Performance Analysis of Cognitive Unmanned Aerial Vehicle

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A thesis submitted to the Department of Electrical and Electronic Engineering in partial fulfillment of the requirements for the degree of
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Declaration

It is hereby declared that

1. The thesis submitted is our own original work while completing degree at Brac University.
2. The thesis does not contain material previously published or written by a third party, except where this is appropriately cited through full and accurate referencing.
3. The thesis does not contain material which has been accepted, or submitted, for any other degree or diploma at a university or other institution.
4. We have acknowledged all main sources of help.

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Approval

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Abstract

The traditional spectrum sensing by cognitive radio sometimes decreases as to the effects of fading and shadowing. Besides, Cognitive radio (CR) based unmanned aerial vehicle (UAV) will provide a higher data transmission without severely being affected by the multipath fading and shadowing. In this paper our goal is to maximize the energy efficiency and throughput by minimizing the power consumption of the UAV. We have designed an analytical model where we worked on the air to ground and ground to ground channel gain. For this paper we also have just considered the downlink communication between the UAV and the ground objects. For improving the energy efficiency of the UAV transmission power is reduced and it is done by two mathematical approaches. Firstly, with the Lambert W function we find the optimal transmission power that is later used to increase the energy efficiency. Secondly, two multi-objective optimization problem (MOP) is introduced and with Lagrangian approach we solve the MOPs to find the maximum power transmitted which is to be used for increasing energy efficiency.

Keywords: Unmanned aerial vehicle (UAV), Cognitive radio network, Energy efficiency, Spectrum sensing, Throughput, Power allocation, Multi-objective optimization.

DEDICATED TO OUR PARENTS

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List of Acronyms

UAV	Unmanned Aerial Vehicle
CR	Cognitive Radio
LoS	Line-of-Sight
NLoS	Nonline-of-Sight
4G	Fourth Generation
5G	Fifth Generation
IoT	Internet of things
D2D	Device-to-Device
AWGN	Additive White Gaussian Noise
SNR	Signal-to-Noise Ratio
AtG	Air-to-Ground
GtG	Ground-to-Ground
BS	Base Station
PTs	Primary transmitters
MOP	Multi-objective Optimization Problem
SOP	Single Optimization Problem
EE	Energy Efficiency
ABS	Aerial Base Station

List of Symbols

PT_x	Primary transmitter
SR_x	Secondary receiver
ST_x	UAV or secondary transmitter
PR_x	Primary receiver
P_S	Transmitted power from ST_x to SR_x
P_P	Transmitted power from PT_x to SR_x
γ_p	SNR of PT_x
γ_t	SNR of the threshold
g_p	Channel gain between PT_x and SR_x
g_s	Channel gain between ST_x and SR_x
d_{xy}	Linear distance between nodes x and y
$PL_{xy}(d_{xy})$	Path loss in linear form
p_{xy}^{LoS}	Probability of available LoS between AtG nodes
θ	Angle between the UAV and other nodes (BSs and PTs)
Φ	Constant value dependent on the environment
φ	Constant value dependent on the environment
n	Pathloss exponent
f	Carrier frequency
c	Speed of light
λ	Wavelength
L_{LoS}	Average additional loss due to LoS link

Ω	Indicator for detecting the status of PT_x
γ_s	SNR of ST_x
σ^2	Noise power
t_d	Transmission time
t_s	Sensing time
p_f	Probability of a false alarm
p_d	Probability of detection
$p_r(\Omega = 0)\left(1 - p_f\right)$	Detection probability on the absence of the PT_x
$p_r(\Omega = 1)\,(1 - p_d)$	Detection probability on the presence of the PT_x
P_{cir}	Circuit power
P_s^0	Optimal Transmitted power $(\Omega = 0)$ (Without Constraints Case)
P_s^1	Optimal Transmitted power $(\Omega = 0)$ (Without Constraints Case)
W	Lambert W function
EE_D	Energy efficiency $(\Omega = 0)$
EE_I	Energy efficiency $(\Omega = 1)$
S_D	Effective throughput $(\Omega = 0)$
S_I	Interference throughput $(\Omega = 1)$
$P_{S.min}$	Minimum transmission power required to transmit signals to the SR_x
$S_{D.min}$	Minimum required effective throughput for SR_x
$S_{I.min}$	Minimum required interference throughput at SR_x

$l(P_s, \delta)$	Lagrange Function for optimization problem, where P_s and δ are the variables considered
δ_a	Positive variable mutual to the constraints of the MOP
δ_b	Positive variable mutual to the constraints of the MOP
P_s^*	Transmitted Power from ST_x to SR_x (With Constraints case for both '$\Omega = 0$' and '$\Omega = 1$'

Chapter 1

Introduction

1.1 Introduction to UAV

The civil utilization of unmanned aerial vehicle (UAV), also commonly known as drones or remotely piloted aircraft have gained increasing significance in the last few decades because of their vast utilities and cost efficiency. At the very beginning, UAVs were only used for military purpose as they were deployed in hostile territory to minimize pilot casualty. As the cost is reducing in the global market and device's architectural design is getting miniaturized, they are now more popular among the general public and that's why outstanding applications in civilian and commercial domains have been emerging. Some of the typical examples are communication relaying, weather monitoring, traffic management, surveillance over agricultural fields and coastal areas, collecting images and data for research purpose, emergency search and rescue during natural disasters etc.

UAVs are generally classified into two types: static wing versus rotational wing. Each of them have their own specification. Static-wing UAVs have high speed and can carry heavy payload and to achieve that they have to maintain a persistent motion to remain airborne. That's why they are not suitable for stationary assignments like close observation. On the other hand, rotational-wing UAVs such as quad copters have limited mobility and payload. That's why they are not only capable of moving in any direction but also are able to stay stationary in the air.

It is expected that in the near future UAVs are going to play a vital role for high speed wireless communication. In fact, UAV based wireless communication can offer promising solution to provide wireless connectivity for devices without infrastructure coverage. The concept of using UAVs as flying base stations can boost the capacity and coverage of existing wireless networks.

Another significant feature of an UAV is if it has line-of-sight (LOS) connections towards the users then it can enhance the coverage and transmission rate. Moreover, because of the high mobility, UAVs can easily be deployed to support cellular networks and enhance their quality-of-service (QoS).

For Devices lacking infrastructure coverage, wireless connections is provide to them by implementing an UAV. In addition to a UAVs capability, they also provide wireless connection seamlessly over a coverage to ground users, which is a favorable solution to IoT networks. Moreover, IoT applications provides devices operated by both machine and human with a connection over the internet. As for efficiency factor of these applications, high information rate with low latency is mandatory. As a result, locations with poor coverage area, IoT applications may not perform seamlessly. Therefore as to the aerial advantage of an UAV, it can provide innovative and functional solution such as an UAV based IoT platform. Additionally an UAV based IoT platform will provide operational functionality over the sky at the same time mitigating the coverage issue and ensuring high information rate with low latency. In the coming day's implementation of 5G networks are getting popular and on the other hand providing 5G connectivity with proper coverage is getting difficult. With UAV implementation the coverage issue is maintained, as the aerial advantage will provide the UAV with option to perform as a cluster head so that the coverage area is always high as to aerial mobility.

There are some limitations that sometimes make UAV based communication challenging. Among them two commonly faced challenges are limited energy availability and spectrum scarcity.

1.2 Review of Previous Works and Observation

Over the years researchers have been working on how to diversify the applications of UAV based wireless communication system by minimizing the energy consumption and maximizing the effective throughput, Spectrum sensing, incorporation with IoT and so on. In this section we have mentioned some significant research that has been done on UAVs.

Huang *et al.* (2016) considered a cognitive UAV based communication system where UAV trajectory and transmit power is optimized jointly so that the received interference power at each primary receiver cannot exceed interference temperature threshold. However, this model was non-convex and was challenging to solve optimally. To overcome this they came up with an algorithm which efficiently helped them to achieve an optimal solution. They found promising result that joint UAV trajectory and power control scheme improves the achievable rate compared to the previous results [1].

Liu *et al.* (2018) worked on maximizing the effective throughput by optimizing the spectrum sensing. In cognitive radio (CR) system, shadowing effect and multipath fading may reduce spectrum sensing performance. However, UAV has the ability to detect and receive a higher strength signal without getting disturbed by shadowing and fading. They proposed an improved spectrum sensing method for UAV based CR system to easily access an idle spectrum. The spectrum sensing optimization is designed as an optimization problem. This maximizes the effective throughput of the UAV by optimizing the sensing radian subject to the constraint of the interference throughput. To improve sensing performance in fading channel they have also proposed virtual cooperative spectrum sensing. Here the effective throughput is maximized by jointly optimizing local the number of sensing slots and sensing radian. We can see from their simulation results that by optimizing spectrum sensing, and the virtual cooperative spectrum sensing in deep fading channel better UAV transmission performance can be obtained [2].

Sboui *et al.* (2017) gave the insights of how to integrate an underlay CR solution with an energy efficient power allocation scheme in order to ensure efficient and long term operations is illustrated. They studied the deployment of UAV incorporated with CR system in an area surrounded by primary network (PN). Here, to maximize the energy efficiency (EE) by optimizing the transmitting power UAV shares the spectrum with PN. They worked on a model where the UAV continuously keep communicating with the ground receiver. They designed a framework based on interference and minimal rate constraints which not only improves the power allocation but also maximizes the EE. They also focused on the presence of optimal altitude based the location of the UAV with respect to the other terminals. Their results showed that at low power budget value a transmission may occur due to the low minimal rate [3].

Zeng *et al.* (2016) researched on how to provide cost effective wireless connectivity for devices without infrastructure coverage using UAVs. Wireless systems with low-altitude UAVs are in general faster to deploy, more flexible to re-configured, and due to the presence of short-range line-of-sight (LoS) links have better communication channels. They provided an overview of UAV based wireless communication system, by introducing the basic networking architecture and main channel characteristics, highlighting the key design considerations as well as the new opportunities to be exploited [4].

Khuwaja *et al.* (2018) mentioned that UAV communication has its own distinctive channel characteristics compared with widely used cellular and satellite systems. That’s why, accurate channel characterization is a must for the performance optimization and designing an efficient UAV communication systems. Their paper provided an extensive survey on the measurement campaigns launched for UAV channel modeling using low altitude platforms and elaborately discussed various channel characterization efforts. They also reviewed the contemporary perspective of UAV channel modeling approaches [5].

Mili *et al.* (2016) investigated the maximum achievable energy efficiency of UAV based cognitive network by considering the spectrum sharing and spectrum sensing. In order to maximize the energy efficiency they have designed an optimization problem that not only maximizes the erjodic capacity but also limiting the interference power of the primary user minimizes the transmission power of the secondary user [6].

Azari *et al.* (2018) have provided a remarkable analysis on the data collected from the network performance of a cellular tower and the users. They calculated and analyzed the throughput based on the data collected from flying the UAV in different places with varying the interference. They have found some promising results by using microcells when the network is dense limits the UAV performance. They expressed their concern regarding the use of UAV based cellular network saying that will be a constraint on the way of development of ultra dense network [7].

Shakhatreh *et al.* (2018) conducted a survey on the applications of UAVs that are going to play a game changing role in the near future. Besides, they brought up the key challenges for civil applications of UAVs that includes swarming, collision avoidance and security concerns. Based on their research they elaborately discussed how UAVs with cognitive radio system can be an emerging public communication technology with spectrum allocation [8].

Zhou *et al.* (2018) proposed that by using UAVs as aerial base station (ABS) by providing coverage to the users located at ground can be a promising solution to IoT network. In their paper they have considered a model where communication network incorporated with underlay ABS are able to provide coverage for a limited period of time in limited areas. To calculate the uplink and downlink coverage probability they have proposed a framework using stochastic geometry and this framework is applicable for line of sight (LoS) and non-line of sight (NLoS). They verified their results by using Monte Carlo simulation. Their results showed that nontrivial impact of different aerial channel environments on the uplink and downlink coverage probability [9].

Wang *et al.* (2018) suggested where the ground base stations (BS) are not available at that places UAVs enabled BSs can highly enhance device-to-device (D2D) communication and overall system performance. But in that scenario the challenging thing is the interference among UAVs and D2D pairs. Here, the authors have investigated power control optimization for underlayed communication between the UAVs and D2D pairs. In this case there are multiple users for UAV based BSs. Their goal was to maximizing the throughput. The authors have developed an algorithm for their model on low complexity power control which is based on Hessian matrix. Their simulation results were promising and the systems throughput was outstanding [10].

Ghazzai *et al.* (2017) took their work to one step forward than the others. They have worked on micro unmanned aerial vehicles (MUAVs). The main feature of MUAVs are they need a limited time access to the spectrum for data transmission and the reason behind this is the limited battery capacity. That's why during using UAVs mainly two problem arises and they are energy management of the battery and spectrum sensing for limited access. To minimize the energy usage the authors have incorporated cognitive radio network. Here, their goal was to discover a three-dimensional point and also finding out solution for power control to successfully transmit a data. To deal with the optimization problem they have structured an algorithm based on Weber formula. From their simulation results it was concluded that cognitive radio technology constitutes a promising solution to cope with the spectrum problems existing in UAV applications [11].

Mozaffari *et al.* (2016) created a model to analyze the coverage and rate performance of UAV based wireless communication in the presence of underlaid D2D communication links by considering a network where UAVs provide downlink transmission support. Here for the sake of deriving the coverage and rate analysis they concentrated on two cases. For the first case as a function of users and height of the UAV two things were derived. One was average sum rate of the users. Another one was the average coverage probability. And for the second case the number of points that the UAV is going to stop was calculated by using disk covering problem. From the simulation results it was analyzed that output was maximized for the existing optimal value of the UAV altitude. Besides, if the function of moving independently is incorporated into the UAV then the overall coverage probability tremendously enhances [12].

Motlagh *et al.* (2017) discussed about the future aspects of UAV incorporated with technologies like 4G and 5G networks which are going to dominate the communication networks for the next few years. They also focused on the use of UAV equipped with IoT. To implement this model they designed a framework which contains a MEC node and another node that processes the local data collected from during surveillance using UAVs [13].

Saleem *et al.* (2014) mentioned the day by day increasing demand of UAVs and their way of operation. As the scarcity of spectrum is becoming a concern for UAVs because the traffic in the IEEE bands allocated for UAVs are increasing, they proposed an alternative. They developed a model UAV where it uses the cognitive radio network by spectrum sensing. They also discussed about the future aspects and challenges for CR based network [14].

Lagkas *et al.* (2018) worked on how to create an IoT domain which will be incorporated with UAV. According to their paper they designed a model where UAVs were equipped with cameras, sensors and GPS and thus they would be able to collect data more efficiently [15].

1.3 Motivation

UAVs have attracted a great deal of attention from various sectors because of their vast range of application. While integrated with wireless mobile network they can offer a wide range of solutions to many communication challenges. Besides, it can also bring out some innovative technologies for the modern world. Because of their ability to get easily airborne and easy mobility they are convenient to operate. Also one of the great feature of UAVs are their reliable line-of-sight communication link. By using UAV based wireless communication link a promising but temporary solution can be provided to the network congestion problem. But there are some challenges that might be a problem to all of these innovative ideas. One of the challenges are energy management of the UAV and another one is spectrum scarcity. Our main goal in this thesis was to design an analytical model that will maximize the energy efficiency and also bring out the desired throughput. In our model we have integrated cognitive radio based solution. Usually, analytical methods are faster than simulation and drastically reduce the computational time. Simulations are simpler but slower and appropriate techniques must be used to reduce the number of simulation required to get a satisfactory result. Accurate models are needed to describe the propagation and performance evaluation of system for the transmitted signal through a transmission media like wireless. Analytical models are always very helpful for a deeper comprehension and overall view of the system can be understood and they require rigorous statistical analysis of both the phase noise and frequency offset behavior.

1.4 Objective of the Thesis

The goal of the thesis is to design an analytical model to maximize the energy efficiency and throughput by applying optimal power. To meet the goal, the following objectives have been identified.

1. To develop an analytical model using Cognitive unmanned aerial vehicle.

2. To derive expression for energy efficiency so that we can get the optimal and maximum power for both secondary user cases (present and absent).

3. To compare and analyze the effective and interference throughput with respect to transmitted power and distance.

4. To acquire the enhanced throughput by applying optimal power.

1.5 Organization of the Thesis

This thesis is organized in four chapters as follows:

Chapter-1 is the introductory chapter. It contains the basic introduction of an UAV, advantages and challenges of UAV based communication. It was followed by literature that contained significant previous works on UAV, motivation and our objective of the thesis.

Chapter-2 presents four parts. Firstly, we discussed the network description of our model with detailed figure. Secondly, we discussed about sensing and transmission period for our model. Thirdly, there is our path-loss model with necessary equation. Here we focused on the air to ground channel and ground to ground channel. Finally, showed our mathematical model. There is four segments in our mathematical model. We discussed about throughput, energy efficiency, and maximum energy efficiency with constraints and without constraints.

Chapter-3 we analyzed the graphs that we found from out simulation results. We compared the effective and interference throughput with respect to transmitted power and distance.

Chapter-4 presents the concluding remarks of all the chapters and highlights some possible promising avenues of further development.

Chapter 2

System Model

2.1 Introduction

Unmanned Arial vehicle (UAV) is very advantageous when it comes to implementing aerial based communication systems. Therefore, in our paper, we consider ta UAV taking a flight for indicating the presence or absent states of a primary user (PU). Here the UAV performs as the secondary transmitter (ST_x), which communicates with the secondary receiver (SR_x). In addition, the advantage of aerial mobility lets the UAV to communicate with the receiver over a dominant, non-fading line of sight link (LoS). Furthermore, the following sections provides the depth discussion of our proposed model consisting of the network description, the sensing and transmission period description followed by the path-loss and mathematical model. In addition, the mathematical model provides throughput for the presence and absent states of PU with that, energy efficiency is be also measured mathematically. Henceforth, using the Lambert W method we acquire the optimal power transmission for both PU states and for similar cases Lagrangian approach is taken to acquire the highest transmission power.

2.2 Network Description

We assume a downlink scenario of cognitive UAV-based system displayed in Figure 1 where a secondary receiver (SR_x) and a primary transmitter (PT_x) is located at the ground. Moreover (SR_x) works as a base station (BS) and receives a signal from UAV transmitter (ST_x) which is located at the air. P_s is the power transmitted from ST_x to SR_x and P_p is the power transmitted from PT_x to SR_x.

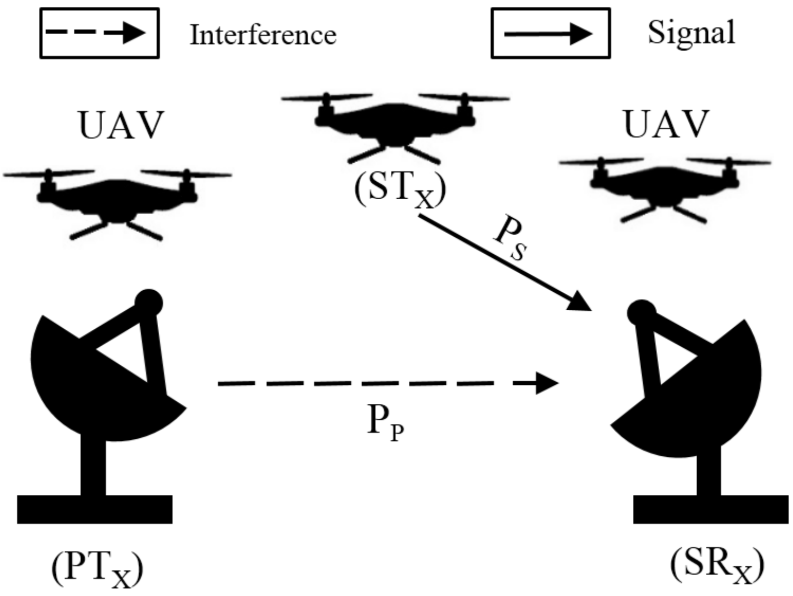

Figure 1: Communication link between ST_x and SR_x

2.3 Sensing and transmission Period

Initially before transmission initiates ST_x searches for unused and engaged channels by calculating the signal to noise ratio (SNR) of licensed spectrum. This mainly works as each ST_x just picks a channel and checks whether it is in an unused state or an engaged one. As usual, the PT_x also sends its usual data to the designated primary receiver(PR_x) . The SNR of PT_x is measured by the ST_x. After that, the received SNR is compared to the threshold SNR. If the threshold SNR is, lower than the received SNR then the channel is declare to be unused and if the opposite then the channel is engaged. The theoretical expression is defined as [19].

$$\Omega(\gamma_p) = \begin{cases} 1, & \gamma_p \geq \gamma_t \\ 0, & \gamma_p < \gamma_t \end{cases} \tag{2.1}$$

where γ_p is the SNR of PT_x and γ_t is the threshold SNR.

2.4 Path Loss Model

We consider in this paper the channel gains between PT_x and SR_x as g_p and between ST_x and SR_x as g_s. Here, all channel gains are assume to be independent and constant while the transmission is going on and all the information about the state of the channel are available. According to the characteristics of the system, we have acknowledged two types of channels, which are the air-to-ground (AtG) channel and ground-to-ground (GtG) channel.

2.4.1 AtG Channel

In this case, both the non-fading line of sight (LoS) component and the fading non-line of sight (NLoS) component are present. We have assumed that LoS links between UAVs and BSs with a certain probability p_{xy}^{LoS} the pathloss between AtG is expressed as [17]:

$$PL_{xy}(d_{xy}) = p_{xy}^{LoS}\, PL_{xy}^{LoS} + (1 - p_{xy}^{LoS}) PL_{xy}^{NLoS} [dB] \tag{2.2}$$

where p_{xy}^{LoS} is the probability of available LoS between AtG nodes, which is expressed as:

$$p_{xy}^{LoS} = \frac{1}{1 + \Phi \exp(-\varphi[\theta - \Phi])} \tag{2.3}$$

where θ is the angle between the UAV and other nodes (BS's and PT's) which depends on the distance d_{xy}. Next, Φ and φ are the constant values that are dependent on the environment. In addition, the LoS free pathloss is expressed as:

$$PL_{xy}^{LoS} = 10n\ \log_{10}\left(\frac{4\pi d_{xy}}{\lambda}\right) + L_{LoS}\ [dB] \tag{2.4}$$

where n is the pathloss exponent and from $c = f\lambda$ where f is the frequency of the carrier and c represents light speed. Hence L_{LoS} is the average supplementary loss as a result of LoS link.

2.4.2 GtG Channel

In this scenario, we have considered only NLoS between the ground nodes. Therefore, the pathloss in free space is expressed as follows [18]:

$$PL_{xy}^{NLoS} = 10n\ \log_{10}\left(\frac{4\pi d_{xy}}{\lambda}\right) + L_{NLoS}\ [dB] \quad (2.5)$$

where L_{NLoS} is the average supplementary loss as a result of NLoS link and it depends on the environment. In addition, the other parameters have similar representations. Therefore, we write the GtG pathloss as:

$$PL_{xy}(d_{xy}) = PL_{xy}^{NLoS}\ [dB] \quad (2.6)$$

2.5 Mathematical Model

2.5.1 Throughput

In our proposed model UAVs initially senses the presence or absence states of the PT_x. For distinguishing the status of the PT_x, we use 'Ω' as a marker. Now when $\Omega = 1$ it means the PT_x is active on transmission and when $\Omega = 0$ it means the PT_x is not active on transmission. Now we have expressed the effective throughput (S_D) when $\Omega = 0$ as follows:

$$S_D = \left(\frac{t_d}{t_s + t_d}\right) p_r(\Omega = 0) \left(1 - p_f\right) \log_2\left(1 + \frac{P_s|g_s|^2}{\sigma^2}\right)$$

$$= \left(\frac{t_d}{t_s + t_d}\right) p_r(\Omega = 0) \left(1 - p_f\right) \log_2(1 + \gamma_s|g_s|^2) \qquad (2.7)$$

where $\gamma_s = \frac{P_s}{\sigma^2}$. Moreover t_d and t_s are transmission time and sensing time respectively, $p_r(\Omega = 0) \left(1 - p_f\right)$ is accurate detection probability when PT_x is absent and finally p_f is probability of a false alarm.

In addition, we have expressed the equation for interference throughput (S_I) when ($\Omega = 1$) as follows:

$$S_I = \left(\frac{t_d}{t_s + t_d}\right) p_r(\Omega = 1) (1 - p_d) \log_2\left(1 + \frac{P_s|g_s|^2}{\sigma^2 + P_p\left|g_p\right|^2}\right)$$

$$= \left(\frac{t_d}{t_s + t_d}\right) p_r(\Omega = 1) (1 - p_d) \log_2\left(1 + \frac{\gamma_s|g_s|^2}{1 + \gamma_p\left|g_p\right|^2}\right) \qquad (2.8)$$

Where $\gamma_p = \frac{P_p}{\sigma^2}$ and $p_r(\Omega = 1) (1 - p_d)$ is the fail detection probability on the presence of the PT_x. p_d is probability of detection.

2.5.2 Energy Efficiency

It is can be characterized as the ratio of data rate to the total power consumption by the UAV device [6]. The expression of energy efficiency considering the effective throughput and interference throughput is expressed as:

$$EE_D = \frac{S_D}{P_s + P_{cir}} \tag{2.9}$$

$$EE_I = \frac{S_I}{P_s + P_{cir}} \tag{2.10}$$

where P_s is the power transmitted from ST_x to $SR_x$and P_{cir} is the circuit power.

2.5.3 Maximum Energy Efficiency without Constraints

In this case we have derived the expression of optimal power transmission without constraints for the presence of interference and without interference. From the equation of energy efficiency we have derived the expression of optimal power by using Lambert W function.

$$\max_{P_s} \; EE_D(P_s) \tag{2.11}$$

In case of $(\Omega = 0)$ we get the energy efficiency as follows:

$$EE_D(P_s^0) = \frac{\left(\frac{t_d}{t_s + t_d}\right) p_r(\Omega = 0)(1 - p_f) \log_2\left(1 + \frac{P_s^0 {g_s}^2}{\sigma^2}\right)}{P_s^0 + P_{cir}} \tag{2.12}$$

and the optimal transmitted power P_s^0 is obtained as follows:

$$P_s^0 = \frac{\sigma^2 \left(\frac{(P_{cir} g_s^2 - \sigma^2)}{\sigma^2 W\left(0, \frac{exp(-1)\,(P_{cir}\, g_s^2 - \sigma^2)}{\sigma^2}\right)} - 1 \right)}{g_s^2} \tag{2.13}$$

here W represents the Lambert W function

Proof: Given in Appendix A.

Next considering $(\Omega = 1)$ we get the following energy efficiency as:

$$EE_I(P_s^1) = \frac{\left(\frac{t_d}{t_s + t_d}\right) p_r(\Omega = 1)(1 - p_d) \log_2\left(1 + \frac{\frac{P_s^1 {g_s}^2}{\sigma^2}}{1 + \gamma_p g_p^2}\right)}{P_s^1 + P_{cir}} \tag{2.14}$$

and the optimal transmitted power P_s^1 is obtained as follows:

$$P_s^1 = \left(\frac{\left(P_{cir} {g_s}^2 - \left(1 + \gamma_p g_p^2\right)\sigma^2\right)\left(1 + \gamma_p g_p^2\right)\sigma^2}{W\left(0, \left(\frac{P_{cir} {g_s}^2 - \left(1 + \gamma_p g_p^2\right)\sigma^2}{\left(1 + \gamma_p g_p^2\right)\sigma^2}\right)\exp(-1)\right) {g_s}^4} - 1\right) * \left(\frac{\left(1 + \gamma_p g_p^2\right)\sigma^2}{{g_s}^2}\right) \tag{2.15}$$

Proof: Given in Appendix B.

2.5.4 Maximum Energy Efficiency with Constraints

To increase energy efficiency provided in eq. (2.9) and (2.10) we have constructed two multi objective optimization problem (MOP) for the cases where PT_x is inactive in eq. (2.7) and active in eq. (2.8), respectively. Moreover, through this MOP we collectively get the maximized value of the effective throughput and interference throughput and at the same time minimizing the power, for both eq. (2.9) and (2.10). Thus, the following MOP for when PT_x is inactive $(\Omega = 1)$ satisfies a desired average power constraint received at the SR_x [21]:

$$\min_{P_S} \quad P_S \tag{2.16a}$$

$$\max_{P_S} \quad S_D \tag{2.16b}$$

$$s.t. \quad P_S \geq P_{S.min} \tag{2.16c}$$

here $P_{S.min}$ is the minimal power for transmission required to transmit signals to the SR_x. Form the above MOP we see that only one objective is reduced and the rest are contemplated as constraints. Thus, the resulting single optimization problem (SOP) with respect to the above MOP is as follows:

$$\min_{P_s} \quad P_s \tag{2.17a}$$

$$s.t. \quad S_D \geq S_{D.min} \tag{2.17b}$$

$$s.t. \quad P_s \geq P_{s.min} \tag{2.17c}$$

here $S_{D.min}$ is the minimum required effective throughput for SR_x . Similarly in case where PT_x is active $(\Omega = 1)$ thus, the equivalent MOP and SOP is devised as:

$$\min_{P_S} \quad P_S \tag{2.18a}$$

$$\max_{P_S} \quad S_i \tag{2.18b}$$

$$s.t. \quad P_S \geq P_{S.min} \tag{2.18c}$$

hence the following SOP is:

$$\min_{P_S} \quad P_S \tag{2.19a}$$

$$s.t. \quad S_I \geq S_{I.min} \tag{2.19b}$$

$$s.t. \quad P_S \geq P_{S.min} \tag{2.19c}$$

where $S_{I.min}$ is the minimum required interference throughput at the SR_x.

The Lagrangian approach [20] providing the solution to the optimization problem stated above in eq. (2.16) and (2.17) for the inactive PT_x $(\Omega = 0)$ is as follows:

$$l(P_s, \delta) = P_s + \delta_a(S_{D.min} - S_D) + \delta_b(P_{s.min} - P_s) \tag{2.20}$$

here δ_a and δ_b are positive variables mutual to the constraints given in eq. (2.17b) and (2.17c).

Now deriving $l(P_s, \delta)$ in eq. (2.20) with respect to P_s, we get:

$$\frac{dl(P_S,\delta)}{dP_S} = -\delta_b + \frac{g_s^2\, \delta_a\, p_r\, t_d\, (p_f - 1)}{\sigma^2\, \log(2)\, (t_s + t_d)\left(1 + \frac{g_s^2\, P_S}{\sigma^2}\right)} + 1 \tag{2.21}$$

Next taking eq. (2.21) and equaling it to zero, we acquire the transmitted power P_s^* as:

$$P_s^* = \sigma^2\, \frac{\left(\frac{g_s^2\, \delta_a\, p_r\, t_d\, (p_f - 1)}{\sigma^2\, \log(2)\, (t_s + t_d)\, (\delta_b - 1)}\right) - 1}{g_s^2} \tag{2.22}$$

Proof: Given in Appendix C.

Similarly deriving eq. (2.22) with respect to δ_a we get:

$$\frac{dl(P_S,\delta)}{d\delta_a} = S_{D.min} + \frac{p_r\, t_d\, log\left(1 + \frac{g_S^2\, P_S}{\sigma^2}\right)(p_f - 1)}{log(2)\, (t_s + t_d)} \tag{2.23}$$

Now taking eq. (2.23) and equaling it to zero, corresponding P_s^* is:

$$P_s^* = \frac{\sigma^2 \left(exp\left(- \frac{S_{D.min} \log(2)\ (t_s\ +\ t_d)}{p_r\ t_d (p_f - 1)} \right) - 1 \right)}{g_s^2} \tag{2.24}$$

Proof: Given in Appendix C.

Finally deriving eq. (2.20) with respect to δ_b, we get:

$$\frac{dl(P_s, \delta)}{d\delta_b} = P_{s.min} - P_s \tag{2.25}$$

Hence taking eq. (2.25) and equaling it to zero, corresponding P_s^* is:

$$P_s^* = P_{s.min} \tag{2.26}$$

Likewise, the solution for the optimization problem stated in eq. (2.18) and (2.19) through Lagrangian approach for when PT_x is active $(\Omega = 1)$ is:

$$l(P_s, \delta) = P_s + \delta_a(S_{I.min} - S_I) + \delta_b(P_{s.min} - P_s) \tag{2.27}$$

Now deriving $l(P_s, \delta)$ in eq. (2.27) with respect to P_s, we get:

$$\frac{dl(P_s,\delta)}{dP_s} = -\delta_b + \frac{g_s^2\ \delta_a\ p_r\ t_d\ (p_d - 1)}{\sigma^2\ log(2)\ (t_s + t_d) \left(1 + \frac{g_s^2 P_s}{\sigma^2 (1+\gamma_p g_p^2)} \right) (1+\gamma_p g_p^2)} + 1 \tag{2.28}$$

Now taking eq. (2.28) and equaling it to zero, we get transmitted power P_s^* as:

$$P_s^* = \sigma^2 \left(\frac{\left(\frac{g_s^2\ \delta_a\ p_r\ t_d\ (p_d - 1)}{\sigma^2\ log(2)\ (t_s\ +\ t_d)\ (\delta_b - 1)(1 + \gamma_p g_p^2)} \right) - 1}{g_s^2} \right) (1 + \gamma_p g_p^2) \tag{2.29}$$

Proof: Given in Appendix D.

Similarly, deriving eq. (2.27) with respect to δ_a, we get:

$$\frac{dl(P_s,\delta)}{d\,\delta_a} = S_{I.min} + \frac{p_r\, t_d \log\left(1 + \frac{g_s^2\, P_s}{\sigma^2\left(1 + \gamma_p\, g_p^2\right)}\right)(p_d - 1)}{log(2)\ (t_s + t_d)} \tag{2.30}$$

Now taking eq. (2.30) and equaling it to zero, resulting P_s^* is:

$$P_s^* = \frac{\sigma^2\left(1 + \gamma_p g_p^2\right)\left(exp\left(-\frac{S_{I.min}\, log(2)\ (t_s + t_d)}{p_r\, t_d\, (p_d - 1)}\right) - 1\right)}{g_s^2} \tag{2.31}$$

Proof: Given in Appendix D.

Finally, deriving (2.27) with respect to δ_b, we get:

$$\frac{dl(P_s,\delta)}{d\,\delta_b} = P_{s.min} - P_s \tag{2.32}$$

Henceforth taking eq. (2.32) and equaling it to zero, corresponding P_s^* is:

$$P_s^* = P_{s.min} \tag{2.33}$$

Chapter 3

Result and Discussion

3.1 Introduction

In this section, we have identified the effective and interference throughput and energy efficiency of the UAV with plotting through MATLAB. In addition, we have plotted five figures and analyzed the results below.

3.2 Plots and Discussion

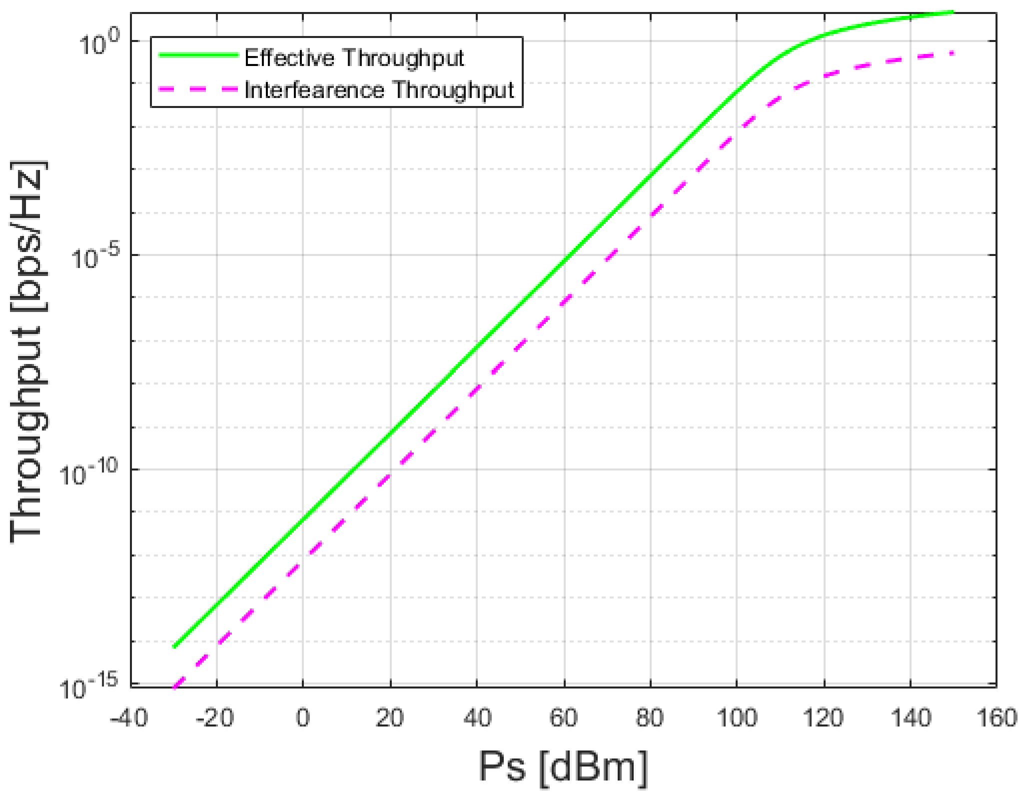

Figure 2: Effective and Interference throughput vs. Transmitted Power

From Figure 2 we can see by comparing both the effective and interference throughput that at 40 dBm transmitted power the effective throughput is 7.025×10^{-08} bps/Hz and the interference throughput 7.025×10^{-09} bps/Hz. Thus the efficiency of the effective throughput is higher.

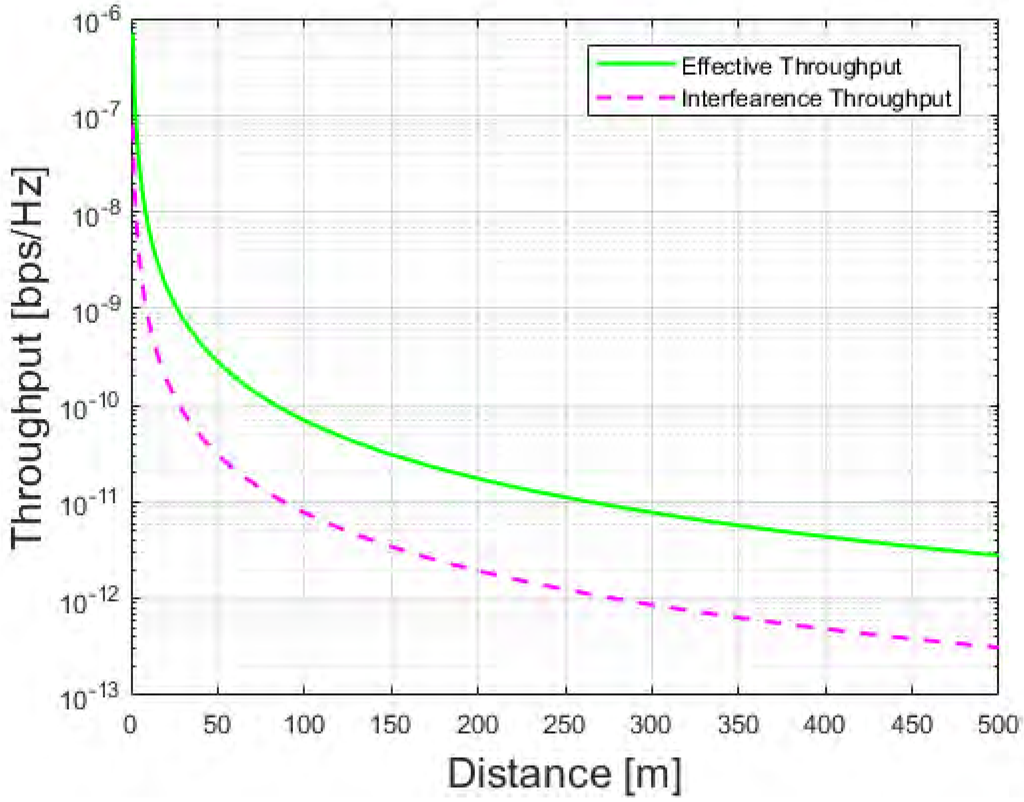

Figure 3: Effective and Interference throughput vs. Distance

From Figure 3 we can see by comparing both the effective and interference throughput that at 200m distance the effective throughput is 1.756×10^{-11} bps/Hz and the interference throughput 1.951×10^{-12} bps/Hz. Thus the efficiency of the effective throughput is higher.

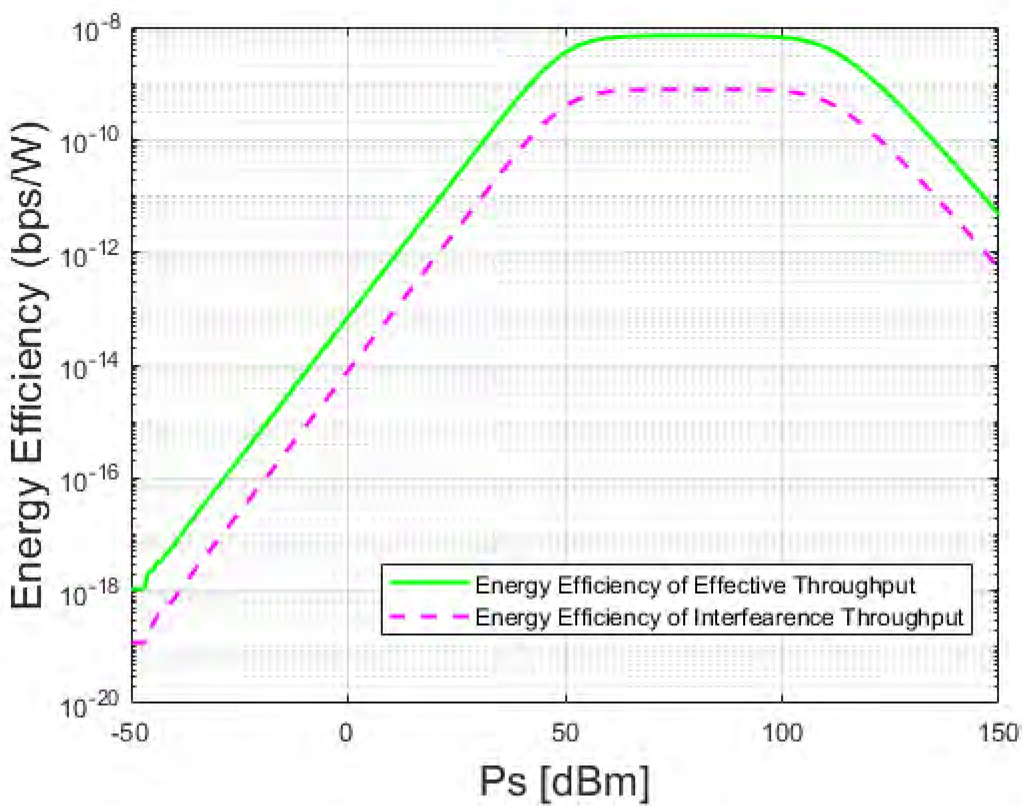

Figure 4: Energy Efficiency vs. Transmitted Power

From Figure 4 we can see by comparing both the energy efficiency of effective and interference that at 75dBm transmitted power the energy efficiency of effective throughput is 7.001×10^{-09} bps/W and energy efficiency of the interference throughput is 7.779×10^{-10} bps/W. Thus the energy efficiency of the effective throughput is higher.

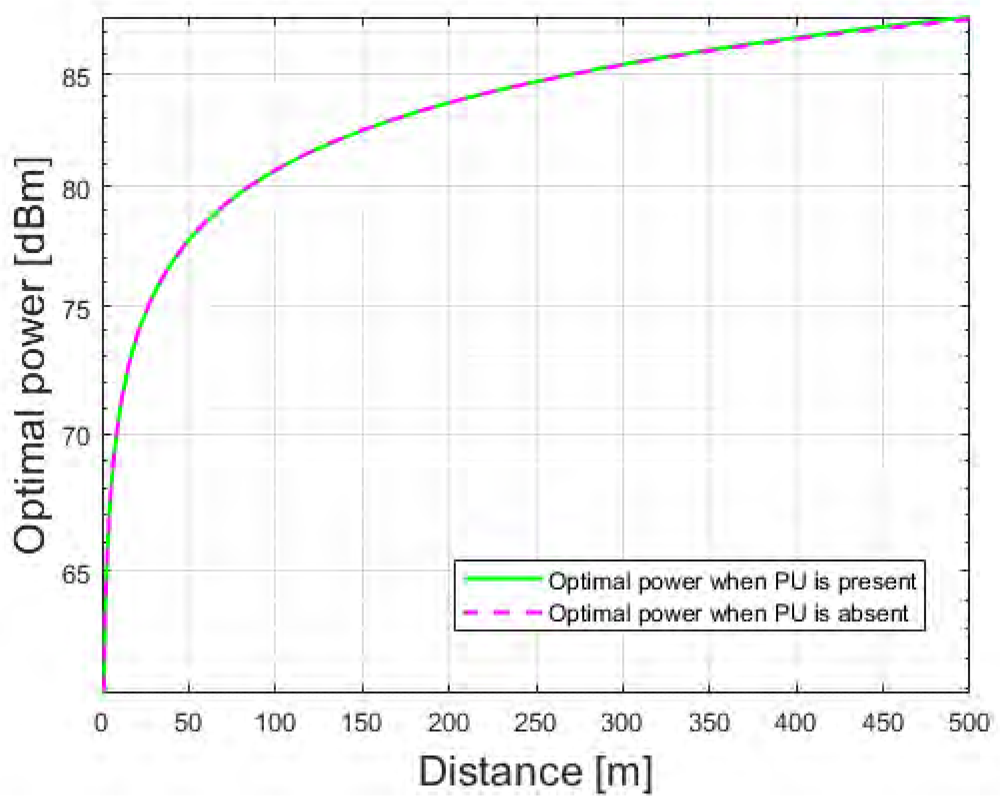

Figure 5: Optimal power vs. Distance

From Figure 5 we can see the optimal power is 83.72 dBm at 200m for both cases where PU is present and absent.

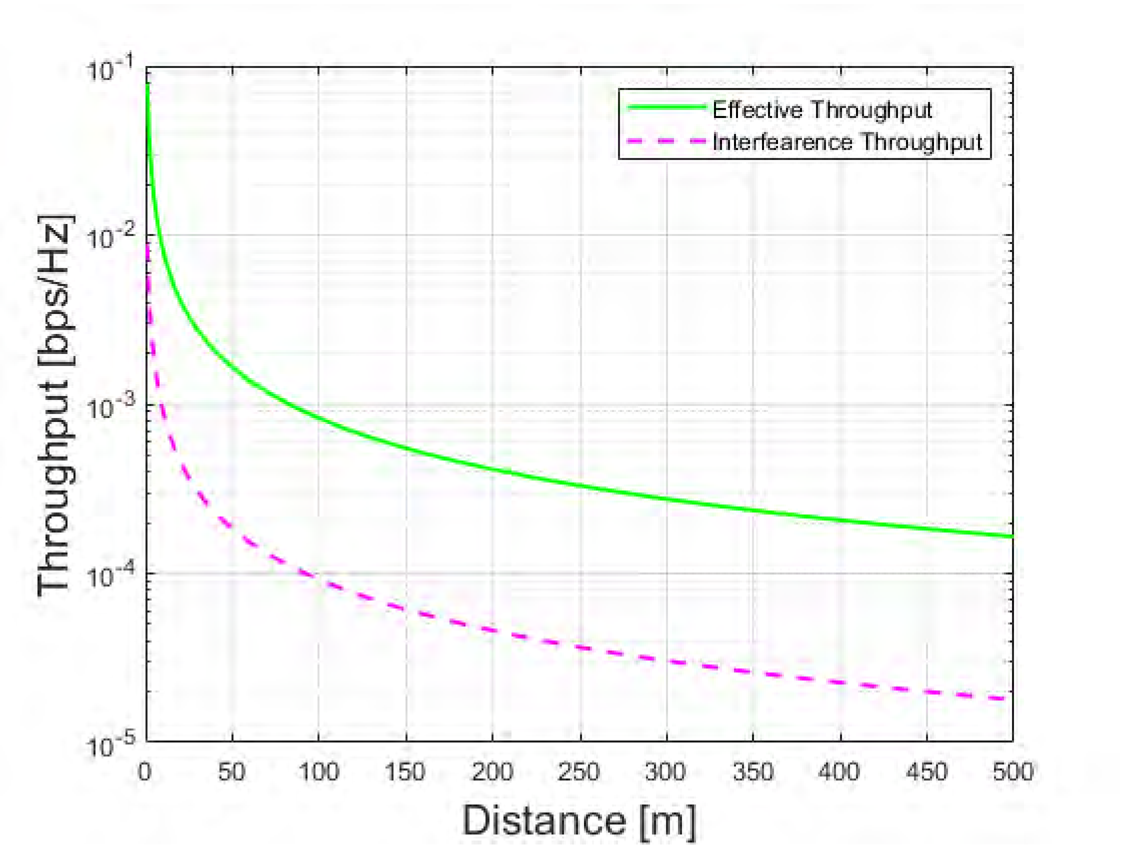

Figure 6: Effective and Interference throughput vs. Distance when optimal power is applied

From figure 6 we see that after optimal power is applied, both the effective and interference throughput that at 200m distance the effective throughput is 4.134×10^{-04} bps/Hz and the interference throughput 4.594×10^{-05} bps/Hz. So, we can jump to the conclusion that after applying the optimal power the throughputs have significantly improved.

Chapter 4

Conclusion

In this paper, an UAV based cognitive radio network is proposed for the purpose of improving spectrum sensing due to higher optimal and maximum transmission power. Here we have minimized the transmission power and evaluated the energy efficiency of the secondary transmitter (UAV).

4.1 Limitation

Here cognitive radio is used as it will sense and provide utilization of the radio spectrum but at near future even those unused spectral bands will be licensed thus reducing the usage of cognitive radios. Moreover, at some point The UAV will need to be recharged as it is battery powered, thus spectrum sensing with at all time is difficult.

4.2 Future Scope

As to the vast implementation of UAVs, there is still a lot of future scope for the UAV related applications. Now the wireless computing system that is used in most application is cloud computing. Moreover, as the number of devices connecting to the internet (IoT) are increasing vastly, many information are need to be processed and preserved over the cloud server. In addition, even a simple command over the internet is processed at the cloud server. Thus relying solely on cloud server for cloud computing will prove to be less efficient as network latency will increase. Therefore, a new way of computing is introduced nowadays known as 'Fog Computing'. The main difference between cloud and fog computing is that cloud computing is a centralized system meaning every single cloud server is responsible for preserving and processing all user information or requests where in fog computing the user information and requests are prioritize based on the various parameters with the addition of low network latency. Henceforth, as our UAV based application uses basic cloud computing for data processing, implementing fog computing surely increase efficiency. Moreover, all the UAV applications will be efficiently utilized as it will provide less delay time for data processing and thus the saved time will ensure the charging time of UAVs. Furthermore, to enhance the performance of a cognitive radio, transmit antenna selection techniques considering signal to interference plus noise ratio (SINR) may be implemented [S.1]. Also for the performance enhancement random cognitive network focusing on two-slope path-loss, function for better understanding of propagation environments may be implemented [S.2].Finally more energy efficient of cognitive radio may be implemented by introducing an optimization problem tasked to maximize energy efficiency for both primary and secondary networks[S.3]

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Appendix A.

PROOF OF OPTIMAL POWER TRANSMISSION WITHOUT CONSTRAINT WHEN $\Omega = 0$

Now deriving EE_D in eq. (13) with respect to P_s we get:

$$\frac{dEE_D}{dP_s} = 0$$

$$\Rightarrow \sigma^2 p_r t_d \, \log\left(1 + \frac{P_s \, g_s^2}{\sigma^2}\right)\left(p_f - 1\right)\left(1 + \frac{P_s \, g_s^2}{\sigma^2}\right) = p_r \, g_s^2 \, t_d\left(p_f - 1\right)\left(P_{cir} + P_s\right)$$

$$\Rightarrow \log\left(\frac{P_s \, g_s^2 + \sigma^2}{\sigma^2}\right) = \frac{P_{cir} g_s^2 + P_s \, g_s^2}{P_s \, g_s^2 + \sigma^2}$$

$$\Rightarrow \log\left(\frac{P_s \, g_s^2 + \sigma^2}{\sigma^2}\right) - 1 = \frac{P_{cir} g_s^2 + P_s \, g_s^2}{P_s \, g_s^2 + \sigma^2} - 1$$

$$\Rightarrow \log\left(\frac{P_s \, g_s^2 + \sigma^2}{\sigma^2}\right) + \ln e^{-1} = \frac{P_{cir} g_s^2 - \sigma^2}{P_s \, g_s^2 + \sigma^2}$$

$$\Rightarrow \left(\frac{P_s \, g_s^2 + \sigma^2}{\sigma^2}\right) \exp(-1) = \exp\left(\frac{P_{cir} g_s^2 - \sigma^2}{P_s \, g_s^2 + \sigma^2}\right)$$

$$\Rightarrow \frac{\exp(-1)\,\left(P_{cir} g_s^2 - \sigma^2\right)}{\sigma^2} = \left(\frac{P_{cir} g_s^2 - \sigma^2}{P_s \, g_s^2 + \sigma^2}\right) \exp\left(\frac{P_{cir} g_s^2 - \sigma^2}{P_s \, g_s^2 + \sigma^2}\right) \qquad \text{(A.1)}$$

Using the Lambert method, we rewrite in eq. (A.1)

$$W\left(\frac{\exp(-1)\,\left(P_{cir} g_s^2 - \sigma^2\right)}{\sigma^2}\right) = \frac{P_{cir} g_s^2 - \sigma^2}{P_s \, g_s^2 + \sigma^2}$$

$$\Rightarrow P_s \, g_s^2 + \sigma^2 = \frac{P_{cir} g_s^2 - \sigma^2}{W\left(\frac{\exp(-1)\,\left(P_{cir} g_s^2 - \sigma^2\right)}{\sigma^2}\right)}$$

$$\therefore P_s^0 = \frac{P_{cir} g_s^2 - \sigma^2}{W\left(0, \frac{\exp(-1)\,\left(P_{cir} g_s^2 - \sigma^2\right)}{\sigma^2}\right) g_s^2} - \frac{\sigma^2}{g_s^2} \qquad \text{(A.2)}$$

Appendix B.

PROOF OF OPTIMAL POWER TRANSMISSION WITHOUT CONSTRAINT WHEN Ω = 1

Now deriving EE_I in eq. (15) with respect to P_s we get:

$$\frac{dEE_I}{dP_s} = 0$$

$$\Rightarrow \sigma^2 p_r t_d \log\left(1 + \frac{P_s\, g_s^2}{\sigma^2(1+\gamma_p g_p^2)}\right)(P_d - 1)\left(1 + \frac{P_s\, g_s^2}{\sigma^2(1+\gamma_p g_p^2)}\right) = p_r\, g_s^2\, t_d(p_d - 1)$$
$$(P_{cir} + P_s)$$

$$\Rightarrow \log\left(1 + \frac{P_s\, g_s^2}{\sigma^2(1+\gamma_p g_p^2)}\right)\left(\sigma^2 + \frac{P_s\, g_s^2}{(1+\gamma_p g_p^2)}\right)\left(1 + \gamma_p g_p^2\right) = P_{cir}\, g_s^2 + P_s\, g_s^2$$

$$\Rightarrow \log\left(1 + \frac{P_s\, g_s^2}{\sigma^2(1+\gamma_p g_p^2)}\right)\left(P_s\, g_s^2 + \left(1 + \gamma_p g_p^2\right)\sigma^2\right) = P_{cir}\, g_s^2 + P_s\, g_s^2$$

$$\Rightarrow \log\left(1 + \frac{P_s\, g_s^2}{\sigma^2(1+\gamma_p g_p^2)}\right) = \frac{P_{cir}\, g_s^2 + P_s\, g_s^2}{P_s\, g_s^2 + (1+\gamma_p g_p^2)\sigma^2}$$

$$\Rightarrow \log\left(1 + \frac{P_s\, g_s^2}{\sigma^2(1+\gamma_p g_p^2)}\right) - 1 = \frac{P_{cir}\, g_s^2 + P_s\, g_s^2}{P_s\, g_s^2 + (1+\gamma_p g_p^2)\sigma^2} - 1$$

$$\Rightarrow \log\left(1 + \frac{P_s\, g_s^2}{\sigma^2(1+\gamma_p g_p^2)}\right) + \ln e^{-1} = \frac{P_{cir}\, g_s^2 + P_s\, g_s^2 - P_s\, g_s^2 - (1+\gamma_p g_p^2)\sigma^2}{P_s\, g_s^2 + (1+\gamma_p g_p^2)\sigma^2}$$

$$\Rightarrow \left(\frac{P_s\, g_s^2 + (1+\gamma_p g_p^2)\sigma^2}{\sigma^2(1+\gamma_p g_p^2)}\right)\exp(-1) = \exp\left(\frac{P_{cir}\, g_s^2 - (1+\gamma_p g_p^2)\sigma^2}{P_s\, g_s^2 + (1+\gamma_p g_p^2)\sigma^2}\right)$$

$$\Rightarrow \left(\frac{P_{cir}\, g_s^2 - (1+\gamma_p g_p^2)\sigma^2}{\sigma^2(1+\gamma_p g_p^2)}\right)\exp(-1) = \left(\frac{P_{cir}\, g_s^2 - (1+\gamma_p g_p^2)\sigma^2}{P_s\, g_s^2 + (1+\gamma_p g_p^2)\sigma^2}\right)\exp\left(\frac{P_{cir}\, g_s^2 - (1+\gamma_p g_p^2)\sigma^2}{P_s\, g_s^2 + (1+\gamma_p g_p^2)\sigma^2}\right) \quad \text{(B.1)}$$

Using the Lambert method, we rewrite in eq. (B.1)

$$W\left(\left(\frac{P_{cir}\, g_s^2 - \left(1 + \gamma_p g_p^2\right)\sigma^2}{\sigma^2\left(1 + \gamma_p g_p^2\right)}\right)\exp(-1)\right) = \frac{P_{cir}\, g_s^2 - \left(1 + \gamma_p g_p^2\right)\sigma^2}{P_s\, g_s^2 + \left(1 + \gamma_p g_p^2\right)\sigma^2}$$

$$\Rightarrow \frac{P_{cir}\, g_s^2 - (1+\gamma_p g_p^2)\sigma^2}{W\left(\left(\frac{P_{cir}\, g_s^2 - (1+\gamma_p g_p^2)\sigma^2}{\sigma^2(1+\gamma_p g_p^2)}\right)\exp(-1)\right)} = P_s\, g_s^2 + \left(1 + \gamma_p g_p^2\right)\sigma^2$$

$$\therefore P_s^1 = \left(\frac{\left(P_{cir}\, g_s^2 - (1+\gamma_p g_p^2)\sigma^2\right)(1+\gamma_p g_p^2)\sigma^2}{W\left(0, \left(\frac{P_{cir}\, g_s^2 - (1+\gamma_p g_p^2)\sigma^2}{\sigma^2(1+\gamma_p g_p^2)}\right)\exp(-1)\right) g_s^4} - 1\right) * \left(\frac{(1+\gamma_p g_p^2)\sigma^2}{g_s^2}\right) \quad \text{(B.2)}$$

Appendix C.

PROOF OF MAXIMUM POWER TRANSMISSION WITH CONSTRAINT WHEN

Ω= 0

Derivative of $l(P_S, \delta)$ in eq. (21) with respect to P_s, provides:

$$\frac{dl(P_S,\delta)}{dP_s} = 0$$

$$\Rightarrow (1 - \delta_b) + \frac{g_s^2\, \delta_a\, p_r\, t_d(p_f - 1)}{\sigma^2 \log(2)\,(t_s + t_d)\left(1 + \frac{g_s^2 P_s}{\sigma^2}\right)} = 0$$

$$\Rightarrow \sigma^2 \log(2)\,(t_s + t_d)\left(1 + \frac{g_s^2 P_s}{\sigma^2}\right)(1 - \delta_b) + g_s^2\, \delta_a\, p_r\, t_d(p_f - 1) = 0$$

$$\Rightarrow \sigma^2 \log(2)\,(t_s + t_d)\,(1 - \delta_b) + \log(2)\,(t_s + t_d)\,(1 - \delta_b)(g_s^2 P_s)$$
$$= - g_s^2\, \delta_a\, p_r\, t_d(p_f - 1)$$

$$\Rightarrow g_s^2 P_s = -\left(\frac{g_s^2\, \delta_a\, p_r\, t_d(p_f - 1)}{\log(2)\,(t_s + t_d)\,(1 - \delta_b)} + \frac{\sigma^2 \log(2)\,(t_s + t_d)\,(1 - \delta_b)}{\log(2)\,(t_s + t_d)\,(1 - \delta_b)}\right)$$

$$\Rightarrow g_s^2 P_s = \left(\frac{g_s^2\, \delta_a\, p_r\, t_d(p_f - 1)}{\log(2)\,(t_s + t_d)\,(\delta_b - 1)} - \sigma^2\right) \qquad [\because 1 - \delta_b = -(\delta_b - 1)] \qquad \text{(C.1)}$$

Thus transmitted power P_s^* from (C.1) is:

$$\therefore P_s^* = \sigma^2\, \frac{\left(\frac{g_s^2\, \delta_a\, p_r\, t_d(p_f - 1)}{\sigma^2 \log(2)\,(t_s + t_d)\,(\delta_b - 1)}\right) - 1}{g_s^2} \qquad \text{(C.2)}$$

Derivative of $l(P_S, \delta)$ in eq. (21) with respect to δ_a, provides:

$$\frac{dl(P_S,\delta)}{d\delta_a} = 0$$

$$\Rightarrow S_{D.min} + \frac{p_r\, t_d \log\left(1 + \frac{g_s^2 P_s}{\sigma^2}\right)(p_f - 1)}{\log(2)\,(t_s + t_d)} = 0$$

$$\Rightarrow S_{D.min} \log(2)\,(t_s + t_d) + p_r\, t_d \log\left(1 + \frac{g_s^2 P_s}{\sigma^2}\right)(p_f - 1) = 0$$

$$\Rightarrow p_r\, t_d \log\left(1 + \frac{g_s^2 P_s}{\sigma^2}\right)(p_f - 1) = - S_{D.min} \log(2)\,(t_s + t_d)$$

$$\Rightarrow \log\left(1 + \frac{g_s^2 P_s}{\sigma^2}\right) = -\left(\frac{S_{D.min} \log(2)\,(t_s + t_d)}{p_r\, t_d(p_f - 1)}\right)$$

$$\Rightarrow \left(1 + \frac{g_s^2 P_s}{\sigma^2}\right) = \exp\left(-\left(\frac{S_{D.min} \log(2)\,(t_s + t_d)}{p_r\, t_d(p_f - 1)}\right)\right)$$

$$\Rightarrow g_s^2 P_s = \sigma^2 \exp\left(-\left(\frac{s_{D.min}\ \log(2)\ (t_s + t_d)}{p_r\ t_d(p_f - 1)}\right)\right) - \sigma^2 \tag{C.3}$$

Thus from (C.3) P_s^* results to:

$$\therefore\ P_s^* = \frac{\sigma^2\left(\exp\left(-\frac{s_{D.min}\ \log(2)\ (t_s + t_d)}{p_r\ t_d(p_f - 1)}\right) - 1\right)}{g_s^2} \tag{C.4}$$

Finally derivative of $l(P_S, \delta)$ in eq. (21) with respect to δ_b gives:

$$\frac{dl(P_S, \delta)}{d\ \delta_b} = 0$$

$$\Rightarrow P_{s.min} - P_s = 0 \tag{C.5}$$

Therefore P_s^* is*:*

$$\therefore P_s^* = P_{s.min} \tag{C.6}$$

Appendix D.

PROOF OF MAXIMUM POWER TRANSMISSION WITH CONSTRAINT WHEN Ω= 1

Derivative of $l(P_S, \delta)$ in eq. (28) with respect to P_s, provides:

$$\frac{dl(P_S,\delta)}{dP_S} = 0$$

$$\Rightarrow (1 - \delta_b) + \frac{g_s^2\, \delta_a\, p_r\, t_d\, (p_d - 1)}{\sigma^2 \log(2)\, (t_s + t_d) \left(1 + \frac{g_s^2 P_s}{\sigma^2 \left(1 + \gamma_p g_p^2\right)}\right)\left(1 + \gamma_p g_p^2\right)} = 0$$

$$\Rightarrow \sigma^2 \log(2)\ (t_s + t_d)(1 - \delta_b)\left(1 + \gamma_p g_p^2\right) + \log(2)\ (t_s + t_d)(1 - \delta_b)(g_s^2 P_s)$$
$$= -g_s^2\, \delta_a\, p_r\, t_d\, (p_d - 1)$$

$$\Rightarrow g_s^2 P_s = -\left(\frac{g_s^2\, \delta_a\, p_r\, t_d (p_d - 1)}{\log(2)\, (t_s + t_d)\, (1 - \delta_b)} + \frac{\sigma^2 \log(2)\, (t_s + t_d)(1 - \delta_b)\left(1 + \gamma_p g_p^2\right)}{\log(2)\, (t_s + t_d)\, (1 - \delta_b)}\right)$$

$$\Rightarrow g_s^2 P_s = \left(\frac{g_s^2\, \delta_a\, p_r\, t_d (p_d - 1)}{\log(2)\, (t_s + t_d)\, (\delta_b - 1)} - \sigma^2\left(1 + \gamma_p g_p^2\right)\right) \quad [\because 1 - \delta_b = -(\delta_b - 1)] \qquad \text{(D.1)}$$

Thus transmitted power P_s^* from (C.1) is:

$$\therefore P_s^* = \sigma^2 \left(\frac{\left(\frac{g_s^2\, \delta_a\, p_r\, t_d (p_d - 1)}{\sigma^2 \log(2)\, (t_s + t_d)(\delta_b - 1)\left(1 + \gamma_p g_p^2\right)}\right)^{-1}}{g_s^2}\right)\left(1 + \gamma_p g_p^2\right) \qquad \text{(D.2)}$$

Next derivative of $l(P_S, \delta)$ in eq. (28) with respect to δ_a, provides:

$$\frac{dl(P_S,\delta)}{d\delta_a} = 0$$

$$\Rightarrow S_{I.min} + \frac{p_r\, t_d \log\left(1 + \frac{g_s^2 P_s}{\sigma^2\left(1 + \gamma_p g_p^2\right)}\right)(p_d - 1)}{\log(2)\, (t_s + t_d)} = 0$$

$$\Rightarrow S_{I.min}\ \log(2)\ (t_s + t_d) + p_r\, t_d \log\left(1 + \frac{g_s^2 P_s}{\sigma^2\left(1 + \gamma_p g_p^2\right)}\right)(p_d - 1) = 0$$

$$\Rightarrow p_r\, t_d \log\left(1 + \frac{g_s^2 P_s}{\sigma^2\left(1 + \gamma_p g_p^2\right)}\right)(p_d - 1) = -S_{I.min}\ \log(2)\ (t_s + t_d)$$

$$\Rightarrow \log\left(1 + \frac{g_s^2 P_s}{\sigma^2\left(1 + \gamma_p g_p^2\right)}\right) = -\left(\frac{S_{I.min}\ \log(2)\, (t_s + t_d)}{p_r\, t_d (p_d - 1)}\right)$$

$$\Rightarrow \left(1 + \frac{g_s^2 P_s}{\sigma^2\left(1 + \gamma_p g_p^2\right)}\right) = \exp\left(-\left(\frac{S_{I.min}\ \log(2)\, (t_s + t_d)}{p_r\, t_d (p_d - 1)}\right)\right)$$

$$\Rightarrow \sigma^2\left(1 + \gamma_p g_p^2\right) + g_s^2 P_s = \left(\sigma^2\left(1 + \gamma_p g_p^2\right)\right) \exp\left(-\left(\frac{S_{I.min}\ \log(2)\, (t_s + t_d)}{p_r\, t_d (p_d - 1)}\right)\right) \qquad \text{(D.3)}$$

Thus from (D.3) P_s^* results to:

$$\therefore\ P_s^* = \frac{\sigma^2\left(1+\gamma_p g_p^2\right)\left(\exp\left(- -\frac{S_{I.min}\log(2)\,(t_S + t_d)}{p_r\, t_d(p_d - 1)}\right)-1\right)}{g_s^2} \quad \text{(D.4)}$$

Finally derivative of $l(P_S, \delta)$ in eq. (28) with respect to δ_b gives:

$$\frac{dl(P_S, \delta)}{d\ \delta_b} = 0$$

$$\Rightarrow P_{s.min} - P_s = 0 \quad \text{(D.5)}$$

Therefore P_s^* is*:*

$$\therefore P_s^* = P_{s.min} \quad \text{(D.6)}$$