BRAC University Journal, vol .VI, no. 2, 2009, pp 11-20

COMBINED EFFECTS OF RADIATION AND HEAT GENERATION ON MHD NATURAL CONVECTION FLOW ALONG A VERTICAL FLAT PLATE IN PRESENCE OF HEAT CONDUCTION

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ABSTRACT

Study of the effects of radiation and heat generation on MHD natural convection flow of an incompressible viscous electrically conducting fluid along a vertically placed flat plate in presence of heat conduction is considered. The governing equations of the flow are transformed into dimensionless form with appropriate transformations and then solved using the implicit finite difference method with Keller-Box scheme. The resulting numerical solutions of transformed governing equations are presented graphically in terms of velocity profile, temperature distribution, local shear stress, local heat transfer rate and surface temperature and the effects of magnetic parameter (*M*), radiation parameter (*R*), Prandtl number (*Pr*) and heat generation parameter (*Q*) on the flow and the graphs are discussed.

Key words: Radiation, heat generation, MHD, finite difference method and vertical flat plate.

Nomenclature

b	Plate thickness	T_b	Temperature at outside surface of the plate
C_{fx}	Local skin friction coefficient	T_f	Temperature of the fluid
C_p	Specific heat at constant pressure	T_w	Average temperature of porous plate
f	Dimensionless stream function	T_∞	Temperature of the ambient fluid
g	Acceleration due to gravity	$\bar{u}, \bar{v}$	Velocity components
G_r	Grash of number	u, v	Dimensionless velocity components
h	Dimensionless temperature	$\bar{x}, \bar{y}$	Cartesian co-ordinates
H_0	Strength of magnetic field	x, y	Dimensionless Cartesian co-ordinate
k_f, k_s	Fluid and solid thermal conductivities	β	Coefficient of thermal expansion
l	Length of the plate	η	Dimensionless similarity variable
M	Magnetic parameter	θ	Dimensionless temperature
N_{ux}	Local Nusselt number	μ	Viscosity of the fluid
p	Conjugate conduction parameter	ν	Kinematic viscosity
Pr	Prandtl number	ρ	Density of the fluid
q_w	Heat flux	σ	Electrical conductivity
Q	Heat generation parameter	τ_w	Shearing stress
R	Radiation parameter	ψ	Stream function

I. INTRODUCTION

Radiative heat transfer on natural convection flow of an incompressible, viscous and electrically conducting fluid in presence of transverse magnetic field plays important roles in nuclear power plants, cooling of transmission lines and electric transformer etc. Also, from technological point of view, MHD natural convection flow has significant applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. Considering of its importance, these flow have been studied several research groups [1-3]. As the engineering processes closely related with temperature, accordingly radiation heat transfer has significant influence on engineering. Due to its wide applications in space technology such as space flights, aerodynamic rockets, propulsion systems, plasma physics, spacecraft re-entry aerodynamics and many researchers studied the effect of radiation on MHD free convection flow. Takhar and Soundalgekar [4] studied the effect of radiation on MHD free convection flow of a gas past a sami-infinite vertical plate using the Cogley-vincenti-Giles equilibrium model . The problem of natural convection-radiation interaction on boundary layer flow with Rosscland diffusion approximation along a vertical thin cylinder has been investigated by Hossain and Alim [5]. Radiation effect on free convection flow of fluid from a porous vertical plate was studied by Hossain et al. [6]. Thermal radiation and buoyancy effects on MHD free convection heat generating flow over an accelerating permeable surface with temperature- dependent viscosity studied by Seddeek [7]. Abdel-naby et al. [8] studied the radiation effects on MHD unsteady free convection flow over a vertical plate with variable surface temperature.

The study of heat generation in moving fluids is important in problems dealing with the chemical reactions and those concerned with dissociating fluids such as heat generation are resistance heating in wires, exothermic chemical reactions in a solid and nuclear reactions in nuclear fuel rods where electrical, chemical and nuclear energies are converted to heat. Experimental and theoretical works on heat generation effect have been done extensively [9-13]. But, the effect of radiation and heat generation under the process of steady natural convection flow is studied less as found in literature survey. Therefore, we consider the study is to the effect of radiation and heat generation on MHD natural convection flow of an incompressible, viscous and electrically conducting fluid along a vertical flat plate under the influence of transverse magnetic field. The governing partial differential equations are reduced to locally non-similar partial differential forms by using appropriate transformations. The transformed boundary layer equations are solved numerically adopting implicit finite difference method together with Keller Box Scheme technique [14, 15]. Here, the assumption is focused on the evaluation of the surface shear stress in terms of local skin friction and the rate of heat transfer in terms of local Nusselt number, velocity profiles and temperature distribution for some selected values of parameters consisting magnetic parameter *M*, radiation parameter *R*, Prandtl number *Pr* and heat generation parameter *Q*.

II. MATHEMATICAL FORMULATION

We consider a steady, laminar, incompressible, viscous and electrically conducting fluid along a vertical flat plate of length l and thickness b . The plate temperature T_b is grater than ambient temperature T_∞ maintained constant at the outer surface of the plate and uniform magnetic field of strength H_0 is imposed along the $\bar{y}$ -axis. The flow configuration and the coordinates system are shown in Fig. 1.

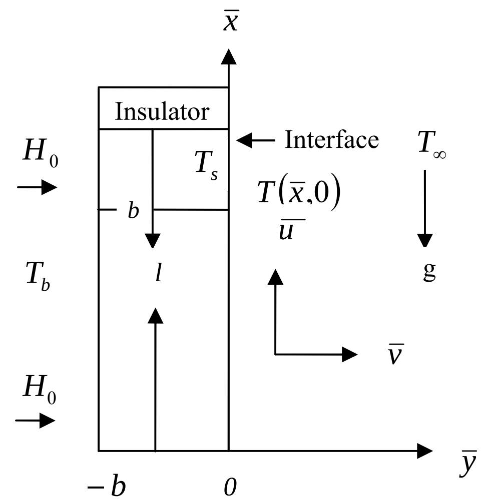


Fig. 1. Physical model and coordinate system.

The governing equation of such flow under the Bousinesq approximation we can be expressed within the usual boundary layer as [8, 13]:

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\nu\frac{\partial^2 \bar{u}}{\partial \bar{y}}+g\beta(T_f-T_\infty)-\frac{\sigma H^2{}_0\bar{u}}{\rho}, \tag{2}$$

$$\bar{u}\frac{\partial T_f}{\partial \bar{x}}+\bar{v}\frac{\partial T_f}{\partial \bar{y}}=\frac{k_f}{\rho C_p}\frac{\partial^2 T_f}{\partial \bar{y}^2}-4\Gamma(T_f-T_b)+\frac{Q_0}{\rho C_p}(T_f-T_\infty) \tag{3}$$

where $\Gamma=\int_0^\infty K_{\lambda w}\left(\frac{\partial e_{b\lambda}}{\partial T}\right)_w d\lambda$ and $K_{\lambda w}=K_\lambda(T_w)$ is the mean absorption coefficient $e_{b\lambda}$ is Plank's function and T is the temperature of the fluid in the boundary layer. Where kinematics viscosity ν, Thermal expression co-efficient β, Electrical conductivity σ, C_p is the specific heat due to constant pressure. The boundary conditions are:

$$\left.\begin{aligned} &\bar{u}=\bar{v}=0,\quad T_f=T(\bar{x},0),\\ &\frac{\partial T_f}{\partial \bar{y}}=\frac{k_s}{b k_f}(T_f-T_b)\, at\ \bar{y}=0, \bar{x}>0\\ &\bar{u}\to 0, T_f\to T_\infty\ at\ \bar{y}\to\infty, \bar{x}>0 \end{aligned}\right\} \tag{4}$$

We observe that the equations (1) to (3) together with the boundary conditions (4) are nonlinear partial differential equations. Now we introduce the following dimensionless dependent and independent variables:

$$\left.\begin{aligned} &x=\frac{\bar{x}}{l},\ y=\frac{\bar{y}}{l},\ u=\frac{\bar{u}\,l}{\nu}Gr^{-1/2},\\ &v=\frac{\bar{v}\,l}{\nu}Gr^{-1/4}, \theta=\frac{T-T_\infty}{T_b-T_\infty}\\ &Gr=\frac{g\beta l^3(T_b-T_\infty)}{\nu^2},\\ &M=\frac{\sigma H_0 l^2}{\mu}Gr^{-1/2},\\ &R=\frac{4\Gamma l^2}{\nu}Gr^{-1/2} \end{aligned}\right\} \tag{5}$$

where $\nu=(\mu/\rho)$ is the kinematic viscosity, Gr is the Grashof number and θ is the non-dimensional temperature. Then equations (1) to (3) can be written in dimensionless form as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+Mu=\frac{\partial^2 u}{\partial y^2}+\theta, \tag{7}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}-R(\theta-1)+Q\theta. \tag{8}$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} &u=v=0,\ \theta-1=p\frac{\partial\theta}{\partial y} && at\ \ y=0, x>0\\ &u\to 0,\ \theta\to 0 && at\ \ y\to\infty, x>0 \end{aligned}\right\} \tag{9}$$

In the above, $M=(\sigma H_0{}^2 l^2/\mu)Gr^{-1/2}$ is the magnetic parameter, $R=(4\Gamma l^2/\nu)Gr^{-1/2}$ is the radiation parameter, $\Pr=(\mu C_p/k_f)$ is the Prandtl number and $Q=(Q_0 l^2/\mu C_p)Gr^{-1/2}$ is the heat generation parameter and $p=(k_f/k_s)(b/l)Gr^{1/4}$ is a conjugate conduction parameter. The value of the conjugate conduction parameter *p* depends on $(b/l),(k_f/k_s)$ and Gr but each of which depends on the types of considered fluid and the solid. Therefore in different cases *p* is different but not always a small number. In the present analysis we have taken *p*=1.The stream function and similarity variable and the dimensionless temperature are considered in the following form to solve the equations (7) and (8) and for the boundary conditions described in equation (9):

$$\begin{aligned} &\psi=x^{4/5}(1+x)^{-1/20}f(x,\eta),\\ &\eta=yx^{-1/5}(1+x)^{-1/20},\\ &\theta=x^{1/5}(1+x)^{-1/5}h(x,\eta), \end{aligned} \tag{10}$$

where ψ is the dimensionless stream function which is related to the velocity components such as $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ and $h(x,\eta)$ is a dimensionless temperature. By substituting equation (10) in equations (7) and (8) and the boundary condition (9) we obtain the transformed equations:

$$f''' + \frac{16+15x}{20(1+x)} f f'' - \frac{6+5x}{10(1+x)} f'^2 - Mx^{2/5}$$
$$(1+x)^{1/10} f' + h = x\left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x}\right), \qquad (11)$$

$$\frac{1}{\Pr} h'' + \frac{16+15x}{20(1+x)} f h' - \frac{1}{5(1+x)} f' h - Rx^{2/5}(1+x)^{1/10} h$$
$$+ Rx^{1/5}(1+x)^{3/10} + Qx^{2/5}(1+x)^{1/10} h = x\left(f' \frac{\partial h}{\partial x} - h' \frac{\partial f}{\partial x}\right) \qquad .(12)$$

and the boundary conditions are

$$\left.\begin{aligned} & f(x,0) = f'(x,0) = 0, h'(x,0) = -(1+x)^{1/4} \\ & + x^{1/5}(1+x)^{1/20} h(x,0) \ at\, y = 0 \\ & f'(x,\infty) \to 0, h(x,\infty) \to 0\, at\, y \to \infty \end{aligned}\right\}. \qquad (13)$$

The set of equations (11) and (12) together with the boundary condition (13) are solved numerically by applying implicit finite difference method with Keller-Box (1978) Scheme. In practical point of view, it is important to calculate the values of the rate of heat transfer and the skin friction co-efficient. This can be written in the dimensionless form as

$$\left.\begin{aligned} C_f &= \left(Gr^{-3/4} l^2 / \mu \nu\right) \tau_w \quad and \\ N_u &= \left(Gr^{-1/4} / k_f (T_b - T_\infty)\right) q_w \end{aligned}\right\}, \qquad (14)$$

where $\tau_w = \mu \left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}$ and $q_w = -k_f \left(\frac{\partial T_f}{\partial \bar{y}}\right)_{\bar{y}=0}$ are t

shearing stress and the heat flux

Thus the local skin friction co-efficient and rate of he transfer are:

$$C_{fx} = x^{2/5}(1+x)^{-3/20} f''(x,0) \qquad (15)$$

$$N_{ux} = -(1+x)^{-1/4} h'(x,0) \qquad (16)$$

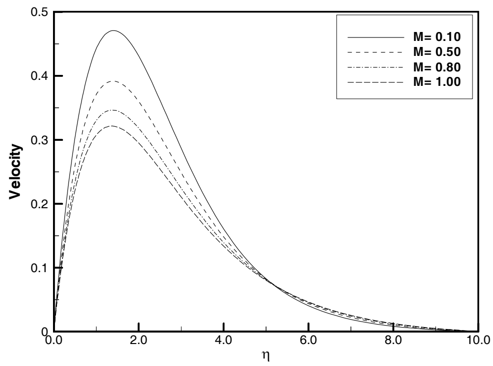
The numerical value of the surface temperature distribution are obtained from the relation

$$\theta(x,0) = x^{1/5}(1+x)^{-1/5} h(x,0) \qquad (17)$$

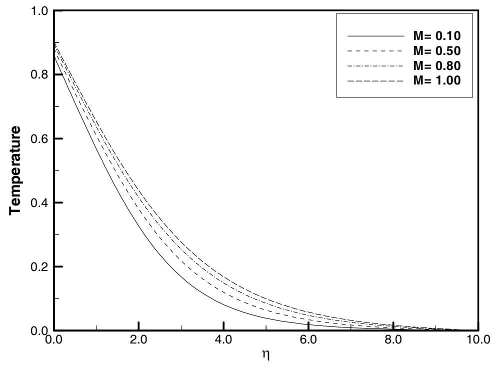
The numerical results obtained for the velocity profiles and temperature distributions for various values of Prandtl number, magnetic parameter, radiation parameter and heat generation parameter are discussed in the following sections.

III. RESULTS AND DISCUSSION

Here we discuss graphically the numerical results obtained from equation (11) and (12) together the boundary condition (13) using the mentioned method and also observed that radiation and heat generation dose effects in the flow region. We focus our attention on the effect of magnetic parameter, Prandtl number, radiation parameter and heat generation parameter on the velocity and temperature field and also the surface shear stress in terms of local skin friction coefficients and heat transfer rate in terms of Nusselt number with in the flow region.



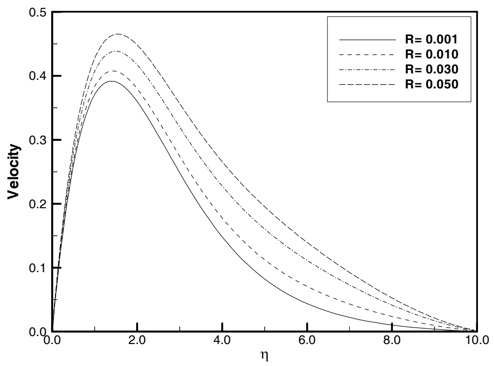
2(a)



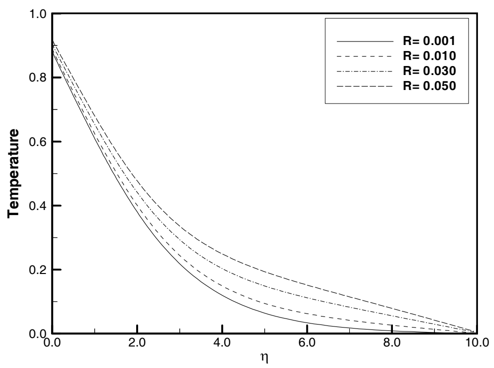
2(b)

Fig.2: (a) Variation of velocity and (b) variation of temperature against η for varying of M with Pr=0.733, R=0.001 and Q=0.01.

The numerical results of velocity and temperature for different values of magnetic parameter M while Pr = 0.733, R = 0.001 and Q = 0.01 are illustrated in Fig. 2(a) and Fig. 2(b), respectively. Fig.2 (a) shows that the velocity profiles decreases with the increase of magnetic parameter due to interaction of the applied magnetic field and flowing fluid particle, which produce Lorentz force that oppose the motion of the fluid. Moreover, for each value of *M*, the velocity is zero at the boundary wall and increase to the peak velocity as η increases and then turn to decrease and finally approach to zero. Furthermore, we have seen that the velocity profiles meet together after certain position of η and cross the side. This is because, the gradient of decreasing of velocity increases with the increasing of magnetic parameter. In Fig. 2 (b), it can be seen that the temperature increases within the boundary layer for the increasing values of magnetic parameter *M* due to the interaction. Moreover, the temperature decreases monotonically with increasing of η for a particular value of *M*. The maximum values of the temperature are 0.8603, 0.8812, 0.8949 and 0.9033 for *M* =0.10, 0.50, 0.80 and 1.00, respectively. Each of which occurs at the surface of the plate.



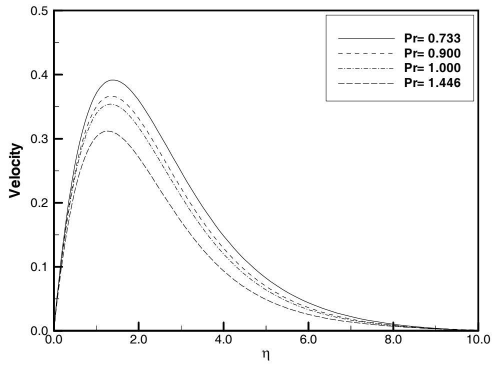
3(a)



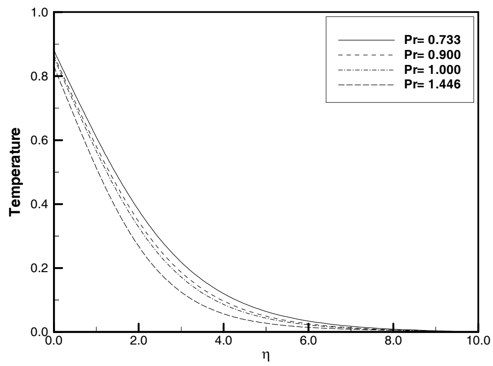
3(b)

Fig.3: (a) Variation of velocity and (b) variation of temperature against η for varying of *R* with *Pr*=0.733, *M*=0.50 and *Q*=0.01.

The effect of radiation parameter on the velocity and the temperature distributions together with a certain value of *Pr*, *M* and *Q* is presented in Fig. 3 (a) and Fig. 3 (b), respectively. Fluid absorbed heat while radiation imitates from the heated plate, as a result the motion and the temperature of the fluid increases with in the flow region. That's why the velocity and the temperature increase with the increasing of *R*. It means that the velocity boundary layer and the thermal boundary layer thickness increase for large value of *R*. From Fig. 3 (a) and also numerical values we have seen that the position of the peak velocity moves toward the boundary layer for the increasing *R*.



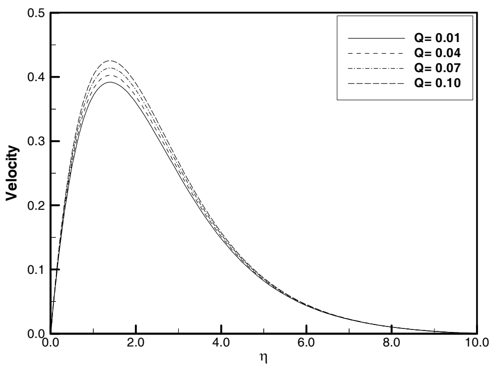
4(a)



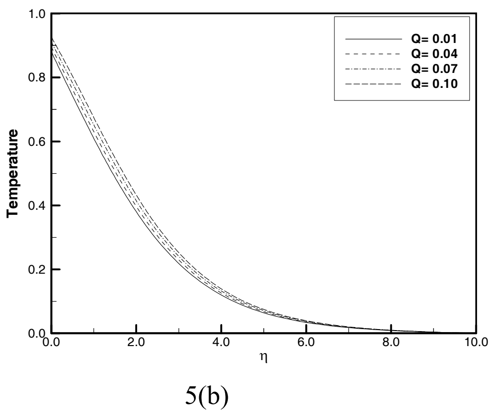
4(b)

Fig.4: (a) Variation of velocity and (b) variation of temperature against η for varying of *Pr* with *M*=0.50, *R*=0.001 and *Q*=0.01.

The variations of velocity profile and temperature distribution for different values of Prandtl number *Pr* with *M* = 0.50, *R* = 0.001 and *Q* = 0.01 shown in Fig. 4(a) and Fig. 4 (b), respectively. The increasing values of *Pr* increase viscosity of the fluid. Viscosity increase means that the density of the fluid increase, which results fluid does not move freely. It can be seen that the velocity decreases gradually and the peak velocity moves towards the interface for the increasing *Pr*. Moreover, the velocity is zero at the wall and increases to the peak as η increases and finally approaches to zero. These are expected behavior because it supports the no-slip condition at the wall and the fluid motion outside the boundary layer. Fig. 4 (b) shows that the temperature distribution over the whole boundary layer decreases due to the increase of *Pr*. It agrees the physical fact that the temperature at the solid fluid interface is reduced because, temperature at the plate considered constant. As a result, the thermal boundary layer thickness as well as velocity boundary layer decreases with the increasing of *Pr*.



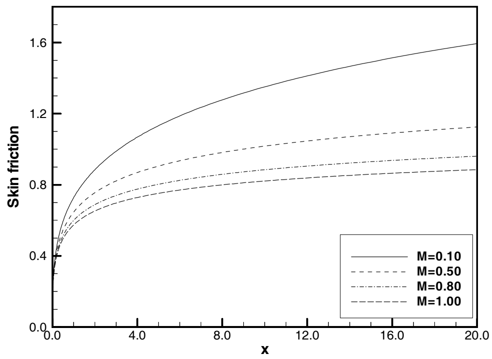
5(a)



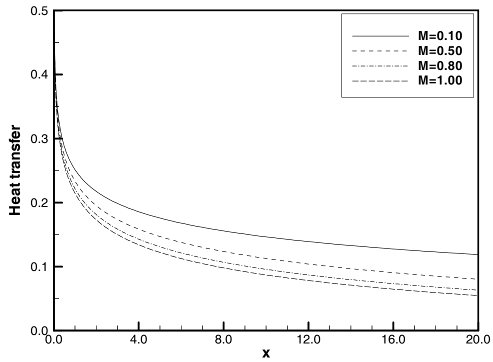
5(b)

Fig.5: (a) Variation of velocity and (b) variation of temperature against η for varying of *Q* with *Pr*=0.733, *M*=0.50 and *R*=0.001.

Fig. 5 (a) and Fig. 5(b), respectively depict the numerical results of the velocity and the temperature for some values of *Q* whereas *Pr* = 0.733, *M* = 0.50 and *R* =0.001. From Fig. 5(a) we conclude that the velocity profiles increases slightly due to the increasing value of heat generation parameter. This is because; increased value of *Q* produces more heat in the solid which increase the motion of the fluid. We have also seen that near the surface of the plate the velocity increases to maximum with increase of heat generation parameter *Q* then after the peak position start to decrease and finally approaches to zero. On the other hand from Fig. 5(b), we observed that the same result holds for temperature distributions within the boundary layer due to increasing of *Q*.



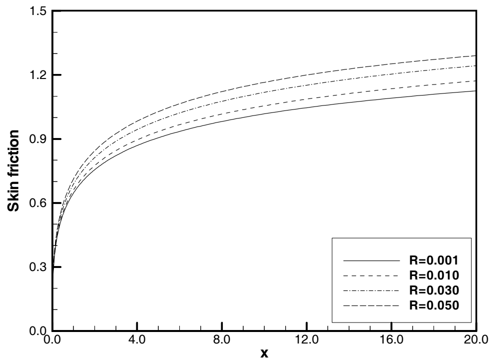
6(a)



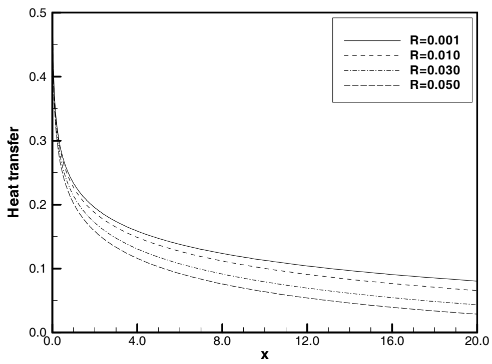
6(b)

Fig.6:(a) variation of skin friction and (b)variation of heat transfer against x for varying of *M* with *Pr*=0.733, *R*=0.001 and *Q*=0.01.

Fig. 6(a) and Fig. 6(b), reveal that the skin friction coefficient and the rate of heat transfer for some values of *M* with *Pr* = 0.733, *R* = 0.001and *Q* =0.01.The velocity decreases as shown in the Fig. 2(a), due to the increasing *M*. Accordingly, the skin friction on the plate decreases as observed in Fig. 6(a). But the temperature within the boundary layer increases (Fig. 2(b)) for the increasing *M*. As a result, the heat transfer rate from the plate to fluid decreases as shown in figure 6(b).



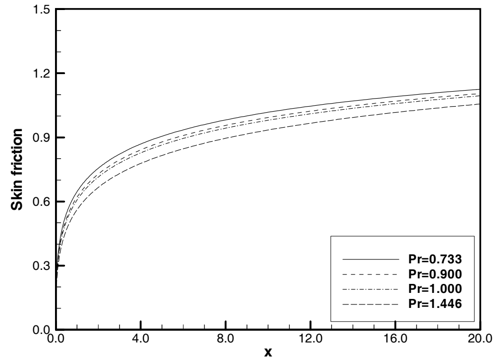
7(a)



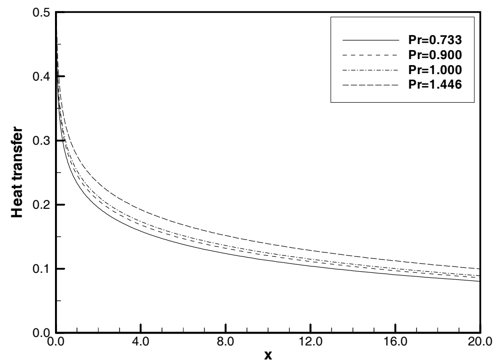
7(b)

Fig.7:(a) variation of skin friction and (b)var -iation of heat transfer against x for varying of *R* with *Pr*=0.733, *M*=0.50 and *Q*=0.01.

The variation of the local skin friction coefficient C_{fx} and local rate of heat transfer N_{ux} for different values of *R* associated with $Pr = 0.733$, $M = 0.50$ and $Q = 0.01$ are illustrated in Fig. 7(a) and Fig. 7(b). Increased value of *R* accelerate the fluid motion as mentioned in Fig. 3(a) and increases the shear stress at the wall, for which local skin friction increase with the increasing of *R*. This phenomenon demonstrated in Fig. 7(a). Similarly increased value of the radiation parameter increases the temperature (Fig. 3(b)) which after decrease the rate of heat transfers along the x-direction.



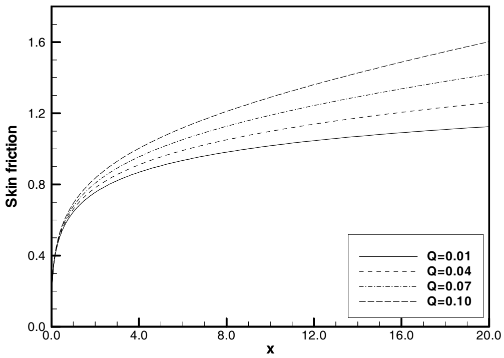
8(a)



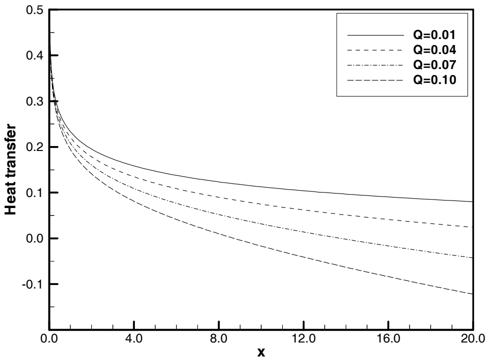
8(b)

Fig.8:(a) variation of skin friction and (b)var -iation of heat transfer against x for varying of *Pr* with *M*=0.50, *R*=0.001 and *Q*=0.01.

The effects of Prandtl number on the skin friction C_{fx} and heat transfer rate N_{ux} with the increasing of axial distance x for the fixed value of magnetic parameter, radiation parameter and heat generation parameter are shown in Fig. 8 (a) and Fig. 8 (b), respectively. The values of *Pr* are proportional to the viscosity of the fluid. So for the increase values of *Pr* the skin friction decreases on the plate which is shown in Fig. 8(a). For a particular value of *Pr* the local skin friction coefficient increases monotonically due to the increasing of *x*. From Fig. 8(b), it is observed that heat transfer rate increases due to increase of *Pr*. Furthermore for a particular value of *Pr* the local heat transfer rate decreases monotonically due to the increasing of *x*.



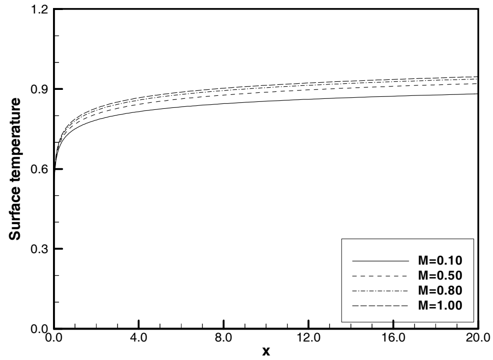
9(a)



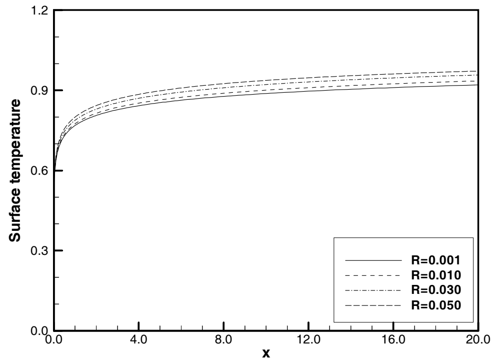
9(b)

Fig.9:(a) variation of skin friction and (b)var -iation of heat transfer against x for varying of *Q* with *Pr*=0.733, *M*=0.50 and *R*=0.001.

Fig. 9(a) and Fig. 9(b) illustrates the local skin friction coefficients and rate of heat transfer for different values of *Q* against x with controlling parameter Prandtl number *Pr* = 0.733, radiation parameter *R* = 0.001 and magnetic parameter *M* = 0.50. It can be seen that an increase heat generation parameter *Q* increase the fluid velocity within the boundary layer that shown in Fig.5 (a). So, the corresponding skin friction coefficients increase with the increasing of *Q*. The opposite result is observed for heat transfer distribution along with the increasing of heat generation parameter.



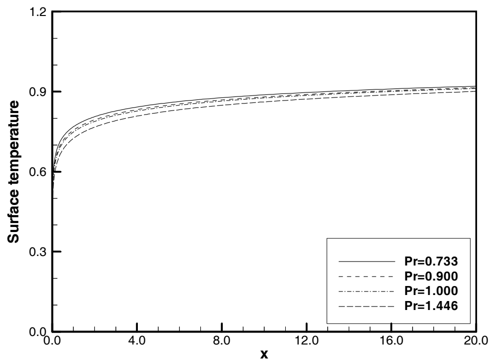
10(a)



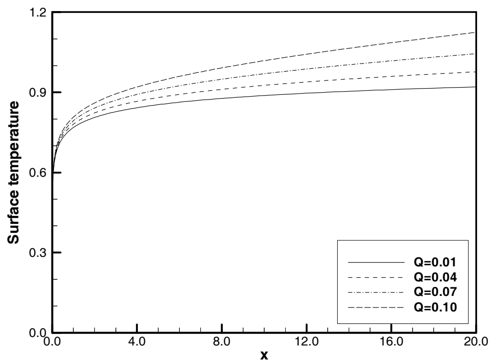
10(b)

Fig.10:(a) Variation of surface temperature against x for varying of *M* and (b) variation of surface temperature against x for varying of *R*.

The influence of magnetic parameter *M*, radiation parameter *R* and heat generation parameter *Q* on surface temperature are depicted in Fig. 10(a), Fig. 10 (b) and 10(d) respectively. It is noted that the surface temperature increase due to the increase value of *M*, *R* and *Q* along the x direction. This is to be expected because the thermal boundary layer thickness rises for the increasing of *M*, *R* and *Q* as observed in Fig.2 (b), Fig. 3(b) and 5(b), respectively.



10(c)



10(d)

Fig.10:(c) Variation of surface temperature against x for varying of *Pr* and (d) variation of surface temperature against x for varying of *Q*.

The temperature within the boundary layer decreases for increasing *Pr* as illustrated in Fig. 4(b) which results a decrease of interfacial temperature as observed in Fig.10 (c). It can be seen that the interfacial temperature increases monotonically for a selected value of *Pr* with the increasing of x along the upward direction.

IV. CONCLUSION

In this analysis the effect of radiation and heat generation on magnetohydrodynamic (MHD) natural convection flow along a vertical flat plate in presence of heat conduction has been investigated for some selected values of pertinent parameters including magnetic parameter, radiation parameter, Prandtl number and heat generation parameter. From the present investigation, it may be concluded that the velocity of the fluid and the skin friction at the interface decrease with the increasing magnetic parameter and Prandtl number while they increase with the increasing of radiation parameter and heat generation parameter .The temperature of the fluid and also the surface temperature increases with the increasing magnetic parameter, radiation parameter and heat generation parameter but decrease for increasing Prandtl number. Moreover, the rate of heat transfer decreases with the increasing of magnetic parameter, radiation parameter and heat generation parameter but increases for increasing of Prandtl number.

REFERENCES

[1] E. M. Sparrow, R. D. Cess: "Effect of magnetic field on free convection heat transfer", *Int. J. Heat and Mass Transfer,* **3**, pp.267-274. (1961)

[2] K. R. Sing, T. G. Cowling: "Thermal conduction in magnetohydrodynamics", *J. Mech. Appl. Math.,* **16**, pp.1-5. (1963)

[3] N. Riley: "Magnetohydrodynamic free convection", *J. Fluid Mech.***18**, pp-577-586.(1964)

[4] H. S. Takhar, V.M. Soundalgekar: "Radiation effects on MHD free convection flow of a gas past a simi- infinite vertical plate", *Applied Scientific Research,* **36**, pp. 163-171. (1980)

[5] M. A. Hossain, M.A. Alim: "Natural convection-radiation interaction on boundary layer flow along a thin vertical cylinder", *J. of Heat and Mass Transfer* **32**, pp. 515-520.(1997)

[6] M. A. Hossain, M.A. Alim, D.A.S. Ress: "The effect of radiation on free convection from a porous vertical plate", *Int. J. of Heat and Mass Transfer*, **42**, pp. 181-191.(1999)

[7] M. A. Seddee: "Thermal radiation buoyancy effects on MHD free convective heat generating flow over an accelerating permeable surface with temperature dependent viscosity", *Canadian Journal of Phys.*, **79**, pp. 725-732.(2001)

[8] Abd El-Naby, Elsayed M. E.Elbarbaryand Nader Y. Abdelazem: "Finite difference solution of radiation effect on MHD

unsteady free convection flow over a vertical plate variable surface temperature", Journal of Applied Mathematics , **2**, pp. 65-86.(2003)

[9] M. M. Molla, M. A. Taher, M. M. K. Chowdhury, M. A Hossain: "Magneto-hydrodynamic natural convection flow on a sphere in presence of heat generation effect", *J. of nonlinear analysis modeling and control,* **10**, pp-349-363.(2005)

[10] M. M. Molla, M. A. Taher, M. A. Hossain: "Magnetohydrodynamic natural convection flow on a sphere with uniform heat flux in presence of heat generation", *Acta Mech*.**186**, pp-75-86 (2006)

[11] M. M. Alam, M. A. Aim, M. M .K. Chowdhury: "Viscous dissipation on MHD natural convection flow over a sphere in presence of heat generation", *J. of Nonlinear Analysis, Modelling and control,***12**, pp. 447-459. (2007)

[12] A. A. Mamun, Z. R. Chowdhury, M.A.Azim, M.A. Maleque: "Conjugate heat transfer for a vertical flat plate with heat generation effect", *J. of nonlinear analysis modeling and control,***13**,pp-1-11(2008)

[13] A. A. Mamun, Z. R. Chowdhury, M. A. Azim, M. M. Molla: "MHD-conjugate heat transfer analysis for a vertical flat plate in presence of viscous dissipation and heat generation", *Int. communications in heat and mass transfer,***35**,pp-1275-1280.(2008)

[14] H. B. Keller: "Numerical methods in the boundary layer theory", *Annu. Rev. Fluid Mech*., **10**, pp. 417-433. (1978)

[15] T. Cebeci, P. Bradshow: *Physical computational aspects of convective heat transfer*, Springer, New York. (1984)