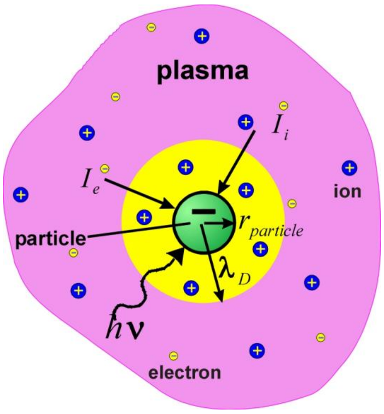

Figure 1.4: Debye length of a plasma (Photo: https://projects.mpe.mpg.de/).

Collective Behavior

All the plasma particles are connecting together through the long range Coulomb force, and they behave collectively when a disturbance acts on them. All the charged particles move together collectively to external disturbance. If a charged particle moves in a plasma, it produces local concentration giving rise to electric fields. Motion of charged particles generates currents which produces magnetic field. These electric and magnetic fields affect the motion of other charged particles far away. This is called the collective behaviour. Thus, we can say that a plasma system is like a network of particles connected by massless springs [4]. Plasma particles exhibit collective behavior i.e. large number of particles co-exist in a Debye sphere. Hence, by the term collective behaviour we mean motions that depend on both local conditions and the state of plasma in remote region.

Debye Shielding

A fundamental characteristics of a plasma is its ability to shield the electric field of an individual charged particle or of a surface that is at some non zero potential. We suppose that a potential is created by disturbances or electric potential is applied externally in a plasma system whose constituents are electrons, positrons, and positively or negatively charged ions. To maintain the quasi-neutrality of the plasma system, the opposite charged particles will surround the potential, and the surrouding particles will take a shape like a sphere or cloud. The radius of the sphere will be long enough to cancel the effect of that potential. This corresponds to a perfect shielding, i.e. no electric field would be present in the body of the plasma outside the cloud. This process of shielding is known as Debye shielding in dusty plasma and the corresponding length is known as Debye length (as shown in 6.5).

The Debye length is mathematically defined as

$$\lambda_{Ds} = \left(\frac{k_B T_s}{4\pi n_s e^2} \right)^{1/2}$$

where k_B is the Boltzmann constant, T_s is the temperature of plasma species s, e is the charge of the electron, and n_s is the number density of plasma species s. Beyond a few Debye lengths, shielding by the plasma is quite effective and the potential due to our charge is negligible. Therefore the conditions we have a plasma or not. (i)

the system must be large enough L >> λ_D and (ii) there must be enough electrons to produce shielding $N_D >>> 1$, where N_D is the number of electrons in a Debye sphere.
Suppose there is a local concentration of charge. If plasma dimensions are much greater than λ_D, then on the whole plasma is still neutral (quasineutral) and we can take $n_e \cong n_i \cong n_o$. If we put an electrode into the plasma, it becomes shielded by a sheath of thickness $\cong \lambda_D$. Now, the expression for the Debye length, λ_D can be expressed as

$$\lambda_D = 69.0\sqrt{\frac{T}{n_e}},$$

where T is in K and n_e in m^{-3}.

Plasma Oscillations

Plasma oscillation, in physics, the organized motion of electrons or ions in a plasma. Each particle in a plasma assumes a position such that the total force resulting from all the particles is zero, thus producing a uniform state with a net charge of zero. If an electron is moved from its equilibrium position, the resulting positive charge exerts an electrostatic attraction on the electron, causing the electron to oscillate about its equilibrium position. Because the interaction between electrons is strong, they all oscillate together at a characteristic frequency that depends on the nature of the particular plasma. There are two different types of characteristics frequency, namely plasma frequency and collision frequency.

Coulomb Coupling Parameter

The Coulomb coupling parameter was suggested [7] as the ratio of potential energy to thermal energy of the constituent particles (viz., electrons, ions, etc.). Coulomb coupling parameter determines the possibility of the formation of plasma crystals. To explain, we consider two particles, both having same charge qs, separated from each other by a distance a. The Coulomb potential energy (ε_c) is

$$\varepsilon_c = \frac{q_s^2}{a}\exp\left(-\frac{a}{\lambda_{Ds}}\right),$$

where s denotes the species of the plasma system, q_s is the charge of the particle, λ_{Ds} is the Debye radius and k_BT_s is the thermal energy of the particle. Now, the Coulomb coupling parameter, Γ_c is defined as

$$\Gamma_c = \frac{Z_s^2 e^2}{a}\frac{\exp\left(-\frac{a}{\lambda_{Ds}}\right)}{k_B T_s}.$$

A plasma system with $\Gamma_c >> 1$ is known as strongly coupled plasma and when $\Gamma_c << 1$ then the plasma is called weakly coupled plasma [2]. It can easily be shown that in several laboratory plasma systems massive particles are strongly coupled because of their huge electric charge and low temperature.

1.1.2 Dusty Plasma

Dusty plasmas are interesting because the presence of particles significantly alters the charged particle equilibrium leading to different phenomena. Remarkable progress has been made in the physics to dusty plasma [8]. Low temperature complex plasmas are partially or fully ionized are considered electrically conducting gases consisting of electrons, ions and charged dust grains. The dust particles are charged owing to the contact with electrons, ions, as well as of surrounding radiation. The size of dust particles is between nanometers to millimeters as billions of times massive than proton [9]. The dust particles are many orders of magnitude heavier than ions, they are a source of ionization and recombination for electrons and their charge is not fixed, but depends on local plasma parameters. Wave propagation in such complex systems is therefore expected to be substantially different from the ordinary two component plasma and the presence of charged dust can have a strong influence on the characteristics of the usual plasma wave modes, even at frequencies where the dust grains do not participate in the wave motion [34]. A research focused on the conditions that described the time of injection of dust into a laboratory negative ion plasma, becomes positively charged for very large values of negative ion density is equal or less than 500 times the electron density [44].

1.1.3 Multi Ion Plasma

The presence of different type of ions in a plasma system is the key factor behind mutli ion plasma. Participation of several different ions help to get better understanding and data about the effects of those ions in that particular plasma system. Multi ion plasma is an remarkable system to study linear and nonlinear wave phenomena. Strong shocks in multi-ion plasmas are key to a number of high-energy density settings. The wave phenomena using the multicomponent plasmas, such as the multi-ion plasmas and dusty plasmas occur in astrophysical plasmas and the industrial plasma [10].

1.1.4 Bohm Quantum Potential

Bohm quantum potential introduced by David Bohm in 1952 is the quantum potential or quantum potentiality is a central concept of the de BroglieBohm formulation of quantum mechanics. This potential acts on a quantum particle also referred to as quantum potential energy, Bohm potential, quantum Bohm potential or Bohm quantum potential. Louis de Broglie had postulated in 1926 that the wave function represents a pilot wave which guides a quantum particle, but had subsequently abandoned his approach due to objections raised by Wolfgang Pauli. The quantum potential approach introduced by Bohm [11] provides a more complete exposition of the idea presented by Louis de Broglie and included answers to the objections which had been raised against the pilot wave theory. The Bhom quantum potential can be mathematically expressed as $Q = -(\hbar/2m)(\bigtriangledown^2 R/R)$. In case of one-body system D. J. Bohm and B. J. Hiley described two essential aspects of the quantum theory. First, there is the well-known phenomenon of interference of electrons (e.g., in a beam that has passed through several slits). The fact that no electrons arrive at points where the wave function is zero is explained simply by the infinite value of the quantum potential Q, which repels particles and keeps them away from points

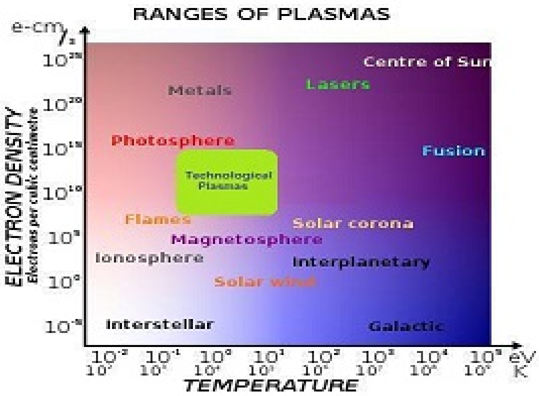

Figure 1.5: The figure showing range of dense plasma based on the electron number density (n_e) and temperature (T) (Photo: https://www.researchgate.net/).

at which R = 0. Second, they considered the phenomenon of barrier penetration and the wave function for such a system is a time-dependent packet. As a result, the quantum potential Q fluctuates in such a way that occasionally it becomes negative enough to cancel the positive barrier potential V , so that from time to time a particle may pass through before the quantum potential changes to a significantly less negative value. The quantum potential approach can be used to model quantum effects without requiring the Schrodinger equation to be explicitly solved and it can be integrated in simulations.

1.1.5 Magnetic Field Effect in Plasma

Magnetic field (magnetic bucket) on the plasma periphery enhances the plasma confinement, causing a rise in the plasma density and an essential improvement in the plasma uniformity. In rf discharges, the application of a magnetic field changes the plasma electrodynamics. Magnetic fields can modify the physical properties of a complex plasma in various different ways. Weak magnetic fields in the mT range affect only the electrons while strong fields in the Tesla regime also magnetize the ions. In a rotating dusty plasma, the Coriolis force substitutes the Lorentz force and can be used to create an effective magnetization for the strongly coupled dust particles while leaving electrons and ions unaffected [12]. Electron drift in the presence of a magnetic field are applied to the low pressure uniform positive column plasma. A longitudinal magnetic field leaves the point-to-point concentration of electrons unchanged and does not alter the relative potentials in the cross section although transverse potential differences in the plasma decrease everywhere in proportion as the magnetic field increases [12]. The transverse plasma fields vanish or even reverse slightly for large enough fields. Plasma exhibits a diamagnetic susceptibility for longitudinal fields, which is proportional to the electron current density to the tube walls. With nonconducting walls electron wall current is automatically adjusted to the ion wall current [12]. The magnetic polarization then increases oppositely to the magnetic field at first, reaches a maximum and then decreases hyperbolically to zero, so that beyond the maximum the plasma is paramagnetic for small variations in the field.

1.1.6 Dense Quantum Plasma

Degenerate matter is extremely high density matter in which pressure no longer depends on temperature due to the quantum mechanical effects. Matter that is so dense, atomic particles are packed together until quantum effects support the material. Material within white dwarfs and neutron stars is degenerate. When the density of a classical plasma increases, or its temperature decreases, it can enter a regime when the quantum nature of its constituent particles starts to affect its macroscopic properties and dynamics. Such plasmas are then called quantum. In quantum plasmas, the mean interparticle distance becomes comparable to the mean de Broglie wavelength of the lightest plasma particles, and the effects of degeneracy e.g., quantum degeneracy of electrons due to Paulis exclusion princinple for fermions become significant. Quantum effects of lighter plasma species (electrons, positrons,

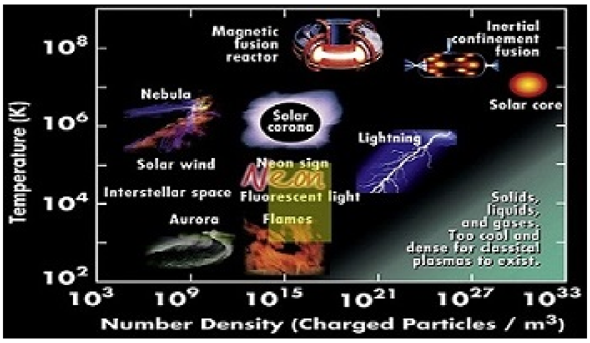

Figure 1.6: The figure showing different regimes of dense plasma based on the number density (n) and temperature (T) (Photo: https://www.researchgate.net/).

etc.) are more effective due to their smaller mass than the heavier plasma species (ions, heavy ions, etc.) which is also depended upon the degeneracy parameter. Quantum effects are predominant in the degenerate quantum plasma, and it is quite consistence considering quantum effects while dealing with quantum plasma. In quantum plasmas, there are new forces associated with i) quantum statistical electron and positron pressures, ii) electron and positron tunneling through the Bohm potential, and iii) electron and positron angular momentum spin. Inclusion of these quantum forces provides possibility of very high-frequency dispersive electrostatic and electromagnetic waves (e.g., in the hard x-ray and gamma rays regimes) having extremely short wavelengths. The degenerate quantum plasma is presumably existed in the early universe [13, 14], in the cometary tails [15], the evolution and star formation [16], solid state physics, condensed matter physics, etc. Unlike a classical ideal gas, whose pressure is proportional to its temperature ($P = nk_BT/V$, where P is pressure, V is volume, n is the number of particles-typically atoms or molecules, k_B is Boltzmann's constant, and T is temperature), the pressure exerted by degenerate matter depends weakly on its temperature and only on the density of the fermions. At extremely high densities, the electron Fermi energy, ϵ_{Fe} becomes much larger than thermal energy $\sim k_BT$. In this situation, the thermal pressure of electrons may be negligible as compared to the Fermi degeneracy pressure. In particular, the pressure remains nonzero even at absolute zero temperature. At relatively low densities, the pressure of a fully degenerate gas is given by $P = K(n/V)^{5/3}$ (non-relativistic), where K depends on the properties of the particles making up the gas. At very high densities, where most of the particles are forced into quantum states with relativistic energies, the pressure is given by $P = K_1(n/V)^{4/3}$ (ultra-relativistic), where K_1 again depends on the properties of the particles making up the gas. Thus, degenerate pressure keeps dense stars in equilibrium independent of the thermal structure of the star.

1.1.7 Formation of Plasma from a Gas

For the existence of plasma, ionization is necessary. When a gas is heated sufficiently hot enough or apply radio frequencies or bombard them with other particles or subjecting it to a strong electromagnetic field applied with a laser or microwave generator then the electrons will have enough energy to escape individual atoms and produce a plasma. Plasmas are not always hot, they can be cold too, as long as matter is ionized. A tubelight, a CFL, are comparatively cool to touch, so is the plasma TV screen. Plasma is believed by scientist to be the most common element in the observable universe, with approximately 99% of the observable matter found in the plasma state. It is naturally occurring and can be found in stars, lightning, the Aurora Borealis, the Sun and common flames. Plasma is also used in neon signs, TVs and fluorescent bulbs because of its characteristic glow when electrically charged.

1.2 Degenerate Compact Objects

Compact objects are generally referring to objects significantly more dense than a star or a planet. For example, white dwarfs or neutron stars are extremely dense stars that have collapsed, no longer able to produce a sufficient amount of pressure within to prevent their outer layers from falling into their centers. Under extreme conditions, these collapses can trigger the formation of a black hole a region of space in which gravity is so strong that even light cannot escape. When normal stars "die," that is, when most of their nuclear fuel has been consumed, compact objects-white dwarfs, neutron stars, and black holes-are "born". All three species of compact object differ from normal stars in two fundamental ways. First, since they do not burn nuclear fuel, they cannot support themselves against gravitational collapse by generating thermal pressure. Instead, white dwarfs are supported by the pressure of degenerate electrons, while neutron stars are supported largely by the pressure of degenerate neutrons. The outward degenerate pressure supports the existence of compact stars counterbalancing the inward gravitational force. Because of the degenerate pressure, a compact star is also referred as a degenerate star [17]. Chandrashekhar limit determines what will be a outlook (i.e. white dwarf, neutron star, and black hole) of a compact star. One can classify a compact star depending on the various value of Chandrashekhar limit. Stellar evolution and fusion are the phenomena that fuel the stars to live.

1.2.1 Stellar Evolution and Fusion

A star begins its life within a nebula. A nebula is a cloud of dust and gas, composed primarily of hydrogen (97%) and helium (3%). Within a nebula, there are varying regions when gravity causes this dust and gas to "clump" together. As these "clumps" gather more atoms, their gravitational attraction to other atoms increases, pulling more atoms into the "clump". This clump is a protostar. Because numerous reactions occur within the mass of forming star material, a protostar is not very stable. In order to achieve life as a star, the protostar will need to achieve and maintain equilibrium. Equilibrium for a star is when the balance between gravity pulling atoms toward the center and gas pressure pushing heat and light away from the center are equal. Achieving and keeping this balance is hard to do. When a star is close to reaching equilibrium, it has two options. Its first option is to become a brown dwarf. A brown dwarf is not quite a real star. It is a case in which a protostar never had enough dust and gas accreted to achieve a temperature hot enough to ignite fusion. It is bigger than a planet, but smaller than a regular star. The second option is for the protostar to build up enough mass to achieve the critical temperature of about 15,000,000 °C, which causes nuclear fusion to begin. Nuclear fusion releases thermal energy, photons, supports the star, and stops the contraction. Thus, a star is born.

1.2.2 Degenerate Pressure

A pressure exerted by dense material consisting of fermions (such as electrons in a white dwarf star) is degenerate pressure. This pressure is explained in terms of the Pauli exclusion principle, which requires that no two fermions be in the same quantum state. The more densely fermions are packed together and must share the same space, the more they must differ from each other in terms of their momentum. In turn, the greater the range in momentum, the greater the fraction of particles with high momentum, and these exert pressure on their surroundings. When this happens the fermions are said to be degenerate. In white dwarf high energy electrons make a significant contribution to the pressure. Because the pressure arises from this quantum mechanical effect, it is insensitive to temperature, i.e. the pressure doesn't go down as the star cools. This pressure is known as electron degeneracy pressure and it is the force that supports white dwarf stars against their own gravity. At high temperatures, particles have lots of energy and many quantum states available to them. On the average, the probability that any quantum state is occupied is rather small ($<< 1$) and the exclusion principle plays little role. At lower temperatures, particles have less energy, fewer quantum states are available and average occupation number of each state increases. Then the exclusion principle becomes essential: the available levels up to some maximum energy (determined by the density) are, on average, nearly filled; higher levels are, on average, nearly empty. Such systems are then termed "degenerate". Actually these statements are strictly true only at zero temperature and when the mutual interactions of the fermions are ignored.

The Pauli exclusion principle states that no two electrons with the same spin can occupy the same energy state in the same volume. Once the lowest energy level is filled, the other electrons are forced into higher and higher energy states resulting in them travelling at progressively faster speeds. These fast moving electrons create a pressure (electron degeneracy pressure) which is capable of supporting a star. This degeneracy pressure originates for all kinds of fermions in a compact stars. We imagine a plasma is cooled and compressed repeatedly. Eventually, we will not be able to compress the plasma any further, because the Pauli exclusion principle states that two fermions cannot share the same quantum state. When in this state, since there is no extra space for any particles, we can also say that a particle's location is extremely defined. Therefore, since according to the Heisenberg's uncertainty principle $\Delta p \Delta x \geq \hbar/2$ where Δp is the uncertainty in the particle's momentum and Δx is the uncertainty in position, then we must say that their momentum is extremely uncertain since the molecules are located in a very confined space. This leads to the conclusion that if we want to compress an object into a very small space, we must use tremendous force to control its particles momentum. This gives the degenerate pressure. Having so much kinetic energy, the electrons exert a tremendous pressure on the walls of the container that holds them. This pressure is called degenerate pressure [18].

1.2.3 Chandrasekhar Limit

Weight is the most crucial property for a white dwarf. When a white dwarf star puts on too much weight (i.e. adds mass), a supernova explosion will be occurred. The greatest mass a white dwarf star can have before it goes supernova is called the Chandrasekhar limit, after astrophysicist Subrahmanyan Chandrasekhar, who worked it in the 1930s. Its value is about 1.4 solar mass $M = 1.44 M_{\odot}$. Chandrasekhar was interested in the final states of collapsed stars as determined by electron degeneracy and had used the work of Arthur S. Eddington and Ralph H. Fowler to begin calculations. He realized that they hadn't included relativity in their calculations. When Chandrasekhar took these relativistic effects into account, something spectacular happened. He found a firm upper limit for the mass of any body which could be supported by electron degeneracy pressure. Once this limit was exceeded the Chandraskehar limit, the object could no longer resist the force of gravity, and it would begin to collapse. The masses above $1.44 M_{\odot}$ there could be no balance between electron degeneracy and the crushing gravitational force [19, 20].

Stars produce energy through nuclear fusion, producing heavier elements from lighter ones. The heat generated from these reactions prevents gravitational collapse of the star. Over time, the star builds up a central core which consists of elements that the temperature at the center of the star is not sufficient to fuse. For main-sequence stars with a mass below approximately 8 solar masses, the mass of this core will remain below the Chandrasekhar limit, and they will eventually lose mass (as planetary nebulae) until only the core, which becomes a white dwarf, remains. Stars with higher mass will develop a degenerate core whose mass will grow until it exceeds the limit. At this point the star will explode in a core-collapse supernova, leaving behind either a neutron star or a black hole [21]. The currently accepted value of the limit is about $1.44 \times (2.864 \times 10^{30})$ Kg [22].

1.2.4 Examples of Compact Objects

White Dwarfs

The knowledge of white dwarfs began in 1850 with the discovery of a companion to Sirius, called Sirius B. It was 10,000 times fainter than Sirius A, however its mass was 0.98 a solar mass. Since its temperature was measured to be 10,000K, its small mass and faint luminosity did not make sense in the context of the mass-luminosity relation for stars. Where a star ends up at the end of its life depends on the mass it was born with. Stars that have a lot of mass may end their lives as black holes

Figure 1.7: A White dwarf with dense stellar corpses (Photo: https://www.space.com/).

or neutron stars. A low or medium mass star (with mass less than about 8 times the mass of our Sun) will become a white dwarf. A typical white dwarf is about as massive as the Sun, yet only slightly bigger than the Earth. This makes white dwarfs one of the densest forms of matter, surpassed only by neutron stars and black holes. Our star, the sun, will die a quiet death. The sun of only average mass, starwise, and after burning through the last of its hydrogen fuel in about five billion years, its outer layers will drift away, and the core will eventually compact to become what's known as a white dwarf, an Earth-size ember of the cosmos. A white dwarf is a very dense solar remnant that is supported by the balance between electron degeneracy pressure and the star's gravitational self-attraction. In a white dwarf, matter is ionized and electrons are free of their atomic orbits around the nuclei. During gravitational collapse, matters density increases and so does the electron concentration within a certain space volume.

Neutron Stars

A neutron star is about 20 km in diameter and has the mass of about 1.4 times that of our Sun. This means that a neutron star is so dense that on Earth. Due to small size and high density, a neutron star possesses a surface gravitational field about 2×10^{11} times that of Earth. Neutron stars can also have magnetic fields a million

Figure 1.8: A new type of neutron star detected by the scientists in Australia have whose existence had only been a hypothesis until now, calling it an "ultra-long period magnetar (Photo: ART-ur / Shutterstock).

times stronger than the strongest magnetic fields produced on Earth. They are one of the possible ends for a star. They result from massive stars which have mass greater than 4 to 8 times that of our Sun. After these stars have finished burning their nuclear fuel, they undergo a supernova explosion. This explosion blows off the outer layers of a star into a beautiful supernova remnant. The central region of the star collapses under gravity. It collapses so that protons and electrons combine to form neutrons. neutron density stops the gravitational collapse, giving the star permanent stability. The remnant star is then a neutron star [23].

Black Holes

In general relativity, a black hole is a region of space in which the gravitational field is so powerful that nothing, including light, can escape its pull. The black hole has a one-way surface, called an event horizon, into which objects can fall, but out of which nothing can come. It is called "black" because it absorbs all the light that hits it, reflecting nothing, just like a perfect blackbody in thermodynamics [24]. Quantum analysis of black holes shows them to possess a temperature and Hawking radiation. At the heart of a black hole is an object called a singularity, a point of zero size, and infinite density. The gravity is so strong because matter has been squeezed into a tiny space. This can happen when a star is dying. Such objects are what we now call black holes, because that is what they are black voids in space [25]. When the body is outside of the gravitational pull, its kinetic energy and potential

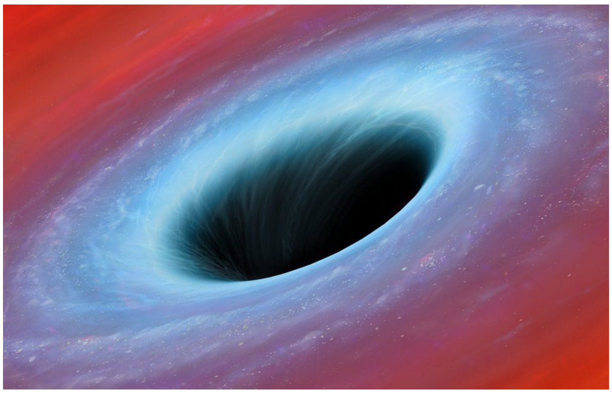

Figure 1.9: A black hole observations were carried out with the European Southern Observatory (ESO) in Chile (Photo: https://www.bbc.com/).

energy will be zero, so if we equate them

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

and the rearrange for v we get an expression for the escape velocity

$$v_e = \sqrt{\frac{2GM}{r}}$$

where M is the mass of the planet or body and r is the radius that are taking off from.

1.3 Nonlinear Theory

A system where the amplitude of the perturbations is just sufficiently large, nonlinearities cannot be ignored. The simplest nonlinear wave is perhaps $u_t + uu_x =$

0. This is a wave in which the speed is the disturbance u itself. The solitary wave solution of a nonlinear dispersive wave equation cleverly balances the nonlinearity against the dispersion so that the wave retains its shape. A remarkable example is the SWs solution of the K-dV equation $u_t + uu_x + u_{xxx} = 0$. The K-dV equation describes the propagation of gravity waves in shallow water and its SW is precisely that observed by Scott Russel in 1834 [26]. The nonlinearities come from the harmonic generation involving fluid advection, the nonlinear Lorentz force, trapping of particles in the wave potential, etc. The nonlinearities in plasmas contribute to the localization of waves, leading to different types of coherent structures (namely DLs, shock structures, etc.) which are important from theoretical, analytical, numerical, and experimental points of view. Superposition principle is not applicable for nonlinear wave equation: two solutions do not add to form another solution. Thus if two solitary waves of the K-dV collide we would expect each to scatter off the other and some new disturbance emerge. In practice this does not happen: the SWs simply pass through each other and emerge essentially unchanged. It is this collisional stability which characterises the soliton. The K-dV is a good approximate equation governing any weakly nonlinear, weakly dispersive system. The K-dV, or a modified form of it, governs shallow waves, ion-acoustic wave, and Alfvén waves in a cold collision-less plasma, and the propagation of sound waves in anharmonic crystal, for example.

1.3.1 Waves in Plasmas

Nonlinear waves are described by nonlinear equations. This means that nonlinear wave equations are more difficult to analyze mathematically, and that no general analytical method for their solution exists. Thus, unfortunately, each particular wave equation has to be treated individually. A detailed aspects of solitary waves are as follows:

1.3.2 Solitary Waves

A solitary wave is a wave which propagates without any temporal evolution in shape or size when viewed in the reference frame moving with the group velocity of the wave. It arises from a balance between nonlinear and dispersive effects. It was first observed by John Scott Russell [26] in 1834, when he saw a round smooth well defined heap of water detach itself from the power of a stopped barge and proceed without change of shape or diminution of speed for over two miles along in the Union Canal, near Edinburgh, supposedly the Edinburgh-Glasgow canal. The envelope of the wave has one global peak and decays far away from the peak. Solitary waves arise in many contexts, including the elevation of the surface of water and the intensity of light in optical fibers.

A soliton is a nonlinear solitary wave with the additional property that the wave retains its permanent structure, even after interacting with another soliton. Solitons form a special class of solutions of model equations, including the K-dV and the Nonlinear Schrodinger (NLS) equations. These model equations are approximations, which hold under a restrictive set of conditions. The soliton solutions obtained from the model equations provide important insight into the dynamics of solitary waves. However, they are limited by the conditions under which the model equations hold.

1.4 Methods for Studying Nonlinear Phenomena

A lot of methods have been employed by different physicists to investigate the nonlinear phenomenon in different plasma systems, e.g., The reductive perturbation method, Quasi-linear Approximation, Pseudo Potential Method [27], Probability Approach, Arbitrary amplitude method, etc. But, amongst of them the reductive perturbation method is the most used and preferable method in studying plasma phenomena.

1.4.1 The Reductive Perturbation Method

The reductive perturbation method is a very powerful way of deriving simplified models describing nonlinear wave propagation and interaction. As the name suggests there are two essential features involved with this technique. Most standard perturbation techniques involve a basic linearisation about some known solution followed by the solution of the resulting linear equations to calculate correction terms. The reductive perturbation method is intimately related to soliton theory. Soliton is a solitary waves, i.e. a single hump of water, which propagates without deformation. In a linear nondispersive medium, propagation without deformation occurs for any wave. The reductive perturbation method, when applied to some integrable equation, could lead to some other equation, again integrable. This has been (and is still) used to construct approximate solutions of integrable equations and to study the mathematical properties.
Furthermore, it is noticed that although $\epsilon << 1$ is required in the reductive perturbation method generally, the reductive perturbation method is also valid for $\epsilon < 1$ in a dusty plasma, which may be extended to branches where the reductive perturbation method is used [46]. The reductive perturbation method, seems to be initially due to Gardner and Morikawa, concerning hydromagnetic waves in a cold plasma in a paper which was unpublished. The method was used again by Washimi and Taniuti [28], applied to the study of ion-acoustic waves. Taniuti and Wei wrote a general theory of the derivation of the K-dV equation [29]. Let us also mention the derivation of the K-dV and Burgers equations, by Gardner and Su [30]. Taniuti and his co-workers initiated the method concerning the envelopes, leading to the nonlinear Schrodinger (NLS) equation. It is remarkable that the first paper in which this equation has been derived using the reductive perturbation method, i.e. a multiscale expansion, does not, properly speaking, study an envelope problem. .

Chapter 2

Nonlinear equation for solitary waves

2.1 Introduction

Collective processes in linear and nonlinear dusty plasma have become special research factor in the past decade mainly due to the realization of their occurrence in both the laboratory and space environments [31]. In different environment of space and astrophysical plasma such as interstellar medium, interplanetary space, interstellar or molecular clouds, comets, planetary rings and the Earth's environments, participation of dust particles are very much common [32]. Shukla and Silin [33] were the first to investigate the dust ion acoustic waves theoretically which was then studied in laboratory experiments. The dust particles are many orders of magnitude heavier than ions, they are a source of ionization and recombination for electrons and their charge is not fixed, but depends on local plasma parameters. Wave propagation in such complex systems is therefore expected to be substantially different from the ordinary two component plasma and the presence of charged dust can have a strong influence on the characteristics of the usual plasma wave modes, even at frequencies where the dust grains do not participate in the wave motion [34]. A research conducted by scientist that considers existing plasma wave spectra has been modified by the presence of static charged dust grains [35]. Dusty plasma which is a mixture of positive ions, negative ions, electrons and highly charged micro particles and nano particles have been treated as an important re-search field [34]. The existence of multi-ion plasmas (containing both positive and negative ion) has already been confirmed in ionosphere and magnetosphere of Earth [36], solar wind [37], bow shock in front of the magneto pause boundary layers [38], Saturn's magnetosphere [39], coma of comet Halley [40], neutral beam sources [41], plasma processing reactors [42] and low-temperature laboratory experiments [43]. Multi-ion plasma is now seems to be an interesting topic to the modern plasma physics researchers for studying the wave phenomena in plasma physics. Recently, Kim and Merlino [44] discussed the conditions that described the time of injection of dust into a laboratory negative ion plasma, becomes positively charged for very large values of negative ion density is equal or less than 500 times the electron density. The presence of the magnetic field (which creates the obliqueness of the wave propagation) plays a vital role in controlling the basic characteristics of the linear and nonlinear waves in space and astrophysical plasmas [45]. Plasma particles are charged particles and the effect of external magnetic field significantly modify the characteristics of propagation of IASWs.

Astrophysical compact objects are examples where relativistic degenerate plasmas are dominant and interesting new phenomena are investigated by several nonlinear effects in such plasmas. The degenerate electron number density is so high (in white dwarfs it can be of the order of 10^{30} cm^{-3}, even more [46-48], such that their cores are composed of strongly coupled non-degenerate ion lattices immersed in degenerate electron fluids that follow the Fermi-Dirac distribution function [49]. An investigation conducted by F. Haans [50] showed that the Bohm potentials associated with the plasma particles significantly modify the basic features of the nonlinear IA waves. Recently, researches of linear and nonlinear [51-59] electrostatic excitation in a collision less dense Fermi plasma (where quantum statistical pressure [51], quantum electron tunneling [60] and relativistic effects [61] become important) have gained a great deal of interest for exploring fundamental properties of linear nonlinear physics as well as developing practical application in plasma based nanotechnology (e.g., ultra-small electronic devices [62], metallic thin films and nano structures [63], quantum free-electron lasers and X-ray sources [64], nano plasmonics [65], intense laser-compressed solid density plasma [66], compact astrophysical objects [67,68], etc.). the equation of state for degenerate electrons are mathematically explained by Chandrasekhar [69]. Chandrasekhar [69,70] presented a general expression for the relativistic electron pressures in his classical papers. The pressure for electron fluid can be given by the following equation

$$P_e = K n_e^{\gamma} \ . \tag{2.1}$$

For non-relativistic limit [69,70]

$$\gamma = \frac{5}{3}; \quad K = \frac{3}{5}\left(\frac{\pi}{3}\right)^{\frac{1}{3}}\frac{\pi\hbar^2}{m} \simeq \frac{3}{5}\Lambda_c\hbar c, \tag{2.2}$$

where $\Lambda_c = \pi\hbar/mc = 1.2 \times 10^{-10}$ cm, and $\hbar$ is the Planck constant divided by 2π. On the other hand, for ultra-relativistic limite[69,70]

$$\gamma = \frac{4}{3}; \ \ K = \frac{3}{4}\left(\frac{\pi^2}{9}\right)^{\frac{1}{3}}\hbar c \simeq \frac{3}{4}\hbar c. \tag{2.3}$$

Analysis of quantum plasma physics started with guiding theoretical works of Klimontovich and Silin [71], and Bohm and Pines [72,73], who studied the dispersive properties of the electron plasma oscillations in a dense quantum plasma with only considering degenerate electrons. Another reserach conducted by Arpan Das, Shreyansh S. Dave, P. S. Saumia and Ajit M. Srivastava that consider the magnetic effect only [74]. In our work, we have considered degenerated quantum plasma with magnetic effect for a four component plasma system consists of magnetized positive and negative ion, degenerate relativistic electron along with stationary dust particle. We derived K-dV, mK-dV and mmK-dV equations along with their SWs solutions which provided better understanding and information about the considered plasma system and it's components. The results of our work can be useful for future advance research and practical purpose related to plasma physics.

2.2 Governing Equations

We have considered four-component plasma system consisting magnetized positive and negative ions, unmagnetized quantum electron with stationary dust particles. The external magnetic field is directed in the z axis, i.e., $B = B_o\hat{z}$, $\hat{z}$ is the unit vector along the z axis. Quasi-neutrality condition is δZ$_d$n$_{d0}$+n_{p0}-n_{n0}-n_{e0}=0, where Z_d is the magnitude of dust charge, $\delta = \pm 1$ (+1 for positive dust particles and -1 for negative dust particles), n_{n0} is the equilibrium number density of negative ions, n_{p0} is the equilibrium number density of positive ions and n_{e0} is the equilibrium number density of electrons. The dynamics of the SWs propagating in such a multi-ion plasma system is governed by the following normalized equations:

$$\frac{\partial n_p}{\partial t} + \frac{\partial}{\partial x}(n_p U_p) = 0, \tag{2.4}$$

$$\frac{\partial n_n}{\partial t} + \frac{\partial}{\partial x}(n_n U_n) = 0, \tag{2.5}$$

$$\frac{\partial U_p}{\partial t} + U_p \frac{\partial U_p}{\partial x} + \frac{\partial \phi}{\partial x} - \omega_p (U_p \times \hat{z}) = 0, \tag{2.6}$$

$$\frac{\partial U_n}{\partial t} + U_n \frac{\partial U_n}{\partial x} - \beta \frac{\partial \phi}{\partial x} + \omega_n (U_n \times \hat{z}) = 0, \tag{2.7}$$

$$n_e \frac{\partial \phi}{\partial x} - K \frac{\partial {n_e}^{\gamma}}{\partial x} + \alpha \frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_e}} \frac{\partial^2}{\partial x^2} \sqrt{n_e}\right) = 0, \tag{2.8}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_n + \mu n_e - \sigma n_p - \delta \eta, \tag{2.9}$$

where n_p is the number density of positive ion, n_n is the number density of negative ion and n_e is the number density of electron which are normalized by n_{n0}. u_p is the fluid speed of positive ion and u_n is the fluid speed of negative ion are normalized by ion-acoustic wave speed $c_i = (K_B T_e/e)^{1/2}$. The normalizing parameters for Poisson,s equation are $\sigma = n_{p0}/n_{n0}$, $\mu = n_{e0}/n_{n0}$, $\eta = Z_d n_{d0}/n_{n0}$ and $\delta\eta = 1 + \mu - \sigma$. The other normalized parameters $\beta = m_p/m_n$ and the non-dimensional quantum parameter for electron $\alpha = n_{n0} H_e^2 / 2n_{e0}$ where $H_e = \hbar\omega_{pe}/K_B T_{Fe}$. ϕ is the electrostatic wave potential normalized by $K_B T_e/e$ with e being the magnitude of the charge of an electron. The positive and negative ion cyclotron frequency $\omega_{p,n} = eB_0/m_{p,n}\,c$ is normalized by the negative ion plasma period $\omega_n^{-1} = (4\pi e^2 n_{n0}/m_n)^{-1/2}$. The time variable ($t$) is normalized by ω_n^{-1}. The space variable (x) is normalized by $\lambda_n = (K_B T_{Fe}/4\pi e^2 n_{n0})^{1/2}$. Here $K = n_{e0}^{\gamma-1} K_e/m_e c^2$, m_p is the mass of positive ion and m_n is the mass of negative ion.

Chapter 3

Derivation of the K-dV Equation

We will derive K-dV equation by employing the reductive perturbation technique in order to examine the characteristics of the SWs propagating in a dense multi-ion plasma system by introducing the stretched coordinates [75] given below as

$$\xi = \epsilon^{1/2}(l_x + l_y + l_z - V_p t), \tag{3.1}$$
$$\tau = \epsilon^{3/2} t, \tag{3.2}$$

where V_p $(= \omega/k)$ is the wave phase speed (ω is angular frequency and k is the wave number) and ϵ is a smallness parameter measuring the weakness of the dispersion $(0 < \epsilon < 1)$. The terms l_x, l_y and l_z are the directional cosines of the wave vector along the x, y and z axes, respectively, so that $l_x^2 + l_y^2 + l_z^2 = 1$. We then expand n_p, n_n, n_e, u_p, u_n and ϕ in power series of ϵ as

$$n_p = 1 + \epsilon n_p^{(1)} + \epsilon^2 n_p^{(2)} + \epsilon^3 n_p^{(3)} + \cdots, \tag{3.3}$$
$$n_n = 1 + \epsilon n_n^{(1)} + \epsilon^2 n_n^{(2)} + \epsilon^3 n_n^{(3)} + \cdots, \tag{3.4}$$
$$n_e = 1 + \epsilon n_e^{(1)} + \epsilon^2 n_e^{(2)} + \epsilon^3 n_e^{(3)} + \cdots, \tag{3.5}$$
$$u_{px,y} = \epsilon^{3/2} {u_{px,y}}^{(1)} + \epsilon^2 {u_{px,y}}^{(2)} + \epsilon^{5/2} {u_{px,y}}^{(3)} + \cdots, \tag{3.6}$$
$$u_{nx,y} = \epsilon^{3/2} {u_{nx,y}}^{(1)} + \epsilon^2 {u_{nx,y}}^{(2)} + \epsilon^{5/2} {u_{nx,y}}^{(3)} + \cdots, \tag{3.7}$$
$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \epsilon^3 \phi^{(3)} + \cdots, \tag{3.8}$$
$$u_{pz} = \epsilon {u_{pz}}^{(1)} + \epsilon^2 {u_{pz}}^{(2)} + \epsilon^3 {u_{pz}}^{(3)} + \cdots, \tag{3.9}$$
$$u_{nz} = \epsilon {u_{nz}}^{(1)} + \epsilon^2 {u_{nz}}^{(2)} + \epsilon^3 {u_{nz}}^{(3)} + \cdots, \tag{3.10}$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, Eqs. (2.4)−(2.9) give

$$\begin{aligned} {u_{pz}}^{(1)} &= \frac{l_z \phi^{(1)}}{V_p}, \\ {u_{nz}}^{(1)} &= -\frac{\beta l_z \phi^{(1)}}{V_p}, \\ n_p^{(1)} &= \frac{l_z^2 \phi^{(1)}}{V_p^2}, \\ n_n^{(1)} &= -\frac{\beta l_z^2 \phi^{(1)}}{V_p^2}, \end{aligned}$$

$$n_e^{(1)} = \frac{\phi^{(1)}}{K},$$
$$V_p = l_z\sqrt{\frac{K(\sigma+\beta)}{\mu}},$$

where V_p represents the dispersion relation for the IA type SWs in the degenerate multi-ion plasma under consideration and $l_z = cos\delta$. Equating the coefficient of $\epsilon^{3/2}$ from Eq. (2.6) and Eq. (2.7) we get

$$u_{py}{}^{(1)} = \frac{l_x}{\omega_p}\frac{\partial\phi^{(1)}}{\partial\xi}(x - component),$$
$$u_{px}{}^{(1)} = -\frac{l_y}{\omega_p}\frac{\partial\phi^{(1)}}{\partial\xi}(y - component),$$
$$u_{ny}{}^{(1)} = \beta\frac{l_x}{\omega_n}\frac{\partial\phi^{(1)}}{\partial\xi}(x - component),$$
$$u_{nx}{}^{(1)} = -\beta\frac{l_y}{\omega_n}\frac{\partial\phi^{(1)}}{\partial\xi}(y - component),$$

and also equating the coefficient of ϵ^2 from Eq. (2.6) and Eq. (2.7) we get

$$u_{py}{}^{(2)} = \frac{V_p l_y}{\omega_p^2}\frac{\partial^2\phi^{(1)}}{\partial\xi^2}(x - component),$$
$$u_{px}{}^{(2)} = \frac{V_p l_x}{\omega_p^2}\frac{\partial^2\phi^{(1)}}{\partial\xi^2}(y - component),$$
$$u_{ny}{}^{(2)} = -\beta\frac{V_p l_y}{\omega_n^2}\frac{\partial^2\phi^{(1)}}{\partial\xi^2}(x - component),$$
$$u_{nx}{}^{(2)} = -\beta\frac{V_p l_x}{\omega_n^2}\frac{\partial^2\phi^{(1)}}{\partial\xi^2}(y - component).$$

Substituting Eqs. (3.1)-(3.10) in Eqs. (2.4)-(2.9) and equating the coefficient of $\epsilon^{3/2}$ from Eqs. (2.4)-(2.5) and also taking the coefficient of $\epsilon^{5/2}$ from Eqs. (2.6)-(2.8), we obtain a set of equations that can be simplified as

$$\frac{\partial n_p^{(2)}}{\partial\xi} = \frac{1}{V_p}\frac{\partial n_p^{(1)}}{\partial\tau} + \frac{l_x}{V_p}\frac{\partial u_{px}{}^{(2)}}{\partial\xi} + \frac{l_y}{V_p}\frac{\partial u_{py}{}^{(2)}}{\partial\xi} + \frac{l_z}{V_p}\frac{\partial}{\partial\xi}(n_p^{(1)}u_{pz}{}^{(1)}) + \frac{l_z}{V_p^2}\frac{\partial u_{pz}{}^{(1)}}{\partial\tau} + \frac{l_z^2}{V_p^2}u_{pz}{}^{(1)}\frac{\partial u_{pz}{}^{(1)}}{\partial\xi} + \frac{l_z^2}{V_p^2}\frac{\partial\phi^{(2)}}{\partial\xi}, \tag{3.11}$$

$$\frac{\partial n_n^{(2)}}{\partial\xi} = \frac{1}{V_p}\frac{\partial n_n^{(1)}}{\partial\tau} + \frac{l_x}{V_p}\frac{\partial u_{nx}{}^{(2)}}{\partial\xi} + \frac{l_y}{V_p}\frac{\partial u_{ny}{}^{(2)}}{\partial\xi} + \frac{l_z}{V_p}\frac{\partial}{\partial\xi}(n_n^{(1)}u_{nz}{}^{(1)}) + \frac{l_z}{V_p^2}\frac{\partial u_{nz}{}^{(1)}}{\partial\tau} + \frac{l_z^2}{V_p^2}u_{nz}{}^{(1)}\frac{\partial u_{nz}{}^{(1)}}{\partial\xi} - \beta\frac{l_z^2}{V_p^2}\frac{\partial\phi^{(2)}}{\partial\xi}, \tag{3.12}$$

$$\frac{\partial n_e^{(2)}}{\partial\xi} = \frac{1}{K}n_e^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi} - (\gamma-1)\frac{1}{K}n_e^{(1)}\frac{\partial n_e^{(1)}}{\partial\xi} + \frac{\alpha}{2K}\frac{\partial^3 n_e^{(1)}}{\partial\xi^3} + \frac{1}{K}\frac{\partial\phi^{(2)}}{\partial\xi}, \tag{3.13}$$

$$\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = \frac{\partial n_n^{(2)}}{\partial\xi} + \mu\frac{\partial n_e^{(2)}}{\partial\xi} - \sigma\frac{\partial n_p^{(2)}}{\partial\xi}. \tag{3.14}$$

Now substituting Eqs. (3.11)-(3.13) into Eq. (3.14), we obtain an equation of the form

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0, \tag{3.15}$$

where

$$A = \left[\frac{\frac{3l_z^4}{V_p^4}(\sigma - \beta^2) - \frac{\mu(2-\gamma)}{K^2}}{\frac{2l_z^2}{V_p^3}(\sigma + \beta)}\right], \tag{3.16}$$

$$B = \left[\frac{(\frac{\sigma}{\omega_p^2} + \frac{\beta}{\omega_n^2}) \times (1 - l_z^2) - \frac{\mu\alpha}{2K^2} + 1}{\frac{2l_z^2}{V_p^3}(\sigma + \beta)}\right]. \tag{3.17}$$

Eq. (3.15) is known as K-dV equation. Now, to investigate the properties of the IASWs, we need to derive the SWs solution of the K-dV equation. So, we introduce another stretched coordinates, $\zeta = \xi - U_0\tau$. After the coordinate transformation, the steady state ($\partial/\partial\tau = 0$) solution of the of K-dV equation can be written as (by taking $\phi^{(1)} = \Phi$)

$$\Phi = \Phi_m sech^2(\frac{\xi}{\Delta_1}), \tag{3.18}$$

where the amplitude Φ_m and width Δ_1 given by

$$\Phi_m = \frac{3U_0}{A}, \tag{3.19}$$

$$\Delta_1 = \sqrt{\frac{4B}{U_0}}. \tag{3.20}$$

The value of μ, σ and β effect both the amplitude, and width. But α, ω_p and ω_n effect the width only.

Chapter 4

Derivation of the mK-dV Equation

The K-dV equation [Eq. 3.15] is the result of the second order calculation in the smallness parameter ϵ, where the quadratic nature has been revealed by the nonlinear term $A\phi^{(1)}\partial\phi^{(1)}/\partial\xi$. For plasmas with more than two species as like our system, however, there can arise cases where A vanishes at a particular value of a certain parameter μ, and Eq. 3.15 fails to describe nonlinear evolution of perturbation. So, higher order calculation is needed at this critical value $\mu = \mu_c$. The higher order calculation must be considered, in which the coefficient of $\partial\Phi/\partial\xi$ vanishes. For this reason we employ the following stretched coordinates.

$$\xi = \epsilon^{1/2}(l_x + l_y + l_z - V_p t), \tag{4.1}$$
$$\tau = \epsilon^{3/2} t. \tag{4.2}$$

We then expand n_p, n_n, n_e, u_p, u_n and ϕ in power series of ϵ:

$$n_p = 1 + \epsilon^{1/2} n_p^{(1)} + \epsilon n_p^{(2)} + \epsilon^{3/2} n_p^{(3)} + \cdots, \tag{4.3}$$
$$n_n = 1 + \epsilon^{1/2} n_n^{(1)} + \epsilon n_n^{(2)} + \epsilon^{3/2} n_n^{(3)} + \cdots, \tag{4.4}$$
$$n_e = 1 + \epsilon^{1/2} n_e^{(1)} + \epsilon n_e^{(2)} + \epsilon^{3/2} n_e^{(3)} + \cdots, \tag{4.5}$$
$$u_{px,y} = \epsilon {u_{px,y}}^{(1)} + \epsilon^{3/2} {u_{px,y}}^{(2)} + \epsilon^2 {u_{px,y}}^{(3)} + \cdots, \tag{4.6}$$
$$u_{nx,y} = \epsilon {u_{nx,y}}^{(1)} + \epsilon^{3/2} u_{nx,y}^{(2)} + \epsilon^2 {u_{nx,y}}^{(3)} + \cdots, \tag{4.7}$$
$$\phi = \epsilon^{1/2}\phi^{(1)} + \epsilon\phi^{(2)} + \epsilon^{3/2}\phi^{(3)} + \cdots, \tag{4.8}$$
$$u_{pz} = \epsilon^{1/2} {u_{pz}}^{(1)} + \epsilon {u_{pz}}^{(2)} + \epsilon^{3/2} {u_{pz}}^{(3)} + \cdots, \tag{4.9}$$
$$u_{nz} = \epsilon^{1/2} {u_{nz}}^{(1)} + \epsilon {u_{nz}}^{(2)} + \epsilon^{3/2} {u_{nz}}^{(3)} + \cdots. \tag{4.10}$$

By substituting Eqs. (4.1)-(4.10) in Eqs. (2.4)-(2.9), we found the same values of $n_p^{(1)}$, $n_n^{(1)}$, $n_e^{(1)}$, ${u_{pz}}^{(1)}$, ${u_{nz}}^{(1)}$ and V_p as like as that of the K-dV equation. Equating the coefficient of $\epsilon^{3/2}$ from Eqs. (2.6)-(2.8) we get

$$ {u_{pz}}^{(2)} = \frac{lz^3}{2V_p^3} + \frac{lz}{V_p}\phi^{(2)},$$
$$ {u_{nz}}^{(2)} = \beta^2\frac{lz^3}{2V_p^3} - \beta\frac{lz}{V_p}\phi^{(2)},$$
$$ n_p^{(2)} = \frac{3lz^4}{2V_p^4}(\phi^{(1)})^2 + \frac{lz^2}{V_p^2}\phi^{(2)},$$

$$n_n^{(2)} = \frac{3\beta^2 lz^4}{2V_p^4}(\phi^{(1)})^2 - \beta\frac{lz^2}{V_p^2}\phi^{(2)},$$
$$n_e^{(2)} = \frac{2-\gamma}{2K^2}(\phi^{(1)})^2 + \frac{1}{K}\phi^{(2)},$$
$$\rho^{(2)} = -\frac{1}{2}A_1(\phi^{(1)})^2,$$
$$A_1 = \frac{3l_z^4}{V_p^4}(\sigma-\beta^2) + \frac{\mu(\gamma-2)}{K^2}.$$

Now the eqautions can be simplified as to the next higher order of ϵ, we obtain a set of equations:

$$\frac{\partial n_p^{(3)}}{\partial \xi} = \frac{1}{V_p}\frac{\partial n_p^{(1)}}{\partial \tau} + \frac{l_x}{V_p}\frac{\partial u_{px}{}^{(2)}}{\partial \xi} + \frac{l_y}{V_p}\frac{\partial u_{py}{}^{(2)}}{\partial \xi} + \frac{l_z}{V_p^2}\frac{\partial u_{pz}{}^{(1)}}{\partial \tau} + \frac{l_z}{V_p}\frac{\partial}{\partial \xi}(n_p^{(1)}u_{pz}{}^{(2)})$$
$$+\frac{l_z}{V_p}\frac{\partial}{\partial \xi}(n_p^{(2)}u_{pz}{}^{(1)}) + \frac{l_x}{V_p}\frac{\partial}{\partial \xi}(n_p^{(1)}u_{px}{}^{(1)}) + \frac{l_z^2}{V_p^2}u_{pz}{}^{(1)}\frac{\partial u_{pz}{}^{(2)}}{\partial \xi} + \frac{l_y}{V_p}\frac{\partial}{\partial \xi}(n_p^{(1)}u_{py}{}^{(1)})$$
$$+\frac{l_z^2}{V_p^2}\frac{\partial \phi^{(3)}}{\partial \xi}, \tag{4.11}$$

$$\frac{\partial n_n^{(3)}}{\partial \xi} = \frac{1}{V_p}\frac{\partial n_n^{(1)}}{\partial \tau} + \frac{l_x}{V_p}\frac{\partial u_{nx}^{(2)}}{\partial \xi} + \frac{l_y}{V_p}\frac{\partial u_{ny}^{(2)}}{\partial \xi} + \frac{l_z}{V_p^2}\frac{\partial u_{nz}^{(1)}}{\partial \tau} + \frac{l_z}{V_p}\frac{\partial}{\partial \xi}(n_n^{(1)}u_{nz}^{(2)})$$
$$+\frac{l_z}{V_p}\frac{\partial}{\partial \xi}(n_n^{(2)}u_{nz}^{(1)}) + \frac{l_x}{V_p}\frac{\partial}{\partial \xi}(n_n^{(1)}u_{nx}^{(1)}) + \frac{l_z^2}{V_p^2}u_{nz}^{(1)}\frac{\partial u_{nz}^{(2)}}{\partial \xi} + \frac{l_y}{V_p}\frac{\partial}{\partial \xi}(n_n^{(1)}u_{ny}^{(1)})$$
$$-\beta\frac{l_z^2}{V_p^2}\frac{\partial \phi^{(3)}}{\partial \xi}, \tag{4.12}$$

$$\frac{\partial n_e^{(3)}}{\partial \xi} = \frac{1}{k}n_e^{(1)}\frac{\partial \phi^{(2)}}{\partial \xi} + \frac{1}{k}n_e^{(2)}\frac{\partial \phi^{(1)}}{\partial \xi} - (\gamma-1)n_e^{(1)}\frac{\partial n_e^{(2)}}{\partial \xi} + \frac{\alpha}{2K}\frac{\partial^3 n_e^{(1)}}{\partial \xi^3}$$
$$-(\gamma-1)n_e^{(2)}\frac{\partial n_e^{(1)}}{\partial \xi} + \frac{1}{K}\frac{\partial \phi^{(3)}}{\partial \xi} - \frac{(\gamma-1)(\gamma-2)}{2}n_e^{(1)2}\frac{\partial n_e^{(1)}}{\partial \xi}, \tag{4.13}$$

$$\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = \frac{\partial n_n^{(3)}}{\partial \xi} + \mu\frac{\partial n_e^{(3)}}{\partial \xi} - \sigma\frac{\partial n_p^{(3)}}{\partial \xi}. \tag{4.14}$$

Now, combining Eqs. (4.11)-(4.13) into Eq. (4.14), we obtain an equation of the form

$$\frac{\partial \phi^{(1)}}{\partial \tau} + C\left[\phi^{(1)}\right]^2\frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0, \tag{4.15}$$

where

$$C = \left[\frac{\frac{15l_z^6}{V_p^6}\frac{(\sigma+\beta^3)}{2} - \mu\frac{(2-\gamma)-4(\gamma-1)(\gamma-2)}{2K^3}}{\frac{2l_z^2}{V_p^3}(\sigma+\beta)}\right], \tag{4.16}$$

$$B = \left[\frac{(\frac{\sigma}{\omega_p^2}+\frac{\beta}{\omega_n^2})\times(1-l_z^2) - \frac{\mu\alpha}{2K^2} + 1}{\frac{2l_z^2}{V_p^3}(\sigma+\beta)}\right]. \tag{4.17}$$

Eq. (4.15) is known as mK-dV equation. We again introduce a stretched coordinates like K-dV equation, ζ= ξ-$U_0\tau$. The stationary SWs solution of mK-dV equation can be written as (by taking $\phi^{(1)} = \Psi$)

$$\Psi = \Psi_m sech(\frac{\xi}{\Delta_2}), \tag{4.18}$$

where the amplitude Ψ_m and width Δ_2 are given by

$$\Psi_m = \sqrt{\frac{6U_0}{C}}, \tag{4.19}$$

$$\Delta_2 = \sqrt{\frac{B}{U_0}}. \tag{4.20}$$

The value of μ, σ and β effect both the amplitude, and width like K-dV equation. But α, ω_p and ω_n effect the width only.

Chapter 5

Derivation of the mmK-dV Equation

It is obvious from Eq. 3.15 that $A = 0$ since $\phi^{(1)} \neq 0$. One can find that $A = 0$ at its critical value $\mu = (\mu)_c$ (which is a solution of $A = 0$). So, for μ around its critical value μ_c, $A = A_0$ can be expressed as

$$A_0 = s(\frac{\partial A}{\partial \mu})_{\mu=\mu_c}|\mu - \mu_c| = sc_1\epsilon, \tag{5.1}$$

where $|\mu - \mu_c|[= (\mu = \mu_c)]$ is a small and dimensionless parameter and can be taken as the expansion parameter ϵ, i.e. $|\mu - \mu_c| \simeq \epsilon$ and $s = 1$ for $\mu < \mu_c$ and $s = -1$ for $\mu > \mu_c$. c_1 is a constant depending on plasma parameter γ and K is given by

$$c_1 = \frac{\gamma - 2}{K^2}. \tag{5.2}$$

So, $\rho^{(2)}$ can be expressed as

$$\epsilon^2\rho^{(2)} \simeq -\epsilon^3\frac{1}{2}c_1 s(\phi^{(1)})^2. \tag{5.3}$$

Now taking the coefficient of ϵ^3 from Poisson's equation, we get

$$-\frac{\partial \rho^{(3)}}{\partial \xi} = \frac{\partial n_n^{(3)}}{\partial \xi} + \mu\frac{\partial n_e^{(3)}}{\partial \xi} - \sigma\frac{\partial {n_p}^{(3)}}{\partial \xi}. \tag{5.4}$$

Now we can find the value of $\rho^{(3)}$ from Eqs. (5.4), where the values of $n_p^{(3)}$, $n_n^{(3)}$ and $n_e^{(3)}$ can be found from using Eqs. (4.11)-(4.13). Therefore, combining these equations into Eqs. (5.4), we can finally write the following equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + sc_1 D\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + C(\phi^{(1)})^2\frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0, \tag{5.5}$$

where

$$D = \frac{V_p^3}{2l_z^2}. \tag{5.6}$$

Eq. (5.5) is known as mmK-dV equation. It is also called Gardner equation [76]. The stationary SWs solution of mmK-dV equation can be directly written as(by taking $\phi^{(1)} = \Phi$)

$$\Phi = [\frac{1}{\Phi_{m2}} - (\frac{1}{\Phi_{m2}} - \frac{1}{\Phi_{m1}})cosh^2(\frac{\zeta}{\Delta_3})]^{-1}, \tag{5.7}$$

$$\Delta_3 = \frac{2}{\sqrt{K_1\Phi_{m1}\Phi_{m2}}}, \tag{5.8}$$

where the amplitude Φ_m and width Δ_3 are given by

$$\Phi_{m(2,1)} = \Phi_m[1 \pm \sqrt{1 + \frac{U_0}{V_0}}], \tag{5.9}$$

$$\Phi_m = -\frac{sc_1D}{C}, \tag{5.10}$$

$$V_0 = \frac{(sc_1D)^2}{6C}, \tag{5.11}$$

$$K_1 = \frac{C}{6B}. \tag{5.12}$$

The mmK-dV equation is very effective describing the higher order nonlinearity.

Chapter 6

Physical Analysis

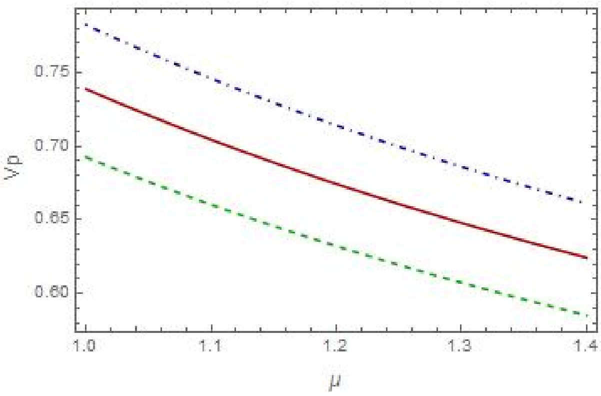

Figure 6.1: Showing the variation of V_p with μ for $U_0 = 0.01$, $\gamma = 4/3$, $\beta = 0.35$ and $n_{e0} = 9.1 \times 10^{29}$. The upper dotdashed blue line is for $\sigma = 1.50$, the middle dotted green line is for $\sigma = 1.30$ and the lower solid red line is for $\sigma = 1.10$

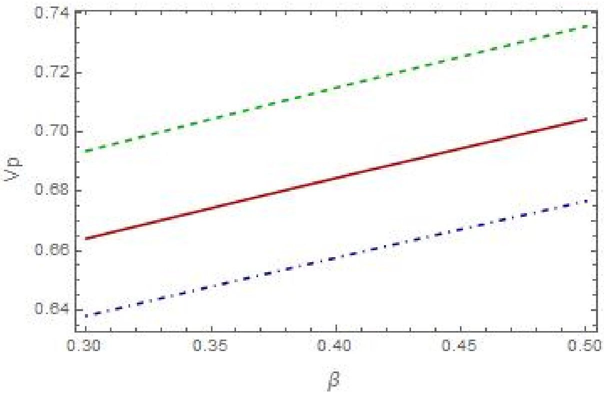

Figure 6.2: Showing the variation of V_p with β for $U_0 = 0.01$, $\gamma = 4/3$, $\sigma = 1.30$ and $n_{e0} = 9.1 \times 10^{29}$. The upper dashed green line is for $\mu = 1.10$, the middle solid red line is for $\mu = 1.20$ and the lower dotdashed blue line is for $\mu = 1.30$

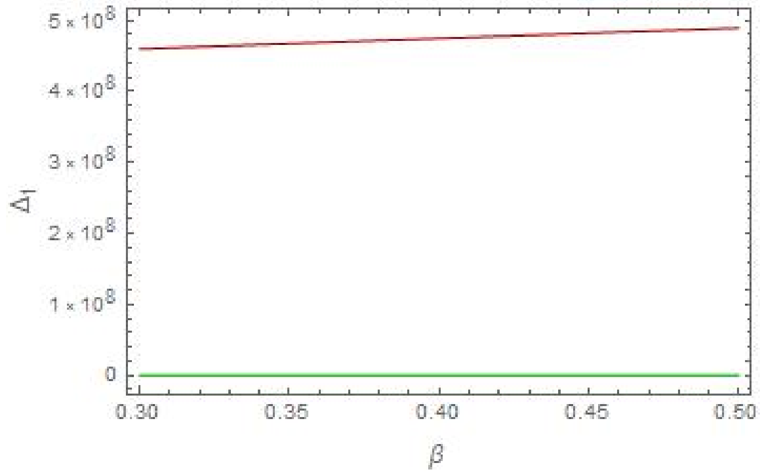

Figure 6.3: Showing the variation of Δ_1 with β (K-dV) for $U_0 = 0.01$, $\sigma = 1.3$, $\mu = 1.20$, $\beta = 0.35$, $\alpha = 0.5$, $\omega_p = 0.25$, $\omega_n = 0.30$ and $n_{e0} = 9.1 \times 10^{29}$. The upper solid red line is for $\gamma = 5/3$ represents non-relativistic limit and the lower solid green line is for $\gamma = 4/3$ represents ultra-relativistic limit.

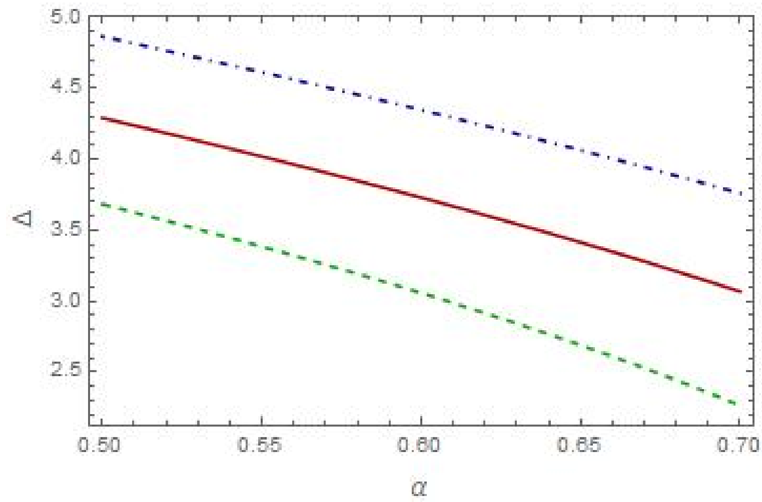

Figure 6.4: Showing the variation of Δ_2 with α (mK-dV)for $U_0 = 0.01$, $\gamma = 4/3$, $\omega_p = 0.25$, $\omega_n = 0.30$ $\beta = 0.35$, and $n_{e0} = 9.1 \times 10^{29}$. The upper dotdashed blue line is for $\sigma = 1.50$, the middle solid red line is for $\sigma = 1.30$ and the lower dashed green line is for $\sigma = 1.10$.

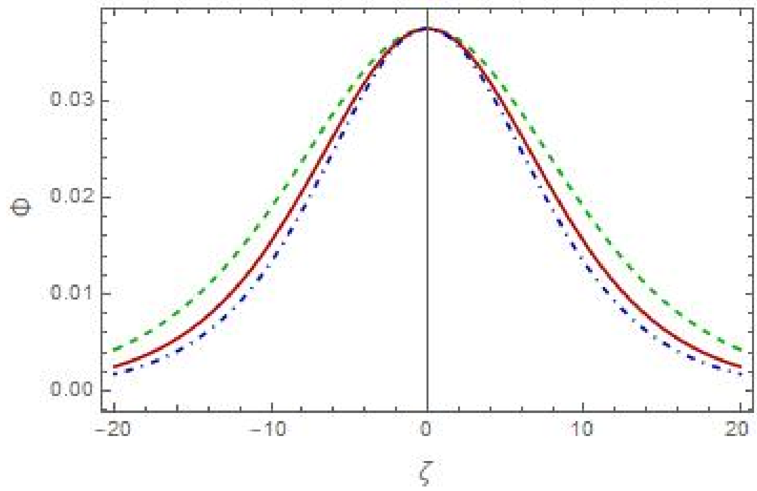

Figure 6.5: Showing the profile of Φ with ζ (K-dV)for different values of ω_n for $U_0 = 0.01$, $\gamma = 4/3$, $\omega_p = 0.25$ $\beta = 0.35$,$\mu = 1.2$ $\sigma = 1.3$, $\alpha = 0.5$ and $n_{e0} = 9.1 \times 10^{29}$. The upper dashed green line is for $\omega_n = 0.15$, the middle solid red line is for $\omega_n = 0.20$ and the lower dotdashed blue line is for $\omega_n = 0.25$.

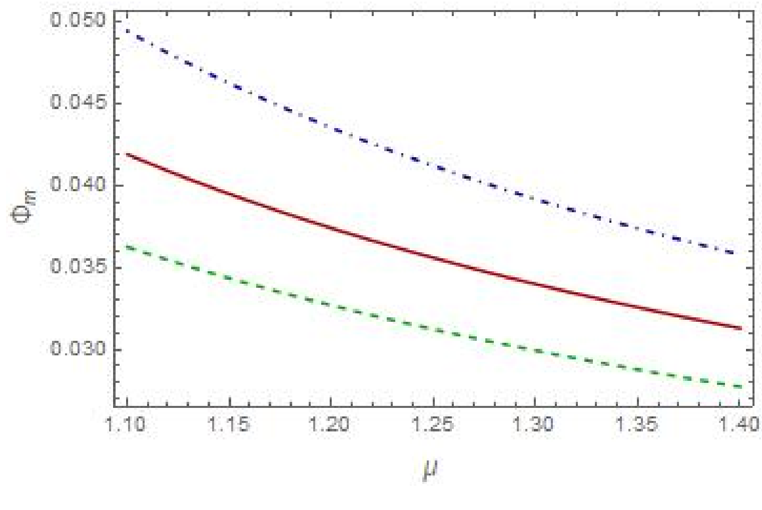

Figure 6.6: Showing the variation of Φ_m with μ (K-dV) for $U_0 = 0.01$, $\gamma = 4/3$, $\sigma = 1.3$, $\alpha = 0.5$ and $n_{e0} = 9.1 \times 10^{29}$. The upper dotdashed blue line is for $\beta = 0.4$, the middle solid red line is for $\beta = 0.35$ and the lower dashed green line is for $\beta = 0.3$.

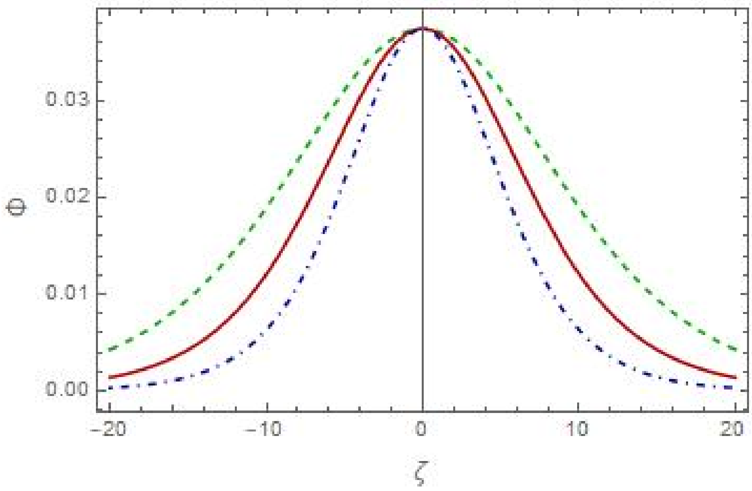

Figure 6.7: Showing the profile of Φ with ζ (K-dV) for $U_0 = 0.01$, $\gamma = 4/3$, $\omega_n = 0.30$ $\beta = 0.35$, $\mu = 1.2$, $\sigma = 1.3$, $\alpha = 0.5$ and $n_{e0} = 9.1 \times 10^{29}$. The upper dashed green line is for $\omega_p = 0.20$, the middle solid red line is for $\omega_p = 0.25$ and the lower dotdashed blue line is for $\omega_p = 0.30$.

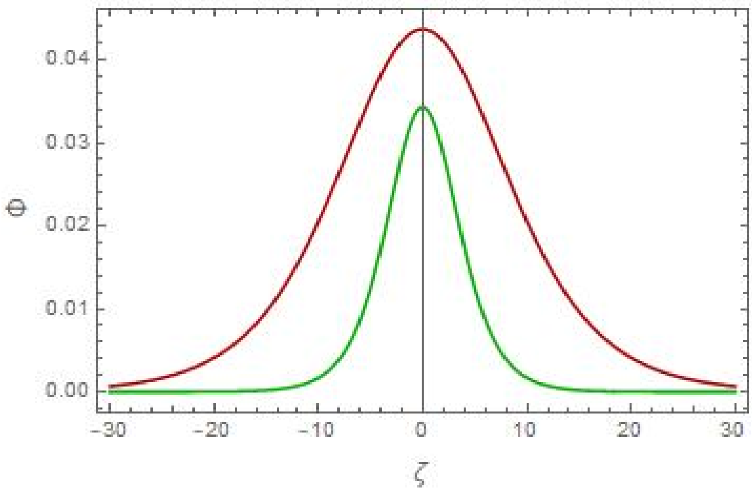

Figure 6.8: Showing the positive dust profile of Φ with ζ (K-dV) for $U_0 = 0.01$, $\gamma = 4/3$, $\omega_p = 0.25$, $\omega_n = 0.30$, $\beta = 0.35$, $\alpha = 0.5$, $\delta = +1$ and $n_{e0} = 9.1 \times 10^{29}$. The upper red is for $\eta = 0.50$ and the lower green line is for $\eta = 1.50$

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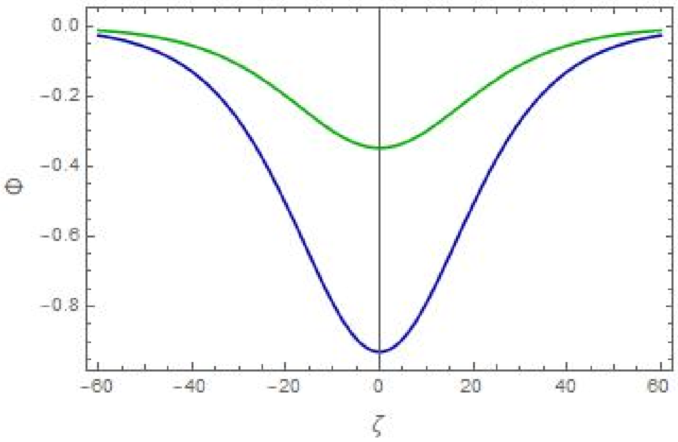

Figure 6.9: Showing the negative dust profile of Φ with ζ (K-dV) for $U_0 = 0.01$, $\gamma = 4/3$, $\omega_p = 0.25$, $\omega_n = 0.30$, $\beta = 0.35$, $\alpha = 0.5$, $\delta = -1$ and $n_{e0} = 9.1 \times 10^{29}$. The blue is for $\eta = 2.50$ and the lower green line is for $\eta = 2.80$

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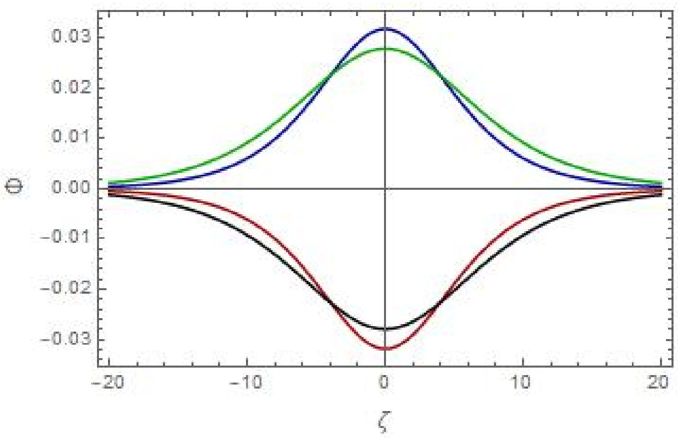

Figure 6.10: Showing the profile of Φ with ζ (mmK-dV) for $U_0 = 0.01$, $\gamma = 4/3$, $\omega_p = 0.25$, $\omega_n = 0.30$, $\beta = 0.35$, $\mu = 1.2$, $\alpha = 0.5$ and $n_{e0} = 9.1 \times 10^{29}$. The upper green and blue lines are of negative profile (s=-1) for $\sigma = 1.40$ and $\sigma = 1.10$ respectively. The bottom red and black lines are of positive profile (s=+1) for $\sigma = 1.10$ and $\sigma = 1.40$ respectively.

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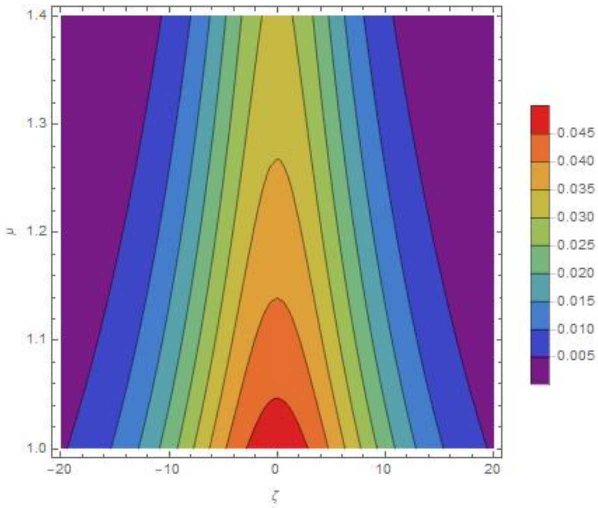

Figure 6.11: Showing the effect of μ on electrostatic potential Ψ with ζ (K-dV)for $U_0 = 0.01$, $\gamma = 4/3$, $\omega_p = 0.25$, $\omega_n = 0.30$ $\beta = 0.35$, $\alpha = 0.5$, $\sigma = 1.30$ and $n_{e0} = 9.1 \times 10^{29}$.

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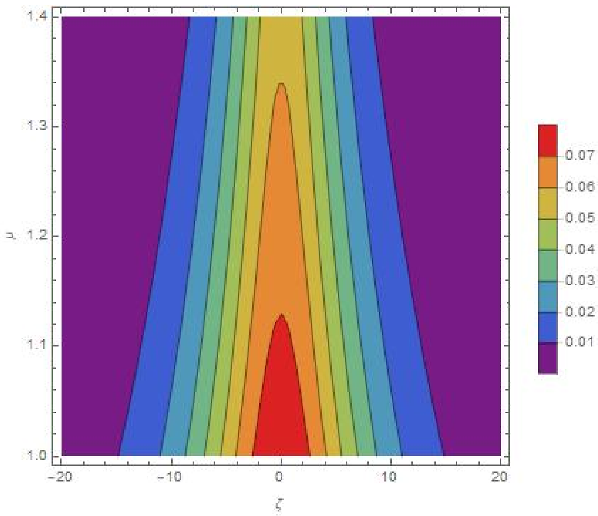

Figure 6.12: Showing the effect of μ on electrostatic potential Ψ with ζ (mK-dV) for $U_0 = 0.01$, $\gamma = 4/3$, $\omega_p = 0.25$, $\omega_n = 0.30$ $\beta = 0.35$, $\alpha = 0.5$, $\sigma = 1.30$ and $n_{e0} = 9.1 \times 10^{29}$.

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Chapter 7

Discussion

The properties and effect of different variables on SWs in magnetized degenerate quantum dusty plasma are briefly discussed in this section. We also emphasize the effects of relativistic factors, quantum parameter, dust charge state and density of plasma components on the IASWs in such degenerate quantum plasmas. To fulfill our purpose, we have mathematically derived K-dV, mK-dV, mmK-dV equation and also analyzed their SWs solutions based with several typical plasma parameters. We consider some plasma species density which is consistent with the relativistic degenerate astrophysical plasmas, e.g., $n_{e0} = 9.1 \times 10^{29} cm^{-3}$, $n_{n0} = 7.5 \times 10^{29} cm^{-3}$, $n_{p0} = 7.0 \times 10^{29} cm^{-3}$. The values of several plasma parameters that we used are, e.g., σ= 1.10 to 1.50, β=0.30 to 0.50, μ=1.10 to 1.40, η=0.5 to 2.80, α=0.50 to 0.70, ω_p=0.15 to 0.30 and ω_n=0.20 to 0.40. Finally, the results that we have found in this investigation can be summarized as follows:

1. The value of phase speed V_p of the the IASWs decreases with the increase of the value of μ but increases with the increase of the value of σ (see Fig. 6.1). The effect of μ and β on the phase speed is shown in Fig. 6.2. We have found same result for the variation of μ but phase speed of the IASWs increases with the increase of the value of β. Positive and negative ions are providing the inertia and degenerate pressure of electrons are providing the restoring force. Due to change of densities of plasma species (positive ions, negative ions and electrons) and the change of the mass ratio of positive to negative ions, inertia, and restoring force are changing. As a result, phase speed is increasing in some case and decreasing in some case for different values of μ, σ and β.

2. The Variations of width (Δ_1) of K-dV SWs with β for non-relativistic ($\gamma = 5/3$) and ultra-relativistic ($\gamma = 4/3$) cases is depicted in Fig. 6.3. It is found that width is much higher for $\gamma = 5/3$ than $\gamma = 4/3$ .This is happening due to the less number density of plasma species in non-relativistic case than the ultra-relativistic case.

3. The effect of quantum parameter(α) on the width (Δ_2) of the mK-dV SWs is displayed in Fig. 6.4. Width is decreasing with the increase of the value of α (i.e, increase of tunneling effect which cause more interaction between electrons and ions).

4. The magnitude of the external magnetic field B_0 has no effect on the amplitude of the SWs but it does have a direct effect on the width of SWs (Fig. 6.5 and

Fig. 6.7). We found that as the magnitude of ω_p and ω_n increases, the width of SWs decreases. Magnetic field makes the solitary structures more spiky. The variation of width is more for the change of ω_p than ω_n.

5. The influences of μ and β on amplitude (Φ_m) of K-dV SWs are found in Fig. 6.6. Amplitude of the K-dV SWs is decreasing with the increase of the value of μ but for β ,we found the opposite scenario. The physics behind this is explained earlier on point 1.

6. The variation of η on the K-dV SWs for positive and negative dust are depicted on Fig. 6.8 and Fig. 6.9. We have found both positive and negative profile for positive and negative charge dust. Both width and amplitude of K-dV SWs are decreasing with the increase of the value of η. It is also found that for negatively charged dust the changing effect of η is more interactive than positively charged dust.

7. The mmK-dV equation contains higher-order nonlinearity which can have different types of solutions depending on the values of σ. We have shown that depending on σ, the SWs with positive (compressive) for $s = -1$ or negative (rarefactive) for $s = +1$ potential exist. As we mentioned before on point 3, the increase of the value of σ similarly results the increase of width and amplitude here in Fig. 6.10.

8. The effect of μ of the mK-dV SWs are shown in Fig. 6.11 and Fig. 6.12. The width and amplitude both are decreasing with increase of the value of μ . The physics behind this scenario is mentioned previously on point 5.

From the analyzed data, we can understand the overall characteristics of the components of the considered plasma system along with the effects of different parameters.

Chapter 8

Conclusion

In my thesis, we considered a plasma system with some specific components and conditions. Through the work, we analyze the effects of the components on that particular plasma system theoretically. We plot graphical representations to provide better understanding about the outcomes of increasing or decreasing the value of any variables related to the plasma system. The results of our work can be useful for future advance research and practical purpose related to plasma physics.

Appendix A

Solution of the K-dV Equation

The K-dV Eq. can be written as

$$\frac{\partial y}{\partial t} + Py\frac{\partial y}{\partial x} + Q\frac{\partial^3 y}{\partial x^3} = 0, \tag{8.1}$$

where P and Q are constants. To obtain a stationary localized solution of this K-dV equation, we first transform the independent variables to $\xi = x - U_0 t$, $\tau = t$:

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} - U_0\frac{\partial}{\partial \xi}, \tag{8.2}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}. \tag{8.3}$$

Now, substituting Eq. (8.2) and Eq. (8.3) into Eq. (8.1), one gets

$$\frac{\partial y}{\partial \tau} - U_0\frac{\partial y}{\partial \xi} + Py\frac{\partial y}{\partial \xi} + Q\frac{\partial^3 y}{\partial \xi^3} = 0. \tag{8.4}$$

For steady state condition Eq. (8.4) reduces to ($\partial y/\partial \tau \to 0$)

$$-U_0\frac{\partial y}{\partial \xi} + Py\frac{\partial y}{\partial \xi} + Q\frac{\partial^3 y}{\partial \xi^3} = 0,$$

$$=> -U_0 y + \frac{1}{2}Py^2 + Q\frac{\partial^2 y}{\partial \xi^2} = R_1, \tag{8.5}$$

where R_1 is an integration constant. Now, under appropriate boundary conditions, viz., $y \to 0$ and $\partial^2 y/\partial \xi^2 \to 0$ at $\xi = \pm\infty$, one can find R_1 as R_1=0. So the Eq. (8.5) becomes

$$\frac{\partial^2 y}{\partial \xi^2} = \frac{1}{Q}(U_0 y - \frac{1}{2}Py^2). \tag{8.6}$$

Now, multiplying both sides of the above Eq. (8.6) by $\partial y/\partial \zeta$, one gets

$$\frac{\partial}{\partial \xi}\left[\frac{1}{2}\left(\frac{\partial y}{\partial \xi}\right)^2\right] = \frac{\partial}{\partial \xi}\left[\frac{1}{2Q}\left(U_0 y^2 - \frac{P}{3}y^3\right)\right],$$

$$=> \left(\frac{\partial y}{\partial \xi}\right)^2 = \frac{U_0}{Q}y^2\left(1 - \frac{P}{3U_0}y\right) + R_2, \tag{8.7}$$

where R_2 is an integration constant. Under the boundary conditions, $y \to 0$ and $\partial^2 y/\partial\xi^2 \to 0$ at $\xi = \pm\infty$, one can express R_2 as $R_2 = 0$. So, the above equation becomes

$$\frac{\partial y}{\partial \xi} = y\sqrt{\frac{U_0}{Q}}\sqrt{1 - \frac{P}{3U_0}y}. \tag{8.8}$$

We now assume

$$\sqrt{1 - \frac{P}{3U_0}y} = Y,$$

$$=> 1 - \frac{P}{3U_0}y = Y^2,$$

$$=> y = \frac{3U_0}{P}(1 - Y^2). \tag{8.9}$$

$$\frac{\partial y}{\partial \xi} = -\frac{6U_0}{P}Y\frac{\partial Y}{\partial \xi}. \tag{8.10}$$

Now, substituting Eq. (8.9) and Eq. (8.10) into Eq. (8.8), we get

$$-\frac{6U_0}{P}Y\frac{\partial Y}{\partial \xi} = \frac{3U_0}{P}(1 - Y^2)\sqrt{\frac{U_0}{Q}}Y,$$

$$=> \frac{2\partial Y}{1 - Y^2} = -\sqrt{\frac{U_0}{Q}}\partial\xi,$$

$$=> \left[\frac{1}{1+Y} + \frac{1}{1-Y}\right]\partial Y = -\sqrt{\frac{U_0}{Q}}\partial\xi,$$

$$=> \ln\left[\frac{1-Y}{1+Y}\right] = \sqrt{\frac{U_0}{Q}}\partial\xi + R_3,$$

$$=> \frac{1-Y}{1+Y} = R_4 e^{\sqrt{\frac{U_0}{Q}}\xi}, \tag{8.11}$$

where R_3 is an integration constant and $R_4 = e^{R_3}$. Now, imposing the condition, y $=3U_0$/P at $\xi = 0$, one can find R_4 as $R_4 = 1$. Substituting $R_4 = 1$ in Eq. (8.11) we obtain

$$Y = \frac{1 - e^{\xi\sqrt{\frac{U_0}{Q}}}}{1 + e^{\xi\sqrt{\frac{U_0}{Q}}}}. \tag{8.12}$$

Now, substituting Eq. (8.12) into Eq. (8.9), we obtain

$$y = \frac{3U_0}{P}\left[1 - \left(\frac{1 - e^{\xi\sqrt{\frac{U_0}{Q}}}}{1 + e^{\xi\sqrt{\frac{U_0}{Q}}}}\right)^2\right],$$

$$=> y = \frac{3U_0}{P}\left[\frac{4e^{\xi\sqrt{\frac{U_0}{Q}}}}{\left(1 + e^{\xi\sqrt{\frac{U_0}{Q}}}\right)^2}\right],$$

$$=> y = \frac{3U_0}{P}\left[\frac{2}{e^{-\xi\sqrt{\frac{U_0}{Q}}} + e^{\xi\sqrt{\frac{U_0}{Q}}}}\right],$$

$$=> y = \left(\frac{3U_0}{P}\right) sech^2\left(\xi\sqrt{\frac{U_0}{4Q}}\right),$$

$$=> y = \left(\frac{3U_0}{P}\right) sech^2\left(\xi\sqrt{\frac{U_0}{4Q}}\right),$$

Therefore, the stationary solitary wave solution of the K-dV equation is

$$y = y_0 sech^2\left[\frac{(x - U_0\tau)}{\Delta_1}\right]. \tag{8.13}$$

where y_0 and Δ_1 are the amplitude and the width of the solitary waves respectively, and are given by

$$y_0 = \frac{3U_0}{P}, \tag{8.14}$$

$$\Delta_1 = \sqrt{\frac{4Q}{U_0}}. \tag{8.15}$$

Appendix B

Solution of the mK-dV Equation

The mK-dV Eq. can be written as

$$\frac{\partial y}{\partial t} + Py^2\frac{\partial y}{\partial x} + Q\frac{\partial^3 y}{\partial x^3} = 0, \tag{8.16}$$

where P and Q are constants. To obtain a stationary localized solution of this mK-dV equation, we first transform the independent variables to $\xi = x - U_0 t$, $\tau = t$:

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} - U_0\frac{\partial}{\partial \xi}, \tag{8.17}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}. \tag{8.18}$$

Now, substituting Eq. (8.17) and Eq. (8.18) into Eq. (8.16), one gets

$$\frac{\partial y}{\partial \tau} - U_0\frac{\partial y}{\partial \xi} + P/3\frac{\partial y}{\partial \xi} + Q\frac{\partial^3 y}{\partial \xi^3} = 0, \tag{8.19}$$

For steady state condition Eq. (8.19) reduces to ($\partial y/\partial \tau \to 0$)

$$-U_0\frac{\partial y}{\partial \xi} + P/3\frac{\partial y^3}{\partial \xi} + Q\frac{\partial^3 y}{\partial \xi^3} = 0,$$

$$=> \frac{\partial}{\partial \xi}\left(-U_0 y + \frac{Py^3}{3} + Q\frac{\partial^2 y}{\partial \xi^3}\right)$$

$$=> -U_0 y + \frac{Py^3}{3} + Q\frac{\partial^2 y}{\partial \xi^3} = R_1, \tag{8.20}$$

where R_1 is an integration constant. Now, under appropriate boundary conditions, viz., $y \to 0$ and $\partial^2 y/\partial \xi^2 \to 0$ at $\xi = \pm\infty$, one can find R_1 as R_1=0. So, the above Eq. becomes

$$-U_0 y + \frac{Py^3}{3} + Q\frac{\partial^2 y}{\partial \xi^3} = 0,$$

$$=> \frac{\partial^2}{\partial \xi^2} = \frac{1}{Q}\left(U_0 y - \frac{Py^3}{3}\right), \tag{8.21}$$

Now, multiplying both sides of the above Eq. (8.21) by $\partial y/\partial \xi$, one gets

$$\left(\frac{\partial^2 y}{\partial \xi^2}\right)\frac{\partial y}{\partial \xi} = \frac{1}{Q}\left(U_0 y - \frac{Py^3}{3}\right)\frac{\partial y}{\partial \xi},$$

$$=> \frac{\partial}{\partial\xi}\left[\frac{1}{2}\left(\frac{\partial y}{\partial\xi}\right)^2\right] = \frac{1}{Q}\frac{\partial}{\partial\xi}\left(\frac{1}{2}U_0 y^2 - \frac{P}{12}y^4\right),$$

$$=> \left(\frac{\partial y}{\partial\xi}\right)^2 = \frac{U_0}{Q}y^2\left(1 - \frac{P}{6U_0}y\right) + R_2, \tag{8.22}$$

where R_2 is an integration constant. Under the boundary conditions, $y \to 0$ and $\partial^2 y/\partial\xi^2 \to 0$ at $\xi = \pm\infty$, one can express R_2 as $R_2 = 0$. So, the above equation becomes

$$\frac{\partial y}{\partial\xi} = y\sqrt{\frac{U_0}{Q}}\sqrt{1 - \frac{P}{6U_0}y^2}. \tag{8.23}$$

We now assume

$$\sqrt{1 - \frac{P}{6U_0}y} = Y,$$

$$=> \frac{P}{6U_0}y = 1 - Y^2,$$

$$=> y^2 = \frac{6U_0}{P}(1 - Y^2). \tag{8.24}$$

$$=> \frac{\partial y}{\partial\xi} = -\frac{6U_0}{Py}Y\frac{\partial Y}{\partial\xi}. \tag{8.25}$$

Now, substituting Eq. (8.24) and Eq. (8.25) into Eq. (8.23), we get

$$-\frac{6U_0}{Py}Y\frac{\partial Y}{\partial\xi} = \sqrt{\frac{U_0}{Q}}y\sqrt{1 - \frac{P}{6U_0}y^2},$$

$$=> -Y\frac{\partial Y}{\partial\xi} = \sqrt{\frac{U_0}{Q}}Y(1 - Y^2),$$

$$=> \frac{\partial Y}{1 - Y^2} = -\sqrt{\frac{U_0}{Q}}\partial\xi, \tag{8.26}$$

,

$$-\sqrt{\frac{U_0}{Q}}\partial\xi = \frac{1}{2}\left[\frac{1}{1+Y} + \frac{1}{1-Y}\right]\partial Y$$

$$-\sqrt{\frac{U_0}{Q}}\partial\xi = \frac{1}{2}\left[\ln 1 + Y + \ln 1 - Y\right] + R_3$$

$$=> \ln\left(\frac{1-Y}{1+Y}\right) = \sqrt{\frac{4U_0}{Q}}\xi + R_3,$$

$$=> \frac{1-Y}{1+Y} = R_4 e^{\sqrt{\frac{4U_0}{Q}}\xi}, \tag{8.27}$$

where R_3 is an integration constant and $R_4 = e^{R_3}$. Now, imposing the condition, y $=(4U_0/P)^2$ at $\xi = 0$, one can find R_4 as $R_4 = 1$. Substituting $R_4 = 1$ in Eq. (8.27) we obtain

$$Y = \frac{1 - e^{\xi\sqrt{\frac{4U_0}{Q}}}}{1 + e^{\xi\sqrt{\frac{4U_0}{Q}}}}. \tag{8.28}$$

Now, substituting Eq. (8.28) into Eq. (8.24), we obtain

$$y^2 = \frac{6U_0}{P}\left[1 - \left(\frac{1 - e^{\xi\sqrt{\frac{4U_0}{Q}}}}{1 + e^{\xi\sqrt{\frac{4U_0}{Q}}}}\right)^2\right],$$

$$=> y^2 = \frac{6U_0}{P}\left[\frac{4e^{\xi\sqrt{\frac{4U_0}{Q}}}}{\left(1 + e^{\xi\sqrt{\frac{4U_0}{Q}}}\right)^2}\right],$$

$$=> y^2 = \frac{6U_0}{P}\left[\frac{2}{e^{-\frac{1}{2}\xi\sqrt{\frac{4U_0}{Q}}} + e^{\frac{1}{2}\xi\sqrt{\frac{4U_0}{Q}}}}\right],$$

$$=> y^2 = \left(\frac{6U_0}{P}\right) sech^2\left(\xi\sqrt{\frac{U_0}{Q}}\right),$$

$$=> y = \sqrt{\frac{6U_0}{P}} sech\left(\xi\sqrt{\frac{U_0}{Q}}\right),$$

$$=> y = y_0 sech\left(\frac{\xi}{\Delta_2}\right). \tag{8.29}$$

Therefore, the stationary solitary wave solution of the mK-dV equation is

$$y = y_0 sech\left[\frac{(x - U_0\tau)}{\Delta_2}\right], \tag{8.30}$$

where y_0 and Δ_1 are the amplitude and the width of the solitary waves respectively, and are given by

$$y_0 = \sqrt{\frac{6U_0}{P}}, \tag{8.31}$$

$$\Delta_2 = \sqrt{\frac{Q}{U_0}}. \tag{8.32}$$

Appendix C

Solution of mmK-dV Equation

The mmK-dV Eq. can be written as

$$\frac{\partial y}{\partial t} + Py\frac{\partial y}{\partial x} + Qy^2\frac{\partial y}{\partial x} + R\frac{\partial^3 y}{\partial x^3} = 0, \tag{8.33}$$

where P, Q and R are constants. To obtain a stationary localized solution of this mmK-dV equation, we first transform the independent variables to $\xi = x - U_0 t$, $\tau = t$:

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} - U_0\frac{\partial}{\partial \xi}, \tag{8.34}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}. \tag{8.35}$$

Now, substituting Eq. (8.34) and Eq. (8.35) into Eq. (8.33), one gets

$$\frac{\partial y}{\partial \tau} - U_0\frac{\partial y}{\partial \xi} + Py\frac{\partial y}{\partial \xi} + Qy^2\frac{\partial y}{\partial \xi} + R\frac{\partial^3 y}{\partial \xi^3} = 0, \tag{8.36}$$

For steady state condition Eq. (8.36) reduces to ($\partial y/\partial \tau \to 0$)

$$-U_0\frac{\partial y}{\partial \xi} + Py\frac{\partial y}{\partial \xi} + Qy^2\frac{\partial y}{\partial \xi} + R\frac{\partial^3 y}{\partial \xi^3} = 0,$$

$$=> -U_0 y + \frac{1}{2}Py^2 + \frac{1}{3}Qy^3 + R\frac{\partial^2 y}{\partial \xi^2} = R_1, \tag{8.37}$$

where R_1 is an integration constant. Now, under appropriate boundary conditions, viz., $y \to 0$ and $\partial^2 y/\partial \xi^2 \to 0$ at $\xi = \pm\infty$, one can find R_1 as R_1=0. So, the above Eq. becomes

$$=> \frac{\partial^2}{\partial \xi^2} = \frac{U_0 y}{R} - \frac{1}{2R}Py^2 - \frac{1}{3R}Qy^3, \tag{8.38}$$

Now, multiplying both sides of the above Eq. (8.38) by $\partial y/\partial \xi$, one gets

$$\frac{\partial}{\partial \xi}\left[\frac{1}{2}\left(\frac{\partial y}{\partial \xi}\right)^2\right] = \frac{\partial}{\partial \xi}\left[\frac{1}{2R}\left(U_0 y^2 - \frac{P}{3R}y^3 - \frac{Q}{6R}y^4\right)\right],$$

$$=> \frac{1}{2}\left(\frac{\partial y}{\partial \xi}\right)^2 + \left(-\frac{U_0}{2R}y^2 + \frac{P}{6R}y^3 + \frac{Q}{12R}y^4\right) = R_2, \tag{8.39}$$

where R_2 is an integration constant. Under the boundary conditions, $y \to 0$ and $\partial^2 y/\partial\xi^2 \to 0$ at $\xi = \pm\infty$, one can express R_2 as $R_2 = 0$. So, the above equation becomes

$$\frac{1}{2}\left(\frac{\partial y}{\partial \xi}\right) + V(y) = 0, \tag{8.40}$$

where the pseudo-potential V(y) is

$$V(y) = -\frac{U_0}{2R}y^2 + \frac{P}{6R}y^3 + \frac{Q}{12R}y^4, \tag{8.41}$$

It is obvious from Eq. (8.41) that

$$V(y)|_{y=0} = \frac{\partial V(y)}{\partial y}|_{y=0} = 0, \tag{8.42}$$

$$\frac{\partial^2 V(y)}{\partial y^2}|_{y=0} < 0. \tag{8.43}$$

We now consider two types of solutions (namely DL and GS), which are explained below: The conditions Eq. (8.42) and Eq. (8.43) imply that the GS solution of Eq. (8.40) exists if and only if

$$V(y)|_{y=y_m} = 0, \tag{8.44}$$

The condition can be expressed as

$$-\frac{U_0}{2R} + \frac{P}{6R}y_m + \frac{Q}{12R}y_m^2 = 0, \tag{8.45}$$

$$=> U_0 = \frac{P}{3}y_m + \frac{Q}{6}y_m^2. \tag{8.46}$$

Now, using Eq. (8.45) we have

$$y_{m2,1} = y_m\left[1 \pm \sqrt{1 + \frac{U_0}{V_0}}\right], \tag{8.47}$$

where y_m = -P/Q and $V_0 = P^2/6Q$. Substituting Eq. (8.47) in Eq. (8.46), one obtains

$$U_0 = \frac{P}{3}y_{m2,1} + \frac{Q}{6}y_{m2,1}^2. \tag{8.48}$$

Now, using Eq (8.47) and Eq. (8.48) into Eq. (8.40) we have

$$\left(\frac{\partial y}{\partial \zeta}\right)^2 - \gamma y^2(y - y_{m1})(y - y_{m2}) = 0, \tag{8.49}$$

where

$$y_{m1} = y_m\left[1 - \sqrt{1 + \frac{U_0}{V_0}}\right], \tag{8.50}$$

$$y_{m2} = y_m\left[1 + \sqrt{1 + \frac{U_0}{V_0}}\right]. \tag{8.51}$$

Therefore the stationary GS solution of the sG Eq. is

$$y = [\frac{1}{y_{m2}} - (\frac{1}{y_{m2}} - \frac{1}{y_{m1}})cosh^2(\frac{\zeta}{\Delta_3})]^{-1}, \tag{8.52}$$

where Δ_3 is the width of the GSs and is given by

$$\Delta_3 = \frac{2}{\sqrt{K_1 y_{m1} y_{m2}}}. \tag{8.53}$$

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