Performance analysis of a MC-OFDM DS-CDMA system over a faded channel

Thesis Report

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Abstract

Performance Analysis will be carried out for a multi-carrier Direct Sequence CDMA (MC-DS-CDMA) system with orthogonal frequency division multiplexing (OFDM) in the presence of fading and delay spread. The analysis will be made considering a RAKE receiver to combat the effect of channel fading and delay spread. The expressions for the output signals add interfering terms at the output of the Rake receiver will be divided considering the channel effect. The probability density function (PDF) of the SNR at the output of rake receiver and BER expression will be found considering equal gain combining technique. The performance results will be evaluated based on the theoretical analysis. The limitation on the system BER will be evaluated and the optimum system design parameters will be determined.
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Chapter-1:

1.1 Background:

In the era of CDMA application the schemes combining OFDM and Code division were proposed in 1993. These schemes were developed by several researchers. [1] N.Yee, J-P. Linnartz and G.Fettweis, A.Chouly, A.Brajal and S.Jourdan developed the MC-CDMA; V.DaSilva and E.S.Sousa developed Multicarrier Ds-CDMA and Mt-CDMA was developed by L.Vandendorpe.

1.2 Introduction:

In this paper we are going to present an analytical expression of a system having multi-carrier Direct Sequence CDMA (MC-DS-CDMA) in a multipath fading environment using a rake receiver at the receiving end.

Theoretical analysis will be carried out for a multi carrier direct sequence CDMA system where orthogonal frequency division multiplexing (OFDM) multiplexing is used. The performance will be analyzed both for fading and delay spread and a particular receiver (Rake) is going to be used for combating the effects of channel fading and delay spread.

Later in this paper the variation in the performance is going to be analyzed for the changes in number of subcarriers, users, bit error rate and so on. We will find the probability density function (PDF) of the SNR at the output of the rake receiver and then after using corresponding codes we are going to get respective graphs via simulation. We will also find the limitations of the system and figure out the optimum system design parameters.

1.3 System Description:

Code Division Multiple Access (CDMA) is combined with multi carrier modulation (MCM) several times in wireless communications systems. As mentioned earlier it is a form of multiplexing that allows multiple signals to use the same transmission channel for transmission. It hence utilizes the bandwidth to the highest level. Multiplexing actually sends multiple signal
streams on a particular carrier at the same time in the form of a single signal containing all the information’s and then recovers it be decoding at the receiving end.

Our thesis works with techniques such as Direct Sequence (DS) CDMA and OFDM techniques where in Direct sequence CDMA a spreading sequence is applied in the time domain. In this technique the spread spectrum transmission is used where the data user signal is multiplied by the code sequence. The duration of each element in that code is called the CHIP TIME. The ratio between the user symbol time and the chip time is called the spread factor. The bandwidth occupied by the transmitted signal is known as the product of spread factor and bandwidth used by the user data signal. In the receiving end the signal is multiplied again by the same synchronized code. In this step the code is removed by decoding and thus we get our desired signal [2].

![Fig: An example of DS-CDMA](image-url)
OFDM is a technique which allows the signal to split into several narrowband channels at different frequencies called subcarriers. This spread spectrum technique actually describes that the data signal is distributed over a large number of subcarriers that are spread apart at precise frequencies. The space between these frequencies ensures the “orthogonally” of the technique preventing the demodulators from interference. This has a lot of benefits such as high spectral affiancresiliency of RF interference and lower multi-path distortion. But as there are numerous versions of the signal, interference with the other Inter Symbol Interference increases and it makes it difficult for the receiver to obtain the original information.[6]

1.4 Different Schemes of CDMA:

Multimedia services in a mobile radio communication CDMA is a very vital aspirant since it deals extremely well with the asynchronous nature of multimedia data traffic. To rephrase it provides higher capability over all the common access techniques named Time Division Multiple Access (TDMA), Frequency Division Multiple Access (FDMA). It also counters the channel frequency selectivity. There are four new multiple access schemes based on a combination of code division and OFDM. Such as,
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1. Multicarrier (MC) CDMA
2. Multicarrier DS-CDMA
3. Multitone (MT)-CDMA and
4. Multicarrier OFDM DS-CDMA

The above mentioned signals use the Fast Fourier Transform (FFT) for both transmission and reception at the receiving end.[1]

1.4.1 Multicarrier (MC) CDMA:

This scheme involves the original data stream over different subcarriers using a known spreading code in the frequency domain. A portion of the symbol analogous to a chip of the spreading code is transmitted all the way through a different subcarrier. The codes widely used in this scheme are the Hadamard Walsh Codes since the auto-correlation characteristics of the spreading codes are neglected.[1]

1.4.2 Multicarrier DS CDMA:

In this method the transmitter uses the s/p converter to convert the unique data stream using a given spreading code in the time domain. Hence the resultant spectrum of each subcarrier satisfies the orthogonality with the smallest amount of frequency partition or spacing.

This is basically a scheme for the uplink communication channel as the OFDM is initiated in this scheme which is very efficient for launching a quasi-synchronous channel.
1.3.2 Block diagram of MC DS-CDMA Transmitter

The above figure shows the MC-DS-CDMA transmitter of the $j^{th}$ user with the power spectrum. Here $G_{MD}$ is known as the processing gain where $Nc$ and $C_j(t)$ remains the same defining the number of subcarriers and the spreading code respectively.[1]
The above figure shows that respective j-th user’s receiver has same no. of sub-channel where received signal is demodulated and decoded by respective sub frequencies and codes to get wanted signal. LPF are used to mitigate high frequency component of signal.

The multicarrier DS-CDMA has a larger subcarrier partition for an enhanced frequency diversion and narrow band interference suppression. It also transmits the similar data via numerous subcarriers.[1]

1.4.3 Multitone – CDMA:

The MT-CDMA transmitter spreads the converted data in the time domain using the spreading code so that the each subcarrier is orthogonal and has minimum frequency separation. Hence the resulting spectrum of each of them does not satisfy the orthogonality condition. In this scheme spreading codes of bigger length is used to transmit the data if compared with a single carrier DS-CDMA scheme. Therefore it can accommodate more users than DS-CDMA scheme. [1]

1.4.4 Multicarrier OFDM DS-CDMA:

In this method the transmitter uses the s/p converter to convert the unique data stream using a given spreading code in the time domain. OFDM is a technique which allows the signal to split
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into several narrowband channels at different frequencies called subcarriers. The space between these frequencies ensures the “orthogonally” of the technique preventing the demodulators from interference.

1.5 Merits & Limitations:

Merits of Multi-Carrier:

- **Effect of channel will be less**: Since we are using multiple carriers to modulate/demodulate signal, channel effect will be less. As a result, we’ll get better signal in receiving end.

- **Efficient use of bandwidth**: Multi-carrier scheme divide the bandwidth in different frequencies as the system is using multiple carrier in sub-channels for modulation which leads to efficient use of bandwidth.

- **Better quality of received signal**: Multi-carrier scheme provides less noise & interference. As a result, receiver gets better quality signal.

Merits of OFDM:

- **Spectrally efficient method**: It is spectrally efficient because of narrow bandwidth. The merit of narrow band communication is to realize stable long-range communication. In addition to, the carrier purity of transmission spectrum is very good; therefore it is available to manage an operation of many radio devices within same frequency bandwidth at same time.

- **More resistant to fading.**

- **More tolerant to delay spread**: Since OFDM system uses cyclic prefix block, receiving side gets signal which also
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come after delay. By prefix system, the system is adding copy of original data before data as prefix so that receiver can get all delay signals.

- Better fit with today’s high-speed data requirements and operation.

Limitations:

- Timing synchronization error- If MC OFDM DS-CDMA system does not synchronize whole process then proper signal will not be received at receiver side.
- Doppler frequency shift- Due to Doppler Effect, there will be offset in carrier frequency.
- Frequency selective fading.

1.6 Channel Limitations:

1.6.1 Channel Noise: A basic and generally accepted model for thermal noise in communication channels, is the set of assumptions that

- The noise is additive, i.e., the received signal equals the transmit signal plus some noise, where the noise is statistically independent of the signal.
- The noise is white, i.e., the power spectral density is flat, so the autocorrelation of the noise in time domain is zero for any non-zero time offset.
- The noise samples have a Gaussian distribution.

1.6.2 Fading:

In wireless communications, fading is deviation of the attenuation that a carrier-modulated telecommunication signal experiences over certain propagation media. The fading may vary with time, geographical position and/or radio frequency, and
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is often modeled as a random process. A fading channel is a communication channel that experiences fading. In wireless systems, fading may either be due to multipath propagation, referred to as multipath induced fading, or due to shadowing from obstacles affecting the wave propagation, sometimes referred to as shadow fading[12].

1.6.3 Doppler Effect:

When a source generating waves moves relative to an observer, or when an observer moves relative to a source, there is an apparent shift in frequency. If the distance between the observer and the source is increasing, the frequency apparently decreases, whereas the frequency apparently increases if the distance between the observer and the source is decreasing. This relationship is called Doppler Effect (or Doppler Shift) after Austrian Physicist Christian Johann Doppler (1803-1853)[13].

1.6.4 Interference:

Interference is a fundamental nature of wireless communication systems, in which multiple transmissions often take place simultaneously over a common communication medium. In recent years, there has been a rapidly growing interest in developing reliable and spectral efficient wireless communication systems[14].

1.7 Rayleigh Fading:

Rayleigh fading is a statistical model for the effect of a propagation environment on a radio signal, such as that used by wireless devices.

Rayleigh fading models assume that the magnitude of a signal that has passed through such a transmission medium (also called a communications channel) will vary randomly, or fade, according to a Rayleigh distribution — the radial component of the sum of two uncorrelated Gaussian random variables. Rayleigh fading is most applicable when there is no
dominant propagation along a line of sight between the transmitter and receiver. Our analysis is based on Rayleigh fading channel[15].

1.8 Objectives:

- Performance analysis of MC DS-CDMA with OFDM in the presence of fading & delay spread.
- PDF of SNR at output of Rake receiver and BER expression will be found considering equal gain combining technique.
- Limitations on the system BER & the optimum system design parameters will be determine.

1.9.1 Pre-Thesis Work:

1.8.1 Block diagram of MC OFDM DS-CDMA Transmitter
Let us assume an MC-Ds-CDMA system, the block diagrams of which are as shown in the figure. In the specific cell, let for any period of time, there are J-no of users and j is the reference user. Now to have a system model, let us make following assumptions:

a. All the users are active at any time.
b. All the user’s transmitter power levels are equal.
c. All the transmitted signals are suffering equal amount of fading (i.e. flat fading) at any time
d. The bit rate is much larger than chip rate of the coding PN-sequences.

**System Analysis:**

Now let us consider the following the terms in this system analysis,

\[ J = \text{Total no. of user} \]
\[ P = \text{Chip power of each user} \]
\[ N_c = \text{Number of chip code for each subcarrier channel} \]
\[ N = \text{Number of chip of the code for each subcarrier channel} \]
\[ L = \text{Code length of each user} \]
\[ C^j = \text{Code of the j-th user} \]
\[ R_b = \text{Bit rate} \]
\[ m_j(t) = \text{Input data stream of j-th user} \]
\[ B^j_n(t) = n-th bit m_j(t) \]

Thus,

\[ m_j(t) = \sum_{n=-\infty}^{\infty} b^j_n, \text{Where } b = \pm 1 \]

Now the input data of the j-th user are converted into Nc parallel data streams and each of the parallel data is coded by the channel respective code of the j-th user. Thus each data bit is
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speeded in time domain and then is modulated by the respective sub carrier. To write the general expression of the sub carriers, let us consider the following,

\[ \omega_c = \text{Freq of the reference channel} \]

\[ \Delta \omega_c = \text{Freq spacing between two successive channels} \]

\[ \phi_k = \text{Instantaneous phase angle of the k-th sub carrier} \]

So the signal expression of the sub carrier is:

\[
\sum_{k=1}^{N_c} \sqrt{2P} b_{n,k}^j \cos (\omega_c t + k\Delta \omega + \phi_k)
\]

Thus, the expression of transmitted signal of the j-th user is as follows:

\[
S_T(t) = \sum_{k=1}^{N_c} \sqrt{2P} b_{n,k}^j \left( \sum_{x=1}^{N} C_{x,k}^j \cos (\omega_c t + k\Delta \omega + \phi_k) \right)
\]

Here

\[ b_{n,k}^j = n-th \text{ bit of the jth user which is being modulated by the k-th channel} \]

\[ C_{x,k}^j = x-th \text{ chip of the k-th section of the j-th user's code} \]

Generally a single user Ds-CDMA rake receiver contains several correlators. The BER performance is dependent on the number of branches the rake receiver employs. A four branched rake receiver is used mostly due to hardware limitation.

The Nyquist filters are also initiated in here for base band pulse shaping as it may combine the paths wrongly.

This happens because the distortion caused by the noise in the auto-correlation characteristics gives an error.

The system capacity is based on the structure of the rake receiver and the multiple access interference (MAI) which is a resultant of the defective auto-correlation characteristics and spreading code imperfection."
A single user receiver takes the received signals for other active users as inactive interference. In multi user detection the receiver equally detects the signals to lessen the non orthogonal properties of the received signals. Hence the performance is much better. Though it does not have the ability to make the best use of the energy scattered in the time domain in the receiving end.[2]

1.8.2 block diagram of MC OFDM DS-CDMA Receiver
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While reception the receiver of the \( j \)-th user receives signals transmitted by all the \( j \) number of users. Thus the expression of the received signals is

\[
r(t) = \sum_{m=1}^{i} \left[ \sqrt{2P} \sum_{k=1}^{N_c} \frac{b_{n,k}^m}{\sum_{x=1}^{N} C_{x,k}^j} \cos \left( \omega_c t + k\Delta \omega_c t + \theta \right) \right] + \eta(t)
\]

While reception the receiver of the \( j \)-th user receives signals transmitted by all the \( j \) number of users. Thus the expression of the received signals is

\[
r(t) = \sum_{m=1}^{i} \left[ \sqrt{2P} \sum_{k=1}^{N_c} \frac{b_{n,k}^m}{\sum_{x=1}^{N} C_{x,k}^j} \cos \left( \omega_c t + k\Delta \omega_c t + \theta \right) \right] + \eta(t)
\]

Here \( \eta(t) \) is the term AWGN noise, whose spectral density is around \( N_0/2 \). This signal while propagating through the medium suffers frequency selective multi-path Rayleigh Fading. To account the fading consider that \( \alpha \) be the instantaneous amplitude distortion and \( \theta \) be the instantaneous phase distortion due fading, so at the receiver end, the expression will be,[3]

\[
r(t) = \sum_{m=1}^{i} \alpha \left[ \sqrt{2P} \sum_{k=1}^{N_c} \frac{b_{n,k}^m}{\sum_{x=1}^{N} C_{x,k}^j} \cos \left( \omega_c t + k\Delta \omega_c t + \theta \right) \right] + \eta(t)
\]

But due to the Doppler Effect, there will be offset in carrier frequency.

If \( \delta \omega_c \), is the amount of such frequency offset then,

\[
r(t) = \sum_{m=1}^{i} \alpha \left[ \sqrt{2P} \sum_{k=1}^{N_c} \frac{b_{n,k}^m}{\sum_{x=1}^{N} C_{x,k}^j} \cos \left( \omega_c t + k\Delta \omega_c t + \theta + \delta \omega_c t \right) \right] + \eta(t)
\]

After demodulation the equation will be,
The decoded equation looks like,

\[
y(t) = \sum_{m=1}^{j} \alpha \sqrt{\frac{2P}{2}} \sum_{k=1}^{N_c} b_{m,k}^j \left( \sum_{x=1}^{N} C_{x,k}^j \right) \cos(\omega_c t + k\Delta\omega_c t + \theta + \phi_k + \delta\omega_c t) \times \cos(\omega_c t + k\Delta\omega_c t) \\
+ \sum_{k=1}^{N_c} \eta(t) \sum_{x=1}^{N} C_{x,k}^j \cos(\omega_c t + k\Delta\omega_c t)
\]

The decoded equation looks like,

\[
y(t) = \sum_{m=1}^{j} \left[ \frac{\alpha \sqrt{2P}}{2} \sum_{k=1}^{N_c} \left( \sum_{x=1}^{N} \left( C_{x,k}^m \times C_{x,k}^l \right) \right) \cos(2\omega_c t + 2k\Delta\omega_c t + \theta + \phi_k + \delta\omega_c t) \\
+ \cos(\delta\omega_c t + \phi_k + \theta) \right] \sum_{k=1}^{N_c} \sum_{x=1}^{N} C_{x,k}^j \cos(\omega_c t + k\Delta\omega_c t)
\]

Separating it after cross and auto correlation,

\[
y(t) = \frac{\alpha \sqrt{2P}}{2} \sum_{k=1}^{N_c} \left( \sum_{x=1}^{N} (1) \right) \left\{ \cos(2\omega_c t + 2k\Delta\omega_c t + \theta + \phi_k + \delta\omega_c t) + \cos(\delta\omega_c t \\
+ \phi_k + \theta) \right\} \sum_{m=1}^{j-1} \left[ \frac{\alpha \sqrt{2P}}{2} \sum_{k=1}^{N_c} \left( \sum_{x=1}^{N} (\mu) \right) \cos(2\omega_c t + 2k\Delta\omega_c t + \theta \\
+ \phi_k + \delta\omega_c t + \cos(\delta\omega_c t + \phi_k + \theta) \right] \sum_{k=1}^{N_c} \sum_{x=1}^{N} C_{x,k}^j \cos(\omega_c t + k\Delta\omega_c t)
\]

The value of \( \mu \) is “1” when maximum interference occurs where as it is “0” when there is no interference. Now after passing through the LPF the high freq components will be eliminated. The signal then becomes,

\[
y(t) = \frac{N_a \sqrt{2P}}{2} \sum_{k=1}^{N_c} b_{m,k}^j \cos(\delta\omega_c t + \phi_k + \theta) \\
+ \sum_{j=1}^{j-1} \left[ \frac{\alpha \sqrt{2P}}{2} \sum_{k=1}^{N_c} \left( b_{n,k}^m \mu_N \cos(\delta\omega_c t + \phi_k + \theta) \right) \right] + \sum_{k=1}^{N_c} \eta(t) \sum_{x=1}^{N} C_{x,k}^j \cos(\omega_c t + k\Delta\omega_c t)
\]

After integrating it over 0 to \( T_p \),
Then for the time being, we ignore the offset freq and then the equation becomes,

$$y(t) = \frac{Na_0}{2T_b} \sum_{k=1}^{N_c} b_{n,k}^j \cos(\Phi_k + \theta)] \int_0^{T_b} dt$$

$$+ \sum_{m=1}^{j-1} \left[ \frac{e^{j\mu N T F}}{2T_b} \sum_{k=1}^{N_c} \left[b_{n,k}^m \cos(\delta \omega_c t + \Phi_k + \theta) \right] \right] dt$$

$$+ \frac{1}{T_b} \int_0^{T_b} \sum_{k=1}^{N_c} \eta(t) \sum_{x=1}^{N} C_{x,k}^j \cos(\omega_c t + k \Delta \omega_c t) dt$$

Or,

$$y(t) = \frac{Na_0}{2} \sum_{k=1}^{N_c} b_{n,k}^j \cos(\phi_k + \theta) + \sum_{m=1}^{j-1} \left[ \frac{e^{j\mu N T F}}{2} \sum_{k=1}^{N_c} b_{n,k}^m \cos(\phi_k + \theta) \right] + \frac{1}{T_b} \int_0^{T_b} \sum_{k=1}^{N_c} C_{x,k}^j \cos(\omega_c t + k \Delta \omega_c t) dt$$

Or we get,

$$y(t) = y_j(t) + y_{MAI}(t) + \eta(t)$$

Here,

$$y_j(t) = \text{Desired signal of j-th user.}$$

$$y_{MAI}(t) = \text{Multiple access Interference}$$

$$\eta(t) = \text{Noise} [3]$$

### 1.9.2 Work done in Thesis:
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- Power of received signal is derived.
- Evaluation of BER & PDF of SNR at output of Rake receiver.
- Limitations on the system BER & the optimum system design parameters will be determined at a given BER $10^{-6}$.
- The analyzed data will be applied in mat lab simulator and thus the respective results and graphs will be obtained.

Chapter-2

2.1 System Block Diagrams:

2.1.1 Block diagram of MC OFDM DS-CDMA Transmitter
2.1.2 block diagram of MC OFDM DS-CDMA Receiver
2.2 **Transmitter & Receiver:**

**2.2.1 Transmitter:**

MC OFDM DS-CDMA transmitter consists of serial to parallel converter, encoders, modulators, inverse fast Fourier transform and cyclic prefix. We considered a particular cell for analyzing the transmitter’s functions. The expression of transmitted signal which has been derived before of the j-th user is as follows:

\[
S_T(t) = \sqrt{2P} \sum_{k=1}^{N} b^j_{n,k} \left( \sum_{x=1}^{N} c^j_{x,k} \cos(\omega_c t + k\Delta\omega + \varphi_k) \right)
\]

Here

- \( b^j_{n,k} = n \)-th bit of the j-th user which is being modulated by the k-th channel\( c^j_{x,k} = x \)-th chip of the k-th section of the j-th user code \[3\] \( J \) = Total no. of user
- \( P \) = Chip power of each user
- \( Nc \) = Number of chip code for each subcarrier channel
- \( N \) = Number of chip of the code for each subcarrier channel
- \( L \) = Code length of each user
- \( C^j \) = Code of the j-th user
- \( R_b \) = Bit rate
- \( m^j(t) \) = Input data stream of j-th user
- \( B^j_{n}(t) = n \)-th bit \( m^j(t) \)

Thus,

\[
m^j(t) = \sum_{n=-\infty}^{\infty} b^j_{n}, \text{Where } b = \pm 1
\]
2.2.2 Receiver:

MC OFDM DS-CDMA consists of delete cyclic prefix, serial to parallel converter, demodulators, Rake receivers, integrators, parallel to serial converter.

The received signal which has been derived before of j-th user is as follows:

\[ y(t) = \frac{N_{\text{c}} \mu \sqrt{E_b}}{2 \tau_b} \sum_{k=1}^{N_{\text{c}}} b_{n,k}^j \cos (\varphi_k + \theta)] \int_0^{T_b} dt \]

\[ + \sum_{m=1}^{j-1} \left[ \frac{N_{\text{c}} \mu \sqrt{E_b}}{2 \tau_b} \sum_{k=1}^{N_{\text{c}}} b_{n,k}^m \cos (\varphi_k + \theta) \right] \int_0^{T_b} dt \]

\[ + \frac{1}{T_b} \int_0^{T_b} \sum_{k=1}^{N_{\text{c}}} \eta(t) \sum_{x=1}^{N} \mathcal{C}_{x,k} \cos (\omega_c + k\Delta\omega_c t) dt \]

Or,

\[ y(t) = \frac{N_{\text{c}} \mu \sqrt{E_b}}{2} \sum_{k=1}^{N_{\text{c}}} b_{n,k}^j \cos (\varphi_k + \theta) + \sum_{m=1}^{j-1} \left[ \frac{N_{\text{c}} \mu \sqrt{E_b}}{2} \sum_{k=1}^{N_{\text{c}}} b_{n,k}^m \cos (\varphi_k + \theta) \right] + \frac{1}{T_b} \int_0^{T_b} \sum_{k=1}^{N_{\text{c}}} \mathcal{C}_{x,k}^j \cos (\omega_c t + k\Delta\omega_c t) dt \]

Or we get,

\[ y(t) = y_j(t) + y_{\text{MAI}}(t) + \eta(t) \]

Here,

\[ y_j(t) = \text{Desired signal of j-th user}. \]

\[ y_{\text{MAI}}(t) = \text{Multiple access Interference} \]

\[ \eta(t) = \text{Noise [3]} \]

**Power of Desired Signal:**

The desired signal of the j-th user is \( y_j(t) \) which is given as follows:
\[ y_j(t) = N \alpha \sqrt{P/2} \sum_{k=1}^{N_c} b_{n,k}^j \cos(\varphi_k + \theta) \]

Since, \( b_{n,k}^j = \pm 1 \),

\[ \sum_{k=1}^{N_c} b_{n,k}^j = N_c \]

Thus, the above term becomes:

\[ y_j(t) = NN_c \alpha \sqrt{P/2} \sum_{k=1}^{N_c} \cos(\varphi_k + \theta) \]

Thus, for the \( j \)-th user, the desired signal power, \( P_s \) is given as

\[ P_s = \frac{1}{2} \alpha^2 \left( N N_c \sqrt{P/2} \right)^2 \]

\[ = \frac{1}{4} \left( \alpha^2 N^2 N_c^2 P \right) \]

**Power of Multiple Access Interference:**

\[ y_{MAI}(t) = \sum_{m=1}^{j-1} \left[ \frac{\alpha N \mu \sqrt{2P}}{2} \sum_{k=1}^{N_c} b_{n,k}^m \cos(\varphi_k + \theta) \right] \]

So the interference power is given as,

\[ P_{MAI}(t) = \sum_{m=0}^{j-1} \left[ \frac{1}{2} \alpha^2 \mu^2 N^2 N_c^2 \frac{P}{2} \right] \]

\[ = \sum_{m=0}^{j-1} \left[ \frac{1}{4} \alpha^2 \mu^2 N^2 N_c^2 P \right] \]

\[ = \frac{1}{4} \alpha^2 \mu^2 N^2 N_c^2 P (j - 1) \]
Power of Noise:

\[ n(t) = \frac{1}{T_b} \int_{0}^{T_b} w(t) \sum_{k=1}^{N} C_k^j \cos(\omega_c t + k\Delta\omega_c t) dt \]

Now, it can be evaluated that, the noise power is:

\[ \sigma_n^2 = \frac{N_0}{4T_b}, \text{where } N_0 = KTR_b \]

Signal to Interference and Noise Ratio (SINR):

Instantaneous SINR = \[ \frac{P_s}{P_{MAI} + \sigma_n^2} \]

Or,

\[ \zeta = \frac{\frac{1}{4} (\alpha^2 N^2 N_c^2 P)}{\frac{1}{4} \alpha^2 \mu^2 N^2 N_c^2 P (j - 1) + \frac{N_0}{4T_b}} \]

\[ = \frac{(\alpha^2 N^2 N_c^2 P T_b)}{\alpha^2 \mu^2 N^2 N_c^2 P (j - 1)T_b + N_o} \]

\[ = \frac{(\alpha^2 N^2 N_c^2 E_b)}{\alpha^2 \mu^2 N^2 N_c^2 E_b (j - 1)T_b + N_o} \]

\[ = \frac{(\alpha^2 N^2 N_c^2)}{\alpha^2 \mu^2 N^2 N_c^2 (j - 1) + (\frac{E_b}{N_o})^{-1}} \]

BER without Diversity (i.e. Rake Receiver)

When CNR increases transmission power increases and the bit error rate (BER) decreases. For an example if we have 100000 bits and there are only 100 erroneous bits then BER is 100/100000 = 10^-3.
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If Noise power and carrier power is known we can easily find the CNR and the receiver Power. The level of BER is dependent on the CNR. We increase CNR BER decreases. But there is an optimum level up to which it can be decreased.

BER is a function of CNR. While designing an RF we select a modulating mode in such a way that it has a low Bandwidth demand but a high bitrate and good spectral density. After comparing all these we finally get an effective mode of modulation.

For an example if we think about a communication system where the transmitted signal is being passed to the receiving end and then it is passed through a band pass filter. The received signal when passed by a coherent demodulator has a noise attached to it which is known as the Additive White Gaussian Noise (AWGN). Additive because noise never cancels each other out it always adds to the previous one. White because it has all color frequency components.

If we increase BW, noise will become really high. Therefore to keep the bandwidth at an optimum level is better. Optimum bandwidth is BW = 2Rb

\[ y(t) = r(t) \times A_L \cos \omega_c t \]

\[ = A_c \cos \omega_c t \times \cos \omega_c t + n_0(t) \]

\[ = \pm \frac{A_c A_L}{2} [2\cos^2 \omega_c t] + n_0(t) \]

\[ = +A_0 [1 + \cos 2\omega_c t] + n_0(t) \]

After the low pass filter in the coherent demodulator we can pass the signal through the sampler and multiply it with the clock pulse and then we can observe that 1st sampler is giving positive output (1) and second sample is giving negative output (0).

Error occurs only then when 1 is transmitted and 0 arrives. And vice versa.

Probability of 1 transmitted 0 received = \( P_r \{ V_{0,1} < V_{th/1} \} \); output voltage when 1 is transmitted.

\[ BER = P_r (V_{01} < V_{th/1}) \times P_r (1) + (V_{00} > V_{th/0}) P_r (0) \]
Therefore,

\[ = \int_{-\infty}^{0} P(V_{01}) \, dV_{01} \times \frac{1}{2} + \int_{0}^{\infty} P(V_{00}) \, dV_{00} \times \frac{1}{2} \]

\[ \downarrow \hspace{1cm} \downarrow \]

\[ (I_1) \hspace{1cm} (I_2) \]

Where,

\[ P(V_{01}) = \frac{1}{\sqrt{2\pi 6x^2}} e^{-\frac{(V_{01} - A_0)^2}{26x^2}} \]

\[ P(V_{00}) = \frac{1}{\sqrt{2\pi 6x^2}} e^{-\frac{(V_{00} - A_0)^2}{26x^2}} \]

\[ I_1 = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi 6x^2}} e^{-\frac{(V_{01} - A_0)^2}{26x^2}} \, dV_{01} \]

Let, \( \frac{V_{01} - A_0}{\sqrt{26x}} = \chi \rightarrow \frac{1}{\sqrt{26x}} \, dV_{01} = d\chi \)

\[ I_1 = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{0} e^{-\chi^2} \, d\chi \]

\[ = \frac{1}{\sqrt{\pi}} \int_{A_0/\sqrt{26x}}^{\infty} e^{-x^2} \, dx \]
Therefore,

\[
I_1 = \frac{1}{2} \text{erfc}\left[\frac{A_0}{\sqrt{2} \sigma_n}\right]
\]

\[
I_2 = \frac{1}{2} \text{erfc}\left[\frac{A_0}{\sqrt{2} \sigma_n}\right]
\]

BER = \(I_1 + I_2\)

\[
= \frac{1}{2} \times \frac{1}{2} \text{erfc}\left[\frac{A_0}{\sqrt{2} \sigma_n}\right] + \frac{1}{2} \times \frac{1}{2} \text{erfc}\left[\frac{A_0}{\sqrt{2} \sigma_n}\right]
\]

\[
= \frac{1}{2} \text{erfc}\left[\frac{A_0}{\sqrt{2} \sigma_n}\right]
\]

\[
= \frac{1}{2} \text{erfc}\left[\sqrt{\frac{\text{SNR}}{2}}\right]
\]

Now, this \(\zeta\) is a function of \(\alpha\). So, for an instantaneous value of \(\alpha\), the instantaneous Bit Error Rate is,

\[
P_b(\alpha) = f(\zeta)
\]

\[
= \frac{1}{2} \text{erfc}\left[\frac{\zeta}{\sqrt{2}}\right]
\]

2.2.2

Thus,

\[
P_b(\alpha) = 0.5 \times \text{erfc}\left[\frac{\frac{\alpha^2 \mu^2 N_c^2 N_c^2 (1/\sqrt{2})}{\alpha^2 + \mu^2 N_c^2 N_c^2 (j-1) + (\frac{E_b}{N_0})^{-1}}}{\sqrt{2}}\right]
\]

2.2.3

Here, \(\alpha\) is considered 1 which means no fading & \(\mu\) is varied.

Now, the average bit error rate of the system without rake (with fading) can be obtained as follows:
Here, $\sigma$ is amplitude variance.

### 2.3 Evaluation of Bit Error Rate with Rake Receiver:

A receiver technique which uses several baseband correlators to individually process several signal multipath components. The correlator outputs are combined to achieve improved communications reliability and performance [9]. The rake receiver consists of multiple correlators, in which the receive signal is multiplied by time-shifted versions of a locally generated code sequence. The intention is to separate signals such that each finger only sees signals coming in over a single (resolvable) path. The spreading code is chosen to have a very small autocorrelation value for any nonzero time offset. This avoids crosstalk between fingers.

![Rake Receiver Diagram]

**Figure: Rake Receiver** [10]
Different reflected waves arrive with different delays. A rake receiver can detect these different signals separately. These signals are then combined, using the diversity technique called maximum ratio combining.

In a multipath channel, delayed reflections interfere with the direct signal. However, a DS-CDMA signal suffering from multipath dispersion can be detected by a rake receiver. This receiver optimally combines signals received over multiple paths.

![Diagram of Rake Receiver](image)

Figure: Rake receiver with 5 fingers [10]

Like a garden rake, the rake receiver gathers the energy received over the various delayed propagation paths. According to the maximum ratio combining principle, the SNR at the output is the sum of the SNRs in the individual branches, provided that

- we assume that only AWGN is present (no interference)
- codes with a time offset are truly orthogonal

Let us consider, there are rake receivers in each sub-carrier channel which have L no of fingers and they are considered for maximum gain combining. If $\zeta_1, \ldots, \zeta_L$ be the instantaneous bit SINR for different branches then the probability of bit error rate at the output of an SSDR is given as follows:
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\[ P_b(y) = \frac{1}{2^{L-1}} e^{-\gamma} \sum_{k=0}^{L-1} b_k y_k \]

Where, \[ b_k = \frac{1}{k!} \sum_{k=0}^{L-1-k} \binom{2L-1}{n} \]

L = no. of fingers of Rake receiver.

Chapter-3:

For simulation, MATLAB has been used. We used the equations that we got from our analysis & put them in mat Lab code to get results. We got several graphs by varying parameters. From those graphs it can be seen that parameters can be changed based on demand in communication.

Graph Analysis:

3.1 Without Fading:

The graph below is determined from 2.2.1 & 2.2.2 equations which means analysis is based without fading.
In the graph above we can see for different numbers of chip of code (N), the relation between no. of users (j) & BER. As no of chips of code increase, BER decreases for a certain no of users. As an example, for seven users, for N = 8, 16, 32, 64 respective BER are $10^{-2}, 10^{-3}, 10^{-4}, 10^{-6}$. So for a certain no of users we can see BER improvement as N increases. To get better signal in receiver, BER should be less. BER is taken $10^{-6}$ for our analysis. BER $10^{-6}$ is the maximum tolerable value for signal in wireless communication. Considering this value we’ll proceed our analysis. In above graph, it is also visible that as no. of chips of code increases, no. of users also increases. And based on that our next graph is determined.
In above graph, as no. of chip of the code increases, no. of users also increases. For fewer amounts of users less no. of chips of code is required to transmit signal. If there are few users in transmission side, then MC OFDM DS-CDMA system transmitter uses less no. of chips for certain code to encode the transmitted signal before transmission. In receiver side, the signal is decoded by same amount of chips of code. But when there is large no. of users then transmitter has to encode signal by few more no. of chips of code. Because as users increases in a cell probability of interference is increases. To avoid interference large no. of chips of code is required which make the signal strong. Here, graph is varied by no. of sub-carriers, Nc. For a certain amount of chips of code, more users can transmit signal at a time in this system by increasing no. of sub-carriers.
In the graph above we can see that taking the same numbers of chip of the code, we can easily vary the parameters as per demand. For an example If we take values of Nc & j for N=16, 32, 64 from above graph, we’ll get the graph below. Above for N=16, values of Nc & j has been shown.

In above graph, by varying the no. of chips of code, we can determine no. of users for a certain no. of sub-carriers. As an example, for N=64, MC OFDMDS-CDMA system can support fifty users by modulating/demodulating signals with more than fifty sub-carriers. The system can also
vary these parameters as per demand of a certain cell. We can see in the graph above, using the same N=64, the system provide less no. of sub-carriers for less users.

In above graph, as no. of chips of code increase, Eb/No dB is decreases. Here we are varying no. of users (j). This graph is also determined by BER=10^-6. It is also visible that as no. of users increase, Eb/No dB increases. As an example, for N=16, for j = 6,10,20,30 respective Eb/No are 24 dB,27 dB,32 dB,42 dB. So for a certain no of chips of code , Eb/No dB decrease with the
In the graph above we can see that taking the same numbers of chip of the code, we can easily vary the parameters as per demand. For an example If we take values of Eb/No dB & j for N=16, 32, 64 from above graph, we’ll get the graph below. Above for N=16, values of Eb/No dB & j has been shown.
In above graph, by varying the no. of chips of code, we can determine no. of users for a certain value of Eb/No dB. As an example, for j=10, for N=16, 32, 64 in MC OFDM-DPS-CDMA system respective EB/No are 22, 23 & 24. As Eb/No increases, the system needs more amount of power which is not required. So We can have less power for less no. of chips of code.

In the Graph above .We kept a fixed number of users (j) and Number of chip of the code is fixed at 8.Here we varied the number of subcarriers (Nc = 8,16,32,64) and observed that with the increase of Eb/No dB ,bit error rate (BER) decreases .For an example ,for BER 10^-6 and Nc = 8,16,32,64 we get Eb/No dB = 5,7,8,10 respectively. And with the BER value of 10^-6 we developed the graph below.
In the graph above, we can clearly see the improvement of BER. Since Eb/No is increasing for a certain number of users BER is decreasing.

### 3.2 With Fading:

The graph below has been determined from 2.2.4 equations where we varied $\alpha$.

![Graph showing BER improvement with varying Eb/No dB and $\sigma^2$.](image)

In the Graph above, We kept a fixed number of users ($j$) and Number of chip of the code & no. of sub-carriers is fixed at 8. Here we varied the value of variance ($\sigma^2 = 0.1, 0.2, 0.5$) and observed that with the increase of Eb/No dB, bit error rate (BER) decreases. For an example, for BER $10^{-6}$ and $\sigma^2 = 0.1, 0.2, 0.5$ we get Eb/No dB = 7, 8.5, 11 respectively.

### 3.3 No of Fingers in Rake Receiver:

Rake receivers contains no of fingers which has delay block, encoder, LPF & summation block. Rake receivers are used to receive multipath propagation delay signals. MC OFDM DS-CDMA uses fingers as per demand. Here, we considered no. of fingers as $L$. We took optimum value of $L$ for fixed no. of users ($j=10$), no. of chip of code ($N=16$) & no. of sub-carriers ($Nc=16$) in the graph below. This graph shows how BER improves as no. of fingers increase.
In the graph above, with the increment of L, BER decreases. It is also visible that as Eb/No increases, BER decreases.

Chapter 4

4.1 Future Work:

Third generation cdma wireless system technology has been arrived and now the focus is on them than that of traditional systems in terms of speed efficiency and so on.

Cdma high data rate system is also proposed to address wireless access to data network.

The internet traffic pattern demands much higher throughput on the forward link so in the future more research is going to be done on this model.

4.2 Conclusion:

In this chapter we elaborated on what works we have done. We analyzed the performances of different schemes of Multicarrier DS-CDMA. The expressions for the transmitted and received signal using Rake receiver was also analyzed. We determined the expressions of power of received signals. The graph analysis has been shown using MAT Lab. Graph shows how the
parameters can be varied according to the demand of the system. We did our analysis by taking the value of BER $10^{-6}$.

**Chapter 5**

**REFERENCES:**


