Research on Best Peer Finding Using Q-Learning
Using Hotspot Network

by

ANU PRIA ROY

ID: 13201062

Department of Computer Science & Engineering
BRAC UNIVERSITY

Supervised by: MOIN MOSTAKIM
Co-supervisor: SURAIYA TARIN

Thesis report submitted to the BRAC University in accordance with the requirements of the degree of BACHELOR IN COMPUTER SCIENCE AND ENGINEERING in the Dept. of Engineering & Computer Science.
Abstract

Peers can share internet by creating hotspot networks. The main driver device needs to decide when and which device it needs to connect and gives more internet to perform the operation. By the help of the Q-learning we will try to make a candidate set of best peers so that it can minimize the internet sharing. Q-learning method not only learns from the network parameters as processing capacity, number of resources in the peers but also learns from their state of congestion. According to the proposed method, the data stored in the P2P network is spread across a large number of nodes as the distribution of data can be random and the desired resources are located more quickly without being forwarded and at the same time the network traffic will be minimized.
Acknowledgement

Before starting to write this paper, I would like to express my utmost gratitude to Almighty God who gave us the zeal, determination, strength and intelligence to complete my thesis. I want to thank my parent for their support and my respected faculties and dear classmates for their constant support and motivation. Most importantly, I would like to thank my supervisor Moin Mostakim Sir and co-advisor Suraiya Tarin Miss for their consistent supervision, guidance and unflinching encouragement in accomplishing my work and help me to execute my idea with great success.
Author’s Declaration

I, hereby declare that this thesis paper is based on the proposed method found by myself. Materials of work found by other researcher are mentioned by reference. This thesis, neither in whole or in part, has been previously submitted for any degree.

SIGNATURE OF THE AUTHOR:

..............................
ANU PRIA ROY
ID: 13201062

SIGNATURE OF THE SUPERVISOR:

..............................
MOIN MOSTAKIM
LECTURER
DEPT. OF COMPUTER SCIENCE, BRAC UNIVERSITY

SIGNATURE OF THE CO-SUPERVISOR:

..............................
SURAIYA TARIN
LECTURER
DEPT. OF COMPUTER SCIENCE, BRAC UNIVERSITY
# Table of Contents

Chapter 1 Introduction 1
  1.1. Motivation 1
  1.2. Objective 1

Chapter 2 Literature Review 3
  2.1. Overview 3
  2.2. Previous work 3
  2.3. System implementation 4
    2.3.1. Q-Learning 4
    2.3.2. Hotspot Networking 5
    2.3.3. Analytic Hierarchy Process (AHP) 8
    2.3.4. Grey Relational Analysis (GRA) 8
    2.3.5 A Network Selection Scheme using AHP and GRA 9

Chapter 3 Work and Analysis 13
  3.1 K random walks 13
  3.2 K nearest neighbor 13
  3.3 Routing control in unstructured P2P networks 15
  3.4 Comparison between KNN and Q Learning 16
  3.5 Proposed Simulation Method 18

Chapter 4 Conclusion 21
  4.1. Conclusion 21
  4.2. Future Work 21

References 22
List of Figures

Figure 2.1 WLAN 5
Figure 2.2 Router connection 6
Figure 2.3 Identifying Jammer Location 7
Figure 2.4 AHP and GRA based selection model 11
Figure 3.1 KNN algorithm process 1 14
Figure 3.2 KNN algorithm process 2 14
Figure 3.3 KNN algorithm process 3 15
Figure 3.4 GRC comparison between Device A and Device B 20
Chapter 1
Introduction

1.1 Motivation

Peer to peer (P2P) networks are growing popularity and are being employed in a wide range of popular internet applications, such as content delivery, file sharing and multimedia streaming etc. P2P systems distribute the load of data storage, computation, communications and management among thousands of peers. Peer can join and depart the network at any time, at their will. The work of serving files in virtually all current P2P systems is performed for free by its users. Since users do not benefit from serving files to others, many users decline to perform this altruistic act. A recent study of the Gnutella network found that more than 70% of its users contribute nothing to the system. The phenomenon of selfish individuals who opt out of a voluntary contribution to a group of common welfare has been widely studied and is known as the free rider problem. The communal sharing of information good is “Discretionary Database” and the resulting free rider problem received study long before the advent of P2P systems.

1.2 Objectives

In this paper, we propose a new method for unstructured P2P networks in order to overcome the issues of query routing overheads and congestion is peer through congestion control which is Q-learning using hotspot networks. The detection of “Hotspot” or areas of unusual overcome has matured greatly and now relatively fast computation available there is considerable access to the technology of hotspot detection. The proposed Q-learning learns the networks parameters such as processing capacity, number of connections and number of resources in the peers, along with their state of congestion. In this technique peers are avoided to forward
queries to the congested peers. Our research result show that the desired resources are located more quickly and queries in the whole network are balanced than that random walk method.
Chapter 2
Literature Review

2.1 Overview

Initially our aim is to identify and detect good peer networking builds on strategies that include both using good practices that include both using good practices that other peer networks have found helpful and responding to challenge that appear regularly for such activities. Our aim to compare different types of algorithm based on the performance from the previous researches and try to match up to their accuracy mark.

Peer to peer connection seeks to promote the users in finding and support users in their effort to follow up on the peer review and ensure that it has a positive effect by providing framework. Peer connections have been used in the field of updated technology and industries. So, it is our goal to find out the least peer connection among so many connections using Q-learning networking through hotspot network so that we can get some advantage.

2.2 Previous Work

We are studying various types of algorithms on the peer finding previous research paper. Comparing them based on performance, time complexity, medium, cost, environment. Goal is to get best peer among many networks using hotspot networking using Q-learning.

The idea of P2P system was first established in 1969, in the first request for comments, RFC 1. The RFC implements a “host to host” connection, indiscriminate of a client server categorization which provides responses in the first true implantation of a P2P network was
‘Usenet’ developed in clients still access resources through servers, servers themselves peer with each other in the fashion of a P2P network, sending messages to each other on demand without a central authority.

2.3 System Implementation

Proposed Method

For finding best peer to peer network we mainly focused on two concepts. One is Q-learning and the another one is Hotspot network. We will also focus on another term which is AHP and GRA.

2.3.1 Q-learning:

For identifying the congested peers and avoid forwarding queries to them, Q-learning method which is a method of reinforce learning, is applied to monitor the state of the peers in the network [9]. A policy is a rule that the agent follows in selecting actions, given the state it is in. When such an action value function is learned, the optimal policy can be constructed by simply selecting the action with the highest value in each state. One of the strengths of Q-learnings is that it is able to compare the expected utility of the available actions without requiring a model of the environment.

Additionally, Q to directly approximate the optical action value function for an arbitrary target policy. The one step Q-learning model if defined as follows:

\[ Q_{\text{local}}(s,a) = R(s) + \gamma \max_{a'} Q(s',a') \]

\[ Q_{\text{new}}(s,a) = Q(s,a) + \alpha Q_{\text{local}}(s,a) \]

Where Q(s,a) is an action value function R(s) is the reward, \( \alpha \) is the learning rate which is set between 0 and 1. \( \gamma \) is the discount factor, also set between 0 and 1. And this parameter of \( \gamma \) that
future rewards are worthless than immediate rewards. And maxₐ is the maximum reward that can be achieved in the next state.

2.3.2 Hotspot networking:

It is important to define two conceptual focus which is clustering and hotspot [3]. Clustering of spatially referenced feature is broadly defined by the term “Unusual aggression” of events. The events are assumed to be spatially distributed and can assume an aggregated form whereby within “cluster” the inter point distance is much less than between clusters [3]. Usually, the term clustering is applied to random events, rather than continuous field. Intensity essentially describes the local density of events on the map and can be displayed as a contour surface. Figure 1 display two contour surfaces generated by simulator that could represent different degrees of clustering in the underlying event process. In general, the definition of cluster location, cluster size, cluster shape and possibly some measure of minimum intensity.

Figure 2.1 WLAN
Table 2.1 Recording router table based on efficiency of the Devices

<table>
<thead>
<tr>
<th>Devices</th>
<th>Ipad</th>
<th>Laptop</th>
<th>Smartphone1</th>
<th>PC</th>
<th>Smartphone2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Router</td>
<td>150</td>
<td>270</td>
<td>120</td>
<td>250</td>
<td>190</td>
</tr>
</tbody>
</table>

Decision:

<table>
<thead>
<tr>
<th>Device</th>
<th>Laptop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Router</td>
<td>270</td>
</tr>
</tbody>
</table>

Table 2.2 Laptop has the high efficiency

Figure 2.2 Router connection with laptop
A hotspot is a physical location where people may obtain internet access, typically using Wi-Fi technology, via a wireless local area network (WLAN) using a router connected to an internet service provider. Public hotspot may be found in a number of business for use of customers in many developed urban areas throughout the world. Hotspots differ from wireless access points, which are the hardware devices used to provide a wireless network service. Private hotspots allow internet access to a device via another device which may have data access.

Figure 2.3 Identifying Jammer Location

Our approach to identify jammer location integrates Q-learning with the underlining routing protocol. Routing protocol such as OLSR, include unused message fields in broadcast control messages that may be utilized to convey additional information between nodes. Specifically, in order to support the dynamic operation of Q-learning at each node. Using the distributed Q-learning frame work, we integrated Q-learning related messages into the routing protocol by attaching them to hello messages. These messages are propagated to all ne hop neighbors. When its neighbors receive those messages, it detaches the Q-learning messages and stores them in a corresponding V table in reverse time order for the calculation of future Q values.
For each time slot, the calculation function of Q-learning is called, which searches the old Q-value and V table to decide the optimal action for next time slot. The receiving node stores the above information into V and PL lists. The asynchronous feature of the form of Q-learning we employ is well suited for congestion integrating into our algorithm is a traffic control method whereby nodes in the network adjust their outgoing traffic rate when the network suspects there is a jammer location. Such adjustment would reduce the likelihood of congestion being fast declared as jamming by own algorithm.

2.3.3 Analytic Hierarchy Process (AHP)

AHP is defined as a procedure to divide a complex problem into a number of deciding factors and integrate the relative dominances of the factors with the solution alternatives to find the optimal one[8]. AHP is carried out in five steps:

Step 1 Structuring a problem as a decision hierarchy of independent decision elements
Step 2 Collecting information about the decision elements
Step 3 Comparing the decision elements pairwise on each level in the matter of their importance to the elements in the level above
Step 4 Calculating the relative priorities of decision elements in each level
Step 5 Synthesizing the above results to achieve the overall weight of each decision alternative

2.3.4 Grey Relational Analysis (GRA):

GRA builds grey relationships between elements of two series to compare each member quantitatively[8]. One of the series is composed of best quality entities, while the other series contains comparative entities. The less difference between the two series, the more preferable the comparative series. A grey relational coefficient (GRC) is used to describe the relationship
between them and is calculated according to the level of similarity and variability. GRA is usually implemented following six steps:

Step 1 Classifying the elements of series by three situations: larger-the-better, smaller-the-better, and nominal-the-best
Step 2 Defining the lower, moderate, or upper bounds of series elements
Step 3 Normalizing individual entities
Step 4 Defining the ideal series
Step 5 Calculating the GRCs
Step 6 Selecting the alternative with the largest GRC

2.3.5 A network selection scheme using AHP and GRA

In this section we apply AHP and GRA to network selection [8]. The factors that decide the network selection and the relationship among the factors are defined. The whole selection process is presented by a model and detailed explanations. In the network selection scenario, users are always trying to seamlessly access high-quality wireless service at any speed, any location, and any time through selecting the optimal network. Therefore, ensuring a specific QoS is the objective in the process of network selection. As a result, QoS is in the topmost level of the AHP hierarchy for network selection. According to a survey of QoS components in mobile communications, the main QoS components in a network are defined as throughput ($\mathcal{Q}$), timeliness ($\mathcal{T}$), reliability ($\mathcal{R}$), security ($\mathcal{S}$), and cost ($\mathcal{C}$), which are in the second level of the hierarchy. In consequent levels, received signal strength (RSS) and coverage area are used to present availability. The adoption of coverage area is in order to avoid frequent handoffs for high-speed users.

Three parameters, delay ($\mathcal{D}$), response time ($\mathcal{R}$), and jitter decide timeliness. Bit error rate (BER, $\mathcal{E}$), burst error ($\mathcal{B}$), and average number of retransmissions per packet ($\mathcal{P}$) are used to define reliability. In our scheme, UMTS and WLAN are considered as available network alternatives (in the bottom level of the hierarchy), and have various parameters in these QoS factors and subfactors. The whole network selection model is shown in figure. Since
availability is the precondition to other QoS deciding factors, in order to severe sources we use network availability as a trigger for the QoS data collector. Only after a network is detected as available, the network performance, service class, and user preference are estimated and collected. Because UMTS could be always on, deciding the availability of WLAN becomes the main problem. Once the RSS of WLAN is larger than the RSS threshold (e.g., ~80 dBm), which allows communication service for a period of time, and the user is estimated to be in the coverage of WLAN for more than a time limit (e.g., 1 min), the network selector begins to collect other QoS information from the network and user to determine whether to hand off to WLAN; otherwise, the GRCs of WLAN and UMTS are assigned 0 and 1, respectively, allowing the decision maker to keep UMTS connected.

The process of deciding is actually a trade-off between network performance and user profile specification [7]. Therefore, two types of data, user based and network-based, need to be collected for comparison. Meanwhile, users themselves have different requirements for service; for instance, some people are concerned about cost, others about security. The questionnaire (shown in Figure 2.4) is actually the database containing all user-based information. User preferences are filled into the questionnaire before accessing any network, and the current service class is detected and mapped into a number of specific QoS attributes. User preferences might be some certain ranges or generic terms, such as strict, bounded, tolerable, and unbounded. Because we deal with pairwise comparison intangibly, and the results only express the intensities of importance of the factors, the above two types of user based parameters are both acceptable. There are two types of network-based parameters, quantitative and qualitative. Quantitative parameters can be processed directly by GRA, while qualitative parameters are evaluated with a rating from 1 to 10; the larger the number, the better.

Once all the information on the QoS parameters is collected, pairwise comparisons are performed at each level (step 3 [AHP], Figure 2.4). Three AHP matrices are constructed. One of
them is used to compare QoS factors, and the other two are used to compare timeliness and reliability subfactors, respectively. The priorities of these elements are derived by the method mentioned earlier (step 4 [AHP], Figure 2.4). The global priorities of subfactors are achieved through multiplying priorities of subfactors by the global priorities of the corresponding parent (step 5 [AHP], Figure 2.4).

The performances of UMTS and WLAN are evaluated by deciding the differences from the ideal network condition ($S_0$). The network condition data are first normalized using the method introduced earlier (step 3 [GRA], Figure 2.4). There are only two situations, larger-and smaller-the better, in the network selection scenario; therefore, the reference $S_0$ can be defined as step 4 (GRA, Figure). Since the effect of each factor on the final goal is different, Eq. 1 can be modified as step 5 (GRA Figure 2.4)
where $s^*_{\text{umts/wlan}}(q)$ is the normalization of the $q$th UMTS QoS data or WLAN OoS data and $w_q$ is the $q$th OoS parameter weights.

The GRCs of UMTS and WLAN are compared in the handoff decision maker. The larger the coefficient, the more ability the network has to fulfill the requirements of user and service.

Therefore, the network alternative with the largest GRC is selected as the next network service provider.
3.1 K-Random Walks Algorithm

K-random walks algorithm is a kind of popular random search method that each node uses k walkers, when a intermediate node receives a random walker, it checks whether the is available or not[1]. If the object is not found it forwards the query to a random selected neighbor. Local flooding with k Independent random walks starts the search with flooding the queries until the discovering k neighbors which value of k is determinate before. Search is successful when one of the neighbors has the object otherwise each of k nodes begins an independent random walk.

It is advantageous of both flooding and random walk. The message complexity is small, if flooding occurs locally.

Its disadvantages is when more hops traveling walkers occurs high message overhead.

3.2 K Nearest Neighbor (KNN) Algorithm:

KNN is an algorithm based on machine learning where traditional methods such as inner product[6] , cosine and Euclidean are all based on the vector space model, which did not consider the full length of the sample. When the chosen K value is small , the similar recordings are put in the same cluster , while choosing the k value too big may cause misclassification of similar recordings. It is used to improve the classification accuracy of marginal data which fall outside the regions of representatives. Research using KNN is a subsequent based research which is related to
Data Reduction (DR)[6]. The experimental result shows that DR can obtain relatively higher reduction rate which preserving its classification accuracy. It is relatively slow in its basic form of model construction, since much time is spent in trying probable merge.

K Nearest-Neighbors classifier requires storing the whole training set and may be too costly when this set is large, many researchers have attempted to get rid of the redundancy of the training set to alleviate this problem [7]. KNN is simply based on the idea that "objects" that are "near" each other will also have similar characteristics.

Figure 3.1 KNN algorithm process 1

Figure 3.2 KNN algorithm process 2
To overcome the problems of low efficiency and dependency on $k$, a few representatives are needed from training dataset with some extra information to represent the whole training dataset. In the selection of each representative, use of the optimal but different $k$ decided by dataset itself to eliminate the dependency on $k$ without user’s intervention.

Using KNN in the research suffers from the quality of the composition as it is not sufficiently heterogeneous [6]. Because of heterogeneity in the respondents’ characteristics could affected with both the nature and the extent of the predictor.

### 3.3 Routing control in Unstructured P2P networks

A new method for unstructured P2P networks in order to overcome the issues of query routing overheads and congestions in peers through congestion control [5]. The proposed collaborative Q-Learning learns the networks parameter such as processing capacity, number of connections, number of resources in the peers along with their state of congestion. By this technique, peers are avoided to forward queries to the congested peers [5]. From the simulation result it shows that the
desired resources are located more quickly and also queries in the whole network are balanced, by proposed protocol. It is another efficient search by using links or state information.

3.4 Comparison between K Nearest Neighbor (KNN) and Q-Learning:

<table>
<thead>
<tr>
<th>K-Nearest Neighbor</th>
<th>Q-Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>KNN is an algorithm based on machine learning there are not many training parameters, the computational complexity is not high and the performance is satisfactory.</td>
<td>In order to identifying congested peers and avoid forwarding queries to them, learning method which is a method of reinforcement learning is applied to monitor the state of the peers in the network.</td>
</tr>
<tr>
<td>KNN algorithm firstly to select pre k sample when the similarity values are started in descending order then to determine the categories of test sample with class mapping method.</td>
<td>Since the distribution of the data can be random, the stored in a P2P network is spread across a large number of nodes. One of the most popular applications of P2P network is file sharing.</td>
</tr>
<tr>
<td>It gives another new method, the similarity summing algorithm based on shared nearest algorithm, specific details are found.</td>
<td>Q-Learning gives another algorithm which is known as Nash Stackelberg Q-learning, a hierarchical distributed teaching frame in heterogeneous cognitive network with the network as leader that aims at maximizing its utility and the mobiles as follows that have their individual objectives.</td>
</tr>
<tr>
<td>KNN is simply based on the idea that &quot;objects&quot; that are &quot;near&quot; each other will also have similar characteristics.</td>
<td>Q Learning focuses on each and every peer that are near and far from it.</td>
</tr>
<tr>
<td>Proposed simulation method:</td>
<td></td>
</tr>
<tr>
<td>---------------------------</td>
<td>--</td>
</tr>
<tr>
<td>Here we consider a simulation of model through which we can find the best peer in the file sharing purpose. The simulation results reveal the proposed network scheme can efficiently decide the</td>
<td></td>
</tr>
</tbody>
</table>

### 3.5 Proposed simulation method:

One of the most strengths of the Q-Learning is it is able to compare the expected utility of the available action without a model of the environment.

| KNN is one of the most simple and straight forward data mining techniques. | Q-Learning deals with continuous attributes, discrete attributes as well as dichotomous attributes. |
| KNN usually deals with continuous attributes however it also deals with discrete attributes. |  |

| Limitations: | Limitations: |
| Using KNN in the research suffers from the quality of the composition. | Agent learns action value function which is expected utility of acting in give state. So, it can restrict ability to learn. |
| Such type of algorithm is not sufficiently heterogeneous. |  |
| The limited heterogeneity in respondents’ characteristics could have affected with both the nature and the extent of the predictor variables. |  |
| The large scope of KNN and the complex multidisciplinary nature of the study is also a major challenge. |  |
tradeoff among user preference, service application and network connection. In addition, the priorities of options can be decided based on approximate comparisons among the QoS parameters instead of exact values of the QoS parameters in the heterogeneous system with only two network alternatives, which means simpler implementation. The simulation steps are described below.

**Step 1**

In step 1 only the signal from device A is sensed; therefore, device A is directly selected [8]. This is an example of a simple case with no selection process involved.

**Step 2**

In step 2 the user is trying to receive meeting video and speech components in the office. After confirming that device B is available for transmission and the user is estimated to be in the office for at least 5 min, the network selector begins to determine the optimum network using AHP and GRA [8]. It is clear that the performance of device B is closer to the optimal criteria. The decision maker then selects device B as service provider based on the comparison results. It is observed that the exact values of the QoS parameters become much less important after normalization. Hence, in the situation of two alternatives, we only need to know which alternative is higher or larger with respect to a certain QoS parameter without estimating the exact value. It significantly reduces the complexity of implementation.

It shows the weights of QoS factors and the corresponding selection decisions on the assumption that the user changes requirements for **security or cost**, and the requirements for throughput, timeliness, and reliability are fixed based on the service class.

When GRC of device is less than that of device B (i.e., the comparison result is negative), device B is selected; otherwise, device A is selected. It is observed that the selection result would change when the priority of security is increased to around 0.06 but cost does not play a key role during selection. Even though the user has no requirement on cost, device B is still
selected due to high bandwidth. The perceived quality has to be sacrificed for high security, however, which is an advantage of device A.

After the meeting, the user begins to transmit some files. Once the network selector discovers that the current service class changes to background class, it maps the service class into a series of QoS characteristics. The network detector then reevaluates the weights of QoS factors and subfactors. Given that the conditions of device A and device B are the same as estimated while transmitting the meeting components, the results show that device B is still the optimum option for the user, and additionally device B is more desirable in the scenario of providing background-class service. This is a more complex example than case 1.

The network selection scheme is executed twice. It happens once when a new network alternative is sensed, and again when the service class changes.

**Step 3**

In step 3, the signal from device B starts decaying when the user leaves the office. Device B is kept connected for as long as possible until RSS [8] from device B is detected as lower than the threshold for a period of time. Consequently, the remaining files must be transferred through device A.

**Step 4**

In step 4, three networks (device A, device B, and device C) are available to the user. The GRCs are calculated and then compared in the decision maker. Device C with the largest GRC is selected.

In this case, device C provides a little higher throughput, which is advantageous to background class service; the network selector however selects lower-throughput device C, which has the merits of higher reliability, higher security and lower cost. This example illustrates that the network selection mechanism is a trade-off among user preference, service application, and network condition.
In the above simulation steps, the user enjoys either real-time or non-real-time service during movement. The delay-sensitive network alternative is selected for real-time applications, and the high-throughput high-reliability network alternative is selected for non-real-time applications. It reveals that the proposed scheme balances more comprehensive QoS decision factors than the aforementioned papers, and efficiently makes a handoff decision more favorable for the user.

3.6 Results:

![Graph](image)

Figure 3.4: Network selection with decreased weight of cost

The prediction of different parameters has been showed in figure 3.4 and it is predicted that with the proposed method the quality increased around 2-3% for each feature.
Chapter 4
Conclusion

4.1 Conclusion

P2P networks have been witnessed in many applications in the past few decades. Through this network still suffer various limitations during load balancing and decentralized during resource locating approaches. In this paper, we developed a framework in an unstructured peer to peer system. Our goal is to increase the data availability and also increase the search performance. The main advantage of the Q-learning based implementation using hotspot network is reduction in number of hops visited by a search query. As a result, the network traffic has been reduced. Q-learning method not only learns from the network parameters as processing capacity, number of resources in the peers but also learns from their state of congestion. According to this method, queries are suggested to avoid being forwarded and balanced among the peers. Since the distribution of data can be random, the data stored in the P2P network is spread across a large number of nodes and the proposed algorithm will show that the desired resources are located more quickly and also queries in the whole framework are balanced and many consume network bandwidth when the node checks objects presence in the nodes listed in Q-table.

4.2 Future Work

In the future, I hope to implement the Q-learning model in code and then search for the best and most efficient peer finding network by comparing space and time complexities of the paths. During the process, there will be many network jammers found and these create problem so I hope to remove them using the Q-learning algorithm.
References


[7] G. Guo, H. Wang, D. Bell, Y. Bi and K. Greer. “KNN Model-Based Approach in Classification” School of Computing and Mathematics, University of Ulster Newtownabbey, BT37 0QB, Northern Ireland, UK.