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# GENERATION OF SUPERSONIC SOLITONS BY EARTHQUAKE EMISSION IN THE VAN ALLEN RADIATION BELT

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ABSTRACT

The parametric coupling between earthquake emitted circularly polarized electromagnetic radiation and ponderomotively driven ion-acoustic perturbations in the Van Allen Radiation Belt is considered. A cubic nonlinear Schrödinger equation for the modulated radiation envelope is derived, and then solved analytically. Supersonic solitons are found to be generated in the Van Allen Belt which can be a tool for the prediction of a massive earthquake to be followed.

**Key words:** Earthquake, ULF emission, Radiation Belt, Magnetic Bottle, Solitons.

## I. INTRODUCTION

An Italian experiment that aims to predict earthquakes by studying charged particles in space has begun on the International Space Station. Italian astronaut Roberto Vittori arrived at the space station aboard a Russian Soyuz spacecraft and will set up the experiment during his eight-day stay on the station. The experiment, called LAZIO- SIRAD (Low Altitude Zone Ionization Observatory--Self-indicating Instant Radiation Dosimeter) will run at least six months and test a hypothesis: *electromagnetic waves are released before an earthquake* [ 1].

Various theories explain the waves. One suggests that before the ground ruptures and shakes in a full- blown earthquake, the stresses that long been accumulating in the fault zone start to crush granite rock. This could excite electrons in the rock's crystal structure, which then emit electromagnetic radiation when they drop back to their normal energies.

According to the hypothesis underpinning LAZIO-SIRAD, the electromagnetic waves move towards Earth's surface, and the longest keep going into the atmosphere. Eventually they reach the Van Allen radiation belts, two concentric rings of charged particles trapped by Earth's magnetic fields. LAZIO-SIRAD will attempt to detect any earthquake related electromagnetic waves by measuring changes in the number of charged particles and the strength of magnetic fields in the Van Allen belts.

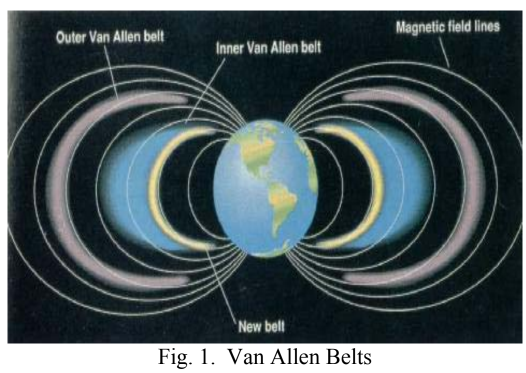



Fig. 1. Van Allen Belts

The Van Allen Radiation Belt is a toroid of energetic charged particles around earth's magnetic field. It was discovered by James Van Allen of University of IOWA by Explorer1 on January 31 1958.

It extends above the equator at an altitude of about

4,000 miles (6437 kilometers). This belt is populated by very energetic protons in the 10-100 MeV range (a byproduct of collisions of cosmic rays with atoms of the atmosphere). The cosmic radiation has a rather low intensity (comparable to starlight) and only by accumulating particles over the span of years does the inner belt reach its high intensity.

Recently [2], it is observed that a possible mechanism for the generation of electromagnetic waves in the lithosphere is to be identified with the micro-fracturing process of rocks. These micro-fractures in turn, involve the breaking of a large number of atomic bonds, i.e. the covalent or ionic bonds giving rise to the crystalline structure of the rocks. The formation of unbalanced charge distributions, of opposite sign, on the opposite sides of the micro-fractures is to be expected and, consequently, the onset of strong local electric fields. The existence of such fields was demonstrated in experiments, performed under vacuum, on the breaking of adhesion bonds. Potential differences reaching $10^4$ V were estimated from the spectrum of x –rays emissions with a flux of $10^9$ protons / $cm^2$.

Another mechanism has also been suggested which could be effective in the fracture of rocks which are poor conductors of electricity. The electric charges of opposite sign, created on opposite sides of micro-fractures, constitute electric dipoles which have lifetimes determined by the electrical relaxation time of the material. Such dipoles have a linear dimension corresponding to the width of the micro-fracture. This dimension can be modulated by the damped mechanical vibrations, originated by rupture of the atomic bonds, of its walls giving rise to dipole oscillations.

The very first space probe sent into orbit by America was in 1958. One of the devices it carried with it was one that searched for charged particles in the upper atmosphere. When the data were collected and reviewed under the direction of James A. Van Allen (University of Iowa), a region of high energy charged particles was detected. The charged particles in the Van Allen belts are trapped in the Earth's magnetic field. The force on the charged particles is normal to the magnetic field lines. These highly energetic charged particles actually move in helical paths around the magnetic field lines. As the charged particles move closer to the earth's magnetic poles, the force increases until the particle can no longer move any farther forward and gets repelled back down to repeat the process with the opposite pole.

If a charged particle has a velocity component parallel to a magnetic field line, it will, as mentioned earlier, travel in a helical path. The parallel component determines the pitch of the helix and the normal component determines the radius of the helix.

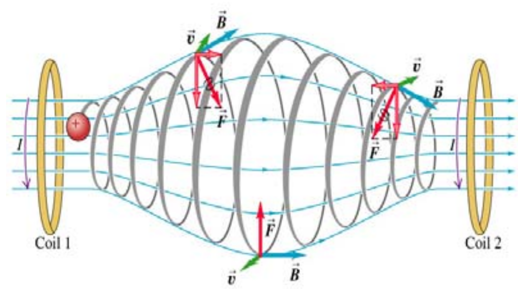



Fig. 2. Magnetic Bottle

The spacing between magnetic field lines (and any other field lines for that matter) indicates the strength of the field. The closer the field lines are, the stronger the field is. When a charged particle enters a non uniform magnetic field, such as the Earth's, it will start to oscillate back and forth. As the magnetic field lines get closer, the strength of the field increases. When the particle reaches a point where the field is too strong to continue forward, it gets "reflected" back. If said particle gets reflected at both ends of the magnetic field, it is said to be trapped in a magnetic bottle.

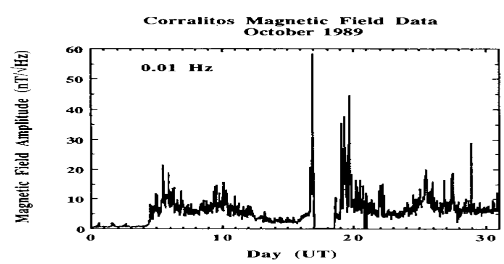



Fig. 3. Magnetic field anomaly. Loma Prieta earthquake, October 18, 1989, M=7.

Bäthe 1973 [3] explained the physics of seismic emissions. An empirical relation had been found between the magnitude of an earthquake and the total amount of energy released. The energy E, corresponding to a magnitude M from a body waves has been found to follow, to a first approximation, the law

Log E= 5.24 + 1.44 M

It is valid only for M >5. A magnitude increase of of one unit in the M-scale corresponds to an energy increase by 27.

For M= 7 , E ~ 2 x $10^{15}$ J. This energy accumulates as elastic wave associated with compression and deformation of rocks.This mechanical energy is then released in short time during which largest shocks are produced. Meloni et. al. 2001[ 4], Teisserye, 2002 [5]explain high noise level in VLF and ULF bands frequently observed before earthquake. This is due to dipole oscillations with the beginning of micro- fracturing process. Features of the emissions are: it has non-vanishing components for VLF region ( 30 KHz - 3 KHz ), ELF region ( 3 KHz - 3 Hz) ,ULF region ( < 3 Hz ). Experimental evidence of EME before the fracture and at the fracture of rocks are well documented in literature.[6,7]. Earthquake electromagnetic precursors were defined by Dea et. al. (1993)[ 8] as " Events with elevated broad-band signals in the .1 - 20 Hz region appearing several hours to several days in advance of moderate to large earthquake". The same authors have published recordings and data on increase of power spectra noise in the 1- 5 Hz that were taken one or two days before many moderate magnitude earthquakes in California. Thus the following scenario may be assumed:

Earthquake emissions are Ultra Low Frequency Electromagnetic Emissions (ULF EME). The ULF EME waves are generated in earthquake preparation zone several hours before the main shock. At a certain altitude the EME can propagate as Alfvén wave along geomagnetic field lines. It can resonantly interact with high energy particles trapped in the Van Allen belts. When ULF EME interacts with particles in the Radiation Belts, they may generate spiky pulses which are observed in space stations before a massive earthquake. Thus a massive earthquake may be predicted from space through plasma dynamics of Van Allen Radiation Belt.

## II. PLASMA DYNAMIC MODEL OF RADIATION BELT

The earth's magnetic field is considered to be a dipole with the components:

$$B_r = \frac{2\mu}{r^3}\cos\theta , \qquad (1)$$

$$B_\theta = \frac{\mu}{r^3}\sin\theta , \qquad (2)$$

$$B_\phi = 0 . \qquad (3)$$

The magnitude of the field is then

$$B = \frac{\mu}{r^3}\sqrt{1+3\cos^2\theta} . \qquad (4)$$

We consider the field of the Van Allen Belt at the equatorial region. Therefore, we consider

$$r = const. \quad \theta \approx \pi/2 \quad B \sim B_\theta \approx \frac{\mu}{r^3} = const.$$

where $\mu = B_0 R_E{}^3$ and $B_0$ - field at the equator. We calculate the plasma parameters in Van Allen Belt. Earth's magnetic field is in the order of $B \sim 0.3 \div .5G$. Therefore, the ion cyclotron frequency

$$\omega_{ci} = \frac{zeB}{m_i c} = 9.58\times10^3 \times z\times\eta^{-1}\times B rad/\sec \qquad (5)$$

$$= 9.58\times10^3 \times1\times1\times.5 rad/\sec$$

$$= 4.79\times10^3 rad/\sec.$$

where $\eta = \frac{m_i}{m_p}$ .

Therefore,

$$f_{ci} = \frac{\omega_{ci}}{2\pi} = 1.52\times10^3 \times z\times\eta^{-1}\times B Hz$$

$$= 1.52\times10^3 \times1\times1\times.5 Hz \qquad (6)$$

$$= 760 Hz.$$

Seismic Emission observed by Dea et al. (1993 ) recorded $f \sim .1 \div 20 Hz$. We see that earthquake electromagnetic emission frequency is much less than the ion cyclotron frequency i.e. $\omega << \omega_{ci}$ (ULF emission.)

## III. TWO FLUID MHD

We consider a uniform collisionless plasma consisting of ions and electrons (s=i,e) embedded in the magnetic field of the Radiation Belt. Then two fluid description of the plasma is given by the following system of equations:

$$\partial_t n_s + \nabla \cdot (n_s \vec{v}_s) = 0, \quad (7)$$

$$\partial_t \vec{v}_s + \vec{v}_s \cdot \nabla \vec{v}_s = \frac{q_s}{m_s}(\vec{E} + \frac{1}{c}\vec{v}_s \times \vec{B}) - \frac{T_s}{m_s}\nabla n_s, \quad (8)$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \quad (9)$$

$$\nabla \times \vec{B} = \sum_s \frac{4\pi}{c} q_s n_s \vec{v}_s + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}, \quad (10)$$

$$\nabla \cdot \vec{E} = \sum_s \frac{4\pi}{c} q_s n_s, \quad (11)$$

$$\nabla \cdot \vec{B} = 0. \quad (12)$$

where,

$q_s$ -is the charge, $n_s$ -is the density, $\vec{v}_s$ -is the velocity, $T_s$ -is the temperature of plasma species and $\vec{E}, \vec{B}$ -are the electric and magnetic fields, respectively.

## IV. LINEAR PERTURBATIONS

To derive the linear dispersion relation, we consider

$n_s = n_0 + n_{s1}e^{i\omega t}$, $\vec{v}_s = \vec{v}_{s1}e^{i\omega t}$, $\vec{E} = \vec{E}_1 e^{i\omega t}$, and $\vec{B} = \vec{B}_0 + \vec{B}_1 e^{i\omega t}$. (13)

with $\vec{B}_0 = \{B_{0r,} B_{0\theta,} 0\}$, given by the equations (1 ) and (2 ) respectively.

Considering $\tilde{v}_{s1} = v_{s1r} + iv_{s1\phi}$, $\tilde{E}_1 = E_{1r} + iE_{1\phi}$, with $\vec{k}$ parallel to $\hat{\theta}$ . We find

$$\tilde{v}_{s1} = \frac{-iq_s \tilde{E}_1}{m_s(\omega - \omega_{cs})}; \quad \omega_{cs} = \frac{q_s B_{0\theta}}{m_s c} \ . \quad (14)$$

Then the system of equations (7 ) – (12 ) yield the following wave equation

$$(\nabla^2 - \frac{\omega}{c^2}\sum_s \frac{\omega_{ps}{}^2}{\omega - \omega_{cs}})\tilde{E}_1 = 0, \quad (15)$$

with

$$\omega_{ps}{}^2 = \frac{4\pi n_0 q_s{}^2}{m_s}, \quad (16)$$

gives the general dispersion relation

$$\frac{k^2c^2}{\omega^2} = 1 - \sum_s \frac{\omega_{ps}{}^2}{\omega(\omega - \omega_{cs})} \ . \quad (17)$$

For ULF earthquake emission,

$\omega < \omega_{ci} << \omega_{ce}$, $\frac{v_A{}^2}{c^2} << 1$, where

$v_A = \frac{B_{0\theta}}{\sqrt{4\pi\rho}}$, is the Alfvén velocity with

$\rho = n_0(m_i + m_e)$.

Then from equation (17 ) we find the following dispersion relation for the electromagnetic wave in the Radiation Belt

$$k^2 v_A{}^2 = \frac{\omega^2 \omega_{ci}}{\omega_{ci} - \omega} \quad (18)$$

## V. NLS EQUATION

From the dispersion relation (18 ) we derive the group velocity and group dispersion of the radiation

$$v_g = \frac{2kv_A{}^2(\omega_{ci} - \omega)^2}{\omega\omega_{ci}(2\omega_{ci} - \omega)}, \quad (19)$$

$$\frac{1}{2}v_g{}' = \frac{\left[\omega(1 - \frac{2kv_g}{\omega_{ci} - \omega}) - \frac{k^2 v_g{}^2}{\omega}\right](\omega_{ci} - \omega)}{k^2(2\omega_{ci} - \omega)} \ . \quad (20)$$

The nonlinear interaction between electromagnetic emission and ion acoustic perturbations produces an electric field envelope, which obeys (Karpman and Washimi, 1976, 1977 [9,10], Shukla and Stenflo, 1984, 1985 [11,12])

$$i(\partial_t E_1 + \frac{v_g}{r}\frac{\partial E_1}{\partial \theta}) + \frac{1}{2}v_g{}' \frac{1}{r^2}\frac{\partial^2 E_1}{\partial \theta^2} - \Delta E_1 = 0, \quad (21)$$

where the nonlinear frequency shift

$$\Delta = \frac{v_g}{2kc^2}\sum_s \frac{\omega\omega_{ps}{}^2}{\omega - \omega_{cs}}\left[\begin{array}{l}\frac{\delta n_s}{n_0} + \frac{kv_{s\theta}\omega_{cs}}{\omega(\omega - \omega_{cs})} \\ -\frac{\omega}{(\omega - \omega_{cs})^3}\frac{q_s{}^2|E_1|^2}{2m_s{}^2c^2}\end{array}\right] . \quad (22)$$

For $\omega < \omega_{ci} << \omega_{ce}$,

$$\frac{\delta n_e}{n_0} \approx \frac{\delta n_i}{n_0} = N\,, \quad \Delta = -\frac{v_g \omega^2}{2kv_A^{\ 2}} N, \qquad (23)$$

The dynamics of the low frequency IA perturbations is governed by the ion continuity equation

$$\frac{\partial N}{\partial t} + \frac{1}{r}\frac{\partial v_{i\theta}}{\partial \theta} = 0, \qquad (24)$$

and the ion momentum equation

$$\frac{\partial v_{i\theta}}{\partial t} + \frac{c_s^{\ 2}}{r}\frac{\partial N}{\partial \theta} = f\,, \qquad (25)$$

where we have defined the sound velocity $c_s = [(T_e + \gamma_i T_i)/m_i] \equiv [T_0/m_i]^{1/2}$, Here $\gamma_i$ is the ion adiabatic index. The force appearing in the right hand side of Eq.(25 ) is the ponderomotive force associated with electromagnetic emission.

$$f = -\frac{q_i^{\ 2}}{2m_i\ (\omega - \omega\ )^2}\left[(1 - \frac{\omega_{ci}}{\omega})\frac{1}{r}\frac{\partial}{\partial \theta} - \frac{k\omega_{ci}}{\omega^2}\frac{\partial}{\partial t}\right]|E_1|^2. \qquad (26)$$

Combining Eqs. (24 ) and (25 ) we obtain the equation for the density perturbation

$$\frac{\partial^2 N}{\partial t^2} - \frac{c_s^{\ 2}}{r^2}\frac{\partial^2 N}{\partial \theta^2} = \frac{q_i^{\ 2}}{2m_i^{\ 2}(\omega - \omega_{ci})^2}\left[(1 - \frac{\omega_{ci}}{\omega})\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} - \frac{k\omega_{ci}}{\omega^2}\frac{1}{r}\frac{\partial^2}{\partial \theta \partial t}\right]|E_1|^2. \qquad (27)$$

Eqs. (21 ) and ( 27 ) via Eq.(22) form a closed system which describes the simultaneous evolution of the density perturbation $N$ and the electric field amplitude $E_1$.

## VI. STATIONARY ENVELOPE SOLITONS

We are looking for the slow dynamics of the emission envelope. Therefore we may move to frame which moves at the group velocity $v_g$. Thus we define the moving coordinates : $\xi = r\theta - v_g t, \quad \tau = t,$ Furthermore, we neglect $\partial^2{}_{\tau^2}, \partial_{\tau\xi}{}^2 -> 0$ considering the slow time variation of the parameters. As for seismic emission $\omega << \omega_{ci}$, we find

$$N = \frac{1}{2}\frac{q_i^{\ 2}}{m_i^{\ 2}\omega_{ci}\omega}\frac{1}{(v_g^{\ 2} - c_s^{\ 2})}(1 - \frac{kv_g}{\omega}) \times (|E_1|^2 - |E_{1\infty}|^2), \qquad (28)$$

and

$$\Delta = -Q\left(|E_1|^2 - |E_{1\infty}|^2\right), \qquad (29)$$

where

$$Q = \frac{v_g \omega^2}{kv_A^{\ 2}}\frac{1}{4}\frac{q_i^{\ 2}}{m_i^{\ 2}\omega_{ci}\omega}\frac{1}{(v_g^{\ 2} - c_s^{\ 2})}(1 - \frac{kv_g}{\omega}). \qquad (30)$$

The constant contribution to the right hand side of Eq. (28) may eliminated by introducing $E_1 \to E_1 e^{-iQ|E_{1\infty}|^2 \tau}$.

Then equation (21 ), in combination with Eq. (29 ), may now be cast in the reduced form of a Nonlinear Schrödinger Equation (NLSE)

$$i\partial_\tau E_1 + \frac{1}{2}v_g{}' \frac{\partial^2 E_1}{\partial \xi^2} + Q|E_1|^2 E_1 = 0. \qquad (31)$$

We seek the solution of Eq. (31) in the form

$$E_1(\xi,\tau) = E(\xi,\tau)e^{i\psi(\xi,\tau)}, \text{ with } \partial_\xi \psi = \kappa, \ \partial_\tau \psi = -\varpi,$$

Then the solution of the NLS equation can be written as (Mofiz ,1990, 1993[13,14], Mofiz et al. 1993[15]).

$$E(\xi,\tau) = E_0 \operatorname{sec}h\left[\left|\frac{Q}{2v_g{}'}\right|^{1/2} E_0 \xi\right] e^{-iQ|E_{1\infty}|^2 \tau} \qquad (32)$$

where

$$E_0 = \left|\frac{A}{Q}\right|^{1/2}, \text{ with } A = \kappa v_g - \varpi - \frac{1}{2}v_g{}' \kappa^2, \qquad (33)$$

and

$$\delta = \left| \frac{2 v_g{}'}{Q E_0{}^2} \right|^{1/2} . \qquad (34)$$

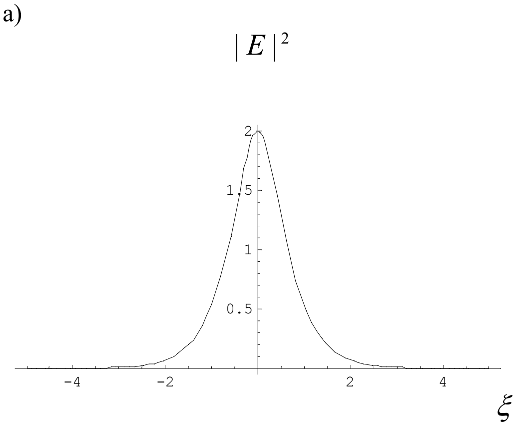
For $A \approx \kappa v_g$, the dimensionless wave amplitude is

$$\bar{E} = \frac{e E_0}{m_i \omega v_g} = 2\left[ \frac{\omega_{ci}}{\omega} \frac{\kappa k v_A{}^2}{\omega^2} \frac{1}{1 - \frac{v_g}{v_A}} \right]^{1/2} . \qquad (35)$$

For the case of earthquake emission, $\frac{\omega_{ci}}{\omega} >> 1$, and $v_g \sim v_A$, so we see that $\bar{E}$ may be quite large. Secondly, since $\frac{1}{2} v_g{}' \approx -\frac{v_A{}^2}{\omega_{ci}} = -\frac{B_0 c}{4\pi n_0 e} < 0$, and $\frac{1}{2} v_g{}' Q > 0$, for $v_g > c_s$, the solitonic pulse is supersonic.

Again $\delta \cong \left| \frac{2c}{e\kappa} \sqrt{\frac{m_i}{\pi n_0}} \right|^{1/2} \sim \frac{1}{n_0{}^{1/4}}$. which shows that for a low dense plasma, the soliton's pulse width may also very large. Thus we see that a low frequency earthquake emission may generate a supersonic wide pulse in the Van Allen Radiation Belt, which may be a precursor of a massive earthquake to be followed later.

a)




b)

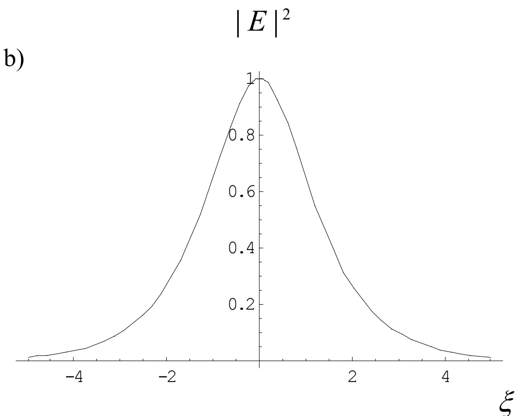



Figure 4 Supersonic solitons generated by ULF earthquake emission a) $\delta = .5$ . b) $\delta = 1$.

## VII. CONCLUSIONS

We have studied the parametric interaction between ultra-low frequency circularly polarized earthquake emission with the ponderomotively driven non-resonant ion-acoustic density perturbation in the Van Allen Radiation Belt. A cubic nonlinear Schrödinger equation is derived and solved numerically. It is shown that the emission is modulated and it produces supersonic solitary pulses. For a low dense plasma and the propagating speed closer to the Alfvén speed, the generated pulse amplitude and the width are sufficiently large to detect. The detection of such solitary pulses may be a precursor of a massive earthquake to be followed later.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