Study of Soliton Propagation Inside Optical Fiber for ultra-short pulse

A Thesis Presented

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Declaration

We do hereby declare that the thesis titled “Study of Soliton Propagation Inside Optical Fiber for ultra-short pulse” is submitted to the Department of Electrical and Electronic Engineering of BRAC University in partial fulfillment of the Bachelor of Science in Electrical and Electronic Engineering. This is our original work and was not submitted elsewhere for the award of any other degree or any other publication.

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Thesis Abstract

We will study the pulse propagation in optical fiber. The effect of non-linearity and dispersion of the pulse propagation causes temporal spreading of pulse and it can be compensated by non-linear effect. When the effects are combined they can generate stable, undistorted pulses called soliton over long distances. Numerical analysis can be carried out using different types of method including Fourier series analysis technique, Split Step Fourier Method. An extensive study has been carried out to find the appropriate analysis method. Different types of pulse was used including Hyperbolic secant, Gaussian and Super-Gaussian pulses.
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CHAPTER 1:

Introduction

With the advance of the information technology and the explosive growth of the graphics driven World Wide Web, the demand for high bit rate communication systems has been raising exponentially. In recent time, the intense desire to exchange the information technology has refueled extensive research efforts worldwide to develop and improve all optical fiber based transmission systems. For this reason, optical soliton pulse which is the subject of this thesis has involved.

Optical solitons are pulse of light which are considered the natural mode of an optical fiber. Solitons are able to propagate for long distance in optical fiber, because it can maintain its shapes when propagating through fibers. We are just at the beginning of what will likely be known as the photonics. One of the keys of success is ensuring photonics revolution and use the optical solitons in fiber optic communications system. Solitons are a special type of optical pulses that can propagate through an optical fiber undistorted for tens of thousands of km. the key of solitons formation is the careful balance of the opposing forces of dispersion and self-phase modulation. In this paper, we will discuss the origin of optical solitons starting with the basic concepts of optical pulse propagation. In this paper, we will discuss about theory of soliton, pulse dispersion, self phase modulation and nonlinear Schrödinger equation (NLSE) for pulse propagation through optical fiber. The next chapter contains the analyzing method, split step fourier method. Accordingly, in next two chapter, we study different pulses and implement them using NLSE. In the last part, we will show our research result regarding different pulses to generate soliton.
CHAPTER 2:

Literature Review

In this chapter basic theory of soliton will be discussed. Generation of soliton takes place due to dispersion light when it propagating along optical fiber and self-phase modulation effect. Nonlinear Schrödinger equation is also explained which defines mathematically the amplitude variation of ultra-short pulse of light.

2.1 Soliton

In mathematics and physics a soliton is a self-reinforcing solitary wave. It is also a wave packet or pulse that maintains its shape while it travels at constant speed. Solitons are caused by a cancellation of nonlinear and dispersive effect in the medium. Dispersive effects mean a certain systems where the speed of the waves varies according to frequency. Solitons arise as the solutions of a widespread class of weakly nonlinear dispersive partial differential equation describing physical systems. Soliton is an isolated particle like wave that is a solution of certain equation for propagating, acquiring when two solitary waves do not change their form after collision and subsequently travel for considerable distance. Moreover, soliton is a quantum of energy or quasi particle that can be propagated as a travelling wave in non-linear system and cannot be followed other disturbance. This process does not obey the superposition principal and does not dissipate. Soliton wave can travel long distance with little loss of energy or structure.
2.2 Pulse Dispersion

In digital communication systems, information is encoded in the form of pulses and then these light pulses are transmitted from the transmitter to the receiver. The larger the number of pulses that can be sent per unit time and still be resolvable at the receiver end, the larger is the capacity of the system.

However, when the light pulses travel down the fiber, the pulses spread out, and this phenomenon is called Pulse Dispersion. Pulse dispersion is shown in the following figure.

![Fig 2.1: Pulse dispersion](image)

Pulse dispersion is one of the two most important factors that limit a fiber’s capacity (the other is fiber’s losses) [1]. Pulse dispersion happens because of four main reasons:

i. Intermodal Dispersion
ii. Material Dispersion
iii. Waveguide Dispersion
iv. Polarization Mode Dispersion (PMD)

An electromagnetic wave, such as the light sent through an optical fiber is actually a combination of electric and magnetic fields oscillating perpendicular to each other. When an electromagnetic wave propagates through free space, it travels at the constant speed of $3.0 \times 10^8$ meters.
However, when light propagates through a material rather than through free space, the electric and magnetic fields of the light induce a polarization in the electron clouds of the material. This polarization makes it more difficult for the light to travel through the material, so the light must slow down to a speed less than its original $3.0 \times 10^8$ meters per second. The degree to which the light is slowed down is given by the material’s refractive index $n$. The speed of light within material is then $v = 3.0 \times 10^8$ meters per second/n.

This equation shows that a high refractive index means a slow light propagation speed. Higher refractive indices generally occur in materials with higher densities, since a high density implies a high concentration of electron clouds to slow the light.

Since the interaction of the light with the material depends on the frequency of the propagating light, the refractive index is also dependent on the light frequency. This, in turn, dictates that the speed of light in the material depends on the light’s frequency, a phenomenon known as chromatic dispersion.

Optical pulses are often characterized by their shape. We consider a typical pulse shape named Gaussian, shown in Figure 2.2. In a Gaussian pulse, the constituent photons are concentrated toward the center of the pulse, making it more intense than the outer tails.

![Fig 2.2: A Gaussian pulse](image)

Optical pulses are generated by a near-monochromatic light source such as a laser or an LED. If the light source were completely monochromatic, then it would generate photons at a single fre-
frequency only, and all of the photons would travel through the fiber at the same speed. In reality, small thermal fluctuations and quantum uncertainties prevent any light source from being truly monochromatic. This means that the photons in an optical pulse actually include a range of different frequencies. Since the speed of a photon in an optical fiber depends on its frequency, the photons within a pulse will travel at slightly different speeds from each other. The result is that the slower photons will lag further and further behind the faster photons, and the pulse will broaden. An example of this broadening is given in Figure 2.3.

![Figure 2.3: A pulse broadens due to chromatic dispersion](image)

Chromatic dispersion may be classified into two different regimes: normal and anomalous. With normal dispersion, the lower frequency components of an optical pulse travel faster than the higher frequency components. The opposite is true with anomalous dispersion. The type of dispersion a pulse experiences depends on its wavelength; a typical fiber optic communication system uses a pulse wavelength of 1.55 µm, which falls within the anomalous dispersion regime of most optical fiber.

Pulse broadening, and hence chromatic dispersion, can be a major problem in fiber optic communication systems for obvious reasons. A broadened pulse has much lower peak intensity than the initial pulse launched into the fiber, making it more difficult to detect. Worse yet, the broadening of two neighboring pulses may cause them to overlap, leading to errors at the receiving end of the system.

However, chromatic dispersion is not always a harmful occurrence. As we shall soon see, when combined with self-phase modulation, chromatic dispersion in the anomalous regime may lead to the formation of optical solitons [1].
2.3 Kerr Effect

The Kerr effect, also called the quadratic electro-optic effect (QEO effect), is a change in the refractive index of a material in response to an applied electric field. The Kerr electro-optic effect, or DC Kerr effect, is the special case in which a slowly varying external electric field is applied by, for instance, a voltage on electrodes across the sample material. Under this influence, the sample becomes birefringent with different indices of refraction for light polarizer parallel to or perpendicular to the applied field. The difference in index of refraction, $\Delta n$, is given by

$$\Delta n = \lambda K E^2,$$

Where $\lambda$ is the wavelength of the light, $K$ is the Kerr constant, and $E$ is the strength of the electric field. This difference in index of refraction causes the material to act like a wave plate when light is incident on it in a direction perpendicular to the electric field. If the material is placed between two "crossed" (perpendicular) linear polarizer, no light will be transmitted when the electric field is turned off, while nearly all of the light will be transmitted for some optimum value of the electric field. Higher values of the Kerr constant allow complete transmission to be achieved with a smaller applied electric field.

Some polar liquids, such as nitro toluene ($C_7H_7NO_2$) and nitrobenzene ($C_6H_5NO_2$) exhibit very large Kerr constants. A glass cell filled with one of these liquids is called a Kerr cell. These are frequently used to modulate light, since the Kerr effect responds very quickly to changes in electric field. Light can be modulated with these devices at frequencies as high as 10 GHZ. Because the Kerr effect is relatively weak, a typical Kerr cell may require voltages as high as 31 KV to achieve complete transparency. Another disadvantage of Kerr cells is that the best available material, nitrobenzene, is poisonous. Some transparent crystals have also been used for Kerr modulation, although they have smaller Kerr constants [2].

The optical Kerr effect or AC Kerr effect is the case in which the electric field is due to the light itself. This causes a variation in index of refraction which is proportional to the local irradiance of the light. This refractive index variation is responsible for the nonlinear optical effects this effect only become significant with very intense beams such as those from laser.
2.4 Self-Phase Modulation

Self-phase modulation (SPM) is a nonlinear effect of light-matter interaction. With self-phase modulation, the optical pulse exhibits a phase shift induced by the intensity-dependent refractive index. An ultra short pulse light, when travelling in a medium, will induce a varying refractive index in the medium due to the optical Kerr effect. This variation in refractive index will produce a phase shift in the pulse, leading to a change of the pulse's frequency spectrum. The refractive index is also dependent on the intensity of the light. This is due to the fact that the induced electron cloud polarization in a material is not actually a linear function of the light intensity. The degree of polarization increases non-linearly with light intensity, so the material exerts greater slowing forces on more intense light. The result is that the refractive index of a material increases with the increasing light intensity.

Phenomenological consequences of this intensity dependence of refractive index in fiber optic are known as fiber nonlinearities.

![Fig 2.4: Self-Phase Modulation](image)

In this figure, a pulse (top curve) propagating through a nonlinear medium undergoes a self-frequency shift (bottom curve) due to self-phase modulation. The front of the pulse is shifted to lower frequencies, the back to higher frequencies. In the centre of the pulse the frequency shift is approximately linear.
For an ultra short pulse with a Gaussian shape and constant phase, the intensity at time $t$ is given by $I(t)$ [3]:

$$I(t) = I_0 \exp\left(\frac{-t^2}{\tau^2}\right)$$  \hspace{1cm} (2.1)

Where $I_0$ is the peak intensity and $\tau$ is half the pulse duration.

If the pulse is travelling in a medium, the optical Kerr effect produces a refractive index change with intensity [3]:

$$n(I) = n_0 + n_2 \cdot I$$  \hspace{1cm} (2.2)

Where $n_0$ is the linear refractive index and $n_2$ is the second-order nonlinear refractive index of the medium.

As the pulse propagates, the intensity at any one point in the medium rises and then falls as the pulse goes past. This will produce a time-varying refractive index [3]:

$$\frac{dn(I)}{dt} = n_2 \frac{dI}{dt} = n_2 \cdot I_0 \cdot \frac{-2i}{\tau^2} \cdot \exp\left(\frac{-t^2}{\tau^2}\right).$$

This variation in refractive index produces a shift in the instantaneous phase of the pulse [8]

$$\phi(t) = \omega_0 t - \frac{2\pi}{\lambda_0} \cdot n(I) L$$

Where $\omega_0$ and $\lambda_0$ are the carrier frequency and (vacuum) wavelength of the pulse, and $L$ is the distance the pulse has propagated. The phase shift results in a frequency shift of the pulse. The instantaneous frequency $\omega(t)$ is given by [3]:

$$\omega(t) = \frac{d\phi(t)}{dt} = \omega_0 - \frac{2\pi L}{\lambda_0} \frac{dn(I)}{dt},$$

And from the equation for $dn/dt$ above, this is [8]:

$$\omega(t) = \omega_0 + \frac{4\pi L n_2 I_0}{\lambda_0 \tau^2} \cdot t \cdot \exp\left(\frac{-t^2}{\tau^2}\right).$$

Plotting $\omega(t)$ shows the frequency shift of each part of the pulse. The leading edge shifts to lower frequencies ("redder" wavelengths), trailing edge to higher frequencies ("bluer") and the very
peak of the pulse is not shifted. For the centre portion of the pulse (between \( t = \pm \tau/2 \)), there is an approximately linear frequency shift (chirp) given by [3]:

\[
\omega(t) = \omega_0 + \alpha \cdot t
\]  

(2.3)

Where \( \alpha \) is:

\[
\alpha = \frac{d\omega}{dt} \bigg|_0 = \frac{4\pi L n_2 I_0}{\lambda_0 \tau^2}.
\]

**2.5 Nonlinear Schrödinger Equation (NLSE)**

The general nonlinear Schrödinger equation (NLSE) is given below. The generalized NLSE can be described as a complete form of nonlinear Schrödinger equation in optical fiber because it contains all relevant parameters for solving pulse propagation in nonlinear media.

\[
\frac{\partial A}{\partial z} = \left( -\frac{i \beta_2}{2} \frac{\partial^2 A}{\partial T^2} + \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} - \frac{\alpha}{2} A \right) + \left( i \gamma |A|^2 A - \frac{\gamma}{\omega_0} \frac{1}{A} \frac{\partial}{\partial T} (|A|^2 A - i \gamma T_R \frac{\partial |A|^2}{\partial T}) \right)
\]

Group Velocity Dispersion  
Self Steepening  
Nonlinearity  
Attenuation  
Raman Scattering  
Third Order Dispersion
\[
\frac{\partial A}{\partial z} = \left(-\frac{i\beta_2}{2} \frac{\partial^2 A}{\partial T^2} + \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} - \frac{\alpha}{2} A\right) + i\gamma \left(|A|^2 A + \frac{i}{\omega_0} \frac{1}{A} \frac{\partial}{\partial T} (|A|^2 A - T_R \frac{\partial |A|^2}{\partial T})\right)
\]

(2.4)

**Simplified NLSE**

In this equation, we have removed the nonlinear effects like self steepening and Raman scattering as this study focuses on the interaction between \(\beta_2\) and \(\gamma\).

\[
\frac{\partial A}{\partial z} = \left(-\frac{i\beta_2}{2} \frac{\partial^2 A}{\partial T^2} + \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} - \frac{\alpha}{2} A\right) + \left(\gamma |A|^2 A\right)
\]

(2.5)

The third order dispersion term is removed because \(\beta_3\) becomes negligible for picoseconds pulses.

\[
\frac{\partial A}{\partial z} = \left(-\frac{i\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \frac{\alpha}{2} A\right) + \left(\gamma |A|^2 A\right)
\]

(2.6)

\[
\gamma = \frac{2n_2}{\lambda A_{eff}};
\]

(2.7)

Where,

\(\gamma\) is the nonlinear parameter

\(A_{eff}\) = Effective core area of the fiber;

\(n_2\) = nonlinear index coefficient;

\(\lambda\) = optical wavelength

Here, \(\tau\) is the normalized time parameter, \(Z\) is the normalized length, and \(U\) is the normalized input power used in the normalized NLSE.
\[ \tau = \frac{T}{T_0} \]  \hspace{2cm} (2.8)

\[ Z = \frac{z}{L_D} \]  \hspace{2cm} (2.9)

\[ U = \frac{A}{\sqrt{P_0}} \]  \hspace{2cm} (2.10)

The previous normalization parameters are used in transforming the simplified NLSE to normalized NLSE.

### 2.5.1 Group velocity dispersion

Group velocity dispersion is the fact that the group velocity of light in a transparent medium depends on the optical frequency or wavelength. This term can also be used to define quantity, which is the derivation of the inverse group velocity with respect to the angular frequency (or sometimes the wavelength):

\[ G\text{VD} = \frac{\partial}{\partial \omega} \left( \frac{1}{v_g} \right) = \frac{\partial}{\partial \omega} \left( \frac{\partial k}{\partial \omega} \right) = \frac{\partial^2 k}{\partial \omega^2} \]

The group velocity dispersion is the group delay dispersion per unit length. The basic units are \( s^2/m \). For example, the group velocity dispersion of silica is +35 fs\(^2\)/mm at 800 nm and −26 fs\(^2\)/mm at 1500 nm [4].

For optical fiber communications, the group velocity dispersion is usually defined as a derivative with respect to wavelength rather than the angular frequency of the signal. This can be calculated from the above-mentioned GVD parameter:

\[ D_4 = -\frac{2\pi c}{\lambda^2} \cdot G\text{VD} = -\frac{2\pi c}{\lambda^2} \cdot \frac{\partial^2 k}{\partial \omega^2} \]
This quantity is usually specified with units of ps/(nm.km) (picoseconds per nanometer wavelength change and kilometer propagation distance). For example, 20 ps/(nm.km) at 1550 nm (a typical value for telecom fibers) corresponds to −25509 fs2/m [4].

In parametric nonlinear interactions, Group velocity dispersion is responsible for dispersive broadening of pulses and also for the group velocity variance of different waves.

### 2.5.2 Third-order dispersion

Third-order dispersion has come from the frequency dependence of the group delay dispersion. The Taylor expansion of the spectral phase versus angular frequency offset is related to the third-order term. It can be written as

\[
k'''' = \frac{\partial^4 T}{\partial \omega^4} = \frac{\partial^2 \varphi}{\partial \omega^2}
\]

And the corresponding change in the spectral phase within a propagation length \( L \) is

\[
\Delta \varphi(\omega) = \frac{1}{6} \frac{\partial^3 k}{\partial \omega^3} (\omega - \omega_0)^3 L
\]

The third-order dispersion of an optical element is usually specified in units of fs³, whereas the units of \( k'''' \) are fs³/m [5].

In mode-locked lasers for pulse durations below 30 fs, it is necessary to provide dispersion compensation for not only the average group delay dispersion which is also the second-order dispersion, but also for the third-order dispersion and possibly for even higher orders. Sometimes the effect of third-order dispersion requires numerical pulse propagation modeling.

### 2.5.3 Attenuation

Attenuation of light intensity is caused by the absorption, scattering and bending losses in an optical fiber. It is also a loss of optical power when light travels in a fiber. Signal attenuation is defined as the ratio of optical input power (\( P_i \)) to the optical output power (\( P_o \)). Optical input power
is the power injected into the fiber from an optical source. Optical output power is the power received at the fiber end or optical detector. The following equation defines signal attenuation as a unit of length:

$$\text{attenuation} = \left( \frac{10}{L} \right) \log_{10} \left( \frac{P_i}{P_0} \right)$$

Signal attenuation is a log relationship. Length (L) is expressed in kilometers. Therefore, the unit of attenuation is decibels/kilometer (dB/km) [6]. The losses caused by absorption, scattering and bending losses are influenced by fiber-material properties and fiber structure. Attenuation occurs with any type of signal, whether digital or analog. Attenuation is also a natural consequence of signal transmission over long distances.

Efficiency of the optical fiber increases as the attenuation per unit distance is less. One or more repeaters are generally inserted along the length of the cable in order to transmit the signals over long distances. The repeaters boost the signal strength to overcome attenuation. This greatly increases the maximum attainable range of communication [7].

2.5.4 Nonlinearity

In an optical fiber when, the data rates, transmission lengths, number of wavelengths, and optical power levels increased then nonlinearity effects arise. Those dispersions are easily dealt with using a variety of dispersion avoidance and cancellation techniques. Fiber nonlinearities have presented a new area of obstacle. Fiber nonlinearities represent the fundamental limiting mechanisms to the amount of data that can be transmitted on a single optic fiber. System designers must be aware of these limitations and the steps that can be taken to minimize the detrimental effects of fiber nonlinearities [8].

There are two basic mechanisms for arising fiber nonlinearities. One of them is the refractive index of glass which dependents on the optical power going through the material and the other mechanisms is the scattering phenomena. The most detrimental mechanism arises from the refractive index of glass. The general equation for the refractive index of the core in an optical fiber is:
\[ n = n_0 + n_2 \cdot \frac{P}{A_{\text{eff}}} \]

Where, \( n_0 \) = the refractive index of the fiber core at low optical power levels, \( n_2 \) = the nonlinear refractive index coefficient (\( 2.35 \times 10^{-20} \text{ m}^2/\text{W} \) for silica), \( P \) = the optical power in Watts, \( A_{\text{eff}} \) = the effective area of the fiber core in square meters.

The equation shows that minimizing the amount of power, \( P \); launched and maximizing the effective area of the fiber, \( A_{\text{eff}} \); eliminates the nonlinearities produced by refractive index power. Minimizing the power goes against the current approach to eliminating the detrimental effects; however, maximizing the effective area remains the most common approach in the latest fiber designs [8].

2.5.5 Self-steepening

Self-steepening means the change in shape of light pulses, which propagate in a medium with an intensity-dependent index of refraction. The time required for the pulse to steepen into an optical fiber is measured, and the time development of the pulses is calculated for both zero and nonzero times of relaxation of the index of refraction [9]. In our calculation, we assume self-steepening is zero.

2.5.6 Raman scattering

The Raman Effect is the inelastic scattering of photons. When a monochromatic light beam propagates in an optical fiber, spontaneous Raman scattering transfers some of the photons to new frequencies. The probability of a photon scattering to a particular frequency shift depends on that frequency shift, forming a characteristic spectrum. The scattered photons may lose energy or gain energy. If the light beam is linearly polarized, then the polarization of scattered photons may be the same (parallel scattering) or orthogonal (perpendicular scattering). If photons are already present at other frequencies then the probability of scattering to those frequencies is enhanced (stimulated scattering) [10]. In our calculation, we also assume Raman Scattering is zero.
CHAPTER 3:

Analyzing Method

There are many methods to solve NLSE equation. In this paper, we have used split step Fourier method to solve nonlinear Schrödinger equation. It is applied because of greater computation speed and increased accuracy compared to other numerical techniques.

3.1 Split Step Fourier Method

The split step Fourier method is a pseudo-spectral numerical method for solving partial differential equations such as the nonlinear Schrödinger equation. Dispersion and nonlinear effects act simultaneously on propagating pulses during nonlinear pulse propagation in optical fibers. However, analytic solution cannot be employed to solve the NLSE with both dispersive and nonlinear terms present. Hence the numerical split step Fourier method is utilized, which breaks the entire length of the fiber into small step sizes of length ‘h’ and then solves the nonlinear Schrödinger equation by splitting it into two halves, the linear part (dispersive part) and the nonlinear part over z to z + h [7].

Fig 3.1: Split step Fourier method
Each part is solved individually and then combined together afterwards to obtain the aggregate output of the traversed pulse. It solves the linear dispersive part first, in the Fourier domain using the fast Fourier transforms and then inverse Fourier transforms to the time domain where it solves the equation for the nonlinear term before combining them. The process is repeated over the entire span of the fiber to approximate nonlinear pulse propagation. The equations describing them are offered below \[11\].

The value of $h$ is chosen for $\phi_{\text{max}} = \gamma \left| A_p \right|^2 h$, where $\phi_{\text{max}} = 0.05\text{rad}$ and $A_p =$ peak power of $A(z,t)$: $\phi_{\text{max}} =$ maximum phase shift

In the following part the solution of the generalized Schrödinger equation is described using this method.

\[
\frac{\partial A}{\partial z} = \left( - \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial T^2} + \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} - \frac{\alpha}{2} A \right) + \left( i\gamma |A|^2 A - \frac{\gamma}{\omega_0} \frac{1}{A \frac{\partial}{\partial T}} (|A|^2 A) - i\gamma T_R \frac{\partial |A|^2}{\partial T} \right) \tag{3.1}
\]

\[
\frac{\partial A}{\partial t} = (\hat{L} + \hat{N})A \tag{3.2}
\]

The linear part (dispersive part) and the nonlinear part are separated.

**Linear part**

\[
\hat{L} = \left( - \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial T^2} + \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} - \frac{\alpha}{2} A \right) \tag{3.3}
\]

**Nonlinear part**

\[
\hat{N} = \left( i\gamma |A|^2 A - \frac{\gamma}{\omega_0} \frac{1}{A \frac{\partial}{\partial T}} (|A|^2 A) - i\gamma T_R \frac{\partial |A|^2}{\partial T} \right) \tag{3.4}
\]
Linear part solution

The solution of the linear part $\exp(h\hat{L})$ is done in the Fourier domain by using the identity

$$\frac{\partial}{\partial T} = i\omega \cdot \omega.$$ 

Since in the Fourier domain $\omega$ is simply a numerical sequence of digits, the calculation that would otherwise be complicated in the time domain due to computation of the differential terms is mitigated in the Fourier domain.

$$\exp(h\hat{L}) \leftrightarrow \exp[h\hat{L}(i\omega)] \quad \text{and} \quad A(z, t) \leftrightarrow A(z, \omega - \omega_0)$$

Linear Solution  =  $F_T^{-1}\left[\exp[h\hat{L}(i\omega)] \times A(z, \omega - \omega_0)\right]$  \hspace{1cm} (3.5)

Nonlinear part solution

$$A(z + h, T) = \exp(h\hat{N}) \times \text{Linear Solution}$$  \hspace{1cm} (3.6)

In our study as will be shown later, we have considered $\beta_3 = 0$, $\alpha = 0$ and higher order nonlinear effects of self steepening and Raman scattering (reserved for ultra short pulses) to be 0 i.e.

$$\frac{\gamma}{\omega_0} \cdot \frac{1}{\omega_0 A} \frac{\partial}{\partial T} (|A|^2 A) - i\gamma T_k \frac{\partial |A|^2}{\partial T} = 0 .$$

We make this assumption because the pulse width chosen is of the order of picoseconds.
CHAPTER 4:

Analysis of Different Pulses

Pulse is a rapid change in some characteristic of a signal. The characteristic can be phase or frequency from a baseline value to a higher or lower value, followed by a rapid return to the baseline value. Here we have analyzed three types of pulses which are Gaussian Pulse, Super-Gaussian Pulse and Hyperbolic Secant Pulse.

4.1 Gaussian Pulse

![Gaussian Pulse](image)

**Fig 4.1: Gaussian Pulse**

In theory, Gaussian pulses while propagating maintain their fundamental shape, however their amplitude width and phase varies over given distance [12].
Many quantitative equations can be followed to study the properties of Gaussian pulses as it propagates over a distance of \( z \).

The incident field for Gaussian pulses can be written as

\[
U(0, \tau) = Ao^* \exp \left( -\frac{\tau^2}{2T_0^2} \right)
\]

(4.1)

\( T_0 \) is the initial pulse width of the pulse.

Amplitude of a Gaussian pulse varies with the equation

\[
U(0, T) = \frac{T_0}{\left( T_0^2 - i\beta_2 \right)^{1/2}} \exp \left( -\frac{T^2}{2\left( T_0^2 - i\beta_2 \right)} \right)
\]

(4.2)

Width of a Gaussian pulse varies with this equation

\[
\tau_1(z) = T_0 \left[ 1 + \left( \frac{z}{L_D} \right)^2 \right]^{1/2}
\]

(4.3)

Under the effect of dispersion this equation shows how the Gaussian pulse width broadens over \( z \).

### 4.2 Super-Gaussian Pulse

The input field for such pulse is described by

\[
U(0, \tau) = Ao^* \exp \left( -\frac{(1 + iC)\tau^{2m}}{2T_0^2} \right)
\]

(4.4)

Here \( m \) is the order of the super-Gaussian pulse and determines the sharpness of the edges of the input. The higher the value of \( m \) steeper is the leading and trailing edges of the pulse.
As we continue increasing $m$, we eventually get a rectangular pulse shape which evidently has very sharp edges. In case of $m=1$ we get the Gaussian chirped pulse.

The sharpness of the edges plays an important part in the broadening ratio because broadening caused by dispersion is sensitive to such a quality [13].

**4.2.1 Chirp**

Chirp is a signal in which the frequency increases (up-chirp) or decreases (down-chirp) with time. It is commonly used in sonar and radar, but has other applications, such as in spread spectrum communications. In optics, ultra short laser pulses also exhibit chirp. In optical transmission systems chirp interacts with the dispersion properties of the materials, increasing or decreasing total pulse dispersion as the signal propagates.

**4.2.1.1 Types of chirp**

Three types of chirps are discussed in this chapter – Group velocity dispersion induced chirp (down chirp), self phase modulation induced chirp (up chirp) and pre-induced chirp (initial chirp).
GVD induced chirp

GVD induced chirp occurs due to the group velocity dispersion effect while pulse propagates through an optical fiber. The chirp is linear and negative over a large central region. It is positive near the leading edge and becomes negative near the trailing edge of the pulse. Essentially, it causes spreading of pulses [12]. GVD induced chirp is down chirp or negative chirp. Here, high frequency components of a pulse travels at greater a velocity than low frequency components. This difference results in induced negative chirp which signifies the GVD effect.

![GVD induced chirp effect](image)

*Fig 4.3: GVD induced chirp effect*

SPM induced chirp

SPM induced chirp is up-chirp or positive-chirp that occurs due to the nonlinear self phase modulation effect inside an optical fiber. The chirp is linear and positive over a large central region of the pulse. It is negative near the leading edge and becomes positive near the trailing edge of the pulse. SPM induced chirp makes the pulse width narrower from the original pulse width as pulses propagate through an optical fiber. Here, low frequency components of the pulse travel at larger velocity than the high frequency components. This results in induction of positive chirp resulting in narrowing of pulses.
Fig 4.4: SPM induced chirp effect

Fig 4.5: (a) Gaussian pulse phase (b) SPM induced chirp (c) GVD induced chirp
Pre induced chirp

Pre induced chirp is the chirp added to an input pulse before sending it through an optical fiber. Pre induced chirp or initial chirp results from the laser source itself. It can be either positive or negative. It is used to balance the GVD induced chirp and SPM induced chirp. Initial chirp can be useful in certain cases-

- Suppression of four wave mixing
- Lowering pulse width fluctuations
- Spectral compression
- And generating transform limited output pulses

4.3 Hyperbolic Secant Pulse

Pulse propagation for hyperbolic secant pulse

![Hyperbolic Secant Input pulse](image)

*Fig 4.6: Hyperbolic secant Input pulse*
4.3.1 Conditions for soliton

The conditions for soliton are

1) The dispersion region must be anomalous. That is $\beta_2 < 0$.
2) The input pulse must be an un-chirped hyperbolic secant pulse. In our simulation we used the following pulse:

$$U(0, \tau) = \sec h \left( \frac{\tau}{T_0} \right)$$

(4.5)

3) The dispersion length must be approximately the same as the nonlinear length.
4) The GVD induced chirp should exactly cancel the SPM induced chirp.

4.3.2 Higher order soliton

Higher order soliton are soliton with higher energy. More specifically, the energy of a higher order soliton is square of an integer number times higher than a fundamental soliton.

In our simulation we used –

$$U(0, \tau) = N \sec h \left( \frac{\tau}{T_0} \right)$$

(4.6)

Higher order soliton do not have a fixed pulse shape like fundamental soliton. But they gain their shape periodically. The order of the soliton is described by the parameter N.
CHAPTER 5:

Implementation and Optimization

In the previous chapter we have discussed about Gaussian Pulse, Super-Gaussian Pulse and hyperbolic secant pulse. In this chapter, we have implemented those pulses using NLSE according to split step Fourier method (SSFM).

5.1 Gaussian Pulse Implementation

<table>
<thead>
<tr>
<th>GAUSSIAN PULSE INPUT</th>
<th>SIMPLIFIED NLSE</th>
<th>METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U(0, \tau) = \text{Ao} \ast \exp\left(-\frac{(1+iC)(\tau)^2}{2To^2}\right)$</td>
<td>$\frac{\partial A}{\partial z} = \left(-\frac{i\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \frac{\alpha}{2} A\right) + \left(\gamma</td>
<td>A</td>
</tr>
</tbody>
</table>

Table 5.1: Gaussian input

Input pulse

Here, the input pulse is Fourier transformed. Fourier spectrum = $F_r(U(0, \tau))$

The nonlinear Schrodinger equation is solved in two dispersion steps with a nonlinear step in the middle.

Linear dispersive step (1st half)

This is the first step of the linear solution. Here, the Fourier domain input pulse is multiplied with the first part of the nonlinear Schrodinger equation solution that considers the GVD effect.
only without the nonlinear effect. (The loss term is included and can be used in calculation but for this purpose it is considered zero)

\[
\text{Linear solution (1)} = F_T(U(0,\tau)) \times \exp \left( -\frac{\alpha}{2} + \frac{i\beta_2\omega^2}{2} \left( \frac{h}{2} \right) \right)
\]

**Inverse Fourier transform**

The linear solution is transformed into the time domain by inverse Fourier transform using the fast Fourier transform algorithm.

**Linear solution (1) in time domain = \( F_T^{-1} [\text{Linear solution (1)}] \)**

**Nonlinear step**

The nonlinear step is solved in the time domain. The output from the previous step is subjected to the nonlinear solution of the nonlinear Schrödinger equation using the following equations. This step considers the nonlinear effects only without any linear effect term.

The following is the equation for the nonlinear solution.

**Nonlinear solution in time domain = \( \exp i\gamma | \text{Linear solution (1) in time domain} \|^2 h \)**

The nonlinear solution is multiplied with the linear solution from the previous step.

**Nonlinear step output = \( \text{Linear solution (1) in time domain} \times \text{Nonlinear solution in time domain} \)**

The resulting output is the combined output for the first linear step and the nonlinear step in time domain.

**Fourier transform**

To solve for the second linear step of the split step Fourier method we must transform the output of the previous step into time domain.

**Nonlinear step output Fourier domain = \( F_T [\text{Nonlinear step output}] \)**
**Linear dispersive step (2nd half)**

The second step of the linear solution is accomplished by multiplying the Fourier domain output of the previous step and the linear part of the nonlinear Schrodinger equation solution just like in the first linear step.

**Linear solution (2) = Nonlinear step output Fourier domain × exp\( \left( -\frac{\alpha}{2} + \frac{i\beta_2\omega^2}{2}\left( \frac{h}{2} \right) \right) \)**

**Inverse Fourier transform**

Inverse Fourier transform is performed on the output of the second linear part.

**Linear solution (2) in time domain = \( F^{-1}_T \) [Linear solution (2)]**

**Solution for NLSE Over 1 Step size h = Linear solution (2) in time domain**

This previous output is the final output to the split step Fourier method over one step where each step is of size \( h \).

This process is repeated over the length of the fiber.

### 5.2 Super-Gaussian Pulse Implementation

<table>
<thead>
<tr>
<th>SUPER GAUSSIAN PULSE INPUT</th>
<th>SIMPLIFIED NLSE</th>
<th>METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U(0, \tau) = A_0^* \exp\left( -\frac{(1 + iC)T^{2m}}{2To^3} \right) )</td>
<td>( \frac{\partial A}{\partial z} = \left( -\frac{i\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \frac{\alpha}{2} A \right) + \left( \gamma</td>
<td>A</td>
</tr>
</tbody>
</table>

**Table 5.2: Super-Gaussian input**

The Gaussian implementation method can also be applied for super-Gaussian pulse inputs.
5.3 Hyperbolic Secant Pulse Implementation

<table>
<thead>
<tr>
<th>HYPERBOLIC SECANT PULSE INPUT</th>
<th>NORMALIZED NLSE</th>
<th>METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U(0, \tau) = N \sec h \left( \frac{\tau}{T_0} \right)$</td>
<td>$\frac{\partial U}{\partial Z} - i \frac{s}{2} \frac{\partial^2 U}{\partial \tau^2} + i N^2</td>
<td>U</td>
</tr>
</tbody>
</table>

Table 5.3: Hyperbolic Secant input

Fourier transform of input pulse

The input pulse is Fourier transformed.

Fourier spectrum = $F_T(U(0, \tau))$

Linear dispersive step (1st half)

The linear step is solved by multiplying the Fourier domain input hyperbolic secant pulse with the normalized Schrodinger equation solution for the first linear part which consists of only dispersive terms (loss term considered is zero for this purpose)

Linear solution (1) = $F_T(U(0, \tau)) \times \exp \left( -\frac{\alpha}{2} + \frac{i \omega^2}{2} \left( \frac{h}{2} \right) \right)$

Inverse Fourier transform

Inverse Fourier transform is performed on the previous step output.

Linear solution (1) in time domain = $F_T^{-1} [\text{Linear solution (1)}]$

The solution term of the linear part of the nonlinear Schrodinger equation is differ

Nonlinear step

The nonlinear step is solved in the time domain and the process is shown below.
Nonlinear solution in time domain \(= \exp(iN^2) \cdot \text{Linear solution(1) in time domain} \cdot h^2 \)

Nonlinear step output \(=\) Linear solution (1) in time domain \(\times\) Nonlinear solution in time domain

Fourier transform

Nonlinear step output is once again Fourier transformed for the second half of the linear step.

Nonlinear step output Fourier domain \(= F_T [\text{Nonlinear step output}] \)

Linear dispersive step (2\textsuperscript{nd} half)

The second half of the linear step is solved in the following manner similar to the first half.

Linear solution (2) \(=\) Nonlinear step output Fourier domain \(\times\) exp \(\left( -\frac{\alpha}{2} + \frac{i \omega^2}{2} \left( \frac{h}{2} \right) \right) \)

Inverse Fourier transform

Inverse Fourier of the previous output results in time domain output of the split step Fourier method for one step.

Linear solution (2) in time domain \(= F_T^{-1} [\text{Linear solution (2)}] \)

Solution for NLSE over 1 step size \(h = \) Linear solution (2) in time domain

This process is repeated over the length of the fiber.
CHAPTER 6:

Result and Analysis

In this chapter we have used Gaussian pulse, Hyperbolic Secant pulse and Super-Gaussian pulse varying chirp, gamma, input power, soliton order in Matlab simulation to analysis pulse broadening ratio. Pulse broadening ratio should be one throughout all steps of pulse propagation in order to generate soliton. In this thesis, analysis is done by using the pulse broadening ratio of the evolved pulses. Pulse broadening ratio is calculated by using the Full Width at Half Maximum (FWHM).

Pulse broadening ratio = FWHM of propagating pulse / FWHM of First pulse.

Pulse broadening ratio signifies the change of the propagating pulse width compared to the pulse width at the very beginning of the pulse propagation. At the half or middle of the pulse amplitude, the power of the pulse reaches maximum. The width of the pulse at that point is called full width half maximum.

Fig 6.1: Full Widths at Half Maximum
6.1 Gaussian Pulse

In this section of chapter, we will vary different nonlinear and dispersive parameters to find pulse broadening ratio through optical fiber. Pulse broadening ratio of Gaussian pulse with chirp, C= -1, -0.5, 0, 0.5, 1.

![Graph showing pulse broadening ratio with different chirps](image)

**Fig 6.2: Pulse Broadening Ratio of Gaussian Pulse with different chirp**

Here, both GVD and SPM act simultaneously on the Gaussian pulse with initial negative chirp. The evolution pattern shows that pulse broadens at first for a small period of length. But gradually the rate at which it broadens slowly declines and the pulse broadening ratio seems to reach a constant value. This means that the pulse moves at a slightly larger but constant width as it propagates along the length of the fiber. Although the width of the pulse seems constant, it does not completely resemble a hyperbolic secant pulse evolution. We compare the pulse evolution of the Gaussian pulse with no initial chirp and the negative chirped Gaussian pulse evolution to see the difference in shape and width of each of these evolutions. As we previously established GVD and SPM effects cancel each other out when the GVD induced negative chirp equals the SPM induced positive chirp. But in this case the initial chirp affects the way both GVD and SPM behave. The chirp parameter of value -1 adds to the negative chirp of the GVD and deducts from the positive chirp of SPM causing the net value of chirp to be negative. This means that GVD is dominant during the early stages of propagation causing broadening of the pulse. But as the...
propagation distance increases the effect of the initial chirp decreases while the induced chirp effect of both GVD and SPM regains control. The difference between positive and negative induced is lessened and just like in the case of Gaussian pulse propagation without initial chirp the GVD and SPM effects eventually cancels out each other to propagate at constant width.

If both GVD and SPM act simultaneously on the propagating Gaussian pulse with no initial chirp then the pulse shrinks initially for a very small period of propagating length. After that the broadening ratio reaches a constant value and a stable pulse is seemed to propagate.

GVD acting individually results in the pulse to spread gradually before it loses shape. SPM acting individually results in the narrowing of pulses and losing its intended shape. The combined effect of GVD and SPM leads to the eventual generation of constant pulse propagation emulating a hyperbolic secant pulse.

Pulse broadening ratio for various nonlinear parameter $\gamma$ is given below:

![Fig 6.3: Pulse Broadening Ratio for different values of Nonlinearity](image)

In Figure, we study the importance of magnitude of nonlinear parameter $\gamma$ on nonlinear optical fiber.
By keeping the input power and the GVD parameter constant, we generate curves for various values of $\gamma$. The ideal value of the nonlinear parameter is one, where GVD effect cancels out SPM effect to obtain constant pulse width.

Values chosen for this study are $\gamma = 0.002, 0.0025, 0.003, 0.0035$ and $0.004 /W/m$. The purpose is to observe the effect of increasing and decreasing nonlinear parameter on pulse broadening ratio. For $\gamma = 0.003$, the SPM induced positive chirp and GVD induced negative chirp gradually cancels out. This results in the pulse propagating at a constant width throughout a given length of fiber. For $\gamma = 0.0035$, the pulse appears initially more narrow than the previous case. This is because of increasing nonlinearity which results in increased SPM effect. For $\gamma = 0.004$, the pulse broadening ratio initially decreases to a minimum value. For $\gamma = 0.002$, it is obvious that the SPM effect is not large enough to counter the larger GVD effect. For this reason pulse broadens.

Figure of pulse broadening ratio for Gaussian pulse with input power = 0.00056W, 0.0006W, 0.00064W, 0.00068W and 0.00072W.

**Fig 6.4: Pulse Broadening Ratio for different values of Power**
In Figure, we study the importance of magnitude of power on nonlinear optical fiber. By keeping the nonlinear parameter and the GVD parameter constant, we generate curves for various values of input power. It is observed from this plot that, the pulse broadening ratio is more for curves with smaller input power than for those with larger input power.

This property can be explained by the following equation

\[ L_N = \frac{1}{\gamma P_o} \]; Where, \( P_o \) is the input power and \( L_N \) is the nonlinear length.

This equation shows that the nonlinear length is inversely proportional to the input power. As a result \( L_N \) decreases for higher values of \( P_o \). For \( P_o = 0.00072 \)W it is observed that the pulse broadening ratio decreases, meaning narrowing of pulses. Here, the nonlinear parameter \( \gamma \) is also constant so, narrowing of pulses continues to occur. The reason is that the same amount of nonlinear effect occurs, but it manifests itself over \( L_N \). Reducing \( P_o \) has the opposite effect. Here, \( \gamma \) stays constant but \( L_N \) is larger. So the same SPM effect occurs but over a greater nonlinear length. This means that GVD effect occurs at faster rate when dispersion length is comparatively smaller than the nonlinear length. As a result, GVD effect become more dominant for lower input powers, it results in spreading of pulses.
Previously we plotted the graph by varying one parameter and kept constant other two parameters. Here we varied all three parameters and found the optimum values for pulse broadening ratio close to one which are $g=0.003$, $p=0.00064$, $c=-0.05$. 

**Fig 6.5: Pulse Broadening Ratio for different parameters**
We got this figure by using these parameters $g=0.003$, $p=0.00064$, $c=-0.05$. Here, we can see that after 30 steps the amplitude of the output pulse is nearly equal to the amplitude of the input pulse which is 0.025.
6.2 Hyperbolic Secant Pulse

![Fig 6.7: Pulse evolution](image)

Pulse broadening Ratio for soliton propagation

![Fig 6.8: Pulse Broadening Ratio for Soliton Pulse Propagation](image)
Hyperbolic secant pulse propagation without initial chirp is simulated by analytically solving the normalized Schrodinger equation using the split step Fourier method. From Figure 5.6 we observe that the pulse propagates at seemingly constant width. From the curve in Figure 5.7 the pulse broadening ratio is found to be a steady, horizontal line confirming that the pulse travels at a constant width which is equal to the input width of the pulse. This propagation of a constant width hyperbolic secant pulse means that we have obtained soliton propagation in nonlinear optical fiber. Soliton propagation is possible under certain conditions. In general, the dispersion length \((L_d)\) must be approximately equal to the nonlinear length \((L_n)\). This would mean that the group velocity dispersion would take effect over the same length as the nonlinear effects. Under such circumstances, the GVD effect matches the SPM effects entirely and cancels each other out to obtain steady pulse width throughout the length of the fiber. We assume that no attenuation is present for simplification of solution. The GVD and the SPM manifest themselves through their induced chirp effects. For GVD, the high frequency components of the pulse travel at higher velocity than the low frequencies which induces negative chirp causing dispersion. SPM on the other hand, induces positive chirp during propagation causing the pulse to narrow as it evolves.

The net outcome is that the negative induced chirp of GVD cancels out the positive induced chirp of SPM equally producing fundamental soliton \((N=1)\).
6.2.1 Higher order soliton propagation

Pulse Broadening Ratio for Higher Order Soliton Pulse Propagation with N=1, 2 and 3:

**Fig 6.9: Pulse Broadening Ratio for Higher Order Soliton with different N**

**Fig 6.10: Input pulse for different soliton order**
Effect of increasing soliton order on pulse propagation

Here, the power of the soliton order $N$ is increased gradually to obtain pulse broadening ratios of higher order solitons. These ratios are plotted on the same axis for comparison of pulse propagation of each of the higher order solitons for a given length of fiber. The values chosen for $N$ are 2, 3, 4, 5 and 10.

$$N^2 = \gamma P_0 L_D = \gamma P_0 \left( \frac{T_0^2}{\beta_2} \right)$$

The previous equation shows $N^2$, which is called the nonlinear factor. This factor is used for the solution of the normalized nonlinear Schrödinger equation (Refer to Chapter 4). The input pulse to the NLSE for higher order soliton is

$$U(0, \tau) = \text{Nsech} \left( \frac{\tau}{T_0} \right)$$

So increasing the value of $N$ affects the magnitude of the pulse. But the nonlinear factor $N^2$ is also an important part in the solution of the normalized NLSE. These two facts cause the variation in behavior of the higher order soliton since $N$ is not same in each case.

The second order soliton ($N=2$, green line) produces a periodic outcome. It is observed that after a certain period of traversed length the initial pulse shape is re-acquired. This pattern continues to occur at regular intervals. The length of propagating distance after which the initial pulse shape is re-obtained is called the soliton period and is given by the following equation:

$$z_0 = \frac{\pi}{2} L_D = \frac{\pi}{2} \left( \frac{T_0^2}{|\beta_2|} \right)$$

The third order soliton ($N=3$, Blue line) also produces a periodic pattern in the pulse broadening ratio curve. At regular intervals, the width of the pulse oscillates; however it does not seem to re-acquire the original pulse shape but slowly heads towards acquiring a different pulse width. The fourth (red line) and fifth order (black line) oscillates for shorter and shorter period of length. At the end of the chosen propagating distance, the pulse broadening ratio seems to appear...
less oscillatory and heading towards achieving constancy at a much lower value. However, this event is characterized by splitting of pulses along the length of propagation.

The reason for the behavior of higher order soliton is that the pulse splits into several small pulses. The net effect is the eventual destruction of the intended information due to fragmentation of the pulses.

6.3 Super-Gaussian Pulse

![Graph showing pulse broadening ratio for different values of chirp.](image)

**Fig 6.11: Pulse Broadening Ratio for different values of chirp**

In this case, chirp =0, 1, 2, 3, -1, -2 are studied. The pulse broadening ratio curves reveal that as the magnitude of pulse broadening ratio is close to 1 where chirp is from 0 to 1, the effect of dispersion seems to increase. But as chirp is increased from 1 to 2 and then 2 to 3, dispersion effects increases largely.
Fig 6.12: Pulse Broadening Ratio for different values of \( \gamma \)

In this case, \( \gamma = 0.001, 0.002, 0.003, 0.004, 0.005 \) are studied. The pulse broadening ratio curves reveal that as the magnitude of pulse broadening ratio is close to 1 where \( \gamma \) is from 0.002 to 0.003, the effect of dispersion seems to increase.

Fig 6.13: Pulse Broadening Ratio for different values of \( m \)
Super-Gaussian pulse broadening ratios for various powers of Super-Gaussian pulses are studied. From here we obtain pulse broadening ratio curves for \( m = 1, 2, 3, 4 \) and 5. The behavior of the pulses is easily viewed. As we continue increasing the power \( m \) of the Super-Gaussian, the slopes of the straight lines of each of the curves increase elsewhere.

![Graph showing pulse broadening ratio for different values of power](Image)

**Fig 6.14: Pulse Broadening Ratio for different values of power**

Super-Gaussian pulse broadening ratios for various input powers of Super-Gaussian pulses are studied. From here we obtain pulse broadening ratio curves for \( Po = 0.00060, 0.00062, 0.00064, 0.00066 \) and 0.00068. The behavior of the pulses is easily viewed. As we continue increasing the power \( Po \) of the Super-Gaussian, it goes far away from the value pulse broadening ratio 1.
Previously we plotted the graph by varying one parameter and kept constant other two parameters. Here we varied all three parameters and found the optimum values for pulse broadening ratio close to one, which are $g=0.0025, p=0.00058, c=0, m=1$.
**Fig 6.16: Super-Gaussian pulse evolution**

We got this figure by using these parameters $g=0.0025$, $p=0.00058$, $c=0$, $m=1$. Here, we can see that after 15 steps the amplitude of the output pulse is nearly equal to the amplitude of the input pulse which is 0.025.
CHAPTER 7:

Conclusion

In this dissertation we explored the combined effects of various types of pulses including hyperbolic secant pulses, Gaussian pulses and Super-Gaussian pulses. At first, a Gaussian pulse is launched into the optical fiber and we observed the results for variable nonlinearity, variable group velocity dispersion and variable input power in three separate studies. We find that for low nonlinear parameter values the pulse regains initial shape for a given input power.

Hyperbolic pulses are propagated as a constant width pulse called soliton. The perfect disharmonious interaction of the GVD and SPM induced chirps result in diminishing of both dispersive and nonlinear narrowing effects and hence soliton is obtained. Gaussian pulses are also propagated with or without pre-induced (initial) chirp to study the pattern of propagation. It is found that in the case of chirp 0 and chirp -1, the Gaussian pulse acquires a hyperbolic secant pulse shape and travels as a pseudo-soliton. However, higher values of initial chirp leads to indefinite dispersion and pulse shape is not retained; a fact that can be attributed to the critical chirp, a chirp value beyond which no constant width pulse propagation is possible. For the Super-Gaussian pulse propagation we first considered an un-chirped input with zero nonlinearity parameter to understand the effects of the power of the Super-Gaussian pulse \( m \) on the pulse width and found that pulse broadening ratio curve becomes steeper for higher powered Super-Gaussians. We then applied initial chirp on the Super-Gaussian pulses and found that for values of chirp 2 and -2 or higher, the high power Super-Gaussian pulse broadening steadies signifying a decrease in dispersive effects. We also generated pulse broadening ratio curves and evolution patterns for higher order solitons. Here, we demonstrated that as we increase the soliton order, for \( N=2 \) the pulse width periodically varies and regains the original pulse after soliton period \( z_o \).

However, increasing the soliton order further results in initial periodic behavior of pulse width before settling at a much lower value indicating that pulse splitting has occurred.
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17. E. Iannone, et al., Nonlinear Optical Communication Networks, John Wiley &
Appendix A

**Fig: Gaussian pulse flowchart**

START

loop = 1

INPUT

\[ A(0, \tau) = A_0 \exp \left( -\frac{(1 + iC)\tau^2}{2To^2} \right) \]

fourier.spectrum = fft(A)

for splitstep, from h to Z by h
(splitstep = h:h:Z)

\[ \text{fourier.spectrum} = \text{fourier.spectrum} \times \exp \left( -\frac{\alpha}{2} + \frac{i\beta_2\omega^2}{2} \left( \frac{h}{2} \right) \right) \]

f = ifft(fourier.spectrum)

\[ f = f \times \exp \left( \frac{i\gamma}{2} |f|^2 h \right) \]

fourier.spectrum = fft(f)

\[ \text{fourier.spectrum} = \text{fourier.spectrum} \times \exp \left( -\alpha + \frac{i\beta_2\omega^2}{2} \left( \frac{h}{2} \right) \right) \]

STOP

YES

splitstep = Z

NO

INPUT FWHM

RATIO

FWHM saved at every interval

SAVING PULSE AT EVERY INTERVAL

loop = loop + 1

f = fft(fourier.spectrum)
Fig: Hyperbolic secant pulse flowchart
INPUT

\[ A(0, \tau) = A_0 \exp\left(-\frac{(1+iC)^2 \tau^2}{2T_0^2}\right) \]

fourier.spectrum = fft(A)

for splitstep, from h to Z by h
(splitstep=h:h:Z)

\[ \text{fourier.spectrum} = \text{fourier.spectrum} \ast \exp\left(-\frac{\alpha}{2} + \frac{i\beta_0 \omega^2}{2} \left(\frac{h}{2}\right)\right) \]

f = ifft(fourier.spectrum)

\[ f = f \ast \exp i\gamma |f|^2 h \]

fourier.spectrum = fft(f)

\[ \text{fourier.spectrum} = \text{fourier.spectrum} \ast \exp\left(-\alpha + \frac{i\beta_0 \omega^2}{2} \left(\frac{h}{2}\right)\right) \]

STOP

splitstep=Z

YES

NO

INPUT FWHM

RATIO

FWHM

SAVING PULSE AT EVERY INTERVAL

loop = loop + 1

f=fft(fourier.spectrum)